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Physical Interpretations of Relativity Theory

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This volume contains papers which accepted for inclusion in the programme of lectures of meeting "Physical Interpretation of Relativity Theory" which is organized by the Bauman Moscow State Technical University, School of Computing and Technology, University of Sunderland, Liverpool University and British Society for Philosophy of Science.

The most important single objective of the meeting in Summer 2003 is including the advantages of the various physical, geometrical and mathematical interpretations of the formal structure of Relativity Theory; and to examine the philosophical, historical and epistemological questions associated with the various interpretations of the accepted mathematical expression of the Relativity Principle and its development.

The conference is called to examine the various interpretations of the (mathematical) formal structure of Relativity Theory, and the several kinds of physical and mathematical models which accompany these interpretations.

The programme timetable, giving authors and titles of papers as presented and other details of the Moscow Meeting "Physical Interpretation of Relativity Theory" are given on the web site maintained by the Bauman Moscow State Technical University http://fn.bmstu.ru/phys/nov/konf/pirt/pirt_main.html.

The meeting is intended to be of interest to physicists, mathematicians, engineers, philosophers and historians of science, post-graduate students.

The conference is sponsored and supported by the

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THREE PROBLEMS OF BIG G

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Main problems of modern physics and of absolute G measurements, its temporal and range variations from both experimental and theoretical points of view are discussed, and a new next generation multipurpose space project for measuring G, $G(r)$ and G-dot promising an improvement of our knowledge of these quantities by 2-4 orders is advocated.

1 Introduction

If earlier the main trend in physics was the study of different physical interactions - gravitational, electromagnetic, strong and week ones, after 60's and especially during last decades we see the outbreak of investigations along the unification of all four known interactions. This is due mainly to problems related to the gravitational interaction: its quantization, singularities, early universe description etc. We have now a good model unifying week and electromagnetic interactions, more or less acceptable model unifying them with the strong interaction (GUT) and many attempts to construct theories unifying all four interactions (supergravity, superstring, M-theory...). But we had to admit that up till now we have no definite and reliable variant of the unified theory. That is why many studies are being done within models incorporating some common features of these theories, in particular ideas of extra dimensions, dilatonic fields and fields of forms (p-branes) etc.[3,4,16]

Of course, there are still many hot spots within the gravitational interaction itself. Among them one may point out such problems as early universe behavior, description and detection of strong field objects (black holes, wormholes etc.) and gravitational waves, near zone experiments, such as equivalence principle tests, verification of second order, rotational and torsional effects of general relativity etc. Within the last block we want to stress the special role of excrements to measure the value of the gravitational (or Einstein) constant and its possible variations. This experiments are already the new generation ones as they are testing not only gravitational interaction, but also some predictions of unified models and theories. Special role in these activities are played already by space experiments and this role will increase in future. Modern cosmology and its observational part already became an arena for testing predictions of high energy physics. All this causes the fundamental role of gravity in present investigations, which still is the missing link of unified theories[17]. Fundamental physical constants and possible relations between them are the reflection of the situation with unification.

Among fundamental physical constants the Newton gravitational constant G (as well as other fundamental macroscopic parameters H - the Hubble constant, ρ, or Ω, - mean density of the Universe, Λ - the cosmological constant) is known with the least accuracy.[3,4] Moreover, there are other problems related to its possible variations with time[11,12] and range[5] coming mainly from predictions of unified models of the four known physical interactions.

Here we dwell upon the problems of absolute G measurements, its temporal and range variations from both experimental and theoretical points of view and advocate a new universal space project for measuring G, $G(r)$ and G-dot, promising an improvement of our knowledge

of these quantities by 2-4 orders.

Why are we interested in an absolute value of G? It is known in celestial mechanics that we can determine only the product GM, where M is the mass of a body. (GM is known now with the accuracy 10^{-9} which is much better than 10^{-3} for G and correspondingly for masses of planets.) The knowledge of this product is enough for solving most problems in celestial mechanics and space dynamics. But there are other areas where we need separate values of G and M. First, we need knowing much better masses of planets for construction of their precise models. Second, G enters indirectly in some basic standards and is necessary for conversion from mechanical to electromagnetic units, for calibration of gradiometers etc. More precise value of G will be important for future discrimination between unified (GUT, supergravity, strings, M-theory etc.) models as they usually predict certain relations between G and other fundamental constants.[3,13,1,2,14]

There are three problems connected with the Newtonian gravitational constant G:

1. Absolute value of G.
2. Possible time variations of G.
3. Possible range variations of G, or new interactions (forces).

2 Problem of G Stability

2.1 Absolute G measurements

The value of the Newton gravitational constant G as adopted by CODATA in 1986 was based on the Luther and Towler (1982, USA) measurements.

Even at that time other 2 existing on 100ppm level measurements of Facy and Pontikis (1972, France) and Sagitov (1979, Russia) deviated from this value more than their uncertainties.[6] During recent years the situation, after very precise measurements of G in Germany, New Zealand, BIPM, Switzerland, China, Russia etc., became much more vague. The results deviate from the official CODATA value and from each other drastically.

As it is seen from the data announced in November 1998 at the Cavendish conference in London the situation with terrestrial absolute G measurements is not improving. The reported values for G (in units of 10^{11} SI) and their estimated error in ppm are as follows:

Fitzgerald and Armstrong	6.6742	90 ppm
(NZ)	6.6746	134
Nolting et al. (Zurich)	6.6749	210
Meyer et al. (Wupperthal)	6.6735	240
Karagioz et al. (Moscow)	6.6729	75
Richman et al.	6.683	1700
Schwarz et al.	6.6873	1400
CODATA (1986, Luther)	6.67259	128

This situation forced CODATA to increase the uncertainty of G value by one order - now it is officially at 10^{-3} level:

$$G = 6.670 \quad (+/-) \quad 0.010$$

so, we are better now by only one order in knowledge of absolute G value than Cavendish (after more than 200 years). Last 4 years very precise measurements continue to contribute to discrepancies in G values:

Gundlach et al., USA	6.674215 (+/-)	0.000092
Armstrong et al., NZ	6.6742	0.0007
Luo Jun et al., China	6.6699	0.0007
Quinn et al., BIPM (I)	6.6693	0.0009
Quinn et al., BIPM (II)	6.6689	0.0014
Schlamminger et al., CH,	6.7404	33ppm
Karagioz, Russia	6.6729	0.0005

This means that either the limit of terrestrial accuracies is reached or we have some new physics entering the measurement procedure.[7,8,14,17,16] The first means that we should address to space experiments to measure G [9,15] and the second means that a more thorough study of theories generalizing Einstein's general relativity is necessary [16]. One of the attempts to explain different values, obtained in different labs, belongs to Mbelek and Lachieze-Rey [18]. Using simple multidimensional model of Kaluza-Klein type they show that measured G-values depend on the latitude and longitude of the laboratory. Sure, this approach had to be tested independently.

2.2 Data on temporal variations of G

Dirac's prediction based on his Large Numbers Hypothesis is $\dot{G}/G = (-5)10^{-11}$ year^{-1}. Other hypotheses and theories, in particular some scalar-tensor or multidimensional ones, predict these variations on the level of $10^{-12} - 10^{-14}$ per year depending on exact cosmological solutions. Our first calculations based on general relativity with conformal scalar field gave in 1973-77 the level of 10^{-13} per year. Our last calculations within the general class of scalar-tensor theories [19] and simple multidimensional model with p-branes [20] gave for present values of cosmological parameters $10^{-13} - 10^{-14}$ and 10^{-13} per year correspondingly. Similar estimations were made by Miyazaki (2001) within the Machian theories [21] giving for G-dot the estimation 10^{-13} per year and by Fujii (200) on the level $10^{-14} - 10^{-15}$ per year [22].

As to experimental or observational data, the results are rather inconclusive. The most reliable ones are based on Mars orbiters and landers (Hellings, 1983) and on lunar laser ranging (LLR) (Muller et al., 1993; Williams et al., 1996). They are not better than 10^{-12} per year.[7,14] Here are some data on $\dot{G}$:

1. Van Flandern, 1976-1981: (ancient eclipses) $\dot{G}/G = -5 \cdot 10^{-11} y^{-1}$
2. Hellings, 1983-1987: (Viking) $|\dot{G}/G| < 5 \cdot 10^{-12} y^{-1}$
3. Reasenberg, 1987: (Viking) $|\dot{G}/G| < 5 \cdot 10^{-11} y^{-1}$
4. Acceta et al., 1992: (Nucleosyntheses) $|\dot{G}/G| < 10^{-12} y^{-1}$
5. Anderson et al., 1992: (ranging to Mercury and Venus) $|\dot{G}/G| \leq 2 \cdot 10^{-12} y^{-1}$
6. Muller et al., 1993: (lunar laser ranging) $|\dot{G}/G| \leq 5 \cdot 10^{-13} y^{-1}$
7. Kaspi et al., 1994: (timing of pulsars) $|\dot{G}/G| \leq 5 \cdot 10^{-12} y^{-1}$
8. Williams et al., 1996: (lunar laser ranging) $|\dot{G}/G| \leq 8 \cdot 10^{-12} y^{-1}$

So, once more we see that there is a need for corresponding theoretical and experimental studies. Probably, future space missions to other planets and/or LLR will be a decisive step in solving the problem of temporal variations of G and determining the fate of different theories which predict them, as the larger is the time interval between successive measurements and, of course, the more precise they are, the more stringent results will be obtained. Unfortunately, many last missions to closely planets failed during last decades and we had no new data on G-dot.

2.3 Nonnewtonian interactions (EP and ISL tests)

Nearly all modified theories of gravity and unified theories predict also some deviations from the Newton law (ISL) or composite-dependent violation of the Equivalence Principle (EP) due to appearance of new possible massive particles (partners).[5]

In the Einstein theory $G = \text{const}$. If $G = G(t)$ is possible, then, ¿from the relativistic point of view $G \to G(t, r)$. In more detail: Einstein's theory corresponds to massless gravitons which are mediators of the gravitational interaction, obey the 2nd order differential equations and interact with matter with a constant strength G.

Any violation of these conditions leads to deviations from the Newton Law. Here are some classes of theories (generalized gravitational and unified models) which exhibit such deviations.

1. Massive gravitons: theories with Λ and bimetric ones.
2. Effective $G(x, t)$: scalar-tensor theories.
3. Theories with torsion.
4. Theories with higher derivatives (4th order etc.). Here massive modes appear: short and long range forces.
5. Other mediators besides gravitons (partners) appear: $SuperGravity$, $SuperStrings$, $M - theory$ etc. (massive ones).
6. Theories with nonlinearities induced by any of the known interactions (electromagnetic or gravitational or other). Then, some effective mass of the mediators appears.
7. Phenomenological models where the mechanism of deviation from the Newton law is not known (fifth force or so). For describing the possible deviation from the Newton law the usual parametrization $\Delta \sim \alpha e^{-r/\lambda}$ of the Yukawa-type is used.

Here are some estimations of masses, ranges and also strengths for $G(r)$ predicted by various models.

1. Pseudoscalar particle leads to attraction between macro bodies with range $2 \cdot 10^{-4}\,\text{cm} < \lambda < 20\,\text{cm}$ (Moody, Wilizek, 1984), the variable α (strength) from 1 to 10^{-10} in this range is predicted.
2. Supersymmetry: spin-1 partner of the massive spin-3/2 gravitino leads to repulsion in the range: $\lambda \sim 10^3$ km, $\alpha \sim 10^{-13}$ (Fayet, 1986, 1990).
3. Scalar field to adjust Λ (Weinberg, 1989): $m \leq 10^{-3}\,eV/c^2$ or range $\lambda \geq 0.1$ mm. Another variant (Peccei, Sola, Wetterich, 1987) leads to $\lambda \leq 10$ km (attraction).
4. Supergravity (Scherk, 1979); graviton is accompanied by a spin-1 graviphoton: here a repulsion is predicted also.
5. Strings, p-branes, M-theory: dilaton (other scalar fields) and antisymmetric tensor fields appear.
6. Different braneworld models: power-law additional interactions appear on the brane at mm and less ranges.

Conclusion: there is no reliable theory or model of unified type, but all predict new additional attractive or repulsive interactions of a non-Newtonian character in different ranges (composition dependent EP-violation or independent).

Experimental data exclude the existence of these particles on some good level at nearly all ranges except less than a millimeter and also at meters and hundreds of meters ranges.

It is a real challenge to experimentalists!

It should be noted that the only known positive result in the range of 20-500 m was obtained by Achilli et al.[10] They found a deviation from the Newton law with the Yukawa potential strength α between 0,13 and 0,25. Of course, these results need to be verified in other independent experiments, probably in space ones, as some experiments contradict their results in the same range.

3 SEE Project

We saw that there are three problems connected with G: absolute value measurements and possible variations with time and range. There is a promising new space experiment SEE - Satellite Energy Exchange[9] which addresses all these problems and may be more effective in solving them than other laboratory or space experiments.

We studied some aspects of the SEE-Project:[15]

1. Wide range of trajectories with the aim of finding optimal ones:
– circular in the spherical field of the Earth;
– circular in spherical field + earth quadruple modes;
– elliptic, with $e \leq 0.05$.
2. Estimations of other celestial bodies influence.
3. Estimations of relative influence of trajectories to δG, $\delta\alpha$.
4. Modelling measurement procedure of G and α.
5. Estimations of some sources of errors:
– radial oscillations of the shepherd's surface;
– longitudal oscillations of the capsule;
– transversal oscillations of the capsule;
– shepherd nonsphesicity;
– limits on shepherd J_2.
– effects of charging by high energy particles and Earth magnetic belts particles.
– inhomogeneities of masses and of the capsule etc.
6. Error budgets for G, G-dot and $G(r)$. The general conclusion is that the Project SEE may improve our knowledge of G, limits on G-dot and $G(r)$ by 2-4 orders of magnitude.
7. Variation of the SEE method – trajectories near libration points.
8. Different altitudes up to ISS (500 km), short capsule up to 5 m (instead of original 15-20 m).

General conclusion: it is possible to improve (G, α) by 2-4 orders at a range of 1-100 m.

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References

1. V.N. Melnikov, *Multidimensional Classical and Quantum Cosmology and Gravitation.Exact Solutions and Variations of Constants.* CBPF-NF-051/93, Rio de Janeiro, 1993;
   V.N. Melnikov. In: *Cosmology and Gravitation*, ed. M.Novello (Editions Frontieres, Singapore, 1994) p. 147.
2. V.N. Melnikov, *Multidimensional Cosmology and Gravitation,* CBPF-MO-002/95, Rio de Janeiro, 1995, 210 p.
   V.N. Melnikov. In *Cosmology and Gravitation.II* ed. M. Novello (Editions Frontieres, Singapore, 1996) p. 465.
3. K.P. Staniukovich and V.N. Melnikov, *Hydrodynamics, Fields and Constants in the Theory of Gravitation*, (Energoatomizdat, Moscow, 1983), (in Russian).
   V.N. Melnikov, *Fields and Constants in the Theory of Gravitation.* CBPF-MO-02/02, Rio de Janeiro, 2002, 134pp.
4. V.N. Melnikov, *Int. J. Theor.Phys.* **33**, N7, 1569 (1994).
5. V. de Sabbata, V.N. Melnikov and P.I. Pronin, *Prog. Theor. Phys.* **88**, 623 (1992).
6. V.N. Melnikov. In: *Gravitational Measurements, Fundamental Metrology and Constants.* Eds. V. de Sabbata and V.N. Melnikov (Kluwer Academic Publ.) Dordtrecht, 1988, p.283.
7. A.J. Sanders and G.T. Gillies, *Rivista Nuovo Cim.* **19**, N2, 1 (1996).
8. A.J. Sanders and G.T. Gillies, *Grav. and Cosm.* **3**, N4(12), 285 (1997).
9. A.J. Sanders and W.E. Deeds. *Phys. Rev.* **D 46**, 480 (1992).
10. V. Achilli et al., *Nuovo Cim.* **B 12**, 775 (1997).
11. Y.-S. Wu and Z. Wang, *Phys. Rev. Lett.* **57** 1978 (1986).
12. K.A. Bronnikov, V.D. Ivashchuk and V.N. Melnikov, *Nuovo Cimento* **B 102**, 209 (1988).
13. T. Damour, "Gravitation, Experiment and Cosmology", gr-qc/9606079.
14. V.D. Ivashchuk and V.N. Melnikov, "Multidimensional Cosmological and Spherically Symmetric Solutions with Intersecting p-Branes". In *Proc. 2nd Samos Meeting on Cosmology, Geometry and Relativity, September 1998, Springer;* gr-qc/9901001.
    V.D. Ivashchuk and V.N. Melnikov, "Exact Solutions in Multidimensional Gravity with Antisymmetric Forms". Topical Review. *Class. Quant.Grav.,* **18**, pp.R1-R66 (2001)
15. A.D. Alexeev, K.A. Bronnikov, N.I. Kolosnitsyn, M.Yu. Konstantinov, V.N. Melnikov and A.G. Radynov, *Izmeritel'naya Tekhnika,* 1993, No. 8, p.6; No. 9, p.3; No. 10, p.6; 1994, No. 1, p.3;
    Int. J. Mod. Phys. **D 3**, No. 4, p.773 (1994).
    A.D. Alexeyev et al., Measurement of the Gravitational Constant in Space (SEE Project): Sensitivity to orbital parameters and space charge effect. *Metrologia,* **38(5)**, 397-408 (2001).
    V.N. Melnikov et al. Space Project SEE for Measuring G and Its Possible Spatial and Time Variations.*Ciencia Ergo Sum*,**8,** pp.357-367 (2001).
16. V.N. Melnikov, *Exact Solutions in Multidimensional Gravity and Cosmology III.* CBPF-MO-03/02, Rio de Janeiro, 2002, 297 pp.
17. V.N. Melnikov, Gravity as a Key Problem of the Millenium.*Proc. 2000 NASA/JPL Conference on Fundamental Physics in Microgravity,* CD-version, NASA Document D-21522, 2001, p.4.1-4.17, Solvang, CA, USA; gr-qc/0007067.

18. J.P. Mbelek and M. Lachieze-Rey, *Gravit. Cosmol.* **8** 331-338 (2002).
19. K.A. Bronnikov, V.N. Melnikov and M. Novello, Possible time variations of G in scalar-tensor theories of gravity. *Gravit. Cosmol.*, **8**, Suppl.II, p.18-21 (2002).
20. V.D. Ivashchuk and V.N. Melnikov, Problems of G and Multidimensional Models. In *Proc. JGRG11,* Eds. J. Koga et al., Waseda Univ., Tokyo, 2002, pp. 405-409.
21. A. Miyazaki, Varying Cosmological Constant and the Machian Solution in the Generalized Scalar-Tensor Theory, gr-qc/0103003.
22. Y.Fujii, Astrophys.Space Sci. **283** 559-564 (2003).

Genera l relativity as a conformal theory

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The latest astrophysical data on the Supernova luminosity-distance – redshift relations, primordial nucleosynthesis, value of Cosmic Microwave Background-temperature, and baryon asymmetry are considered as an evidence of relative measurement standard, field nature of time, and conformal symmetry of the physical world. We show how these principles of description of the universe help modern quantum field theory to explain the creation of the universe, time, and matter from the physical vacuum as a state with the lowest energy.

From Zero to the Dirac Equation

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A form of the Dirac equation is proposed using algebraic operators instead of gamma matrices. This form of the equation generates a wavefunction that is nilpotent or a square root of zero. This has many advantages, leading to explicit expressions for fermions, antifermions, bosons, baryons, and vacuum; the nilpotent wavefunctions are, in addition, quantum field and supersymmetry operators. The formulation allows an easy demonstration of *CPT* symmetry, a zero sum for the fermion self-interaction energy and a fermion propagator without infrared divergence. On the basis of a fundamental symmetry between space, time, mass and charge, it is proposed that the Dirac state, in this form, packages the entire information relating to these parameters. A computer rewrite mechanism is used as an analogy for the generation from zero of the algebraic and physical structure incorporated into the Dirac state.

1. The nilpotent Dirac equation

While the Dirac equation is conventionally written in terms of the gamma matrix algebra

$$(\gamma^{\mu}\partial_{\mu} + im)\ \psi = \Box\psi = 0 \ , \tag{1}$$

it is possible to create a much more powerful form of the equation using algebraic operators based on a combination of quaternions and multivariate 4-vectors. We assume that the quaternion multiplication follows the rules:

$$\boldsymbol{ij} = -\boldsymbol{ji} = \boldsymbol{k}\ ;\ \boldsymbol{jk} = -\boldsymbol{kj} = \boldsymbol{i}\ ;\ \boldsymbol{ki} = -\boldsymbol{ik} = \boldsymbol{j}\ ;\ \boldsymbol{i}^2 = \boldsymbol{j}^2 = \boldsymbol{k}^2 = \boldsymbol{ijk} = -1 \ ,$$

while the multivariate vector multiplication is of the form:

$$\mathbf{ij} = -\mathbf{ji} = i\mathbf{k}\ ;\ \mathbf{jk} = -\mathbf{kj} = i\mathbf{i}\ ;\ \mathbf{ki} = -\mathbf{ik} = i\mathbf{j}\ ;\ \mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = 1;\ \mathbf{ijk} = i\ .$$

which is isomorphic to that of complex quaternions or Pauli matrices:

$$(i\boldsymbol{i})(i\boldsymbol{j}) = -(i\boldsymbol{j})(i\boldsymbol{i}) = i(i\boldsymbol{k})\ ;\ (i\boldsymbol{j})(i\boldsymbol{k}) = -(i\boldsymbol{k})(i\boldsymbol{j}) = i(i\boldsymbol{i})\ ;\ (i\boldsymbol{k})(i\boldsymbol{i}) = -(i\boldsymbol{i})(i\boldsymbol{k}) = i(i\boldsymbol{j})\ ;$$
$$(i\boldsymbol{i})^2 = (i\boldsymbol{j})^2 = (i\boldsymbol{k})^2 = 1\ ;\ (i\boldsymbol{i})(i\boldsymbol{j})(i\boldsymbol{k}) = i \ ,$$

and allows a full product between multivariate vectors **a** and **b** of the form

$$\mathbf{ab} = \mathbf{a.b} + i\ \mathbf{a} \times \mathbf{b} \ .$$

The quaternion-multivariate 4-vector produces a 32-part algebra, forming a group of order 64 if we take account of + and – terms. This is identical to that of the five gamma matrices if we take a mapping of the form:

$$\gamma^{o} = -i\boldsymbol{i} \qquad \text{or} \qquad \gamma^{o} = i\boldsymbol{k}$$
$$\gamma^{1} = \mathbf{i}\boldsymbol{k} \qquad\qquad \gamma^{1} = \mathbf{i}\boldsymbol{i}$$
$$\gamma^{2} = \mathbf{j}\boldsymbol{k} \qquad\qquad \gamma^{2} = \mathbf{j}\boldsymbol{i}$$
$$\gamma^{3} = \mathbf{k}\boldsymbol{k} \qquad\qquad \gamma^{3} = \mathbf{k}\boldsymbol{i}$$
$$\gamma^{5} = i\boldsymbol{j} \quad (2) \qquad\qquad \gamma^{5} = i\boldsymbol{j}. \quad (3)$$

Substituting the values from (2) into equation (1), we obtain:

$$\psi = 0 \ , \tag{4}$$

Multiplying the equation from the left by $\boldsymbol{j}$ now alters the algebraic representation to (3) and our Dirac equation becomes:

$$\psi = 0 \ , \tag{5}$$

This is not just an algebraic exercise, because, as soon as we apply a free-particle solution, such as

$$\psi = A\, e^{-i(Et - \mathbf{p.r})} \quad .$$

to (5), we find that:

$$(\boldsymbol{k}E + i\boldsymbol{i}\mathbf{i}p_x + i\boldsymbol{i}\mathbf{j}p_y + i\boldsymbol{i}\mathbf{k}p_x + i\boldsymbol{j}\, m)\, A\, e^{-i(Et - \mathbf{p.r})} = 0 \quad ,$$

which may be more conveniently written in the form,

$$(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\, A\, e^{-i(Et - \mathbf{p.r})} = 0 \quad , \tag{6}$$

where $\mathbf{p}$ is a multivariate vector. This equation can only be valid if A is a multiple of $(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$, which means that it must be a *nilpotent* or square root of zero. The same must also be true of ψ, and, although this can only be derived from equation (5), it must also be true for the original form of the Dirac equation, or equation (4). The Dirac wavefunction for a free fermion must *necessarily* be a nilpotent, or contain a nilpotent factor, although this will not be apparent until we write it in the form of equation (5).

2. Properties of the nilpotent structure

Since a multivariate $\mathbf{p}$ or ∇ already incorporates fermionic spin, the four solutions to the nilpotent Dirac equation are easily identified as the four possible combinations of $\pm E$ (fermion and antifermion) and $\pm\,\mathbf{p}$ (spin up and spin down). We may write these in abbreviated form as $(\pm\, \boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\, e^{-i(Et\, -\, \mathbf{p.r})}$. However, we can develop a much more useful form of the equation by transferring the variation in the signs of E and $\mathbf{p}$ from the exponential term to the differential operator, which, instead of being a single expression, now becomes a 4-term row vector, forming a scalar product with the four terms in the column vector representing the Dirac 4-spinor, producing for a free particle the expression:

$$(\pm\, \boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\, e^{-i(Et\, -\, \mathbf{p.r})} = 0. \tag{7}$$

In (7), both the differential operator and the eigenvalue part of the wavefunction are essentially identical, and both are quantized. The equation thus fulfils the basic requirements for automatic second quantization and a quantum field theory, the nilpotent expressions displaying the characteristics of full quantum field operators rather than wavefunctions in the more restricted sense. The four components of the eigenvalue part of the wavefunction are also immediately recognizable as four creation (or annihilation) operators:

$(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$	fermion spin up
$(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$	fermion spin down
$(-\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$	antifermion spin up
$(-\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$	antifermion spin down

Again, if we represent a fermion by the row vector:

$$(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(-\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(-\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$

and an antifermion by the column vector:

$$(-\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(-\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\ ,$$

then the spin 1 boson produced by their combination is simply the scalar product:

$$
\begin{array}{ll}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m)\,,
\end{array}
$$

with numerical value $8E^2$, before normalization. Massless states are represented in the same way:

$$
\begin{array}{ll}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) \\
(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) \\
(-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) \\
(-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p})\,,
\end{array}
$$

and give an identical summation. To represent spin 0 bosons, we simply reverse the sign of **p** in the second column, to give:

$$
\begin{array}{ll}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m)\,,
\end{array}
$$

which has a pre-normalization scalar value of $8m^2$. Massless spin 0 states, however, disappear if represented by nilpotents, since, now

$$
\begin{array}{ll}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) = 0 \\
(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) = 0 \\
(-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) = 0 \\
(-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) & (\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p}) = 0\,.
\end{array}
$$

So, the nilpotent algebra *requires* Goldstone bosons to be unphysical, and Higgs bosons to acquire mass. Pauli exclusion is automatic, because:

$$
\begin{array}{ll}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) = 0 \\
(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) = 0 \\
(-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) = 0 \\
(-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) = 0\,.
\end{array}
$$

However, a two fermion, 'bosonic'-type state is theoretically allowed if the fermions have opposite spins, because

$$
\begin{array}{ll}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(-\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) & (-\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m)
\end{array}
$$

has a non-zero scalar value of $-8p^2$ before normalization. We can think of this, perhaps, as a representation, or at least an idealization, of the unit bosonic state in a Bose-Einstein condensate, or of the Cooper pairs in a superconductor, or even of the combination of electron and magnetic flux line in the quantum Hall phenomenon, or of the single-valued wavefunctions produced in other applications of the Berry phase.

There are many other advantages of the nilpotent structure. A baryon state vectors may be derived using a three-component non-zero structure of the form

$$(\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{i}\,p_x + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{i}\,p_y + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E \pm \boldsymbol{i}\boldsymbol{i}\,p_z + \boldsymbol{i}\boldsymbol{j}\,m)\,,$$

in which we may imagine **p** as having allowed phases in which only *one* of the three components of momentum, p_x, p_y, p_z, is nonzero and represents the total **p**. The products

$$
\begin{array}{l}
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m) \\
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E - \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{j}\,m) \\
(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{i}\,\mathbf{p} + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{j}\,m)\,(\boldsymbol{k}E + \boldsymbol{i}\boldsymbol{j}\,m)
\end{array}
$$

and

$$(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$$
$$(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)$$
$$(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)$$

then become equivalent to the respective fermionic structures, $-p^2(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$ and $p^2(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)$. Using the labels B, G and R to represent the $\mathbf{p}$ variation within the brackets, the total state vector, incorporating all six phases, via the Jacobi identity, becomes

$$\psi \sim (BGR - BRG + GRB - GBR + RBG - RGB)\ ,$$

with mappings

$$\begin{array}{rcl}
BGR & \rightarrow & (\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m) \\
-BRG & \rightarrow & (\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m) \\
GRB & \rightarrow & (\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m) \\
-GBR & \rightarrow & (\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m) \\
RBG & \rightarrow & (\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m) \\
-RGB & \rightarrow & (\boldsymbol{k}E - i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\,(\boldsymbol{k}E + i\boldsymbol{j}\, m)\ .
\end{array}$$

equivalent to the three cyclic and three anticyclic combinations in the standard QCD representation of the baryon wavefunction.

CPT symmetry becomes another natural outcome of the representation, with the individual transformations:

$$\begin{array}{lll}
P & \boldsymbol{i}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,\boldsymbol{i} & = (\pm\,\boldsymbol{k}E +,^{-}\, i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m) \\
T & \boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,\boldsymbol{k} & = (+,^{-}\, \boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m) \\
C & -\boldsymbol{j}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,\boldsymbol{j} & = (+,^{-}\, \boldsymbol{k}E +,^{-}\, i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)
\end{array}$$

combining to give *CPT* or $TCP \equiv$ identity, because:

$$\boldsymbol{k}\,(-\boldsymbol{j}\,(\boldsymbol{i}\,(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,\boldsymbol{i})\,\boldsymbol{j})\,\boldsymbol{k} = -\boldsymbol{kji}\,(\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\,\boldsymbol{ijk} = (\boldsymbol{k}E + i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\, m)\ .$$

A vacuum operator may also be constructed such that its application leaves an original fermion state unchanged. So a term of the form $(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$ remains essentially unchanged if postmultiplied by $\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$, and the process can be continued indefinitely, with the fermion acting continually on the vacuum to reproduce itself:

$$(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\ \ldots$$

However, $\boldsymbol{k}\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,\boldsymbol{k}$ is also the same as the antistate to $(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$, or $(+,^{-}\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$, making this equivalent to

$$(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,(+,^{-}\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\,(+,^{-}\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)\ \ldots$$

So, a real fermion state creates a virtual antifermion mirror image of itself in the vacuum, while a real antifermion state creates a virtual fermion mirror image of itself. The combined real and virtual particle then create a virtual boson state. The process may be continued indefinitely, with $(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$ and $(+,^{-}\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$ acting as virtual supersymmetry operators and creating an infinite series of mutually cancelling fermion and boson loops, exactly as required for renormalization at all orders. The formalism also leads to a perturbation expansion for a first-order QED coupling with a state vector of the form:

$$\Psi_1 = -e\,\Sigma\,[\boldsymbol{k}E + i\boldsymbol{i}\,\boldsymbol{\sigma}.(\mathbf{p} + \mathbf{k}) + i\boldsymbol{j}m]^{-1}\ (i\boldsymbol{k}\phi - \boldsymbol{i}\,\boldsymbol{\sigma}.\mathbf{A})\,(\boldsymbol{k}E + i\boldsymbol{i}\boldsymbol{\sigma}.\mathbf{p} + i\boldsymbol{j}m)\,e^{-i(Et - (\mathbf{p} + \mathbf{k}).\mathbf{r})}\ ,$$

which automatically becomes zero for a self-interacting fermion, and similar cases.[1] In a similar way, the fact that the product of a nilpotent term $(\pm\,\boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m)$ and its complex conjugate is necessarily a nonzero scalar for all values of E and $\mathbf{p}$, unlike the comparable conventional

equivalent ($\gamma^{\mu}p_{\mu} + im$), means that the fermion propagator in the nilpotent formulation has no pole or infrared divergence at any energy.

3. A fundamental group symmetry

The advantages of the nilpotent formalism over any comparable one are so immediately apparent that we may be led to inquire whether there is any fundamental significance in the fact that it squares itself to zero. It is not, in fact, unusual to come across statements that the universe is in some sense a zero totality, for example, in the Machian concept that the total positive rest energy of the universe is exactly countered by the identical amount of negative gravitational energy. Zero also has a special significance as a foundational concept in requiring no arbitrary assumptions. It is a starting point which leaves no unanswered questions. The Dirac equation is, in many ways, the most fundamental equation in physics, and the Dirac or fermion state the most fundamental particle state. Does its nilpotent nature, therefore, suggest a zero origin of some kind, and can we derive the Dirac state itself, and consequently the Dirac equation, directly from the properties of zero?

To answer this, I will draw upon an idea which has been a feature of much of my earlier work. This is the idea that the most fundamental concepts in physics – the parameters space and time, and the sources of the four fundamental interactions, mass (or mass-energy) and three types or 'dimensions' of charge (electromagnetic, strong and weak) – present a zero conceptual totality in being symmetric according to a Klein-4 scheme, with the following properties and exactly opposite 'antiproperties':

mass	conserved	real	continuous nondimensional
time	nonconserved	imaginary	continuous nondimensional
charge	conserved	imaginary	discrete dimensional
space	nonconserved	real	discrete dimensional

The physical justification for this grouping has been explored in earlier papers.[2-5] The structure appears to require a link between discreteness and dimensionality, specifically 3-dimensionality (if we assume that the symmetry-breaking between the three types of charge occurs at a higher level). It also provides a separate algebraic identity for each of the parameters:

parameter	**algebraic identity**	**units**
mass	real scalar	1
time	pseudoscalar	i
charge	quaternion	$\boldsymbol{i}, \boldsymbol{j}, \boldsymbol{k}$
space	vector	$\mathbf{i}, \mathbf{j}, \mathbf{k}$

These units, of course, are exactly those which occur in the nilpotent Dirac equation. If the Dirac state is the most fundamental particle state in nature, we may imagine that it somehow packages the information available from this parameter structure. That it does this in such a way as to produce a direct route to zero is an added bonus, and suggests that a direct examination of the mathematical and physical meaning of the zero state is an immediate priority. This will bring us to questions relevant to the deepest foundations of mathematics, as well as physics. In resolving these, we will find it convenient to use the analogy provided by the computer rewrite system, a piece of software taking an object represented as a string of characters or 'alphabet' and using a set of *rewrite rules* to generate a new string which represents an altered state of the object.[6] Here, our

object will be, in effect, anything other than zero itself, and our rewrite rule will be such that any deviation from the zero state will force a continued attempt to recover the original zero totality.

4. A universal alphabet and rewrite system

The fundamental rule that we need to set up our mathematical system states that any alphabet or set of non-zero states, which generates itself by acting on any subalphabet, will necessarily generate a new zero-totality alphabet by acting on itself. The nature of the specific action need not be specified, and it will not necessarily be mathematical in the first instance.

We may begin by describing deviations from zero by a nonunique term $\mathcal{R}$. This will remain unspecified and undefined, but any action taken in relation to itself will force an attempt at recovering the original zero state. The immediate outcome of the action $(\mathcal{R}) \times (\mathcal{R}) = (\mathcal{R})$ will, therefore, be a *conjugate* (or zero-producing) term, which we may represent by $-\mathcal{R}$, though without implying any assumptions about a specific mathematical meaning for the symbols used. We can represent the process in the form:

$$(\mathcal{R}) \times (\mathcal{R}) = (\mathcal{R}) \rightarrow (\mathcal{R}, -\mathcal{R})$$

and now examine the action of the 'alphabet' $(\mathcal{R}, -\mathcal{R})$ in terms of the 'subalphabets' $(\mathcal{R})$ and $(-\mathcal{R})$ using our rewrite rule, to give:

$$(\mathcal{R}) \times (\mathcal{R}, -\mathcal{R}) = (\mathcal{R}, -\mathcal{R})$$
$$(-\mathcal{R}) \times (\mathcal{R}, -\mathcal{R}) = (\mathcal{R}, -\mathcal{R})$$

However, examining the action of $(\mathcal{R}, -\mathcal{R})$ in relation to itself will produce a *new* conjugated (or zero-totality) alphabet, of the form:

$$(\mathcal{R}, -\mathcal{R}) \times (\mathcal{R}, -\mathcal{R}) = (\mathcal{R}, -\mathcal{R}) \rightarrow (\mathcal{R}, -\mathcal{R}, C,/, -C,/)$$

where

$$(\mathcal{R}) \times (\mathcal{R}, -\mathcal{R}, C,/, -C,/) = (\mathcal{R}, -\mathcal{R}, C,/, -C,/)$$
$$(-\mathcal{R}) \times (\mathcal{R}, -\mathcal{R}, C,/, -C,/) = (\mathcal{R}, -\mathcal{R}, C,/, -C,/)$$
$$(\mathcal{R}, -\mathcal{R}) \times (\mathcal{R}, -\mathcal{R}, C,/, -C,/) = (\mathcal{R}, -\mathcal{R}, C,/, -C,/)$$
$$(C,/, -C,/) \times (\mathcal{R}, -\mathcal{R}, C,/, -C,/) = (\mathcal{R}, -\mathcal{R}, C,/, -C,/) \text{ , etc.}$$

From expressions of this kind, we derive also the results of the 'actions' of subalphabets upon each other. For example:

$$(-\mathcal{R}) \times (-\mathcal{R}) = (\mathcal{R})$$
$$(\mathcal{R}) \times (C,/) = (C,/)$$
$$(C,/) \times (C,/) = (-\mathcal{R})$$
$$(C,/) \times (-C,/) = (\mathcal{R})$$

To construct further conjugated alphabets we need appropriate subalphabets so that the correct rule will automatically apply, for example:

$$(\mathcal{R}, -\mathcal{R}, C,/, -C,/) \times (\mathcal{R}, -\mathcal{R}, C,/, -C,/) = (\mathcal{R}, -\mathcal{R}, C,/, -C,/, C,/C,/', -C,/', C,/C,/', -C,/C,/') \text{ .}$$

The result will be a series of $C,/$-type terms, $C,/$, $C,/'$, $C,/''$, $C,/'''$, etc., and their 'actions' upon each other, such as $C,/C,/'$. The 'self-action' of the $C,/$-type terms will necessarily always result in $(\mathcal{R})$, and the series will continue to infinity. For any pair of terms in the $C,/$ series, however, say X and Y, we have an option, for *either*

$$(XY) \times (XY) = (-\mathcal{R}) \qquad \text{anticommutative}$$

or

$$(XY) \times (XY) = (\mathcal{R}) \qquad \text{commutative}$$

The options are not of equal status, however, for the anticommutative option can be used only once. For each given X, there is only a single Y, XY combination, and the three terms form a *closed cycle*.

In effect, we have generated 3-dimensionality purely through anticommutativity. The commutative option, on the other hand, will remain open to infinity with an unlimited number of Y terms for any given X.

Though there are clearly an infinite set of available options within this process, maximum efficiency (or minimum generation of rules) will occur when we set the default condition at the anticommutative option whenever this is available. The result will be a regular sequence of closed finite-dimensional systems taking us to infinity. This has exactly the same structure as the set of finite integers in conventional enumeration, and can be considered a method of generating it. Significantly, integers, and the entire number system which results from their existence, are created at the same time as (3-)dimensionality, exactly in the way that the properties of the fundamental parameters seem to demand it.

5. An algebraic structure for physics

Up to now, of course, $\mathcal{R}$ has been completely undefined. However, now that we have generated a number system, we are free to *choose* to define it as the Cantorian or non-denumerable set of real numbers. The series of $C,/$-terms will then become an infinite set of complex forms, whose real 'magnitudes', when part of a closed system, will be represented by the constructible real numbers of Robinson's non-standard analysis or Skolem's non-standard arithmetic, like those applied to space, and by the Cantorian reals when the system remains open. The – and × signs can then be allowed to take up their usual algebraic meanings. It will now be convenient to define countable units within $\mathcal{R}$, $C,/$, $C,/'$, $C,/''$, $C,/'''$, ... as, say, 1, $\boldsymbol{i}_1, \boldsymbol{j}_1, \boldsymbol{i}_2, \boldsymbol{j}_2,...$, where $\boldsymbol{i}_n, \boldsymbol{j}_n, \boldsymbol{i}_n \boldsymbol{j}_n = \boldsymbol{k}_n$, and so on, are independent sets of *quaternions*, with anticommutative multiplication:

$$\boldsymbol{i}_n \boldsymbol{j}_n = -\boldsymbol{i}_n \boldsymbol{j}_n = \boldsymbol{k}_n$$
$$\boldsymbol{j}_n \boldsymbol{k}_n = -\boldsymbol{k}_n \boldsymbol{j}_n = \boldsymbol{i}_n$$
$$\boldsymbol{k}_n \boldsymbol{i}_n = -\boldsymbol{i}_n \boldsymbol{k}_n = \boldsymbol{j}_n$$
$$\boldsymbol{i}_n^2 = \boldsymbol{j}_n^2 = \boldsymbol{k}_n^2 = \boldsymbol{i}_n \boldsymbol{j}_n \boldsymbol{k}_n = -1 \quad .$$

The multiplications with other terms, however, will be commutative. For example, when $m \neq n$,

$$\boldsymbol{i}_m \boldsymbol{i}_n = \boldsymbol{i}_n \boldsymbol{i}_m$$
$$\boldsymbol{i}_m \boldsymbol{j}_n = \boldsymbol{j}_n \boldsymbol{i}_m$$
$$\boldsymbol{j}_m \boldsymbol{j}_n = \boldsymbol{j}_n \boldsymbol{j}_m$$

and

$$(\boldsymbol{i}_m \boldsymbol{i}_n)(\boldsymbol{i}_m \boldsymbol{i}_n) = 1$$
$$(\boldsymbol{i}_m \boldsymbol{j}_n)(\boldsymbol{i}_m \boldsymbol{j}_n) = 1$$
$$(\boldsymbol{j}_m \boldsymbol{j}_n)(\boldsymbol{j}_m \boldsymbol{j}_n) = 1$$

though, of course,

$$\boldsymbol{i}_m^2 = \boldsymbol{i}_n^2 = \boldsymbol{j}_m^2 = \boldsymbol{j}_n^2 = -1 \quad .$$

The algebraic series may also be seen as a dualistic doubling of terms in each conjugation process, with the series:

order 2 $(1, -1)$
order 4 $(1, -1) \times (1, \boldsymbol{i}_1)$
order 8 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1)$
order 16 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2)$
order 32 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2)$
order 64 $(1, -1) \times (1, \boldsymbol{i}_1) \times (1, \boldsymbol{j}_1) \times (1, \boldsymbol{i}_2) \times (1, \boldsymbol{j}_2) \times (1, \boldsymbol{i}_3)$,

generating the terms:

order 2 ± 1

order 4 $\pm 1, \pm \boldsymbol{i}_1$

order 8 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1$

order 16 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$

order 32 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
$\pm \boldsymbol{j}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$

order 64 $\pm 1, \pm \boldsymbol{i}_1, \pm \boldsymbol{j}_1, \pm \boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
$\pm \boldsymbol{j}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$
$\pm \boldsymbol{i}_3, \pm \boldsymbol{i}_3\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1,$
$\pm \boldsymbol{i}_3\boldsymbol{j}_2, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_1\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{j}_1, \pm \boldsymbol{i}_3\boldsymbol{j}_2\boldsymbol{i}_2\boldsymbol{i}_1\boldsymbol{j}_1$

….The algebraic groups in this series are clearly recognizable as:

order 2 real scalars

order 4 complex scalars (real scalars plus pseudoscalars)

order 8 quaternions

order 16 complex quaternions or multivariate 4-vectors

order 32 double quaternions

order 64 complex double quaternions or multivariate vector quaternions

The series can also be presented as an endless succession of just three processes: conjugation (which introduces opposite algebraic signs), complexification (which multiplies throughout by a single imaginary term); dimensionalization (which multiplies again by the imaginary term completing the quaternion set – complex numbers are merely incomplete quaternion sets):

order 2	conjugation	$\times (1, -1)$
order 4	complexification	$\times (1, \boldsymbol{i}_1)$
order 8	dimensionalization	$\times (1, \boldsymbol{j}_1)$
order 16	complexification	$\times (1, \boldsymbol{i}_2)$
order 32	dimensionalization	$\times (1, \boldsymbol{j}_2)$
order 64	complexification	$\times (1, \boldsymbol{i}_3)$

The conjugation process is only applied once because further application would make no difference to the character set. The complexification and dimensionalization processes, however, alternate to infinity, with repetition beginning at order 16.

If we equate conjugation with the physical property of conservation, implying that no + value can be acquired by conserved or conjugated quantity unless accompanied by the equivalent – value, then each of these processes is equivalent to one of the property / antiproperty distinctions in the parameter table in section 3. The parameters mass, time, charge and space, being successively real scalar, pseudoscalar, quaternion and multivariate vector (or complex quaternion), also clearly encode the stages in the emergent algebra up to the point of repetition at order 16. However, as we have seen in section 1, to incorporate all these as independent units of a single comprehensive algebra, we need to take the series to order 64, using complex double quaternions or multivariate vector quaternions. This is how we generate the Dirac state.

6. The Dirac state

In purely algebraic terms, the *pentad* structures exemplified by the five terms of (1) and (2) are the most efficient way of compactifying the respective real scalar, pseudoscalar, quaternion and multivariate vector units of mass, time, charge and space into a single package. As we will see, they generate a new infinite series of closed systems, which allow an immediate return to zero totality for the whole. To create a pentad mathematically, it is necessary to take the units of one of the two 3-dimensional components (the quaternion charge or vector space) and impose each onto the units

of the other three parameters. It is most convenient, for this purpose, to use the conserved parameter charge, which, unlike space, has individually identifiable units. So, beginning with

time	space	mass	charge
i	$\mathbf{i}\ \ \mathbf{j}\ \ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$

we may impose each of the three charge units onto one of the algebraic expressions representing time, mass or space:

i	$\mathbf{i}\ \ \mathbf{j}\ \ \mathbf{k}$	1	$\boldsymbol{i}\ \boldsymbol{j}\ \boldsymbol{k}$
$\boldsymbol{k}$	$\boldsymbol{i}$	$\boldsymbol{j}$	

to obtain the following combinations:

$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}\ \ \boldsymbol{i}\mathbf{j}\ \ \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$

although, sometimes, it will be mathematically convenient to multiply all units by an additional i:

$\boldsymbol{k}$	$i\boldsymbol{i}\mathbf{i}\ \ i\boldsymbol{i}\mathbf{j}\ \ i\boldsymbol{i}\mathbf{k}$	$i\boldsymbol{j}$.

The generation of composite units may be expected to generate entirely new physical parameters, which combine the properties of conservation and discrete quantization characteristic of charge with the individual properties which characterize the respective parent parameters, time, space and mass. So, the new composite parameters, which we may call the Dirac energy (E), the Dirac momentum ($\mathbf{p}$) and the Dirac rest mass (m), in addition to being conserved and quantized, will also retain the respective pseudoscalar, multivariate vector, and real scalar properties of time, space and mass:

$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}\ \ \boldsymbol{i}\mathbf{j}\ \ \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$
E	$\mathbf{p}$	m

while, at the same time breaking the symmetry between the three nongravitational interactions by imposing pseudoscalar, multivariate vector, and real scalar properties onto the respective weak, strong and electric charges.

However, it will also be possible to express the same superposition in the context of nonconservation in terms of the quantum (or differential) operators, relating to the parent quantities, time and space:

$i\boldsymbol{k}$	$\boldsymbol{i}\mathbf{i}\ \ \boldsymbol{i}\mathbf{j}\ \ \boldsymbol{i}\mathbf{k}$	$\boldsymbol{j}$
$\partial/\partial t$	∇	m

though the object on which they act must be structured to produce the same conserved state as is represented by E, $\mathbf{p}$ and m. Such a result is obtained by a differential operator acting on the exponential or 'wave' term, $e^{-i(Et\,-\,\mathbf{p.r.})}$, which is, in principle, nothing more than the mathematical representation of the group of space and time translations and rotations, which provide the maximal variation or 'nonconservation' for space and time coordinates in the most idealised or 'free' state. Through this process, E and t, and $\mathbf{p}$ and $\mathbf{r}$, become *conjugate variables*, exchanging statements about conservation into equivalent statements about nonconservation, and vice versa. Quantization, of course, in connecting all four parameters within a single algebraic structure, necessarily establishes direct and inverse numerical relationships between their units. The relationships between the units of E and $\mathbf{p}$, and those of t and $\mathbf{r}$, then lead to the introduction of the fundamental constants $h,^{-}$ and c, while the relationship with m requires a third constant, G, although, for convenience, of course, we tend to choose units such that $h,^{-} = 1$ and $c = 1$. Defining $h,^{-}$ ensures that the system is quantum, and defining c ensures that it is relativistic, and, in fact, it is only with the nilpotent form of the Dirac state that we define a physics which is truly quantum and truly relativistic.

Now, the parent-parameters determine that the three components of the Dirac state, E, $\mathbf{p}$, and m, are specified by unrestricted real number values, and so it becomes possible, using the anticommuting properties of the quaternion and vector operators, and the presence of at least one complex term, to find values of this state, ($\pm\ \boldsymbol{k}E \pm i\boldsymbol{i}\mathbf{p} + i\boldsymbol{j}m$), which square to a *zero numerical*

solution. This produces a new level of closure within our algebraic system allowing a new return to zero, for we can use the square roots of zero to define those states in which the conservation of E, $\mathbf{p}$, and m, applies at the same time as the absolute nonconservation of space and time. The expression which results is the nilpotent Dirac equation, which in its purest or free state form is given by:

$$\psi = (\pm \boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)\, e^{-i(Et - \mathbf{p.r})} = 0\ .$$

The form of the Dirac equation, however, remains the same even when the state is no longer free and the functional term changes from $e^{-i(Et - \mathbf{p.r})}$. In all cases, we define a functional term so that the application of the differential operator produces an eigenvalue of the form $(\pm \boldsymbol{k}E \pm i\boldsymbol{i}\,\mathbf{p} + i\boldsymbol{j}\,m)$, so producing the zero total product we require. Using this technique, we can solve the Dirac equation when the state is constrained by fields which have the characteristics of the electric, strong and weak interactions.[5] Through the Dirac equation also, we can use the natures of E, $\mathbf{p}$ and t, $\mathbf{r}$ to establish a nilpotent structure connecting t and $\mathbf{r}$, with another term τ (described as the 'proper time') in the position occupied by m. This structure may be expressed in the form $(\pm i\boldsymbol{k}t \pm \boldsymbol{i}\mathbf{r} + \boldsymbol{j}\tau)$, and the theory known as 'special relativity' then becomes the working out of its consequences under classical conditions.

Of course, the individual Dirac state or nilpotent is merely a component exhibiting aspects of 'closure' within an infinite process. To determine the rest of the structure, we recall that the series of quaternion systems which produced the original infinite algebra are embedded within an open and infinite series of commutative complex terms of the form $\boldsymbol{i}_s$. So we may consider the nilpotents ψ_1, ψ_2, ψ_3, …, as having coefficients which are unrepeated but arbitrary units or strings of such units. We then generate an infinite-dimensional Grassmann algebra, with successive outer products defined by the Slater determinant, and so requiring $\psi_1 \wedge \psi_1 = 0$ and $\psi_1 \wedge \psi_2 = -\,\psi_2 \wedge \psi_1$, etc. In such an algebra, the state vector units ψ_n must be both nilpotent and antisymmetric, as the Dirac state requires, and each must be unique to avoid an immediate return to zero, which would lead to the trivial case of $\mathfrak{R} = 0$. (This is, of course, Pauli exclusion.) This infinite-dimensional algebra is in every way isomorphic to the complex Hilbert space of conventional quantum theory, which allows an algebraic and nonlocal superposition of fermionic or Dirac states throughout the entire universe. In this sense, then, both the Dirac state and the Dirac equation can be seen as logical consequences of a state of zero universal totality.

References

1 P. Rowlands and J. P. Cullerne, arXiv:quant-ph/0109069.
2 P. Rowlands, *Speculat. Sci. Tech.*, 6, 69-80, 1983.
3 P. Rowlands, arXiv:physics/0106054.
4 P. Rowlands, J. P. Cullerne and B. D. Koberlein, arXiv:physics/0110092.
5 P. Rowlands, arXiv:quant-ph/0301071.
6 P. Rowlands and B. Diaz, arXiv:cs.OH/0209026.

NATURE OF TIME AND PARTICLES-CAUSTICS: PHYSICAL WORLD IN ALGEBRODYNAMICS AND TWISTOR THEORY

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Abstract. In the field theories with twistor structure particles can be identified with caustics of null geodesic congruences defined by the twistor field. As a realization, we consider the "algebrodynamical" approach based on the field equations which originate from noncommutative analysis (over the algebra of biquaternions) and lead to the complex eikonal field and the set of gauge fields associated with solutions of the latter. Related concepts of generating World Function and of multivalued physical fields are discussed. The picture of Lorentz invariant light-formed aether and of Matter born from light arises then quite naturally. Existence of the Time Flow can be also justified and related to the existence of the primodial Light aether.

1 Introduction. The algebrodynamical field theory.

Theoretical physics has arrived to the crucial point at which it should fully reexamine the sense and the interrelations of the three fundamental entities: fields, particles and space-time geometry. *String theory* offers a way to derive the low-energy phenomenology from the unique physics at Plankian scale. However, it doesn't claim to find the *origin* of physical laws, the *Code of Universe* and is in fact nothing but one more attempt to *describe* Nature (in a possibly the most effective way) but not at all to *understand* it.

Twistor program of R. Penrose [1, 2] suggests an alternative to string theory in the framework of which one can hope, in principle, to explain the structure of physical equations. For this, one assumes the existence of the primary *twistor space* $\mathbb{C}P^3$ which underlies the physical space-time and predetermines its Minkowsky geometry and field structure.

However, general concept of twistor program as a unified field theory is not at all clear or formulated up to now. Which equations are really fundamental, in which way can the massive fields be described and in which way the particles' spectrum can be obtained? And, finally, why exactly twistor, a rather refined mathematical object, should be taken as a basis of the fundamental physics?

In the interim, twistor structure arises quite naturally in the so called *algebrodynamics* of physical fields which has been developed in the author's works. From general viewpoint, the paradigm of algebrodynamics can be thought of as a revive of Pithagorean or Platonean ideas about *"Numbers governing physical laws"*. As the only (!) postulate of algebrodynamics one admits the existence of a certain unique and exeptional structure (of purely abstract, algebraic nature) the internal properties of which completely determine both the geometry of physical space-time and the dynamics of physical fields (the latters also of algebraic nature).

The most successful realization of algebrodynamics has been achieved via generalization of complex analysis to the exeptional noncommutative algebras of quaternion ($\mathbb{Q}$) type [11, 12,

13, 17]. It was demonstrated that explicit account of noncommutativity in the very definition of functions "differentiable" in $\mathbb{Q}$ inevitably results in the *non-linearity* of generalized Cauchy-Riemann equations (GCRE). This makes it possible to regard the GCRE as fundamental dynamical equations of *interacting* physical fields.

In the algebra of complex quaternions $\mathbb{B}$ (*biquaternions*) the GCRE acquire the property of Lorentz invariance and, moreover, the gauge and the spinor structures. On this base a self-consistent unified *algebrodynamical* field theory has been constructed in our works [11, 12, 16, 18, 19, 20, 22].

From the most general viewpoint, the GCRE possess a fundamental *light-like* structure which manifests itself in the fact that every (spinor) component $S(x, y, z, t) \in \mathbb{C}$ of the primary $\mathbb{B}$-field must satisfy the *complex eikonal equation* (CEE) [11, 12]

$$\eta^{\mu\nu}\partial_\mu S\partial_\nu S = (\partial_t S)^2 - (\partial_x S)^2 - (\partial_y S)^2 - (\partial_z S)^2 = 0, \tag{1}$$

where $\eta_{\mu\nu} = diag\{1, -1, -1, -1\}$ is the Minkowsky metric and by ∂ the partial derivation by respective coordinate is denoted. The CEE (1) is Lorentz invariant, nonlinear and plays the role similar to that of the *Laplace equation* in complex analysis. Every solution to GCRE can be reconstructed from a set of (four or less) solutions to CEE.

In the meantime, in [22] the general solution to CEE has been presented via investigating of its intrinsic *twistor* structure. It turns out that in this respect every CEE solution belongs to one of two classes, which both can be obtained from a twistor generating function via a simple and purely algebraic procedure. This construction defines also the singular loci of null geodesic congruences related to the eikonal field – the *caustics*. Just at the caustics where the neighbouring rays of the congruence intersect each other the associated physical fields turn to infinity and form, therefore, a particle-like object. Thus, in the algebrodynamical theory *the particles can be considered as (spacially bounded) caustics of related null congruences*.

On the other hand, null congruences naturally defines the "3+1" splitting procedure and points to exceptional role of the time coordinate in the algebrodymamical scheme and in twistor theory in general. Existence of the "Flow of Time" becomes therein a direct consequence of the existence of Lorentz invariant "aether" formed by the primodial light-like congruences ("pre-Light"). We, however, underline the property of *multivaluedness* of the fundamental complex solution to CEE ("World solution") and of the associated physical fields. As a result, at each space-time point one has a *superposition* of a great number of null geodesic congruences and the time flow turns to be *multi-directional*, i.e. consists of a number of different (locally independent and globally unique) "subflows".

In section 2 we consider the structure of CEE and the procedure of algebraic construction of its two classes of solutions. A few simple illustrative examples are presented. In section 3 we discuss the caustic structure of the CEE solutions, in particular of spatially bounded type (*particle-like singular objects*), and the properties of associated physical fields. In section 4, we introduce the "World function" responsible for generating of the "World solution" to CEE and suggest the related concept of multidimensionality of physical fields. Finally, in section 5, we discuss some general issues which bear of the notion of the *primodial light* flow and *light-formed aether* and consider the related problem of *time nature* if the latter is treated in the framework of the twistor theory and of the algebrodynamics especially.

The article is a continuation of our paper [31]. In order to simplify the presentation, we avoid the application of the 2-spinor and of the other refined formalisms refering for this the readers to our recent papers [19, 22, 21, 17, 20].

2 The two classes of solutions to the complex eikonal equation

The eikonal equation describes the process of propagation of wave fronts (field discontinuities) in any relativistic theory, in Maxwell electrodynamics in particular [4, 5]. Physical and mathematical problems related to the eikonal equation were dealt with in a lot of works, see e.g. [6, 7, 8].

The complex eikonal equation (CEE) arises naturally in problems of propagation of restricted light beams [9] and in theory of congruences related to solutions of Einstein or Einstein-Maxwell system [10]. We, however, additionally interpret the complex eikonal as a fundamental *physical field* which describes, in particular, interacting and self-quantized "particle-like" objects formed by singularities of the CEE solutions. By this, the electromagnetic and other conventional fields can be associated with every solution of the CEE and are responsible for description of the process of interaction of singularities.

Let us define, together with Cartesian space-time coordinates $\{t, x, y, z\}$, the so called *spinor* or *null* coordinates $\{u, v, w, \bar{w}\}$ (the light velocity is taken to be unity, $c = 1$)

$$u = t + z, \quad v = t - z, \quad w = x - iy, \quad \bar{w} = x + iy \tag{2}$$

which form the *Hermitian* 2×2 matrix X of coordinates

$$X = X^{+} = \begin{pmatrix} u & w \\ \bar{w} & v \end{pmatrix} \tag{3}$$

In the spinor coordinates representation the CEE (1) looks as follows:

$$\partial_u S \partial_v S - \partial_w S \partial_{\bar{w}} S = 0 \tag{4}$$

and is equivalent to the statement that the complex 4-gradient vector $\partial_\mu S$ is *null*.

CEE possesses a remarkable *functional invariance* [11, 12]: for every $S(X)$ being its solution any (differentiable) function $f(S(X))$ is also a solution. CEE is known [5] to be invariant under transformations of the full 15-parameter *conformal group* of the Minkowsky space-time.

Let us take now an *arbitrary homogeneous* function Π of two pairs of complex variables transforming as *2-spinors* under Lorentz rotations

$$\Pi = \Pi(\xi_0, \xi_1, \tau^0, \tau^1), \tag{5}$$

which are linearly dependent at each space-time point X via the so called *incidence relation*

$$\tau = X\xi \quad \Leftrightarrow \quad \tau^0 = u\xi_0 + w\xi_1, \quad \tau^1 = \bar{w}\xi_0 + v\xi_1. \tag{6}$$

The pair of 2-spinors $\{\xi(x), \tau(x)\}$ linked by Eq.(6) is known as (null projective) *twistor* of the Minkowsky space-time [2].

Let us assume now that one of the components of the spinor $\xi(x)$, say ξ_0, is not zero. Than, by virtue of homogeneity of the function Π we can reduce the number of its arguments to *three projective twistor components*, namely to

$$\Pi = \Pi(G, \tau^0, \tau^1), \qquad G = \xi_1/\xi_0, \quad \tau^0 = u + wG, \quad \tau^1 = \bar{w} + vG \tag{7}$$

With respect to the results of our paper [22],
any (analytical) solution of CEE belongs only to one of two classes and can be obtained from some generating twistor function of the form (7) via a simple algebraical procedure.

For the first class of solutions, let us simply resolve the algebraic equation

$$\Pi(G, u + wG, \bar{w} + vG) = 0 \tag{8}$$

with respect to the only unknown G. Then we come to a complex field $G(X)$ which necessarily satisfies the CEE. Indeed, after substitution $G = G(X)$ Eq.(8) becomes an *identity* and, in particular, can be differentiated with respect to the spinor coordinates $\{u, v, w, \bar{w}\}$. By this we get

$$P\partial_u G = -\Pi_0, \quad P\partial_w G = -G\Pi_0, \quad P\partial_{\bar{w}} G = -\Pi_1, \quad P\partial_v G = -G\Pi_1, \tag{9}$$

where Π_0, Π_1 are the partial derivatives of Π with respect to its twistor arguments τ^0, τ^1 and P is the *total* derivative with respect to G

$$P = \frac{d\Pi}{dG} = \frac{\partial \Pi}{\partial G} + w\Pi_0 + v\Pi_1 \tag{10}$$

which we thus far assume to be nonzero in the space-time domain considered. Multiplying then Eqs.(9) we find that $G(X)$ satisfies the CEE in the form (4). It is easy to check that *arbitrary* twistor function $S = S(G, u + wG, \bar{w} + vG)$ under substitution of the obtained $G = G(X)$ also satisfies the CEE (owing to the functional constraint (8) it depends in fact on only *two* independent twistor variables).

To obtain the second class of CEE solutions, we have from the very beginning to differentiate the generating function Π (7) with respect to G and after this to resolve the resulted algebraic equation

$$P = \frac{d\Pi}{dG} = 0, \tag{11}$$

again with respect to G. Now the function $G(X)$ does not satisfy the CEE; however, if we substitute it into (7) the quantity Π becomes an explicit function of space-time coordinates and necessarily satisfies the CEE (as well as any function of Π by virtue of functional invariance of the CEE). Indeed, differentiating the function Π with respect to the spinor coordinates and taking into account the already satisfied condition (11) we get for this case

$$\partial_u \Pi = \Pi_0, \quad \partial_w \Pi = G\Pi_0, \quad \partial_{\bar{w}} \Pi = \Pi_1, \quad \partial_v \Pi = G\Pi_1, \tag{12}$$

from where the CEE (4) for the function Π follows immegiately.

Solutions of the first type are well known. Indeed, apart from the CEE the field $G(X)$ obtained by the resolution of Eq.(8) satisfies, as it is easily seen from Eqs.(9), the *over-determined* system of differential constraints

$$\partial_u G = G\partial_w G, \quad \partial_{\bar{w}} G = G\partial_v G \tag{13}$$

which define the so called *shear-free (null geodesic) congruences* (SFC). By this, the algebraic Eq.(8) represents *general solution* of Eqs.(13) describing, therefore, the whole of SFC in the Minkowsky space-time. This statement which has been proved in [14] is well known as the *Kerr theorem*.

As to the second class of the CEE solutions obtained through the algebraic constraint (11), to our knowledge it hasn't been considered previously. It is known, however, that condition (11) defines the *singular locus* for SFC, i.e. for the CEE solutions obtained from the Kerr condition (8). It corresponds to the *branching points* of the principal field $G(X)$, i.e. to the space-time

points at which Eq.(8) has *multiple roots*. As to the CEE solutions of the second class themselves, their branching points occure at the locus defined by another obvious condition

$$Q = \frac{d^2\Pi}{dG^2} = 0. \tag{14}$$

The shear-free congruences, as well as their singularities and branching points, play the crucial role in the algebrodynamical theory. They will be discussed below in more details. Here we only repeat that, as it has been proved in [22] and can be considered as a generalization of the Kerr theorem,

the two simple generating procedures described above exhaust all the (analytical) solutions to the CEE representing thus its general solution

(note only that for solutions with $\xi_0 = 0$ another gauge should be used). To make the presentation more clear, we describe now several examples of the above construction.

1. Static solutions. Let the generating function Π depends on the twistor variables in the following way:

$$\Pi = \Pi(G, H), \quad H = G\tau^0 - \tau^1 = wG^2 + 2zG - \bar{w}, \tag{15}$$

where $z = (u - v)/2$ and the time coordinate $t = (u + v)/2$ is thus eliminated. Thus, the ansatz (15) evidently represents the most general type of static solutions to the CEE.

In [15, 10] it was proved that static solutions of the SFC equations (and, therefore, the static solutions to the CEE of the first class) with *spacially bounded* singular locus are exhausted (up to 3D translations and rotations) by the *Kerr solution* [14] which can be obtained from generating function of the form

$$\Pi = H + 2iaG = wG^2 + 2z^*G - \bar{w}, \quad (z^* = z + ia) \tag{16}$$

quadratic in G, with $a = const \in \mathbb{R}$. Explicitly resolving the equation $\Pi = 0$, we obtain the two "modes" of the field $G(X)$

$$G = \frac{\bar{w}}{z^* \pm r^*} = \frac{x + iy}{z + ia \pm \sqrt{x^2 + y^2 + (z + ia)^2}} \tag{17}$$

which in the case $a = 0$ correspond to the ordinary *stereographic projection* $S^2 \mapsto \mathbb{C}$ from the North or South pole respectively. It is easy to check that this solution as well as its twistor counterpartners (and every function of them)

$$\tau^0 = t + r^*, \quad \tau^1 = G\tau^0 \tag{18}$$

satisfy the CEE. Correspondent SFC is in the case $a = 0$ radial with a point singularity; in general case $a \neq 0$ the SFC is formed by the rectilinear constituents of hyperboloids and has a *ring-like* singularity of a radius $R = |a|$. Using this SFC, a Riemannian metric and an electric field can be defined which satisfy the electrovacuum Einstein-Maxwell system. In the case $a = 0$ this is the Reissner-Nordstrëm solution with Coulomb electric field, in general case – the Kerr-Newman solution. In the framework of algebrodynamics, electric charge of the point or of the ring singularity is necessarily fixed in modulus, i.e. "elementary" [11, 12, 20, 21] (see also [17] where the detailed discussion of this solution can be found).

Now let us obtain from the same generating function a solution to CEE of the second class. Differentiating Eq. (16) with respect to G and equating the derivarive to zero, we get $G = -z^*/w$

and, substituting this expression into Eq. (16), obtain finally the following solution to the CEE (which is unique-valued everywhere on 3D-space):

$$\Pi = -\frac{(r^*)^2}{w} = -\frac{x^2+y^2+(z+ia)^2}{x-iy}. \tag{19}$$

It is instructive to note that the equation $\Pi = 0$ being equivalent to two real-valued constraints $z = 0,\ x^2+y^2 = a^2$ defines the ring-like singularity for the Kerr solution (17), as it should be from the general viewpoint.

The static solutions of the II class with spatially bounded singularities are not at all exhausted by the solution (19). For example, the solution obtained from the generating function

$$\Pi = \frac{G^n}{H}, \quad n \in \mathbb{Z}, \quad n > 2 \tag{20}$$

has again the ring-like branching locus $z = 0,\ x^2+y^2 = \frac{(n-1)^2}{n}a^2$ which can be easily obtained from the system of equations $P = 0,\ Q = 0$ (see Eqs.(11),(14)).

Wave solutions. Consider also the generating functions dependent on only one of the twistor variables τ^0, τ^1, say on τ^0:

$$\Pi = \Pi(G, \tau^0) = \Pi(G, u + wG). \tag{21}$$

Both classes of the CEE solutions obtained via functions of the type (21) will then depend on only two spinor coordinates $u = t + z,\ w = x - iy$. This means, in particular, that the fields propagate along the Z-axis with fundamental (light) velocity. Example of a "photon-like" solution of the I class with spacially bounded singularity is presented in [21].

3 Particles as caustics of the primodial light-like congruences

Existence of a null geodesic congruence (NGC) for any solution of the eikonal equation is (at least, in the case of real eikonal) a well-known fact and follows, in particular, from the twistor structure inherent to the CEE. Indeed, according to the theorem above-presented any of the CEE solutions (both of the I and the II classes) is fully determined by a (null projective) twistor field $\{\xi(x), \tau(x)\}$ (in the choosed gauge $\xi_0 = 1, \xi_1 = G(x)$) subject to the incidence relation (6). The latter can be explicitly resolved with respect to the space coorditates $\{x_a,\ a = 1,2,3\}$ as follows:

$$x_a = \frac{\Im(\tau^+\sigma_a\xi)}{\xi^+\xi} - \frac{\xi^+\sigma_a\xi}{\xi^+\xi}t, \tag{22}$$

with σ_a being the Pauli matrices and the time t remaining a free parameter. Eq. (22) manifests that the primodial spinor field $\xi(x)$ reproduces its value along the rays formed by the unit *director vector*

$$\vec{n} = \frac{\xi^+\vec{\sigma}\xi}{\xi^+\xi}, \quad \vec{n}^2 \equiv 1, \tag{23}$$

and propagates along these locally defined directions with fundamental constant velocity $c = 1$. In the choosed gauge we have for Cartesian components of the director vector (23)

$$\vec{n} = \frac{1}{(1+GG^*)}\{\ (G+G^*),\ \ -i(G-G^*),\ (1-GG^*)\ \}, \tag{24}$$

the two its real degrees of freedom being in one-to-one correspondence with the two components of the complex function $G(x)$.

Thus, for every solution of CEE the space is foliated by a congruence of rectilinear light rays, i.e by an NGC. Principal field $G(x)$ of a NGC can be always extracted from one of the two algebraic constraints (8) or (11) which have as a rule not one but rather *a finite set* of different solutions. Suppose that the generating function Π is *irreducible*, i.e. can't be factorized into a number of twistor functions. Then a generic solution of the constraints will be represented by a *multivalued complex function* $G(x)$. Choose locally one continious *branch* of this function. Then a particular NGC and a set of physical fields can be associated with this branch.

Specifically, in the case of the I class of the CEE solutions the spinor $F_{(AB)}$ of *electromagnetic field* is defined explicitly in terms of twistor variables of the solution [19, 21, 22]

$$F_{(AB)} = \frac{1}{P}\left\{\Pi_{AB} - \frac{d}{dG}\left(\frac{\Pi_A \Pi_B}{P}\right)\right\}, \tag{25}$$

where Π_A, Π_{AB} are the first and the second order derivatives of the generating function Π with respect to its two twistor arguments τ^0, τ^1. For every branch of the solution $G(x)$ this field locally satisfies Maxwell vacuum equations. Moreover, as it has been demonsrated in [12, 19, 17], a complex Yang-Mills and a curvature field can be also defined via any CEE solution of the first class.

Consider now analytical continuation of a function $G(X)$ up to one of its branching points which corresponds to the case of multiple roots of Eq.(8) (or of Eq.(11) for the solutions of the II class). At this point the strength of electromagnetic field (25) turns to infinity. The same holds for other associated fields, for curvature field in particular. Thus, the locus of branching points (which can be 0-, 1- or even 2-dimensional, see below) manifests itself as a *common source* of a number of physical fields and can be identified (at least in the case it is bounded in 3-space) with a *particle-like* object.

Such formations are capable of much nontrivial evolution simulating their interactions and even mutual *transmutations* represented by bifurcations of the field singularities. They possess also a realistic set of "quantum numbers", a self-quantized electric charge and a Dirac-like gyromagnetic ratio among them. Numerous examples of such solutions and associated singularities can be found in our works [16, 17, 18, 21] (see also examples presented in the previous section).

On the other hand, for the light-like congruences – NGC – associated with CEE solutions via (24) the locus of branching points coincides with that of the basic G-field and represents itself the familiar *caustic* structure, i.e. the envelope of the system of congruence rays where the neighbouting rays intersect each other ("focusize"). From this viewpoint, *the "particles" are nothing but the caustics of the associated null congruences.*

4 The World function and multivaluedness of physical fields

At this point we have to decide which of the two types of the CEE solutions can be taken in our scheme as a representative for description of the Universe structure as a whole. *As a "World solution" we choose a solution of the I class* because a lot of peculiar geometrical structures and physical fields can be associated with any of the I class solutions [12, 16, 17]. Such a solution can be obtained algebraically from the Kerr constraint (8) and some twistor generating *"World function"* Π; geometrically it gives rise to the NGC with special property – zero shear [2, 3].

Moreover, the CEE solutions of the II class turn then to be also involved into play. In fact, for a solution of the I class singular locus is defined by the joint algebraic system of Eqs.(8),(11). If we resolve Eq.(11) with respect to $G(X)$ and substitute this field into (8), the latter equation $\Pi(G(X)) = 0$ would define then the singular locus (the caustic) for a solution of the first class. On the other hand, the function $\Pi(G(X))$ would necessarily satisfy then the CEE representing its II class solution (with respect to the theorem proved in section 2). Thus,
the eikonal field has here two different meanings being a basic physical field (a CEE solution of the I class) and, at the same time, a characteristic field (a solution of the II class) which describes the branching points of the basic field (i.e., the discontinuties of its 4-gradient).

Let us conjecture now that the World function Π is an *irreducible polinomial of a very high but finite order* so that Eq.(8) is an algebraic (not a transcendental) one. Note that Eq.(8) defines then an algebraic surface in the projective twistor space $\mathbb{C}P^3$.

The World solution of the CEE will have then a finite number of branches of a multivalued complex G-field and a finite number of null directions at any point (represented in 3-space by the director vector (24)) giving rise to equal number of distinct NGC. Any pair of these congruences at some fixed moment of time will, generically, have an *envelope* consisting of a number of connected *one-dimensional* subspaces-caustics some of them being bounded and representing "particles" of generic type. Other types of particle-like structures will be formed at the focal points of *three or more NGC* where Eq.(8) has a root of higher multiplicity. Formations of the latter type would, of course, meet rather rare, and their stability is problematic. Nonetheless, we can model both types of caustics in a simple example based on the generating function [21]:

$$\Pi = G^2(\tau^0)^2 + (\tau^1)^2 - b^2G^2 = 0, \quad b = const \in \mathbb{R} \tag{26}$$

which leads to the 4-th order polinomial equation for G-field. At initial moment of time $t = 0$ the singular locus consists of a pair of point singularities (with "elementary" and opposite electric charges) and a neutral 2-surface (ellipsoidal *cocoon*) covering the charges. The latter corresponds to the intersection of all 4 modes of the multivalued solution while each of the point charges are formed by intersection of a particular pair of (locally radial) congruences [21]. The time evolution of the solution and its singularities is very peculiar: for instance, at $t = b/\sqrt{2}$ the point singularities cancel themselves at the origin $r = 0$ simulating thus the process of *annihilation* of elementary particles.

Thus, we see that the multivalued fields are quite necessary to ensure the self-consistent structure and evolution of a complicated system of particles - singularities. One should not be confused by this unusual property of the primary G-field as well as of other associated fields including the electromagnetic one. "We never deal with fields but only with particles" (F. Dyson). Indeed, in convinient classical theories, the fields are only a tool which serves for description of particle dynamics and for nothing else. In nonlinear theories as well as in our algebrodymical scheme the fields are moreover responsible for creation of particles themselves (as regular *solitons* or *singularities* of fields respectively). In the first, more familiar case we apparently should consider the fields be univalued. The same sutuation occures in the framework of quantum mechanics where many quantization rules result from the requirement for the wave function to be univalued. However, as we have seen above, in our algebrodynamical construction *the field distributions must not necessarily be uni-valued !* As to the process of "measurement" of the field strength, say electromagnetic, it relates directly only to measurement of particles accelerations, currents etc., and the results are then converted into conventional field language.

However, this is not at all necessary (in recall, e.g, of the Wheeler-Feynman electrodynamics and numerous "action-at-a-distance" approaches [23]).

In particular, on the classical (nonstochastic) level we can deal with effective *mean values* of the field modes at a point: similar concept based on purely quantum considerations has been recently developed in the works [25]. In our scheme, the true role of the multivalued field will become clear only after the spectrum and the effective mechanics of particles-singularities will be obtained in a general and explicit form.

We hope that some sort of psycological barrier for acception of the general idea of the field multivaluedness will be get over as it was with possible *multidimensionality* of physical space-time. The advocated concept seems indeed very natural and attractive. In the purely mathematical framework, multivalued solutions of PDEs are the most common in comparison with those represented by δ-type distributions [24]. From physical viewpoint this makes it possible to naturally define a dualistic "corpuscular-field" complex of very rich structure which gather all the particles in the Universe into a unique object. The caustics-singularities are well-defined themselves and undergo a collective motion free of any ambiguity or divergence (the latter can arise here only in result of our unproper discription of the evolution process and of nothing else).

As to the crucial problem of specification of the particular type of generating *World function of the Universe* Π, we have some speculations concerning this problem which will be considered elsewhere.

5 The light-formed relativistic aether and the nature of time

Light-like congruences (NGC) are the basic elements of the picture of physical world which arise in the algebrodynamical scheme as well as in twistor theory in general. They densely fill the space and consist of a great number of branches - components superposed at each space point and moving in different directions with constant in modulus and universal (for any branch of multivalued solution, any point and any system of reference) fundamental velocity. *There is nothing in the Universe exept this primodial light flow ("pre-Light Flow")* because *the whole Matter is born by pre-Light and from pre-Light* in the caustic regions of "condensation" of the pre-light rays.

In a sense, one can speak here about an exeptional form of (relativistic invariant) *aether* which is formed by a flow of pre-Light. Such an aether has nothing in common with old models of *light-carrying* aether which had been treated as a sort of elastic medium. It consists of structureless light elements and is in full correspondence with special theory of relativity. At the same time, the presented notions of aether formed by pre-Light and of the light-born Matter evoke numerous associations with the Bible and with ancient Eastern phylosophy. We have no doubts that many teologists, philosophers or mistics were brought to imagine a similar picture of the World. However, in the framework of successive physical theory this beatiful picture becomes more trushworthy and, to our knowledge, has not yet been discussed in literature [1].

On the other hand, existence of the primodial light-formed aether and manifestation of universal property of local "transfer" of the generating field $G(X)$ with constant fundamental velocity c points to different status of space and time coordinates and offers a new approach to the problem of physical time as a whole. Indeed, it was seemingly a great success in understanding

[1] In a sense similar ideas have been advocated in the works [26, 27, 28, 29]

of time nature when in 1908 H. Minkowsky has joined the space and time into a unique 4-dimensional continuum. However, this synthesis has "shaded" the principal distinction of space and time entities and didn't clarify any of the numerous problems related to the nature of time.

Subjectively, we perceive *time* as a continious latent motion, a *flow*; everybody understands when one speaks about the "River of Time". As a rule, we consider this intrinsic motion to be *uniform* and independent on our will and on material processes. In fact, contrary to arbitrary displacements along a space coordinate which are to much extent ambiguious and different for individual bodies, we are all in a common and unceasing motion together with the Time River. This is a quintessence of the problem of time.

Surprisingly, almost all these consideration are absent in the structure of theoretical physics. Maybe, such situation is partially caused by the fact that the notion of intrinsic flow of time explicitly leads to the notion of its material carrier – the *aether* – which was exiled from physics after confirmation of the Einstein's theory. However, just the conjecture on the existence of the Lorentz invariant aether formed by light-like congruences opens the way to successive inclusion of the notion of time flow into the structure of theoretical physics. In fact, the intrinsic time flow is then materialized and identified with the *Flow of Primodial Light* (pre-Light), and the "River of Time" is then nothing but the "River of Light". Universality of light velocity justifies thus our subjective perception of uniformity of time flow. Note that this concept is in some aspects close to that of *the time generating flows* which has been developed in the works [30].

Locally *the Time Flow is multi-directional*: at each point of 3-dimensional space a great but finite number of directions do exist, and every branch of the multidimensional primary field defines one of these directions and propagates along it forming thus *a branch of the unique Time Flow*. We can think that due just to a great number of generating congruences we are not able to perceive local direction of the Time Flow. Besides, it is natural to suppose that the World Solution possesses a stochastic component which results in irregular chaotic changes of the local directions of the light congruences also impossible for perception. At the same time, the existence of universal velocity c is responsible for our subjective representation of the uniform and homogeneous Flow of Time.

To conclude, we can assert that the algebrodynamical approach based on a purely abstract structure (generalized CR-equations) and on correspondent nonlinear equation for the eikonal field (CEE) results in a striking and unexpected picture of physical World which naturally includes the notions of the Primodial Light Flow and of the Light-formed aether, of multivalued physical fields and of Matter born from the preLight (i.e., of particles – caustics of multivalued preLight congruence). It manifests also deep relations between the existence of universal "light" velocity and the existence of *Time Flow* making it possible to understand in a sense the origin of the Time itself. *Time is nothing but the primodial Light*, and *there is nothing in the World except the preLight Flow* which gives rise to all "dense" Matter in the Universe.

References

[1] R. Penrose, in: *Quantum gravity: an Oxford Symposium*, eds. C.J. Isham, R. Penrose, D.W. Sciama. – Clarendon Press, Oxford, 1975.

[2] R. Penrose and W. Rindler, "Spinors and Space-Time". Vol.2. – Cambridge Univ. Press, Cambridge, 1986.

[3] R. Penrose, *Classical and Quantum Gravity*, **14**, A299 (1997).

[4] V.A. Fock, "Theory of Space, Time and Gravitation". – "GITTL", Moscow, 1955 (in Russian).

[5] H. Bateman, "The Mathematical Analysis of Electrical and Optical Wave-Motion", Dover Publ. Inc., 1955.

[6] S. Fritelli, E.T. Newman and G. Silva-Ortigoza, *J. Math. Phys.*, **40**, 383, 1999;
E.T. Newman and A. Perez, *J. Math. Phys.*, **40**, 1089, 1999.

[7] V.I. Arnold, "Mathematical Methods of Classical Mechanics". – Springer-Verlag, Berlin, NY, 1980.

[8] V.I. Arnold, "Singularities of Caustics and Wave Fronts". – Kluwer, Dordrecht, 1990.

[9] V.P. Maslov, "Complex WKB-Method in Nonlinear Equations". – "Nauka", Moscow, 1977 (in Russian).

[10] R.P. Kerr and W.B. Wilson, *Gen. Rel. Grav.*, **10**, 273 (1979).

[11] V.V. Kassandrov, "Algebraic Structure of Space-Time and Algebrodynamics". – People's Friend. Univ. Press, Moscow, 1992 (in Russian).

[12] V.V. Kassandrov, *Gravitation & Cosmology*, **3**, 216 (1995); (gr-qc / 0007027).

[13] V.V. Kassandrov, *Acta. Applic. Math.*, **50**, 197 (1998);
V.V. Kassandrov, in: *"Quasigroups and Nonassociative Algebras in Physics"*, ed J. Lõhmus and P. Kuusk. – Inst. Phys. Estonia Press, 1990, p 202.

[14] G.C. Debney, R.P. Kerr and A. Schild, *J. Math. Phys.* , **10**, 1842 (1969).

[15] A.Ya. Burinskii, in: *"Problems of Theory of Gravitations and Elementary Particles". No.11*, ed. K.P. Stan'yukovich. – Atomizdat, Moscow, 1980, p. 47 (in Russian).

[16] V.V. Kassandrov and J.A. Rizcallah, in: *"Recent Problems in Field Theory"*, ed A.V Aminova, Kasan Univ. Press, Kasan, 1998, p 176; (gr-qc / 9809078).

[17] V.V. Kassandrov and J.A. Rizcallah, in: *"Fundamental Problems of High Energy Physics and Field Theory"*, ed. V.A. Petrov. – Inst. High Energy Phys., Protvino, 2002, p. 199; *Preprint* gr-qc / 9809056, 1998.

[18] V.V. Kassandrov and V.N. Trishin, *Gravitation & Cosmology*, **5**, 272, 1999; (gr-qc / 0007026).

[19] V.V. Kassandrov and J.A.Rizcallah, *Preprint* gr-qc / 0012109, 2000.

[20] V.V. Kassandrov, *Vestnik People's Friend. Univ.*, Fizika, **8(1)**, 34, 2000 (in Russian).

[21] V.V. Kassandrov, *Preprint* gr-qc /0308045.

[22] V.V. Kassandrov, *Gravitation & Cosmology*, **8**, Suppl. 2, 57 (2002).

[23] *Instantaneous Action-at-a-Distance in Modern Physics. "Pro" and "Contra"*, eds. A. Chubykalo, V. Pope and R. Smirnov-Rueda. – Nova Science Publ. Inc., NY, 1999.

[24] F.Lizzi, G.Marmo, G.Sparano and A.M. Vinogradov, *J. Geom. Phys.*, **14**, 34, 1994.

[25] A.A. Kirillov, *Phys. Lett.*, **B555**, 13 (2003);
A.A. Kirillov and D. Turaev, *Phys. Lett.*, **B532**, 185 (2002).

[26] L.S. Shikhobalov, *Vestnik S.-Petersb. Univ.*, **1(3)**, 109 (1997); **1(4)**, 118 (1999) (in Russian).

[27] I.A. Urusovskii, *Zarubezjnaya radioelectronika*, **3**, 3 (1996); **6**, 64 (1996); **6**, 66, 2000.

[28] V.V. Smolyaninov, *Uspechi Fiz. Nauk*, **170**, 1064 (2000) (in Russian).

[29] I.A. Shelaev, "Introduction to irreversible electrodynamics". – Dubna, 1999 (in Russian).

[30] A.P. Levich, in: *"Construction of Time in Natural Sciences: On the Way to Understanding of Time Phenomenon"*, ed. A.P. Levich. – Moscow State Univ. Press, Moscow, 1996 (in Russian); A.P. Levich, *Gravitation & Cosmology*, **1(3)**, 237 (1995).

[31] V.V. Kassandrov, in: *Mathematics and Practice. Mathematics and Culture. No. 2*, ed. M.Yu. Simakov. – Self-Education, Moscow, 2001, p.67 (in Russian).

Fuzzy phase space /space-time geometry As approach to nonrelativistic quantization

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Novel fuzzy quantization formalism which exploits phase space with fuzzy ordered set (Foset) structure studied [1]. In proposed formalism for discrete Foset A permitted two kinds of relations between its elements – fuzzy points (FP) a_i: standard ordering - $a_j \le a_k$, and weak equality (WE) - $a_j \backslash a_k$ characterized by $\omega_{jk} \ge 0$ [2]. In X^A continious Foset 1-space a_i WE to X^m ordered metricized subset points described by distribution $\omega_i(x) \ge 0$, which corresponds to a_i coordinate principal uncertainty σ_x. In Forset phase space FPS nonrelativistic particle m state $|m\}$ supposedly corresponds to FP characterized by x, p distributions $\omega(x), \omega^P(p)$; ($\omega^Q(q)$ for arbitrary observable Q). For m free and potential $U(x)$ motion FPS weak structure induces correlations between $\omega^{Q_i}, \omega^{Q_j}$ and restricts possible ω^Q evolution. Together with $|m\}$ spectral decomposition in FPS it induces $|m\}$ representation equivalent to rigged Hilbert space H; eventually for m standard quantization ansatz reconstructed. Comparision with Quantum Logic on fuzzy sets discussed [3]. Relativistic fussy space-time and its relation to Connes Noncommutative Geometry discussed [4].

References

[1] S.Mayburov, Proc. Quantum Foundations Conference, Vaxjo, Shweden, 2001 (W.S,Singapour, 2001)
[2] H.Bandmeyer, S.Gottwald 'Introduction in Fuzzy Sets Methods' (Academie Verlag, Berlin, 1993)
[3] J.Pykaz, Found. Phys., 30, 1503 (2000)
[4] S.Mayburov, Proc. VII Marcell Grossman conference (Jerusalem, 1997) (World Scientific, Singapour, 1998)

Gravitational wave's action on the atom of a cosmic maser

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The gravitational wave can affect the behavior of an atom belonging to a cosmic maser in three ways: by reconstructing its levels, by modifying the electromagnetic field of the maser and by changing the metric of the space-time in which the atom exists. All these effects can cause the change in the observable signal of the maser. It is shown that the first effect is negligibly small in comparison to the other two. Besides, the situation of the parametric resonance can take place provided its conditions are fulfilled. If the corresponding astrophysical system is found, the additional low-frequency component would appear in the maser signal. The height of this peak would be such that it could be observed with the help of existing instruments.

1. Introduction

In the earlier papers [1-2] the analysis of the possibility to use a specially chosen cosmic maser as a quantum remote detector of the gravitational waves (GW) was performed.

The idea is based on the recently discussed effect of the optic-mechanical parametric resonance (OMPR) predicted theoretically in [3-5]. The essence of it is that if the two-level atom in the (spectroscopically) strong resonant electromagnetic field (EMW) mechanically vibrates along the wave vector of the field at frequency equal to the Rabi frequency of the field, then the absorption and fluorescence spectra of this atom deforms and obtains the nonstationary periodic component. The amplitude of this component is much higher than the corresponding regular stationary peak that appears when there is no vibration.

Let such an atom belongs to a cosmic maser located near the source of the GW. Then there could be a situation when the OMPR is possible, and the maser radiation could obtain the high nonstationary (low-frequency) component. This signal would be observable with the help of the existing instruments, thus, giving the direct evidence of the GW existence. The set of physical conditions providing this possibility was qualitatively discussed in [1-2]. In this paper the supporting calculations are given.

We will consider a two-level atom in the field of the GW interacting with the spectroscopically strong cosmic maser EMW resonant to the transition. The GW can affect the situation in the three ways. First, it causes the transformation of the atomic levels discussed in [6], and, thus, leads to the change in the dipole moment of the atom in the EMW. Second, it causes the transformation of the electromagnetic field - the effect that discussed in view of the GW detection in [7]. Third, the atom starts to move as a whole along the geodesic in a way leading to the optic-mechanical parametric resonance which in this case could be called optic-metric parametric resonance. These various GW actions on the system will be consequently discussed here. The comparison of the obtained results will make it possible to formulate and solve the OMPR problem directly. After that the physical meaning of the result will be given.

2. Dipole moment's change caused by the GW

The Shroedinger equation for our case is

$$i\hbar \frac{d}{dt}\Psi = H\Psi \tag{1}$$

where the Hamiltonian H is

$$H(t) = H_0 + H_g(t) + H_{em}(t) \tag{2}$$

H_0 is the atomic Hamiltonian which is independent of time, $H_g(t)$ is the perturbation of the atomic Hamiltonian due to GW-atom interaction, $H_{em}(t)$ describes the EMW-atom dipole interaction which can be written as usual

$$H_{em}(t) = \mu E \tag{3}$$

Here μ is a matrix element of the induced dipole momentum operator, $E=E(t)$ is the electric stress of the EMW. The distance between the EMW resonant levels is much larger than the distance between the splitted sublevels that have appeared because of the GW [6]. Then μ can be calculated with the help of the perturbation theory using the corrections to the wave functions appearing with regard to the GW action. Therefore, one should first find the wave functions of the perturbed Hamiltonian

$$i\hbar \frac{d}{dt}\Psi^{(1)} = H_1\Psi^{(1)} = (H_0 + H_g(t))\Psi^{(1)} \tag{4}$$

The relativistic calculation performed in [6] gives[1]

$$H_g = mD\left(\frac{GL}{4c^3}\right)^{1/2}\left[\varepsilon^+{}_{ij} + \varepsilon^\times{}_{ij}\right]X^i X^j \frac{\cos(K_g r_S - Dt)}{r_S}, \tag{5}$$

The expression (5) may be rewritten as follows

$$\begin{aligned} H_g &= Fe^{-iDt} + Ge^{iDt} \\ F &= \frac{mD}{2r_S}\left(\frac{GL}{4c^3}\right)^{1/2}\left[\varepsilon^+{}_{ij} + \varepsilon^\times{}_{ij}\right]X^i X^j e^{iK_g r_S} \\ F_{nm} &= \int \Psi_n^{(0)*} F \Psi_m^{(0)} dq; F_{nm} = G_{mn}^* \end{aligned} \tag{6}$$

where F and G are time independent. Since F is obviously small, let us use the perturbation theory for the case when the perturbations are periodic functions of time [8]. The first approximation for the wave functions gives

$$\Psi_n^{(1)} = \Psi_n^{(0)} + \sum_k a_{kn}(t)\Psi_k^{(0)} \tag{7}$$

where $\Psi_m^{(0)}$ are the wave functions of the unperturbed Hamiltonian, and

$$a_{kn} = -\frac{F_{kn}e^{i(\omega_{kn}-D)t}}{\hbar(\omega_{kn} - D)} - \frac{F_{nk}^* e^{i(\omega_{kn}+D)t}}{\hbar(\omega_{kn} + D)} \tag{8}$$

Expressions (7-8) describe the wave functions of an atom in the GW field.
The expression for the induced dipole momentum is

$$\mu = (1 + a_{11} + a_{22})\mu_0 + e\int \sum_{k\geq 2} a_{k1}\Psi_k^{(0)*} r \Psi_2^{(0)} dq + e\int \sum_{k\neq 2} a_{k2}\Psi_1^{(0)*} r \Psi_k^{(0)} dq, \tag{9}$$

where μ_0 is the dipole momentum calculated with the wave functions $\Psi_1^{(0)}$, $\Psi_2^{(0)}$ of the unperturbed Hamiltonian. As it follows from the equation (8), the expressions for a_{11} and a_{22} are

$$a_{11} = a_{22} = \frac{1}{\hbar D}(F_{11}e^{-iDt} - F_{22}^* e^{iDt}), \tag{10}$$

[1]Here D is the frequency of the GW, m is the electron mass, G is the gravitational constant, L is the full energy flow from a GW source, c is the light speed, X^i is the coordinate components in the atomic center of inertia frame, K_g is the GW wave vector, r_S is the distance from the atom to the GW source, $\varepsilon^+{}_{ij} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \varepsilon^\times{}_{ij} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ are the unity polarization tensors with the nonvanishing components in the directions orthogonal to the GW propagation.

Here the two-level atom model is considered to be applicable, i.e., only one pair of levels takes part in the electromagnetic interaction. Let the frequency of the external monochromatic field correspond to the 1-2 transition. Then the expression (8) means that two last terms in (9) are proportional to the factors oscillating at high frequency, therefore, they may be omitted in the further calculation of the dipole momentum.

The evaluation of the dipole momentum amendment caused by the GW gives

$$a_{11} = \frac{1}{\hbar D} F \cong \frac{1}{\hbar D} \frac{mD}{2r_S} \left(\frac{GL}{4c^3} \right)^{1/2} r_a^{\,2} \tag{11}$$

here r_a is the atom's radius. To calculate the gravitational energy flow L one can use the formula given in [9]

$$L = \frac{288 G I^2 g_e^{\,2} D^6}{45 c^5} \tag{12}$$

It characterizes the neutron star with the gravitational ellipticity g_e and the inertia momentum I. According to [10], the expression for the dimensionless GW amplitude h is

$$h = \frac{GMR^2 D^2 g_e}{c^4 r_S} \tag{13}$$

where R is the star's radius. Then

$$a_{11} = \frac{1}{\hbar D} 12 \sqrt{\frac{2}{5}} D^2 m r_a^{\,2} h = 12 \sqrt{\frac{2}{5}} \frac{m r_a^{\,2}}{\hbar} D h \tag{14}$$

It can be seen that the amendment to the induced dipole momentum is linear in h with the coefficient defined by the equation (14) in which m and r_a are the atomic mass and radius.

3. The GW action on the EMW

To describe the wave process it is convenient to use the eikonal, i.e. the phase of the propagating wave $e^{i\psi}$. The EMW eiconal equation in the flat space-time (no gravitational field) is, thus, can be written as

$$\psi = \omega t - kx \tag{15}$$

In the simplest case - flat space-time, no gravitation - the eikonal equation is

$$\frac{\partial \psi}{\partial x_i} \frac{\partial \psi}{\partial x^i} = 0 \tag{16}$$

Rewriting equation (16) for the case when there is a gravitation field and the space-time is curved, one gets [11]

$$g^{ik} \frac{\partial \psi}{\partial x^i} \frac{\partial \psi}{\partial x^k} = 0 \tag{17}$$

where g^{ik} is the metric tensor. In the case when the GW propagates in the x direction

$$g^{ik} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 + h\cos\frac{D}{c}(x^0 - x^1) & 0 \\ 0 & 0 & 0 & -1 - h\cos\frac{D}{c}(x^0 - x^1) \end{pmatrix} \tag{18}$$

Here $h << 1$ is again the nondimensional amplitude of the GW, and the GW polarization is considered to be circular (not the elliptical) one for simplicity. Writing down the eikonal equation explicitly, one gets

$$(\frac{\partial\psi}{\partial x_0})^2-(\frac{\partial\psi}{\partial x_1})^2-(\frac{\partial\psi}{\partial x_2})^2-(\frac{\partial\psi}{\partial x_3})^2+h[(\frac{\partial\psi}{\partial x_2})^2-(\frac{\partial\psi}{\partial x_3})^2]\cos\frac{D}{c}(x_0-x_1)=0 \quad (19)$$

Since $h << 1$, the eikonal can be expressed as

$$\psi = f_1 + hf_2 \quad (20)$$

where f_1 and f_2 are new functions. Substitute equation (20) into equation (19), neglect all the terms containing h in the powers higher than unity and regard separately the equations with and without h. Then

$$\begin{aligned}&(\frac{\partial f}{\partial x_0})^2-\sum_i(\frac{\partial f}{\partial x_i})^2=0\\&2\frac{\partial f}{\partial x_0}\frac{\partial g}{\partial x_0}-2\frac{\partial f}{\partial x_i}\frac{\partial g}{\partial x_i}+[(\frac{\partial f}{\partial x_2})^2-(\frac{\partial f}{\partial x_3})^2]\cos\frac{D}{c}(x_0-x_1)=0\end{aligned} \quad (21)$$

The first of equations (21) gives

$$\begin{aligned}&f=f(\frac{\omega}{c}(x_0-x_1))\\&\omega^2=c^2k^2\end{aligned} \quad (22)$$

which means that the principal term of the EMW eikonal in the GW field corresponds to the flat wave. Substituting this result into the second equation of (21), one gets the equation for the function f_2

$$2\frac{\omega}{c}\frac{\partial f_2}{\partial x_0}+2k_1\frac{\partial f_2}{\partial x_1}+2k_2\frac{\partial f_2}{\partial x_2}+2k_3\frac{\partial f_2}{\partial x_3}+(k_2^{\ 2}-k_3^{\ 2})\cos\frac{D}{c}(x_0-x_1)=0 \quad (23)$$

This is the first order quasilinear inhomogeneous partial differential equation. Its solution is

$$f_2=-c^2\frac{k_2^{\ 2}-k_3^{\ 2}}{2\omega D}\cos\frac{D}{c}(x_0-x_1)+\varphi(k_1x_0-\frac{\omega}{c}x_1,k_2x_1-k_1x_2,k_3x_2-k_2x_3) \quad (24)$$

where φ is an arbitrary function defined by the boundary conditions. This function can be taken equal to zero for simplicity. Thus, the eikonal of the EMW in the GW field is

$$\psi=\frac{\omega}{c}x_0-k^ix_i-hc^2\frac{k_2^{\ 2}-k_3^{\ 2}}{2\omega D}\cos\frac{D}{c}(x_0-x_1) \quad (25)$$

These calculations show that the GW action upon the EMW is equivalent to the EMW phase modulation with the frequency of the GW and the amplitude

$$A_{em}=h\frac{\omega}{D} \quad (26)$$

4. The free particle in the GW field

Let us now turn to the geodesic equation describing a particle in the GW field. Our aim is to evaluate the amplitude of the particle's periodic displacement orthogonal to the GW wave vector.

To give an introductory example of what influence does a particle's displacement produce on the EMW signal detection, one can easily check the following. If the motionless particle located in the coordinates origin transmits the periodical electromagnetic signals to the remote detector, then the intervals of time between the neighbor emitted signals Δt_p are equal to the intervals of time between the neighbor received signals Δt_d . But if the particle vibrates according to a sine law along the axis parallel to the emitted EMW, the frequency of these vibrations is D and the amplitude is η, then the intervals of time between the neighbor received signals can be expressed by the intervals of time between the neighbor emitted signals in the following way

$$\Delta t_p = \Delta t_d + \frac{k\eta}{\omega}\sin(D\Delta t_p) \tag{27}$$

In terms of eikonal it means that the eikonal obtains the periodic time shift.

Let us now place the atom in the coordinates origin, the x_1 axis is the GW wave vector direction, the x_2 axis is the EMW wave vector direction coinciding with the direction at the Earth. The distance between the atom and the GW source is much less than the distance from any of them to the Earth. The free particle in the gravitation field moves along the geodesic described by the equation [11]

$$\frac{d^2x^i}{ds^2} + \Gamma^i{}_{kl}\frac{dx^k}{ds}\frac{dx^l}{ds} = 0 \tag{28}$$

The Christoffel coefficients $\Gamma^i{}_{kl}$ can be expressed with the help of the metric tensor components

$$\Gamma^i{}_{kl} = \frac{1}{2}g^{im}\left(\frac{\partial g_{mk}}{\partial x^l} + \frac{\partial g_{ml}}{\partial x^k} - \frac{\partial g_{kl}}{\partial x^m}\right) \tag{29}$$

Calculating the coefficients $\Gamma^i{}_{kl}$ with regard to the expression (18) for the GW metric tensor, one can substitute them all into the equation (28), neglect the terms in which h stands in the power higher than unity, and get the following system of equations

$$\begin{aligned}
&t'' + \frac{1}{2}h\frac{D}{c^2}(y'^2 - z'^2)\sin\frac{D}{c}(ct - x) = 0\\
&x'' + \frac{1}{2}h\frac{D}{c}(y'^2 - z'^2)\sin\frac{D}{c}(ct - x) = 0\\
&y'' + \frac{1}{2}h\frac{D}{c}y'(ct' - x')\sin\frac{D}{c}(ct - x) = 0\\
&z'' - \frac{1}{2}h\frac{D}{c}z'(ct' - x')\sin\frac{D}{c}(ct - x) = 0
\end{aligned} \tag{30}$$

where the common notation $x_0 = ct, x_1 = x, x_2 = y, x_3 = z$ was used again and $w' \equiv \frac{dw}{ds}$. The first two equations of the system (30) give

$$ct'' = x'' \tag{31}$$

which leads to

$$x = ct + ns + b \tag{32}$$

where n and b are arbitrary constants.

Let us analyze the meaning of the last expression. At the moment $t = 0$ the motionless atom is in the coordinates origin of the frame motionless relative to the GW source center, i.e. $x = 0$, $s = 0$. That is why $b = 0$. If there were no gravity field, the atom would never leave its place, therefore, $s = -ct/n$, $n = 1$. But since there is a GW, the value of n in view of the linear approximation used might differ from unity by the value proportional to h. That is $n = 1 + hn_1$ where $n_1 = O(1)$, and equation (32) gives

$$x = ct + (1 + hn_1)s \tag{33}$$

Substitute this expression into the third equation of the system (30), corresponding to the direction towards the Earth. Neglecting the terms in which h stands in the power higher than unity, one gets

$$y'' + h\frac{D}{c}y'\sin\left[\frac{D}{c}(1 + hn_1)s\right] = 0 \tag{34}$$

The solution of this equation is

$$y = u_0\int e^{h\frac{1}{1+hn_1}\cos\frac{D}{c}(1+hn_1)s}ds + d \tag{35}$$

where u_0 and d are integration constants. Since $h << 1$, one can get

$$y = u_0 \int \{1 + h\frac{1}{1+hn_1}\cos[\frac{D}{c}(1+hn_1)s]\}ds + d \tag{36}$$

and

$$y = u_0 s + h\frac{1}{(1+hn_1)^2}\frac{c}{D}\sin[\frac{D}{c}(1+hn_1)s] + d \tag{37}$$

Since $s = \frac{x-ct}{1+hn_1}$ and at $t = 0$ the particle is at the coordinates origin

$$y = \frac{u_0(x-ct)}{1+hn_1} - h\frac{1}{(1+hn_1)^2}\frac{c}{D}\sin\frac{D}{c}(ct-x) \tag{38}$$

One can see that there is a sine dependence of the atom's space coordinate corresponding to the direction along the EMW wave vector on the atom's time coordinate. It could be said that the amplitude η of the corresponding oscillations of the particle in the GW field is

$$\eta = h\frac{1}{(1+hn_1)^2}\frac{c}{D} \cong h\frac{c}{D} \tag{39}$$

Comparing these results with the expression (27), one can conclude that the GW determined behavior of a signal transmitting particle can produce the periodical phase shift in the signals received by the detector. The frequency of the shift is equal to the GW frequency, and its amplitude is equal to

$$k\eta = \frac{\omega}{c}h\frac{c}{D} = h\frac{\omega}{D} \tag{40}$$

Comparing the expressions (38), (40) with the expressions (27) and (25,26) one can see that both frequencies and amplitudes of the GW action on the EMW and on the atom's behavior are the same.

5. The possibility of the parametric resonance

To be able to investigate the parametric resonance possibility, let us formulate the obtained results and assumptions that are going to be used.

The formula (14) shows that the GW action on the atomic levels' reconstruction leading to the change in the dipole interaction is negligibly small in comparison to the other two factors, defined by the formulas (26) and (40). Therefore, writing down the Bloch's equations for the density matrix components corresponding to the two-level atom in the strong electromagnetic field in the GW, no additional terms in the expression for the dipole momentum are needed.

The influence of the weak gravitational perturbation on the monochromatic EMW leading to the latter's phase modulation can be described in terms of the amplitude modulation

$$E(t) = E_0(1 - h\frac{\omega}{D}\cos Dt)\cos\Omega t \tag{41}$$

In such interpretation it means that the EMW obtains the second component. The first component is strong and can be described by

$$E_1(t) = E_0\cos(\Omega_1 t - ky) \tag{42}$$

where $\Omega_1 = \Omega \cong \omega$ is the frequency of the external field coinciding with the atomic transition frequency. The second component is weak and can be described by

$$E_2(t) = E_0 h\frac{\omega}{D}\cos(\Omega_2 t - ky) \tag{43}$$

where $\Omega_2 = \Omega - D$.

The GW wave vector is considered to be orthogonal to the EMW wave vector. The GW action on the atomic motion can be interpreted as the appearance of the periodical component V_{osc} in the atomic velocity's component along the EMW wave vector. According to expression (38)

$$\begin{aligned} V_{osc} &= -V_1 \cos Dt \\ V_1 &= hc \end{aligned} . \tag{44}$$

In this case the full time derivative can be written as

$$\frac{d}{dt} = \frac{\partial}{\partial t} + V\frac{\partial}{\partial y}, V = v - V_1 \cos Dt . \tag{45}$$

The nature of the cosmic maser's electromagnetic field is discussed in [12,13]. Here it is important that this field can be described classically. The atom's dynamics can be described by the density matrix $\rho(v, z)$. The lower level of the atom will be considered the ground one for simplicity. Then the Bloch's equations are

$$\begin{aligned} &\frac{d}{dt}\rho_{22} = -\gamma\rho_{22} + 2i[\alpha_1 \cos(\Omega_1 t - k_1 y) + \alpha_2 \cos(\Omega_2 t - k_2 y)](\rho_{21} - \rho_{12}) \\ &\frac{d}{dt}\rho_{12} = -(\gamma_{12} + i\omega)\rho_{12} - 2i[\alpha_1 \cos(\Omega_1 t - k_1 y) + \alpha_2 \cos(\Omega_2 t - k_2 y)](\rho_{22} - \rho_{11}) . \\ &\rho_{22} + \rho_{11} = 1 \end{aligned} \tag{46}$$

Here ρ_{22} and ρ_{11} are the levels' populations, γ and γ_{12} are the longitudinal and transversal decay rates of the atom (since level 1 is the ground one, $\gamma_{12} = \gamma/2$), $\alpha_1 = \frac{\mu E_0}{\hbar}, \alpha_2 = \frac{\mu E_0 h\omega}{D\hbar}$ are Rabi parameters of the strong and weak components of the electromagnetic field; k_1 and k_2 are the wave vectors of the components, in our case they can be considered equal and equal to k. The usual way to deal with the problem like given by the expression (46) is the use of the rotating wave approximation (RWA) and the RWA substitutions

$$\begin{aligned} \rho_{21} &= R_{21} \exp[i(\Omega_1 t - kz)]; \\ \rho_{12} &= R_{12} \exp[-i(\Omega_1 t - kz)]; \\ \rho_{22} &= 2^{-1/2}\tilde{\rho}_{22} \end{aligned} \tag{47}$$

The main goal now is to obtain $Im(R_{12})$ which is proportional to the absorption coefficient of the additional EMW caused by the GW. Rewriting eqs. (46), one gets

$$\begin{aligned} &\frac{d}{dt}2^{-1/2}\tilde{\rho}_{22} = -2^{-1/2}\gamma\tilde{\rho}_{22} - iR_{12}\alpha_1 + iR_{21}\alpha_1 \\ &- iR_{12}\alpha_2 \exp i(-(\Omega_1 - \Omega_2)t - ky) + iR_{21}\alpha_2 \exp i((\Omega_1 - \Omega_2)t + ky) \\ &\frac{\partial}{\partial t}R_{12} = -(\gamma_{12} + i\omega - i\Omega_1 + ik(V_0 - V_1 \cos((\Omega_1 - \Omega_2)t + ky)))R_{12} \\ &- i[\alpha_1 + i\alpha_2 \exp i((\Omega_1 - \Omega_2)t + ky)](2^{1/2}\tilde{\rho}_{22} - 1) \end{aligned} \tag{48}$$

where $V_1 = hc$. Now dividing the equations (48) by α_1 and introducing the notation given in the Appendix one can rewrite (48) in matrix form

$$\frac{\partial}{\partial \tau}W = (Q_0 + \varepsilon Q_1(\tau))W + C(\tau) \tag{49}$$

where

$$Q_0 = i\begin{pmatrix} 0 & -1 & 1 \\ -1 & -\sigma & 0 \\ 1 & 0 & \sigma \end{pmatrix}; W = \begin{pmatrix} \tilde{\rho}_{22} \\ R_{12} \\ R_{21} \end{pmatrix}; C = i\sqrt{2}\begin{pmatrix} 0 \\ 1+\varepsilon a \exp(-i\delta_d \tau) \\ -1-\varepsilon a \exp(i\delta_d \tau) \end{pmatrix};$$
$$Q_1 = \begin{pmatrix} -1 & -ia\exp(i\delta_d\tau) & ia\exp(-i\delta_d\tau) \\ -ia\exp(-i\delta_d\tau) & -\Gamma + i\kappa v_1 \cos(\delta_d \tau) & 0 \\ ia\exp(i\delta_d\tau) & 0 & -\Gamma - i\kappa v_1 \cos(\delta_d\tau) \end{pmatrix}; \sigma = \delta + \kappa v \tag{50}$$

The equation (49) can be solved by the asymptotic expansion method with regard to ε as a small parameter. The goal is to obtain the principal asymptotic term which defines the absorption coefficient behavior. The mathematical methods used in the similar problems are discussed in detail in [3,4]. The main point of the solution is the use of the parametric resonance conditions

$$\begin{aligned} &\sqrt{\sigma^2+2} = |\delta_d| + \varepsilon v \\ &\sqrt{\sigma^2+2} = |\delta| + \varepsilon v \\ &v = O(1) \end{aligned} \tag{51}$$

that must be fulfilled in order to get the result. Finally, the solution gives

$$\mathrm{Im}(R_{21}) \propto aL[-C + A\cos(Dt+\varphi)] \tag{52}$$

The explicit expressions for L, C, A and φ are given in the Appendix.

The meaning of the obtained result is the following.

First, if there is no gravitation radiation, i.e. $a = 0$ (see the notation in the Appendix), no observable effects due to the PR appear in the principal term of the asymptotes. Second, if there is a GW, then the weak additional wave due to the GW action on the EMW produces an amplification of the signal, since the stationary component of the absorption coefficient in (52 has the negative sign. Third, the principal term of the asymptotic expansion used to calculate the absorption coefficient contains the nonstationary term proportional to A. Similarly to the situations discussed in [3-5] and also in [14], it means that the absorption is periodically replaced by the amplification. The frequency in this case is equal to the GW frequency. This third result is the most important and presents the main result of this report. Whatever structure does the stationary and well observed maser signal has, the appearance of the nonstationary component observed with the additional frequency filter will point at the existence of the parametric resonance in the system, that is to the existence of the reasons for this effect. These reasons are the periodical change in the EMW and the periodical change in the atom's velocity.

If the cosmic maser is located in such a vicinity of the GW source that the PR conditions discussed in detail in [1-2] are fulfilled, then there are only two possible mechanisms to make the atoms vibrate. One of them is the interaction of the magnetic momentum of maser's atoms with the alternative magnetic field of the GW source. If the maser's atoms have not got such a momentum, then the only reason for their mechanical vibrations is the gravitation radiation.

We conclude that if the frequency filter used for the observation of the thoroughly selected cosmic maser radiation could provide the possibility to observe the *nonstationary* signal at the GW source frequency, then this gives the direct evidence of the GW existence.

6. Discussion

The solution of the Bloch's equations for the density matrix components of the two-level atom interacting with the strong resonant EMW and the GW should be performed with the help of the regular dipole momentum. The GW action on the monochromatic EMW causing the periodical phase modulation is comparable to the GW action on the time dependence of the atom's coordinate

transversal to the GW wave vector. Provided a suitable maser is chosen, a suitable geometry of the maser-GW source-Earth system is realized, and the observations are modified, both these effects could lead to the parametric resonance the result of which could be hopefully observed.

The direct GW detection is a challenging problem for years [15]. Up to now the main evidence of the GW radiation existence is given in [16], and it is an indirect evidence. The problems of the direct GW observation based on the effect of the opto-metrical parametric resonance discussed above differ essentially from the usual problems already known which are: superhigh sensitivity and superhigh selectivity of the experimental set up. The demands for the astrophysical system are very specific. The neutron stars seem to be the most suitable periodic GW sources. But, on the one hand, the gas clouds in their vicinities usually suffer the powerful radiation that might prevent the existence of the stable maser radiation region. On the other hand, the upper level of the maser atom transition has to be a metastable one to provide the spectroscopically strong field with the Rabi frequency coinciding with the GW frequency in the case of the PR.

As it is discussed in [1-2], the main condition defining the location of the needed cosmic maser is

$$\mathrm{h}\frac{\omega}{D} \approx \frac{\gamma}{\alpha}$$

For example, for the known OH-masers [17] $\omega \sim 10^{10} c^{-1}, \gamma \sim 10^{-1} c^{-1}$. If the intensity of the maser is such that the Rabi frequency of its radiation is $\alpha \sim 10^{1} c^{-1}$ and the pulsar frequency is $D \sim 10^{2} c^{-1}$, then the amplitude of the GW should be $h \sim 10^{-10}$. For the Crab and Vela pulsars this corresponds to the distance close to the border of the wave zone [1-2].

Nevertheless, the needed system can principally exist, and this must be considered in the astronomical and astrophysical investigations. The recent discovery of the laser effect in the star shells [17] essentially broadens the field of search for the suitable astrophysical system.

7. Appendix

The notation used in formulas (49-50)

$$2^{1/2}\alpha_1 t = \tau; \frac{\Omega_2 - \Omega_1}{\alpha_1} = 2^{1/2}\delta_d; \frac{k}{\alpha_1} = 2^{1/2}\kappa; \frac{\gamma}{\alpha_1} = 2^{1/2}\varepsilon; \frac{\gamma_{12}}{\alpha_1} = 2^{1/2}\Gamma\varepsilon;$$

$$\frac{\omega - \Omega_1}{\alpha_1} = 2^{1/2}\delta; \frac{\alpha_2}{\alpha_1} = \varepsilon a; V_1 = \varepsilon v_1$$

Here τ is the dimensionless time, δ_d is the dimensionless distuning of the frequencies of the main and additional waves, ε is the dimensionless small parameter characterizing the strength of the field, Γ is the dimensionless transversal decay rate, δ is the dimensionless distuning of the main wave and the atomic transition frequencies, a is the dimensionless amplitude of the additional (weak) wave, $v_1 = V_1/\varepsilon = O(1)$ is the dimensionless velocity due to the atomic vibrations. The $2^{1/2}$ factor is not obligatory and is not always used. Here it is introduced to make the form of the equations fit the form used in [4].

The notation used in formula (52)

$$L = \frac{F + \sigma}{F^2\sqrt{2}[(\mathrm{Re}(\det K))^2 + (\mathrm{Im}(\det K))^2]}$$

$$A = \sqrt{N_c^{\,2} + N_s^{\,2}}$$

$$\varphi = -\arctan\frac{N_s}{N_c}$$

$$\mathrm{Re}(\det K) = \frac{1}{F^2}\{2B[4\kappa^2 v_1^{\,2} F^2 - \frac{a^2F^2}{(F-\sigma)^2}] - (\sigma^2 + 2\Gamma)$$

$$\mathrm{Im}(\det K) = \frac{1}{F^2}\frac{8a\kappa v_1 BF^2}{F-\sigma}$$

$$F = \sqrt{\sigma^2+2}; B = 1 + \Gamma(\sigma^2+1)$$

$$C = \frac{a\nu(F+\sigma)}{F}\{(\sigma^2+2\Gamma)(B^2+\nu^2) + \frac{2BF^2}{(F-\sigma)^2}[4\kappa^2 v_1^{\,2}(F-\sigma)^2 + a^2]\}$$

$$N_c = \frac{1}{F^2}\{-BF(\sigma^2+2\Gamma)^2(B^2+\nu^2) + 2B^2F(\sigma^2+2\Gamma)(4\kappa^2 v_1^{\,2}F^2 - \frac{a^2F^2}{(F-\sigma)^2}) +$$

$$\sigma(8\kappa^2 v_1^{\,2}F^2 - \frac{a^2F^2}{(F-\sigma)^2})(\sigma^2+2\Gamma)(B^2+\nu^2) - 2\sigma B(8\kappa^2 v_1^{\,2}F^2 - \frac{a^2F^2}{(F-\sigma)^2})(4\kappa^2 v_1^{\,2}F^2 - \frac{a^2F^2}{(F-\sigma)^2}) -$$

$$\frac{8a\kappa v_1 BF^2}{F-\sigma}[\nu F(\sigma^2+2\Gamma) + \frac{8a\kappa v_1 \sigma F^2}{F-\sigma}]\}$$

$$N_s = -\frac{1}{F^2}\{[\nu\sigma(\sigma^2+2\Gamma) + \frac{8a\kappa v_1\sigma F^3}{F-\sigma}][2B(4\kappa^2 v_1^{\,2}F^2 - \frac{a^2F^2}{(F-\sigma)^2}) - (\sigma^2+2\Gamma)(B^2+\nu^2) +$$

$$\frac{8a\kappa v_1 BF^2}{F-\sigma}[B\sigma(\sigma^2+2\Gamma) + F(8\kappa^2 v_1^{\,2}F^2 - \frac{a^2F^2}{(F-\sigma)^2})]\}$$

References

[1] - Siparov S.V. Space maser as an instrument to detect the gravitational waves
Proc. Conf. “Physical Interpretations of Relativity Theory» London, 1998
[2] - Siparov S. V. 2001, Space-time and Substance, 2, 44
[3] - Siparov S. V. 1997, Phys.Rev. A, 55, 3704
[4] - Kazakov A. Ya. & Siparov S. V. 1997, Opt. i Spectrosk., 83, 961
[5] - Siparov S. V. 1998, J.Phys. B, 31, 415
[6] - Leen T. K., Parker L. & Pimentel L. O. 1983, Gen. Relat. and Gravit., 15, 761
[7] - Pegoraro F., Picasso E. & Radicati L. A. 1978, J.Phys. A, 11, 1949
[8] - Landau L. & Lifshits E. 1963, Quantum mechanics, (Moscow, Nauka)
[9] - Amaldi E. & Pizzella G. 1979, in Astrofisika e Cosmologia Gravitazione Quanti e Relativita, (Firenze, Guinti Barbera), (Rus.trans. 1982, Mir, Moscow, p.241)
[10] - Thorne K.S., 1987, in *Three hundred years of gravitation*. Eds. Hawking S.W. and Israel W., Cambrifge University Press.
[11] - Landau L. & LifshitsE. 1967, The theory of field (Moscow, Nauka)
[12] - Cook A. H. 1977, Celestial masers, (London, Cambridge University Press)
[13] - Elitzur M. 1992, Astronomical masers, (Boston, Kluwer Academic Publishers)
[14] - Kazakov A. Ya. 1992, JETP, 102, 1484
[15] - Schutz, B. F. 1999, Class. Quantum Grav., 16, A131
[16] - Hulse, R. A., Taylor, J. H. 1975, Astrophys J., 195, L-51
[17] - Litvak, M. M. 1972, in Atoms and Molecules in Astrophysics, (London and New York, Academic Press)
[18] - S.Johansson, V.S.Letokhov. Pis’ma v ZhETF**, 75**, p.591, 2002

Dynamic stability of a concentrated variable rest mass in a gravitational field of a charged radiative body

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With the help of the obtained functions of Lagrange, equations of Hamilton-Jacobi and equations of a deviation of Weber, are estimated a dynamics stability, in particular, the known results of L.M.Kaplan for circular orbits in theory of Einstein are extended.

Basis are generalization of the solution centrated variable mass in a gravitational field of a massive charged radiative body with usage of a formalism of the theory of Kaluza-Klein-Wesson.

1. Introduction

The formalism of the theory Kaluza-Klein's in a limit gives the Einstein's and Maxwell[1]; the integrating in the metric 5D (five-dimensional Space) gravitational and electromagnetic potentials allows to receive the equations determining to a component of a metric tensor γ_{55}. Assumption about proportionality of in variants in dS^2 5D and in dS^2 4D

$$ds^2 = f dS^2 \tag{1}$$

allows to use the reference solutions in 4D, in particular spherically-symmetric; as an example we used the solution Vaidya-Bonner's[2]. It is used base at the account of a motion conceming a gravitating body of a stream noniteracting concentrated (in sense V.A.Fok[3]) charged points of variable mass $m_0(t,r)$. Their relation to mass m_0 of a gravitating body M_0 is much less than unity

$$\frac{m_0(t,r)}{M_0(t,r)} << 1. \tag{2}$$

At the same time masses taking off δm_0^* and falling in from the outside δm_0^{**} are considered small in comparison to mass m_0:

$$\frac{\delta m_0^*}{m_0} << 1, \quad \frac{\delta m_0^{**}}{m_0} << 1. \tag{3}$$

The count of the indicated stream of charged particles is realized by selection of a Lagrangian. So, we shall consider the solution Vaidya-Bonner[2]:

$$ds_B^2 = \left(1 - \frac{\alpha(u)}{r} + \frac{Q^2(u)}{r^2}\right) du^2 + 2dudr - r^2 d\theta^2 - r\sin^2\theta d\varphi^2, \tag{4}$$

where $u(t,r)$ - function which is included in Finkelstein's transformation[2]

$$du = dx^0 - \frac{dr}{1 - \alpha(u)/r + Q^2(u)/r^2}, \quad x^0 = ct, \tag{5}$$

$\alpha(u)$ - positive decreasing function from u.

From (4) we have

$$L = \left(1 - \frac{\alpha(u)}{r} + \frac{Q^2(u)}{r^2}\right)\dot{u}^2 + 2\dot{u}\dot{r} - r^2\theta^2 - r\sin^2\theta\dot{\varphi}^2, \tag{6}$$

from 6) it is clear that there is energy per unit of mass

$$(\gamma = \left(1 - \frac{\alpha(u)}{r} + \frac{Q^2(u)}{r^2}\right)\dot{u} + \dot{r}, \tag{7}$$

for a stream of noninteracting rest masses with density of charges $\rho_q, \rho_q^*, \rho_q^{**}$ and body with potential Q/r the total energy per unit of mass can be written to a view:

$$\tilde{\gamma} = \gamma + \left(\frac{\rho_q}{\rho_0} + \frac{\rho_q^*}{\rho_0^*}\cdot\frac{\rho_0^*}{\rho_0} + \frac{\rho_q}{\rho_0^{**}}\cdot\frac{\rho_0^{**}}{\rho_0}\right)\frac{Q}{r} + \int\left(\frac{F_0}{\rho_0 c} + \frac{\rho_0^*}{\rho_0}\tilde{A}_0 + \frac{\rho_0^{**}}{\rho_0}\tilde{W}_0\right)ds, \tag{8}$$

where ρ_0 - density of a localized mass of rest, $\rho_0 = \rho_0(s)$, ρ_q/ρ_0 - its specific density of its charge; ρ_q^*/ρ_0^*, ρ_q^{**}/ρ_0^{**} - specific density of particles charge; ρ_0^*, ρ_0^{**} - their mass density; $\tilde{A}_0$, $\tilde{W}_0$ - zero components velocities of particles:

$$\dot{\rho}_0^* = \frac{d\rho_0^*}{ds},\ \dot{\rho}_0^* = \frac{d\rho_0^{**}}{ds},\ \int\frac{F_0}{\rho_0 c} = W.$$

Here W is enthalpy of an active body, equivalent enthalpy of concentrated variable mass; the expression $W + \int\frac{\dot{\rho}_0^{**}}{\rho_0}\tilde{W}_0 ds$ introduces a source of effecting of energy in a localized mass (thermonuclear, chemical, equivalent to oozed heat at the expense of Joule effect, friction and other dissipative processes). The formula (8) under a sign of integral is written correspondingly to a hypothesis Papapetry-Mesherski's (short-range interaction).

It's naturally to suspect proportionality of product of specific densities and potential of a gravitating body to last term in (8), accountable for reproduction of particles:

$$\left(\frac{\rho_q}{\rho_0} + \frac{\rho_q^*}{\rho_0^*}\frac{\rho_0^*}{\rho_0} + \frac{\rho_q}{\rho_0^{**}}\frac{\rho_0^{**}}{\rho_0}\right)\frac{Q}{r} = k\left(W + \int\frac{\rho_0^{**}}{\rho_0}\tilde{W}_0 dss + \int\frac{\rho_0^*}{\rho_0}\tilde{A}_0 ds\right), \tag{9}$$

further we shall identify function of time with a footstep of coordinate in theory Kaluza-Klein's

$$u = x^5, \tag{10}$$

is a private (individual) select (choice) in interpretation of 5-th coordinate in the suggested theories[4]; at the analysis of physics of a black hole (on a hypersurface $r = \alpha(u)$)[5] cases of infinite expansion of time, as his (its) property are studied $u \to \infty$. If to present $x^5 = u$ as product

$$u = \lambda\cdot\tau_5(t,r)c = x^5, \tag{11}$$

where τ_5 - time, c - speed of light, λ - some dimensionless parameter, equal

$$\lambda = \frac{\tilde{\rho}^*}{\tilde{\rho}_0}A_s, \tag{12}$$

where ρ^* - density of a responding stratum of a substance of an attracted body, in which one forms radiation, ρ_0 - initial density of a body prior to the beginning changes of aircraft attitude, $A_s = R_0 / \Delta_0^*$ - geometrical factor – relation of initial radius R_0 of a body with density ρ_0 to thickness of a responding Δ_0^* stratum with density ρ^*. Thus the theoretical aspiration of the relation to indefinitely major quantity ($\Delta_0^* \to 0$) can mean infinite expansion of time $x^5 \to \infty$ and on the contrary. This property will be fixed by (with) frequency of a quantum (complete red bias of light[5]).

2. The equations of Hamilton-Jacobi

As mass of particles is variable, we shall record an equation, using γ - complete density of a particle, receiving his stationary value

$$\gamma = const. \tag{13}$$

The requirement (13) will be demonstrated below. Then Hamiltonian can be recorded as:

$$H_5 = M_0 c\gamma,$$

where $M_0 = \int m_0 ds$ - complete rest mass. So, we have an expression of impulse

$$P_A = M_0 c U_A,\ P^A = M_0 c U^A. \tag{14}$$

Apparently, $U_A U^A = 1$, and we gain

$$P_A P^A = M_0^2 c^2. \tag{15}$$

For activity on definition S we have

$$P_A = -\frac{\partial S}{\partial x^A}. \tag{16}$$

Then, allowing (14), (16) we have the equations of Hamilton-Jacobi by the way

$$\gamma^{AB} \frac{\partial S}{\partial x^A} \cdot \frac{\partial S}{\partial x^B} = M_0^2 c^2. \tag{17}$$

The metric in theory Kaluza-Klein for the signature ++--- will accept a view

$$dS^2 = ds^2 - \gamma_{55}\left(dx^5 + A_\alpha dx^\alpha\right)^2 \tag{18}$$

(for the signature ++--+ we have $dS^2 = ds^2 + \gamma_{55}\left(dx^5 + A_\alpha dx^\alpha\right)^2$). Here

$$ds^2 = g_{\mu\nu} dx^4 dx^\nu. \tag{19}$$

Let's address again to the metric (4). As the new labels for a component of a metric tensor $g_{00} = g_{55} = 1 - \frac{\alpha\left(x^5\right)}{r} + \frac{Q^2\left(x^5\right)}{r^2}$, $g_{22} = -r^2$, $g_{33} = -r^2 \sin^2\theta$, let's assume for (18) are entered

$$ds_B^2 = ds^2. \tag{20}$$

Then we shall designate potential of the basic body by the way

$$A_0 \equiv A_5 = Q/r\,. \tag{21}$$

The scalar function from the theory 5D is determined by equaling to zero point for a component of a tensor of Ricci

$$R_{5\alpha} = 0\,,\; R_{55} = 0\,, \tag{22}$$

$$F^{\lambda}_{\alpha;\lambda} = -3\gamma_{55}^{\frac{\lambda-1}{2}} F_{\lambda\alpha}\,. \tag{23}$$

Here $F_{\lambda\alpha}$ – components of a tensor of intensity of an electromagnetic field; for a case (21) and metric (4) we have the solution (22)

$$\gamma_{55} = \frac{C}{Q^{2/3}}\,,\; C = const\,. \tag{24}$$

With allowance for (4) we shall copy (18)

$$dS^2 = ds_B^2 - \gamma_{55}(Q)\left(dx^5 + A_5 dx^5 + \frac{A_5 dr}{g_{55}}\right)^2, \tag{25}$$

or

$$1 = \frac{ds_B^2}{dS^2} - \gamma_{55}\left(\frac{dx^5}{dS} + A_5\frac{dx^5}{dS} + A_5\frac{dr}{g_{55}dS}\right). \tag{26}$$

Let's assume

$$B = -\gamma_{55}\left(\frac{dx^5}{dS} + A_5\frac{dx^5}{dS} + A_5\frac{dr}{g_{55}dS}\right). \tag{27}$$

Then from (26) follows

$$\frac{dS^2}{ds^2} = \frac{1}{f}\,,\; f = 1 + \frac{B^2}{\gamma_{55}}\,. \tag{28}$$

Let's straighten find out sense B in our case.
Let's record builders of a momentum density in 5D and 4D[1]:

$$P_5 = \rho_0\left(g_{5\beta}U^{\beta} + \frac{B}{\sqrt{1 + B^2/\gamma_{55}}}\frac{Q}{r}\right)\frac{ds}{dS}\,, \tag{29}$$

and with allowance for (9)

$$p_5 = \rho_0\left[g_{5\beta}U^{\beta} + \left(\frac{\rho_q}{\rho_0} + \frac{\rho_q^*}{\rho_0^*}\frac{\rho_0^*}{\rho_0} + \frac{\rho_q}{\rho_0^{**}}\frac{\rho_0^{**}}{\rho_0}\right)\frac{Q}{r}(1+k)\right]. \tag{30}$$

Allowing $P_5 = p_5\, ds/dS$ as comparing (29), (30), we shall receive

$$\left(\frac{\rho_q}{\rho_0} + \frac{\rho_q^*}{\rho_0^*}\frac{\rho_0^*}{\rho_0} + \frac{\rho_q^{**}}{\rho_0^{**}}\frac{\rho_0^{**}}{\rho_0}\right)(1+k) = \frac{B}{\sqrt{1 + B^2/\gamma_{55}}}\,. \tag{31}$$

Thus, expression $B\Big/\sqrt{1 + B^2/\gamma_{55}}$ is proportional to specific charge and source of effecting of energy (through coefficient k). From (31) can be received for B

$$B=\frac{\left(\rho_q/\rho_0+\left(\rho_q^*/\rho_0^*\right)\left(\rho_0^*/\rho_0\right)+\left(\rho_q^{**}/\rho_0^{**}\right)\left(\rho_0^{**}/\rho_0\right)\right)(1+k)}{\sqrt{1-(1+k)^2\left(\rho_q/\rho_0+\left(\rho_q^*/\rho_0^*\right)\left(\rho_0^*/\rho_0\right)+\left(\rho_q^{**}/\rho_0^{**}\right)\left(\rho_0^{**}/\rho_0\right)\right)^2/\gamma_{55}}}. \tag{32}$$

It is necessary to make the note concerning the equation geodetic in 5D

$$\frac{DU^A}{dS}=\frac{dU^A}{dS}+\Gamma^A_{BC}U^BU^C=0. \tag{33}$$

For particles of a stationary value of a rest mass $m_0(\rho_0)=const$ (or the densities) in (33) occur (come up) the quadratic terms $\sim B^2/f$, who are not identifiable in classic electrodynamics[1]. In our case if to accept the relation to B^2/γ_{55} stationary values

$$B^2/\gamma_{55}=k_\gamma=const \tag{34}$$

and, means

$$f=const. \tag{35}$$

The indicated terms will be functions γ_{55}, K_γ and k, determining change of rest masses and their specific charges (see Appendix 1). From a problem Vaidya-Bonner's, allowing the approach in operation[5] for a tensor $T^{\mu\nu}=qK^\mu K^\nu$, where K^μ - null vector, directional along radius; density of energy q in a local frame of reference is equal

$$q=\frac{1}{\pi}\frac{1}{(\gamma+\dot{r})^2}\left\{\frac{Q^2}{r^4}\left(1-\frac{\alpha(x^5)}{r}+\frac{Q^2(x^5)}{r^2}\right)+\frac{2QQ_{,5}}{r^3}+\frac{\alpha_{,5}}{r^2}\right\}. \tag{36}$$

We can see that visible luminosity is far from hypersurface

$$L_\infty(x^5)=\lim_{r\to\infty,\dot{r}\to 0}4\pi r^2q=-\frac{\partial\alpha}{\partial x^5} \tag{37}$$

and

$$L_\infty=L(\gamma+\dot{r})^2,\ L=4\pi r^2q \tag{38}$$

So, the metric (25) can be written to a view

$$dS^2=\gamma_{AB}dx^Adx^B=\frac{1}{f}\left(1-\frac{\alpha(x^5)}{r}+\frac{Q^2(x^5)}{r^2}\right)(dx^5)^2+\frac{2}{f}dx^5dr-\frac{r^2}{f}d\theta^2-\frac{r^2}{f}\sin^2\theta d\varphi^2. \tag{39}$$

Nonzero builders of a metric tensor (in new for 5D) labels from here follow

$$\gamma^4_{55}=\frac{1}{f}\left(1-\frac{\alpha(x^5)}{r}+\frac{Q^2(x^5)}{r^2}\right)$$

(here γ^4_{55} differs from γ_{55})

$$\gamma^{15}=f,\ \gamma^{22}=-\frac{f}{r^2},\ \gamma^{33}=-\frac{f}{r^2\sin^2\theta},\ \gamma^{11}=-\gamma^4_{55}f^2 \tag{40}$$

Then the equations (17) will be written to a view:

$$\gamma^{15}\frac{\partial \mathcal{S}}{\partial r}\cdot\frac{\partial \mathcal{S}}{\partial x^5}+\gamma^{22}\left(\frac{\partial \mathcal{S}}{\partial \theta}\right)^2+\gamma^{33}\left(\frac{\partial \mathcal{S}}{\partial \varphi}\right)^2+\gamma^{11}\left(\frac{\partial \mathcal{S}}{\partial r}\right)^2=\mathcal{M}_0^2c^2. \tag{41}$$

or

$$\frac{\partial \mathcal{S}}{\partial r}\cdot\frac{\partial \mathcal{S}}{\partial x^5}-\frac{1}{r^2}\left(\frac{\partial \mathcal{S}}{\partial \theta}\right)^2-\frac{1}{r^2\sin^2\theta}\left(\frac{\partial \mathcal{S}}{\partial \varphi}\right)^2-\left(1-\frac{\alpha\left(x^5\right)}{r}+\frac{Q^2\left(x^5\right)}{r^2}\right)\left(\frac{\partial \mathcal{S}}{\partial r}\right)^2=\mathcal{M}_0^2c^2/f, \tag{42}$$

here

$$\frac{2}{\sqrt{fc}}H_5=-\partial\mathcal{S}/\partial x^5. \tag{43}$$

For a simplicity we search for the solution for a flat case $\varphi=0$ in the reference shape

$$\mathcal{S}=-\frac{2H_5}{\sqrt{fc}}x^5+l\theta+\mathcal{S}_r(r);\ l=M_0=const. \tag{44}$$

The solution for $\mathcal{S}_r$ looks like:

$$\mathcal{S}_r=\int\left[\frac{H_5}{c\sqrt{f}\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)}+\frac{\sqrt{H_5^2/fc^2-\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)\left(\frac{l^2}{r^2}+\frac{\mathcal{M}_0^2c^2}{f}\right)}}{\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)}\right]dr. \tag{45}$$

Allowing is $\partial\mathcal{S}/\partial H_5=const$, $\partial\mathcal{S}/\partial l=const$ we gain

$$x^5=\frac{c\sqrt{f}}{2}\int\left[\frac{1}{c\sqrt{f}\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)}+\frac{H_5/fc^2}{\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)\sqrt{\frac{H_5^2}{fc^2}-\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)\left(\frac{l^2}{r^2}+\frac{\mathcal{M}_0^2c^2}{f}\right)}}\right]dr. \tag{46}$$

$$\theta=\int\frac{ldr}{r^2\sqrt{\frac{H_5^2}{fc^2}-\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)\left(\frac{l^2}{r^2}+\frac{\mathcal{M}_0^2c^2}{f}\right)}}. \tag{47}$$

Relativity tests for are given in Appendix 2.

3. Requirements of density of energy constance γ

Let's show, that a requirement of a constance γ is the energy communication (connection) of change of rest masses of particles and total energy of an attractive body. Let's record a Lagrangian density of function with allowance of indicated stream particles (flat motion, $\varphi=0$)

$$\breve{L} = \rho_0 \left\{ \frac{1}{2}\left(1 - \frac{\alpha}{r} + \frac{Q^2}{r^2}\right)\left(\dot{x}^5\right)^2 + \dot{x}^5 \dot{r} - \frac{1}{2} r^2 l^2 \left(\frac{\rho_q}{\rho_0} + \frac{\rho_q^*}{\rho_0} + \frac{\rho_q^{**}}{\rho_0} \right) \frac{Q}{r} \dot{x}^5 + \right.$$
$$\left. + \dot{x}^5 \left(W + \int \frac{\dot{\rho}_q^{**}}{\rho_0} W_0 ds + \int \frac{\dot{\rho}_q^*}{\rho_0} A_0 ds \right) \right\}. \tag{48}$$

With allowance for (8), (34), (35) we shall receive

$$\breve{L} = \rho_0 \left\{ \frac{1}{2}\left(1 - \frac{\alpha}{r} + \frac{Q^2}{r^2}\right)\left(\dot{x}^5\right)^2 + \dot{x}^5 \dot{r} - \frac{l^2}{2r^2} + \frac{k_\gamma \sqrt{\gamma_{55}}}{\sqrt{f}\, k} \frac{Q}{r} \right\} . \tag{49}$$

$$\frac{\partial \breve{L}}{\partial \dot{x}^5} = \rho_0 \left\{ \gamma + \frac{k_\gamma \sqrt{\gamma_{55}}}{\sqrt{f} k} \frac{Q}{r} \right\}. \tag{50}$$

Where

$$\gamma + \frac{k_\gamma \sqrt{\gamma_{55}}}{\sqrt{f} k r} \cdot \frac{Q}{r} = \Upsilon . \tag{51}$$

Further

$$\frac{\partial \breve{L}}{\partial \dot{x}^5} = \rho_0 \Upsilon, \quad \frac{d}{ds} \frac{\partial \breve{L}}{\partial \dot{x}^5} = \frac{d(\rho_0 \Upsilon)}{ds} = \rho_0 \frac{d\Upsilon}{ds} + \Upsilon \frac{d\rho_0}{ds} . \tag{52}$$

From (49) we have

$$\frac{\partial \breve{L}}{\partial x^5} = \frac{1}{2}\left(\frac{L_M}{r} - \frac{Q L_Q}{r^2} \right) + \frac{k_\gamma}{\sqrt{f}\, r} \left\{ \frac{\sqrt{\gamma_{55}} Q}{k} \dot{x}^5 \right\}_{,5} . \tag{53}$$

Here $L_M = -\left(d\alpha / 2dx^5\right)\dot{x}^5$ - radiation of a central body because of losses of a rest mass, $L_Q = \left(dQ / 2dx^5\right)\dot{x}^5$ - radiation compensating losses. Equation of Lagrange

$$\frac{d}{ds} \frac{\partial \breve{L}}{\partial \dot{x}^5} - \frac{\partial \breve{L}}{\partial x^5} = 0$$

gives

$$\frac{d\Upsilon}{ds} = -\Upsilon \frac{d \ln \rho_0}{ds} + \frac{k_\gamma}{\sqrt{f}\, r} \left\{ \frac{\sqrt{\gamma_{55}} Q}{k} \dot{x}^5 \right\} + \frac{1}{2}\left(\frac{L_M}{r} - \frac{Q L_Q}{r^2} \right). \tag{54}$$

Requirement of a constance Υ is the expression

$$\frac{d \ln \rho_0}{ds} = \frac{k_\gamma}{\Upsilon \sqrt{f}\, r} \left\{ \frac{\sqrt{\gamma_{55}} Q}{k} \dot{x}^5 \right\}_{,5} + \frac{1}{2}\left(\frac{L_M}{r} - \frac{Q L_Q}{r^2} \right) \frac{1}{\Upsilon}. \tag{55}$$

Thus, the change of mass of a particle flux, including dissipative processes which are included in expression for k, should be compensated by inductive energy, bound with change of rest masses of an attractive body at the expense of radiation and change of his (its) charge $\partial Q / \partial x^5$.

4. Equations of Weber for viewed system

Being $f = const$ we have the equations in 4D

$$\frac{d\dot{\eta}^{\alpha}}{ds}+\Gamma^{\alpha}_{\beta\gamma}\frac{dx^{\beta}}{ds}\cdot\frac{dx^{\gamma}}{ds}=R^{\alpha}_{\beta\gamma\delta}\frac{dx^{\beta}}{ds}\cdot\frac{dx^{\gamma}}{ds}\eta^{\delta}+\frac{1}{\rho_0}\left\{D^{\alpha}_{\beta}\frac{d\eta^{\beta}}{ds}+K^{\alpha}_{\beta}\eta^{\beta}\right\}-\dot{\eta}^{\alpha}\frac{d\ln\rho_0}{ds}. \tag{56}$$

As a result of evaluations we have for perturbations η^1, η^2, η^3 three differential second-kind equation ats $r=R$:

$$\begin{aligned}&\frac{d\dot{\eta}^1}{ds}+\frac{1}{2}\left\{\left(1-\frac{\alpha}{R}+\frac{Q^2}{R^2}\right)\left(\frac{\alpha}{R^2}-\frac{2Q^2}{R^3}\right)-\frac{\alpha_{,5}}{R}+\frac{2Q\cdot Q_{,5}}{R^2}\right\}\left(\dot{x}^5\right)^2=\\&=\left\{\left(\frac{\alpha}{2R}+\frac{Q^2}{R^2}\right)\frac{l^2}{R^4}+\frac{1}{R^2}\left[\frac{\alpha}{R}\left(1-\frac{5}{8}\frac{\alpha}{R}\right)+\frac{5Q^2}{2R^2}\left(\frac{\alpha}{R}-\frac{3}{5}\frac{Q^2}{R^2}\right)\right]\left(\dot{x}^5\right)^2\right\}\eta^1+\\&+\left(\frac{\alpha_{,5}}{2}-\frac{Q\cdot Q_{,5}}{R}\right)\frac{l}{R^2}\dot{x}^5\eta^2+\left(-\frac{\alpha_{,5}}{2}+\frac{Q\cdot Q_{,5}}{R}\right)\frac{l^2}{R^4}\dot{x}^5\eta^5-\dot{\eta}^1\frac{d\ln\rho_0}{ds},\end{aligned} \tag{57}$$

$$\frac{d\dot{\eta}^2}{ds}=\frac{1}{2}\left\{-\left(1-\frac{\alpha}{R}+\frac{Q^2}{R^2}\right)\left(\frac{\alpha}{r^3}-\frac{2Q^2}{R^4}\right)-\frac{\alpha_{,5}}{R^5}+\frac{2Q\cdot Q_{,5}}{R^3}\right\}\dot{x}^5\eta^2-\dot{\eta}^2\frac{\ln\rho_0}{ds}, \tag{58}$$

$$\begin{aligned}&\frac{d\dot{\eta}^5}{ds}-\left(\frac{\alpha}{R^2}-\frac{Q^2}{R^3}\right)\left(\dot{x}^5\right)^2+\frac{l^2}{R^3}=\left(-\frac{\alpha}{R^3}+\frac{3Q^2}{R^4}\right)\left(\dot{x}^5\right)^2\eta^1+\\&+\left\{-\left(\frac{\alpha}{2R}-\frac{Q^2}{R^2}\right)\frac{l\dot{x}^5}{R^2}\right\}\eta^2-\dot{\eta}^5\frac{d\ln\rho_0}{ds}.\end{aligned} \tag{59}$$

Let's consider now requirement of a costance $\dot{x}^5$; the equations of Lagrange for $r,\dot{r}$ from (49) give

$$\frac{d\dot{x}^5}{ds}=-\dot{x}^5\frac{d\ln\rho_0}{ds}+\left(\frac{\alpha}{2r^2}-\frac{Q^2}{r^3}\right)\left(\dot{x}^5\right)^2+\frac{l^2}{r^3}-\frac{k_\gamma\sqrt{\gamma_{55}}Q}{\sqrt{f}\,\gamma r^2k}\dot{x}^5\ . \tag{60}$$

The equalling (55) and equalling to zero point (60) give in the total

$$\left(\dot{x}^5\right)_{r=R}=\frac{\left\{\frac{k_\gamma}{\gamma\sqrt{f}R}\left(\frac{\sqrt{\gamma_{55}}Q}{k}\dot{x}^5\right)\right\}_{,5}+\frac{1}{2}\left(\frac{L_M}{R}-\frac{QL_Q}{R^2}\right)-\frac{l^2}{r^3}+\frac{k_\gamma\sqrt{\gamma_{55}}Q}{\sqrt{f}R^2k}}{\left(\frac{\alpha}{2r^2}-\frac{Q^2}{r^3}\right)}. \tag{61}$$

It is necessary to show equalling to zero point

$$D^1_\beta=0\,,\ K^1_\beta=0\,. \tag{62}$$

Allowing (51), it is possible (54) to write to a view

$$\frac{d\gamma}{ds}=\frac{L_M}{2r}-\frac{QL_Q}{r^2}+\frac{k_\gamma\sqrt{\gamma_{55}}Q\dot{r}}{\sqrt{f}\,r^2k}. \tag{63}$$

But from the metric (4) follows

$$\gamma=\frac{1+l^2/r^2}{\dot{x}^5}-\dot{r}\,. \tag{64}$$

Multiplying (63) on γ, we shall receive

$$\gamma\frac{d\gamma}{ds}=\left(\frac{L_M}{2r}-\frac{QL_Q}{2r^2}\right)\left(\frac{1-l^2/r^2}{\dot{x}^5}\right)-\dot{r}\left(\frac{L_M}{2r}-\frac{QL_Q}{2r^2}\right)+\frac{k_\gamma\sqrt{\gamma_{55}}Q\dot{r}\gamma}{\sqrt{f}\,r^2k}. \tag{65}$$

From metric (4) also have relation for γ^2:

$$\gamma^2=\left(1+\frac{l^2}{r^2}\right)\left(1-\frac{\alpha}{r}+\frac{Q^2}{r^2}\right)+\dot{r}^2. \tag{66}$$

Differentiating (66) and allowing (65), we shall receive expression for radial acceleration

$$\ddot{r}=-\frac{L_M}{2r}+\frac{QL_Q}{2r^2}-\frac{\alpha}{2r^2}+\frac{Q^2+l^2}{r^3}-\frac{3}{2}\frac{\alpha l^2}{r^4}+\frac{2Q^2l^2}{r^5}-\frac{k_\gamma\sqrt{\gamma_{55}}Q\gamma}{\sqrt{f}\,r^2k}. \tag{67}$$

The equalling also give to zero point of a right member (67) a relation (62) according to[2], quantity γ in last term (67) can be presented by the way

$$\gamma=\gamma_0+\frac{\gamma_M\left(x^5\right)}{r}-\frac{\gamma_Q\left(x^5\right)}{r^2}, \tag{68}$$

where

$$\gamma_M\left(x^5\right)=\int_{x_0^5}^{x^5}\frac{L_M}{\dot{x}^5}dx^5, \tag{69}$$

$$\gamma_Q\left(x^5\right)=\int_{x_0^5}^{x^5}\frac{QL_Q}{\dot{x}^5}dx^5. \tag{70}$$

So

$$-\frac{L_M}{2R}+\frac{QL_Q}{2R^2}=\frac{\alpha}{2R^2}-\frac{Q^2+l^2}{R^3}+\frac{3}{2}\frac{\alpha l^2}{R^4}-\frac{2Q^2l^2}{R^5}-\frac{k_\gamma\sqrt{\gamma_{55}}Q\gamma}{\sqrt{f}\,R^2k}. \tag{71}$$

The formulas (55), (61), (71) solve a put problem: equations (57)-(59) can be shown to linear system (under contition of $k=const$)

$$\frac{d\dot{\eta}^1}{ds}+a_1\dot{\eta}^1+a_2\eta^1+a_3\eta^2+a_4\eta^5+a_5=0, \tag{72}$$

$$\frac{d\dot{\eta}^2}{ds}+b_1\eta^1+b_2\dot{\eta}^2=0, \tag{73}$$

$$\frac{d\dot{\eta}^5}{ds}+c_1\dot{\eta}^5+c_2\eta^1+c_3\eta^2+c_4=0. \tag{74}$$

Where a_i, b_i, c_i - constant values, and we have system of the differential equations 2 about with constant values of coefficients. In a case $k=\mathrm{var}$, $k=k(s)$ the system will consist of a linear part and nonlinear, entering in the second term (63), conditioned by change of specific densities of trial particles.

5. Generalization of results of L.M.Kaplan on stability of circular orbits

The results obtained above in the chapters I÷IV for a charged massive body, are compared to deductions [8]. In her the metric of Schwarzschild is taken for a basis; we will use the metric which is taking into account a charge about and integral of energy, which one allows a particle flux of variable charged mass. They also determine permissible coordinates of curvatures describing stability of circular orbits.

Following of L.M.Caplan, as function of Liypunov we shall take a quadrate of effective energy, equal total energy

$$V_{eff}^2 = \gamma^2(\beta) \cong \left(1 + \frac{l^2}{r^2}\right)\beta + \Phi^2 \frac{Q^2}{r^2}, \tag{75}$$

where parameter β is

$$\beta = 1 - \frac{\alpha}{r} + \frac{Q^2}{r^2}. \tag{76}$$

Hereinafter we shall accept

$$r \cong \frac{\alpha}{1-\beta} - \frac{Q^2}{\alpha},$$

$$r^2 \cong \frac{\alpha^2}{(1-\beta)^2} - \frac{Q^2}{1-\beta} = \frac{\alpha^2 - 2Q^2(1-\beta)}{(1-\beta)^2}. \tag{77}$$

So, we have

$$V_{eff}^2 = \left\{1 + \frac{l^2(1-\beta)^2}{\alpha^2} + \frac{2l^2(1-\beta)^3 Q^2}{\alpha^4}\right\}\beta + \frac{\Phi^2 Q^2 (1-\beta)^2}{\alpha^2}. \tag{78}$$

The moment for minimum V_{eff}^2 is identifiable

$$l^2 \cong \frac{\alpha^2\left\{1 - \frac{2\Phi^2 Q^2(1-\beta)}{\alpha^2}\right\}}{(1-\beta)\left\{(3\beta - 1) + \frac{2Q^2(1-\beta)}{\alpha^2}\right\}} \cong \frac{\alpha^2\left\{1 - \frac{2\Phi^2 Q^2(1-\beta)}{\alpha^2}\right\}}{(1-\beta)(3\beta\lambda - 1)}, \tag{79}$$

$$1 \geq \beta \geq \frac{1}{3\lambda}, \ \lambda = \lambda\left(\frac{Q^2}{\alpha^2}\right).$$

So, the moment is converted in perpetuity on $r \to \infty$, so on $r = 3\alpha\lambda$ an orbit with, that is between orbs and feature of Schwarzield there are no circular orbits. Let's substitute (79) in (78)

$$V_{eff}^2 = \left\{1 + \left(1 + \frac{2(1-\beta)Q^2}{\alpha^2}\right)\frac{\left(1 - \frac{2\Phi^2 Q^2(1-\beta)}{\alpha^2}\right)}{(3\lambda\beta - 1)}\right\}\beta + \frac{\Phi^2 Q^2 (1-\beta)^2}{\alpha^2}, \tag{80}$$

$$V_{eff}^2 \to \infty, \ \beta = \frac{1}{3\lambda}, \ v \to c.$$

From here is visible, that af $\beta = \frac{1}{3\lambda}$, $V_{eff}^2 \to \infty$, $v \to c$. Thus, we have af $r = \frac{3}{2}\lambda\alpha$ – an in convertible orbit, inside $r = \frac{3}{2}\lambda\alpha$ – labile; here λ is a function of a charge Q; the stream does not influence stability of orbits.

The equations (72)÷(74) as a whole allow to judge stability on the first approach. However it is important to use method of Liypunov (75); equations of Hamilton-Jacobi allow to estimate stability of radial motions of trial particles.

The operation is executed at support FCP “Federating”.

References

[1]Paul S. Wesson. Space-Time-Matter Modern Kaluza-Klein /World Scientific publishing Co. Pte. Ltd. 1999.

[2]B.D.Koberlein and R.L.Mallet. Particle motion in the gravitation field of charged, radiating body //J. Mth. Phys. 36(2)/February 1995, pp.828-836.

[3]V.A.Vok. Theory of space, time and gravitation. M: ГИТЛ, 1955

[4]D.Yu.Tsipenyik. Transformation of an Electromagnetic Field into Gravitational Field in Model of Extended Space: Prediction and Possible Experimental Check //Gravitation and Cosmology, vol.7, No 3. 2001.

[5]R.W.Linquist, R.A.Schwartz, C.M.Misher. Vaidya’s Radiating Schwarzschild Metric /Physical Review., vol.137, N 5B, March 1965, p.B1364-1368.

[6]J.S.Vladimirov, V.V.Kislov. To a problem on change in the five-measurement theory of a gravitation, electromagnetism and scalar field /Messages Mosk. un-t, ser.3, Physica, Astronomy, 1982, vol.236, pp.18-21.

[7]U.N.Zakirov. Relativistic mechanics of concentrated variable rest mass /Proceedings of School-Seminar “Volga-10” on Recent Problems in Theoretical and Mathematical Physics. Kazan State University, Kazan, 1998, p.93.

[8]L.M.Kaplan //ZHETF, 1949, 10, p.951-952.

Поляритоны и их аналоги в веществе и в физическом вакууме

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Проанализированы закономерности взаимодействия материальных сред с электромагнитными волнами на примерах идеализированных моделей кристаллических решеток диэлектриков и полупроводников. Исследована динамика элементарных возбуждений системы “среда–электромагнитное поле” – поляритонов. Проанализированы свойства поляритонов при наличии свободных носителей в материальной среде. Установлены условия , при которых групповая скорость поляритонов становится предельно малой, т.е. реализуется “остановка “ света в материальной среде. Исследованы особенности закона дисерсии поляритонов в поглощающей и усиливающей среде. Показано, что механическим аналогом поляритонов являются звуковые волны, сильно взаимодействующие с волнами модулированных структур - сверхрешеток. Рассмотрены условия наблюдения явления динамической опалесценции - аномального возрастания эффективности процессов неупругого рассеяния света. Рассмотрены свойства “кристаллической” модели физического вакуума.

1. Введение

В классических работах М.Борна [1,2] была поставлена задача исследования динамики системы «диэлектрическая материальная среда - электромагнитное поле» в условиях сильного взаимодействия составляющих её подсистем. Такое взаимодействие возникает вследствие того, что при ускоренном движении заряженных частиц материальной среды излучаются электромагнитные волны и, в свою очередь, при распространении электромагнитной волны в материальной среде происходит ускорение заряженных частиц. Результатом совместного решения связанных между собой уравнений Максвелла и механических уравнений движения для заряженных частиц среды являются волны смешанного типа («гибридные волны»): частично электромагнитные и частично механические. Элементарные возбуждения, соответствующие такого рода волнам, были названы поляритонами (см. работы [3,4]. В вакууме квантами электромагнитного поля являются фотоны. Фотоны характеризуются известной зависимостью энергии E от импульса p:

$$E=c_0p, \qquad (1.1)$$

где c_0- скорость света в вакууме. При попадании фотона в материальную среду он превращается поляритон, энергия E которого не отличается от энергии соответствующего ему фотона, но зависимость энергии от импульса может существенно отличаться от соотношения (1.1). Кроме того, поляритон характеризуется конечным временем жизни и определенной длиной свободного пробега в материальной среде. Из соотношения (1.1) следует известный закон дисперсии для электромагнитных волн в вакууме:

$$\omega=c_0k. \qquad (1.2)$$

Здесь ω и k - круговая частота и соответствующий волновой вектор электромагнитной волны.

Динамические и кинематические свойства поляритонов в настоящее время являются объектом исследования многих работ. В частности, информация о свойствах поляритонов получается при исследовании свойств неупругого (комбинационного) рассеяния света на поляритонах [5-8].

В данной работе ставилась задача анализа общих закономерностей для поляритонов на основе изучения динамики идеализированных моделей материальных сред с учетом их взаимодействия с электромагнитным полем, поиска аналогов поляритонов в механике сплошной и дискретной среды, установления новых закономерностей неупругого рассеяния света с учетом поляритонных эффектов и анализ явления динамической опалесценции, наблюдаемой вблизи точек фазовых переходов и в ультрадисперсной среде.

2. Динамика идеализированных моделей материальной среды, взаимодействующей с электромагнитным полем

Для теоретического описания свойств электромагнитных волн в материальной среде необходимо исходить из определенной модели вещества. В качестве постейшей идеализированной модели рассмотрим систему, для которой предполагается, что внутри материальной среды имеются равномерно распределенные в пространстве заряженные частицы, колеблющиеся около своих положений равновесия - ***лорентцевы осцилляторы.*** Простейший случай соответствует тому, что такими частицами являются электроны, колеблющиеся около тяжелых, положительно заряженных ионов. Для ряда других задач роль таких осцилляторов выполняют положительно и отрицательно заряженные ионы, формирующие кристаллическую решетку . В полупроводниках в качестве таких осцилляторов могут выступать «дырки», колеблющиеся около отрицательно заряженных ионов. Для того, чтобы среда была электрически нейтральна, необходимо допустить, что в ней присутствуют частицы как отрицательным, так и с положительным зарядом. Для рассматриваемой идеализированной модели будем полагать, что частицы противоположного заряда характеризуются достаточно большой массой и поэтому являются практически неподвижными. Схематически такую модель можно представить в виде кубической кристаллической решетки заряженных частиц, колеблющихся около своих положений равновесия. Одномерным аналогом такой решетки [9,10] может служить кристаллическая цепочка частиц, слабо взаимодействующих друг с другом, но связанных упругими силами с тяжелыми неподвижными ионами, расположенными в узлах кристаллической цепочки (рис. 2.1).

Если период кристаллической цепочки равен a, то число заряженных осцилляторов, приходящихся на единицу длины, есть 1/a; в трехмерном случае для простой кубической решетки концентрация осцилляторов соответственно есть: $N_0 = 1/V$, где V - объем кубической элементарной ячейки.

Важно отметить, что, если равновесные положения заряженных осцилляторов несущественно изменяются в процессе колебаний, то такая модель может быть использована для описания свойств поляритонных волн не только в кристаллах, но и в других, менее упорядоченных средах: аморфных телах, стеклах, жидкостях и, с учетом некоторых ограничений, даже в разреженных средах. Рассматриваемая модель может быть использована также для описания свойств электромагнитных волн в твердых телах, содержащих равномерно распределенные в пространстве примесные центры.

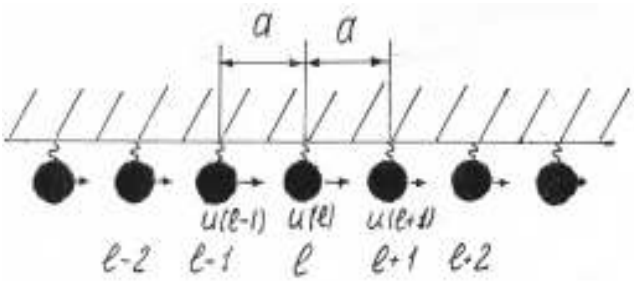

Рис.2.1 Цепочка лорентцевых осцилляторов материальной среды. «Пружинки», показанные на рисунке, соответсвуют учету взимодействия электронов лишь с тяжелыми неподвижными ионами.

Обозначим через **u**(**l**) вектор отклонения заряженной частицы (электрона) от положения равновесия. Здесь $\mathbf{l} = l_1\mathbf{a}_1 + l_2\mathbf{a}_2 + l_3\mathbf{a}_3$ - так называемый вектор трансляции, задающий положение элементарной ячейки в кубической кристаллической решетке (l_1, l_2, l_3 - целые числа).

Запишем уравнение движения заряженной частицы с зарядом $e\sqrt{F}$ (F - так называемая сила осциллятора, характеризующая величину заряда колеблющейся частицы, e - заряд электрона; $F \approx 1$) в поле электромагнитных волн с напряженностью **E**. При этом для простоты будем

полагать, что поле **E** вблизи заряженной частицы (так называемое эффективное поле) совпадает по амплитуде с внешним полем электромагнитной волны в вакууме. При этом получаем:

$$m\frac{d^2\mathbf{u}(\mathbf{I})}{dt^2} = -\gamma_0 \cdot \mathbf{u}(\mathbf{I}) + e\sqrt{F}\mathbf{E}. \qquad (2.1)$$

В правой части уравнения (2.1) присутствует сила, действующая на заряженный осциллятор и обусловленная электрическим полем в области движения заряженной частицы. Уравнение движения (2.1) необходимо рассматривать совместно с уравнениями Максвелла в материальной среде. Запишем эти уравнения в следующем виде:

$$rot\mathbf{E} = -\frac{\partial \mathbf{B}}{dt}; \quad rot\mathbf{H} = -\frac{\partial \mathbf{D}}{\partial t}; \qquad (2.2)$$
$$div\mathbf{D} = 0; \qquad div\mathbf{B} = 0.$$

Система уравнений (2.2) написана для материальной среды, внутри которой нет свободных зарядов ($\rho = 0$) и отсутствуют токи ($j = 0$). Кроме того, для простоты полагается, что среда является немагнитной, т.е. $\mathbf{B} = \mu_0\mathbf{H}$ ($\mu = 1$).

Решение системы уравнений для вещества (2.1) и электромагнитного поля (2.2) ищется в виде плоских монохроматических волн, что является характерным для всех волновых процессов, происходящих в средах с трансляционной симметрией (в частности, в кристаллах): $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{kr}-\omega t)}$; $\mathbf{D} = \mathbf{D}_0 e^{i(\mathbf{kr}-\omega t)}$; $\mathbf{H} = \mathbf{H}_0 e^{i(\mathbf{kr}-\omega t)}$; $\mathbf{B} = \mathbf{B}_0 e^{i(\mathbf{kr}-\omega t)}$; $\mathbf{u} = \mathbf{u}_0 e^{i(\mathbf{kr}-\omega t)}$.

Здесь $\omega = 2\pi/T = 2\pi f$ - круговая частота, характеризующая колебания рассматриваемых величин в процессе распространения волны (f- обычная частота, измеряемая в герцах; $\mathbf{k} = (2\pi/\lambda)\mathbf{n}$ - волновой вектор, направление которого задает направление распространения волны в пространстве; λ - длина волны в материальной среде). Остановимся сначала на анализе первого из уравнений системы (2.2). Применим операцию “rot” к левой и правой части этого уравнения и используем известное тождество векторного анализа: $rot\,rot\mathbf{a} = grad\,div\mathbf{a} - \nabla^2\mathbf{a}$, где ∇ - оператор векторной производной, ($\nabla^2 = \Delta = \frac{\partial^2}{\partial x^2}\boldsymbol{i} + \frac{\partial^2}{\partial y^2}\boldsymbol{j} + \frac{\partial^2}{\partial z^2}\boldsymbol{k}$ - оператор Лапласа). В результате приходим к соотношению:

$$grad \cdot div\mathbf{E} - \Delta\mathbf{E} = -\frac{\partial \mu_0}{\partial t}\frac{\partial \varepsilon\varepsilon_0\mathbf{E}}{\partial t}. \qquad (2.3)$$

При этом мы использовали известные материальные соотношения: $\mathbf{D} = \varepsilon_0\varepsilon\mathbf{E}$ и $\mathbf{B} = \mu_0\mathbf{H}$ ($\mu = 1$), а также третье уравнение Максвелла ($\mathrm{rot}\,\mathbf{H} = \partial\mathbf{D}/\partial t$) системы (2.2).

Подставим во второе уравнение системы (2.2) решение в виде плоских монохроматических волн. В результате, используя правила векторного дифференцирования ($\mathrm{div}\,\mathbf{E}_0 e^{i(\mathbf{kr}-\omega t)} = i\mathbf{k}\mathbf{E}_0 e^{i(\mathbf{kr}-\omega t)}$), получаем соотношение:

$$i\varepsilon_0\varepsilon\mathbf{k}\mathbf{E}_0 e^{i(\mathbf{kr}-\omega t)} = 0. \qquad (2.4)$$

Относительная диэлектрическая проницаемость ε в общем случае предполагается зависящей от частот ω электромагнитной волны, т.е. $\varepsilon = \varepsilon(\omega)$.

Напомним, что в вакууме электромагнитная волна характеризуется только поперечной поляризацией, т.е. имеет место: $\mathbf{E}_0 \perp \mathbf{k}$. В материальной среде необходимо рассмотреть возможность существования как поперечных ($\mathbf{E}_0 \perp \mathbf{k}$), так и продольных ($\mathbf{E}_0 \parallel \mathbf{k}$) волн. Для продольных волн имеет место $\mathbf{E}_0 \cdot \mathbf{k} = \mathbf{E}_0 \cdot \mathbf{k} \cdot \cos 0 \neq 0$. Поэтому для выполнения соотношения (2.4) необходимо, чтобы диэлектрическая проницаемость ε при этом обращалась в нуль для некоторого значения частоты $\omega = \omega_l$: $\varepsilon(\omega_l) = 0$, (т.е. ω_l - нуль диэлектрической проницаемости).

Для поперечной ($\mathbf{E}_0 \perp \mathbf{k}$, $\mathbf{E}_0 \cdot \mathbf{k} = \mathbf{E}_0 \cdot \mathbf{k} \cdot \cos(\pi/2) = 0$) волны соответственно имеем:

$$i\varepsilon_0\varepsilon\ \mathrm{div}\ \mathbf{E} = i\varepsilon_0\varepsilon\mathbf{k}\mathbf{E}_0 e^{i(\mathbf{kr}-\omega t)} = 0; \quad \varepsilon(\omega) \neq 0. \qquad (2.5)$$

Выясним вид зависимости $\varepsilon(\omega)$ для поперечных волн. В этом случае соотношение (2.3.) с учетом (2.5) переходит в волновое уравнение для напряженности поля **E**:

$$(\Delta - \frac{1}{c^2}\frac{\partial^2}{\partial t^2})\mathbf{E} = 0\,. \qquad (2.6)$$

где вводится обозначение: $c = \frac{c_0}{\sqrt{\varepsilon(\omega)}}$ для фазовой скорости поляритонной волны.

При подстановке в уравнение (2.6) решения в виде плоской монохроматической волны ($\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{kr}-\omega t)}$) приходим к следующему алгебраическому соотношению:

$$\omega = \frac{c_0 k}{\sqrt{\varepsilon(\omega)}}\,. \qquad (2.7)$$

Это соотношение в неявном виде задает так называемый закон дисперсии для поляритонной волны: $\omega = \omega(k)$.

Для нахождения вида диэлектрической функции $\varepsilon(\omega)$ и соответствующего показателя преломления $n = \sqrt{\varepsilon(\omega)}$ обратимся к уравнению движения для заряженного осциллятора (2.1). Подставляя решение в виде плоских монохроматических волн в это уравнение, приходим к соотношению для амплитуд $\mathbf{u}_0$ и $\mathbf{E}_0$:

$$-\omega^2\mathbf{u}_0 = -\omega_0^2\,\mathbf{u}_0 + \frac{e\sqrt{\mathrm{F}}\mathbf{E}_0}{\mathrm{m}}, \qquad (2.8)$$

где вводится обозначение: $\omega_0^2 = \gamma_0/\mathrm{m}$ (квадрат собственной частоты колебаний заряженного осциллятора). Отсюда получаем:

$$\mathbf{u}_0 = \frac{e\sqrt{\mathrm{F}}\mathbf{E}_0}{m\left[\omega_0^2 - \omega^2\right]}. \qquad (2.9)$$

В процессе отклонения заряженной частицы от противоположно заряженного иона (рис. 2.1) происходят осцилляции соответствующего дипольного момента: $\mathbf{p} = \mathbf{p}_0 e^{i(\mathbf{kr}-\omega t)} = e\sqrt{\mathrm{F}}\mathbf{u}_0 e^{i(\mathbf{kr}-\omega t)}$. Используем известное из электростатики понятие поляризации $\mathbf{P} = \mathbf{P}_0 e^{i(\mathbf{kr}-\omega t)}$. Физический смысл этой величины состоит в равенстве модуля этого вектора дипольному моменту единицы объема, т.е. для однородной среды:

$$\mathbf{P}_0 = \frac{\mathbf{p}_0}{\mathrm{V}} = \frac{e\sqrt{\mathrm{F}}\mathbf{u}_0}{\mathrm{V}}, \qquad (2.10)$$

где V - объем, занимаемый одним осциллятором. Из соотношения (2.8) следует выражение для амплитуды вектора поляризации имеет вид:

$$\mathbf{P}_0 = \frac{e^2\mathrm{F}\mathbf{E}_0}{\mathrm{mV}(\omega_0^2 - \omega^2)}. \qquad (2.11)$$

С другой стороны, используя соотношение: $\mathbf{D}=\varepsilon_0\mathbf{E}+\mathbf{P}$, ($\varepsilon_0 = 8{,}85\cdot 10^{-12}$ Ф/м), имеем:

$$\varepsilon\varepsilon_0\mathrm{E}_0 = \varepsilon_0\mathrm{E}_0 + \frac{e^2\mathrm{FE}_0}{\mathrm{mV}\left[\omega_0^2 - \omega^2\right]}. \qquad (2.12)$$

Отсюда получаем вид функции $\varepsilon(\omega)$ и квадрата показателя преломления $\mathrm{n}^2(\omega) = \varepsilon(\omega)$:

$$\mathrm{n}^2(\omega) = \varepsilon(\omega) = 1 + \frac{e^2\mathrm{F}}{\mathrm{mV}\varepsilon_0\left[\omega_0^2 - \omega^2\right]} = 1 + \frac{e^2\mathrm{FN}_0}{\mathrm{m}\varepsilon_0\left[\omega_0^2 - \omega^2\right]}. \qquad (2.13)$$

В дальнейшем удобно ввести обозначение для квадрата так называемой плазменной частоты ω_p:

$$\omega_p^2 = e^2F/mV\varepsilon_0\,. \qquad (2.14)$$

С использованием этого обозначения диэлектрическая функция представляется следующим образом:

$$\varepsilon(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2} \quad \text{или} \quad \varepsilon(\omega) = \frac{\omega_l^2 - \omega^2}{\omega_0^2 - \omega^2}, \qquad (2.15)$$

где $\omega_l^2 = \omega_0^2 + \omega_p^2$ (ω_l - значение частоты, при котором $\varepsilon(\omega)$ обращается в нуль).

Вторая формула в (2.15) в литературе известна как соотношение Куросавы Соответственно показатель преломления для рассматриваемой модели материальной среды представляется в виде:

$$n(\omega) = \sqrt{\frac{\omega_l^2 - \omega^2}{\omega_0^2 - \omega^2}}. \qquad (2.16)$$

Как видно из формулы (2.16), на высоких частотах ($\omega >> \omega_0$) показатель преломления стремится к единице, т.е. фазовая скорость света при этом $c \to c_0$. Наоборот, при низких частотах ($\omega \to 0$) имеет место:

$$\varepsilon(0) = \frac{\omega_l^2}{\omega_0^2} \text{ и } n(0) = \frac{\omega_l}{\omega_0}. \qquad (2.17)$$

Если $\omega = \omega_l$, то показатель преломления и диэлектрическая проницаемость согласно (2.16) обращаются в нуль: $\varepsilon(\omega_l) = 0$. Таким образом, на этой частоте ($\omega = \omega_l$) согласно соотношению (2.4) возможно существование продольных электромагнитных волн (рис. 2.2.). Частота продольной электромагнитной волны для рассматриваемой модели вещества не зависит от волнового вектора **k**, т.е. от длины волны.

Если частота $\omega \cong \omega_0$, то показатель преломления и диэлектрическая проницаемость согласно (2.17) резко возрастают и существенно отличаются от единицы.

Таким образом, рассматриваемая идеализированная модель объясняет известное явление дисперсии электромагнитных волн в веществе и дает количественное описание зависимости диэлектрической проницаемости ε и показателя преломления n от частоты ω и от длины волны λ_0 падающего на материальную среду электромагнитного излучения. Из соотношения (2.13) следует:

$$n^2(\omega) = 1 + \frac{\omega_p^2}{\omega_0^2 - \omega^2}. \qquad (2.18)$$

Формулу (2.18) удобно представить также в следующем виде:

$$\varepsilon = n^2 = 1 + \frac{e^2 \mathrm{FN}_0}{m\varepsilon_0\left[\omega_0^2 - \omega^2\right]}, \qquad (2.19)$$

где N_0=1/V - концентрация заряженных осцилляторов (т.е. их число в единице объема). Если частота падающего света ω много больше резонансной частоты ω_0, то величиной ω_0^2 в знаменателе (2.19) можно пренебречь. В этом случае можно использовать более простую формулу для дисперсии диэлектрической проницаемости.

$$\varepsilon(\omega) = \mathrm{n}^2 = 1 - \frac{e^2 \mathrm{FN}_0}{\mathrm{m}\varepsilon_0\omega^2}. \qquad (2.20)$$

Такая формула может быть применима, в частности, для описания дисперсии рентгеновских лучей при их распространении в веществе, так как их частота ω существенно превышает значения собственных частот ω_0 колебаний оптических электронов в атомах и молекулах. Частоты таких колебаний обычно соответствуют видимой области спектра (для окрашенных веществ) и ближнему ультрафиолетовому диапазону спектра. Это выражение для диэлектрической проницаемости может быть использовано также для описания свойств рентгеновского излучения при его прохождении через металлическую среду, прозрачную для рентгеновского излучения.

Остановимся теперь на анализе зависимости $\omega(k)$, т.е. закона дисперсии электромагнитных волн в материальной среде (поляритонных волн), исходя из соотношения (2.7). Используя выражение для диэлектрической проницаемости $\varepsilon(\omega)$, получаем:

$$\omega^2 = \frac{c_0 k^2(\omega_0^2 - \omega^2)}{\omega_l^2 - \omega^2}. \qquad (2.21)$$

Отсюда приходим к биквадратному уравнению для круговой частоты ω:

$$\omega^4 - \omega^2(\omega_l^2 + c_0^2k^2) + c_0^2k^2\omega_0^2 = 0. \qquad (2.22)$$

Таким образом, для рассматриваемой модели материальной среды имеются два типа поляритонных волн, соответствующие двум дисперсионным ветвям, т.е. зависимостям $\omega^2(k)$, являющимися корнями биквадратного уравнения (2.22):

$$\omega_+^2 = \frac{\omega_l^2 + c_0^2k^2}{2}\left(1 + \sqrt{1 - \frac{4c_0^2k^2\omega_0^2}{\omega_l^2 + c_0^2k^2}}\right), \qquad (2.23a)$$

$$\omega_-^2 = \frac{\omega_l^2 + c_0^2k^2}{2}\left(1 - \sqrt{1 - \frac{4c_0^2k^2\omega_0^2}{\omega_l^2 + c_0^2k^2}}\right). \qquad (2.23b)$$

Экспериментальные исследования закона дисперсии поляритонов проводятся на основе анализа частотно-угловых зависимостей комбинационного рассеяния света в нецентросиммметричных кристаллах, для которых процессы комбинационного рассеяния света на поляритонах оказываются разрешеннными согласно правилам отбора. Вид спектров комбинационного рассеяния при углах рассеяния, близких к 90^0, для кристаллов такого типа (фосфид галлия) приведен на рис. 2.2.

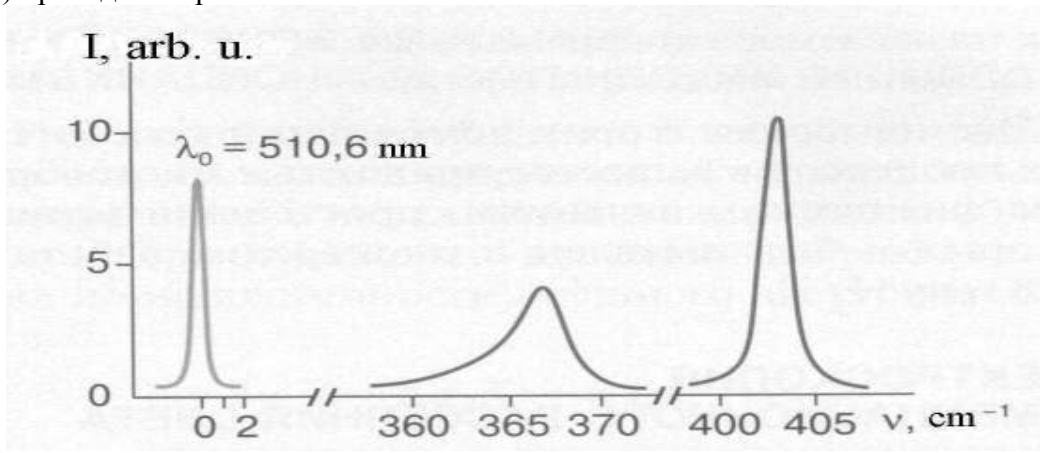

Рис.2.2. Спектр комбинационного рассеяния света в монокристалле фосфида галлия, возбуждаемый лазерным источником света. Максимумы интенсивности в спектре соответствуют поперечным(366 $см^{-1}$) и продольным (402 $см^{-1}$) оптическим колебаниям кристаллической решетки фосфида галлия; ширина полос характеризует соответствующие коэффициенты затухания.

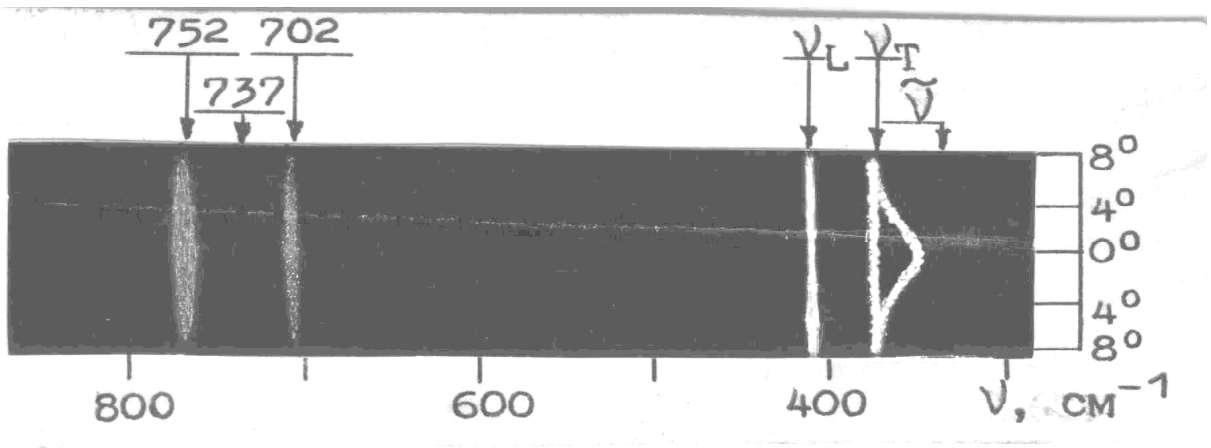

Рис.2.3.Комбинационное рассеяние света на поляритонных волнах в монокристалле фосфида галлия. На вертикальной оси отложен угол рассеяния в градусах; по оси абсцисс указаны- волновые числа в $см^{-1}$; ν_T=366 $см^{-1}$ - волновое число, соответствующее поперечной моде; ν_L=402 $см^{-1}$- волновое число, соответствующее продольной моде оптических колебаний кристаллической решетки фосфида галлия ($\nu=\omega/2\pi c_0$). Слева присутствуют широкие полосы, обусловленные двухфононными процессами.

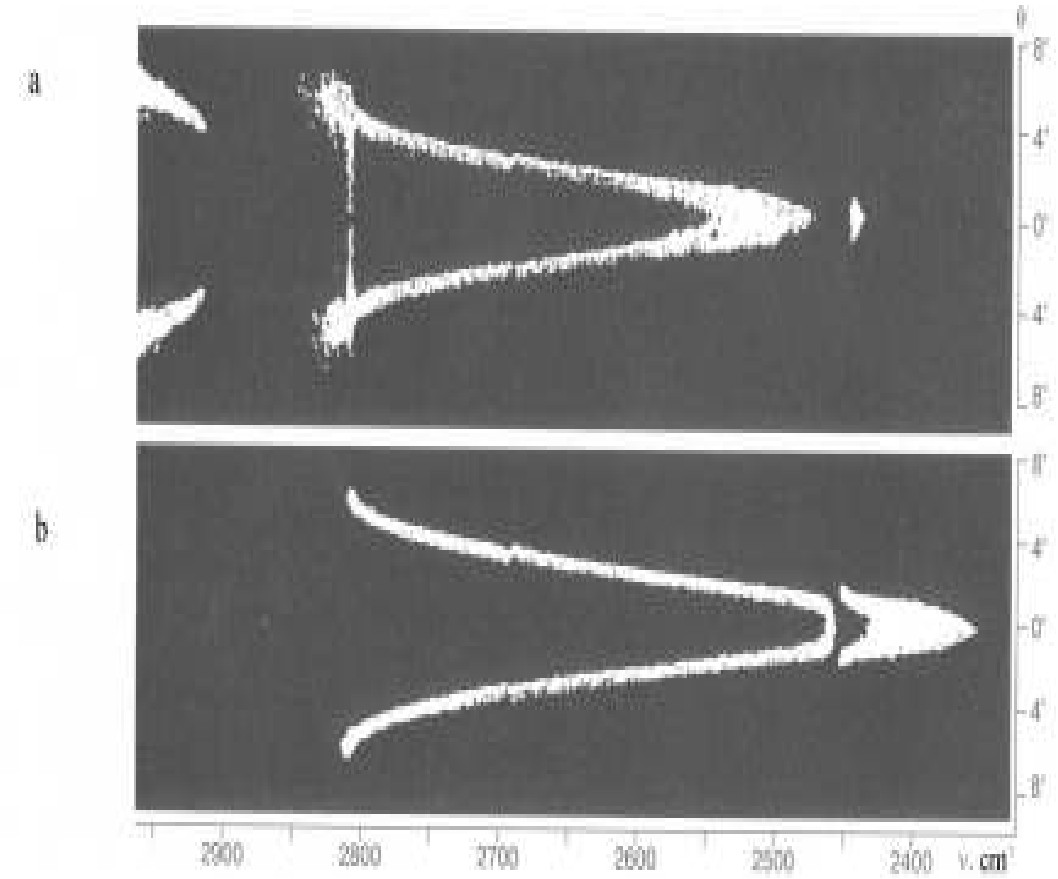

Рис.2.4. Комбинационное рассеяние на поляритонных волнах в монокристалле хлористого аммония. Верхний спектр соответствует возбуждению процесса комбинационного рассеяния “зеленой” линией(510,6 нм.), а нижний - получен при возбуждении желтой (578,2 нм) линией лазера на парах меди.

В этом случае в спектре проявляются поперечные и продольные оптические фононы , соответствующие волновым векторам $к=10^5 см^{-1}$ (согласно закону сохранения импульса в элементарном процессе рассеяния). При малых углах рассеяния света (рассеяние «вперед») в спектрах комбинационного рассеяния обнаруживаются участки поляритонных кривых, проявляющиеся в соответствии с законом сохранения квазиимпульса. Вид спектра комбинационного рассеяния на поляритонных участках для кристаллов фосфида галлия и хлористого аммония приводится на рис. 2.3 и 2.4 соответственно. На спектрах рис. 2.2, наряду с линиями, обусловленными поперечными (366 $см^{-1}$) и продольными (402$см^{-1}$) оптическими фононами обнаруживается поляритонный участок (кривая линия вблизи 366 $см^{-1}$ и полосы второго порядка(702,737 и 752 $см^{-1}$), обусловленные двухчастичными состояниями (парами фононов с противоположными импульсами). В случае хлористого аммония в спектрах малоуглового рассеяния обнаруживаются поляритонные участки с разрывами, обусловленными проявлением двухчастичных и многочастичных состояний. Такой эффект (появление разрывов на поляритонных кривых) известен под названием поляритонного резонанса Ферми.

На основе анализа частотно-угловых спектров комбинационного рассеяния света на поляритонах строятся соответствующие дисперсионные кривые. Вид дисперсионных кривых для поляритонов в кристалле фосфида галлия, полученный из экспериментов [6,7] по комбинационному рассеянию света на поляритонах в этом кристалле, показан на рис. 2.5. При этом устанавливается закон дисперсии поляритонов нижней и верхней ветвей в виде функции $\nu=\nu(k)$ ($\nu=1/\lambda_0$ – волновое число, измеряемое в $см^{-1}$).

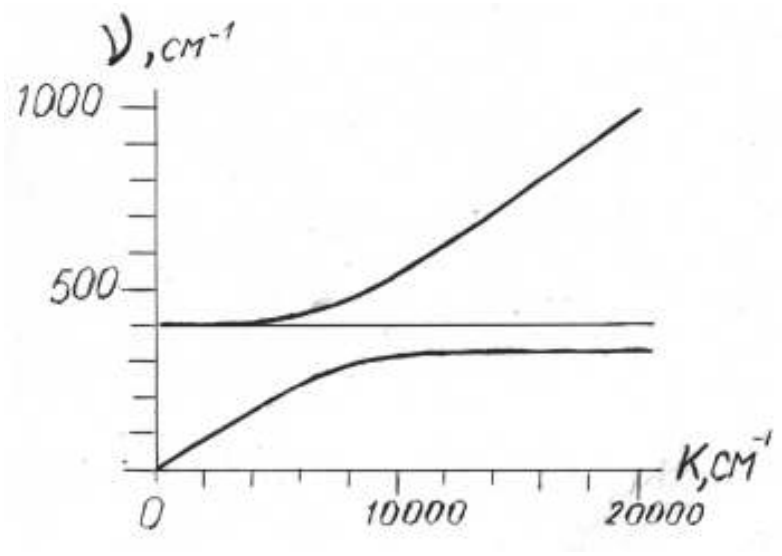

Рис. 2.5. Вид поляритонных кривых, т.е. закон дисперсии для поляритонов в диэлектрическом кристалле фосфиде галлия в инфракрасной области спектра. Нижняя кривая соответствует нижней поляритонной ветви, стремящейся при больших значениях волнового вектора к значению $\nu=\nu_0$, а верхняя кривая – верхней поляритонной ветви , начинающеяся со значения $\nu=\nu_l$; прямая линия на частоте $\nu=\nu_l=\omega_l/2\pi c_0$ соответствует продольным волнам; ν_0=366 см$^{-1}$ и ν_l=402 см$^{-1}$.

В области малых значений волновых векторов можно использовать разложение в ряд Тейлора:

$$\omega_+ = \sqrt{\omega_l^2 + c_0^2\left(1 - \frac{\omega_0^2}{\omega_l^2}\right)k^2}\ , \tag{2.24a}$$

$$\omega_- = c_0 k \frac{\omega_0}{\omega_l}\ . \tag{2.24b}$$

Таким образом, в далекой инфракрасной области спектра, соответствующей низким частотам поляритонных волн, показатель преломления $n = \sqrt{\varepsilon_0} = \frac{\omega_0}{\omega_l} > 1$. Поэтому фазовая и групповая скорости распространения миллиметровых электромагнитных волн в материальной среде может быть существенно меньше (на несколько порядков) скорости распространения света в вакууме. Соответственно статическая диэлектрическая проницаемость ε_0 может быть существенно больше единицы. Особая ситуация имеет место, когда частота собственных колебаний ω_0 лорентцевых осцилляторов становится очень малой. При этом статическая диэлектрическая проницаемость в соответствии с соотношением (2.17), в котором ω_0^2 находится в знаменателе, сильно возрастает: её значение для некоторых веществ достигает 10^4. Отметим, что значение квадрата плазменной частоты в соотношении (2.17),находящееся в числителе, зависит только от заряда и массы осциллятора, т.е. не должно существенно изменяться с температурой. Резкое возрастание низкочастотной диэлектрической проницаемости обнаруживается у ряда твердых тел для узкого интервала температур вблизи так называемой *точки Кюри* ($T=T_c$). Вещества, характеризующиеся аномально высокими значениями статической диэлектрической проницаемости в точке Кюри, ниже точки Кюри ($T < T_c$) оказываются спонтанно поляризованными, т.е. их вектор поляризации $\mathbf{P}_0$ (дипольный момент единицы объема) отличен от нуля даже при отсутствии внешнего электрического поля. При $T > T_c$ спонтанная поляризация такого вещества полностью пропадает ($\mathbf{P}_0 = 0$). Ниже температуры T_c направление вектора спонтанной поляризации может быть изменено на противоположное при приложении к такому веществу внешнего статического (постоянного) электрического поля. Вещества, характеризующиеся существованием спонтанной поляризации,

направление которого может изменяться под действием внешнего статического поля, называются сегнетоэлектриками, или ферроэлектриками (по аналогии с ферромагнетиками). Типичным примером сегнетоэлектрика является титанат бария, для которого статическая диэлектрическая проницаемость возрастает до 10^4 вблизи температуры 120°С. Как выяснилось, роль лорентцевых осцилляторов, обуславливающих аномальное возрастание статической диэлектрической проницаемости в сегнетоэлектриках играют не электроны, а ионы. В случае титаната бария такую роль выполняют ионы титана, бария и кислорода, колеблющиеся с большой амплитудой вблизи равновесных положений (рис. 2.6). В самой точке Кюри частота ω_0 таких колебаний устремляется к нулю, т.е. упругая возвращающая сила практически отсутствует и положения равновесия ионов становятся неустойчивыми. Колебания, частота которых устремляется к нулю при некоторых значениях внешних параметров (температуры, давления), в настоящее время называют ***мягкими модами.*** Появление мягких мод обычно

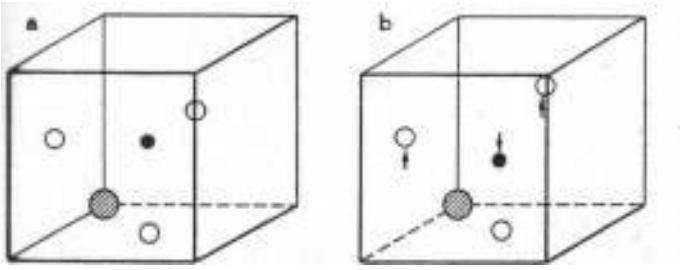

Рис 2.6 Вид элементарной ячейки кристалла титаната бария выше (a) и ниже (b) точки сегнетоэлектрического фазового перехода. Стрелки показывают смещения соответствующих ионов при фазовом переходе; светлые кружки соответствуют кислороду, темные в центре -титану и заштрихованные кружки – барию.

сопровождается фазовым переходом внутри вещества, в частности изменением структуры кристаллической решетки. Так как в точке Кюри возвращающая сила для мягкой моды отсутствует, при небольшом понижении температуры мягкая мода оказывается «замороженной», т.е. атомы не возвращаются в исходное положение равновесия и в ячейке возникает дипольный момент. Вследствие кулоновского взаимодействия такое же направление дипольного момента осуществляется для достаточно большого объема кристалла - сегнетоэлектрического домена. Внутри каждого домена возникает спонтанная поляризация, направление которой можно изменять даже с помощью слабого внешнего электрического поля.

Таким образом, вблизи точек фазовых переходов в твердых телах должно происходить аномальное увеличение низкочастотного показателя преломления и соответственно уменьшение скорости распространения поляритонных волн на низких частотах. Общее выражение для дисперсии диэлектрической проницаемости сегнетоэлектрика в инфракрасной области спектра для температуры $T=T_c$ может быть представлено в виде формулы (2.20), полученной в предположении, что частота лорентцева осциллятора равна нулю ($\omega_0 = 0$). Это и соответствует формированию мягкой моды вблизи точки перехода. При этом для описания закона дисперсии поляритонов вместо биквадратного уравнения (2.22) имеем квадратное уравнение:

$$\omega^2 = \omega_p^2+c_0^2k^2. \qquad (2.25)$$

Соотношение (2.25) задает закон дисперсии для верхней поляритонной ветви в рассматриваемом случае; нижняя ветвь при этом совпадает с осью абсцисс, т.е. должна полностью отсутствовать. При этом величина ω_p^2 задается соотношением (2.14), где m- эффективная масса колеблющихся ионов кристаллической решетки, а F- соответствующая сила осцилляторов. Отметим, что аналогичные соотношения для диэлектрической проницаемости (2.20) и закона дисперсии (2.23) поляритонов имеют место для состояния вещества, называемого плазмой. Плазма - это электрически нейтральная среда; в которой между

электрическими зарядами (положительными и отрицательными) действуют только кулоновские силы, т.е. для рассматриваемой модели плазмы, как и в случае сегнетоэлектрика, $\omega_0 = 0$. Фактически сегнетоэлектрик при температуре Кюри тоже является плазмой, в которой кулоновскими частицами оказываются ионы кристаллической решетки. В случае металлической плазмы кулоновскими частицами являются свободно движущиеся по металлу электроны и «закрепленные» в узлах кристаллической решетки положительные ионы. Если лорентцевыми осцилляторами плазмы являются электроны, то закон дисперсии для поляритонной ветви задается соотношением (2.25), где плазменная частота ω_p вычисляется из соотношения (2.14), в котором масса осциллятора равна массе электрона. Так как диэлектрическая проницаемость плазмы при $\omega = \omega_p$ в соответствии с соотношением (2.20) обращается в нуль, то, наряду с поперечными поляритонными волнами на плазменной частоте существуют также продольные волны, которые в данном случае называются плазменными. Классические частицы, соответствующие плазменным волнам, называются ***плазмонами.*** Закон дисперсии для плазменных волн, характеризующихся продольной поляризации, соответствует нижней дисперсионной кривой на рис. 2.4. Верхняя кривая на этом рисунке соответствует поперечным волнам, элементарные возбуждения которых называют плазмоляритонами. Таким образом, закон дисперсии для электромагнитных волн плазменного типа, соответствующий плазмонам и плазмоляритонам, имеет место для материальных сред различной физической природы, внутри которых находится плазма: разреженных ионизированных газов, металлов, легированных полупроводников, звезд и физического вакуума, в котором присутсвуют свободные заряженные частицы, например, электроны и позитроны. В последнем случае скорсть распространения электромагнитных волн на частотах, близких к плазменным должна быть существенно меньше скорости света При описании свойств поляритонных волн необходимо проанализировать дисперсию, т.е. зависимость от k , не только фазовой, но и групповой скорости волны, которая, как известно, есть: $u = d\omega/dk$. В отличие от вакуума, в материальной среде фазовая ($v = \omega/k$) и групповая ($u = d\omega/dk$) скорости могут существенно отличаться друг от друга. Как видно из приведенных законов дисперсии, групповая скорость поляритонных волн может изменяться в широких пределах: от нуля (верхняя поляритонная ветвь при k=0) до значений, близких к значению скорости света в вакууме. При описании электромагнитного излучения как потока частиц скорость движения таких частиц (фотонов в вакууме и поляритонов в среде) определяется не фазовой, а групповой скоростью. В случае плазмы для поперечных волн из закона дисперсии путем дифференцирования получаем для групповой скорости:

$$u = \frac{d\omega}{dk} = \frac{c_0^2 k}{\sqrt{\omega_p^2 + c_0^2 k^2}}. \qquad (2.26)$$

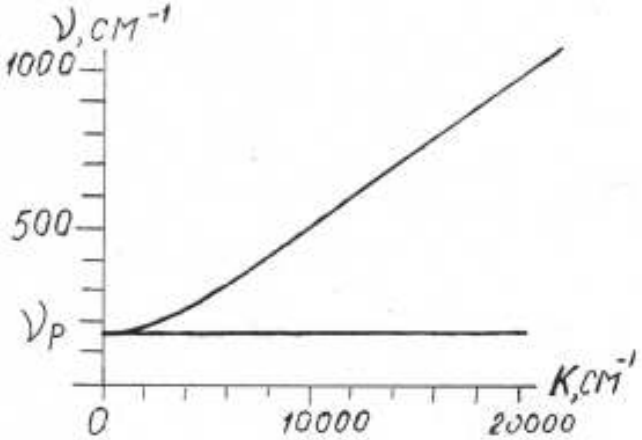

Рис 2.4. Схематический закон дисперсии для поляритонов в плазме. Прямая линия на плазменной частоте ν_p соответствует закону дисперсии плазменных волн; верхняя кривая соответствует возбуждениям, называемых плазмоляритонами.

т.е. при этом фазовая (ω/k) и групповая скорости поляритонных волн существенно отличаются друг от друга.

Вблизи точек структурных фазовых переходов в веществе могут формироваться модулированные фазы, или сверхрешетки, характеризующиеся предельно малыми значениями частоты ω_0 и большими амплитудами соответствующих колебаний – мягких мод. Это приводит к аномальному возрастанию интенсивности рассеяния света на такого рода колебаниях, т.е. к эффекту так называемой динамической или неупругой опалесценции. При этом закон дисперсии для мягкой моды аналогичен закону дисперсии (2.25) для плазмоляритона, в котором вместо скорости света c_0 фигурирует скорость звука s, а, вместо частоты ω_p плазменной моды, частота ω_0 мягкой моды, устремляющаяся к нулю при приближении к точке фазового перезода.

Если в материальной среде имеется не один, а несколько типов заряженных осцилляторов, то формула (2.13) принимает более сложный вид:

$$\varepsilon(\omega) = 1 + \sum_{j=0}^{n} \frac{\varepsilon^2 \mathrm{F_j N_0}}{\mathrm{m_j}\varepsilon_0\left[\omega_{0j}^2 - \omega^2\right]}. \tag{2.27}$$

Здесь индекс j соответствует различным лорентцевым осцилляторам. В этом случае должно иметь место резкое возрастание показателя преломления в нескольких областях спектра, соответствующих резонансным частотам заряженных осцилляторов с частотами ω_{0j}.

Таким образом, использование данной модели позволяет не только объяснить явление дисперсии, но и открывает возможность для расчета значений скоростей распространения поляритонных и плазменных волн в материальных средах. При этом оказывается, что при определенных частотах ω и волновых векторах k групповая скорость волны, соответствующая классической скорости движения поляритона, может аномально уменьшиться по сравнению со скоростью c_0 света в вакууме, т.е. фотоны при определенных энергиях E должны сильно тормозиться на границе “вакуум - материальная среда”. В частности, для плазмы это видно из анализа приведенных формул для закона дисперсии (2.25) и групповой скорости (2.26). Соответствующие поляритонные волны в материальной среде движутся с групповой скоростью, существенно меньшей скорости света c_0 в вакууме, а при $\omega = \omega_p$ когда диэлектрическая проницаемость обращается в нуль, их групповая скорость обращается в нуль, т.е. поляритон, в отличие от фотона в вакууме, может иметь сколь угодно малую скорость.

Такие особенности поляритонных волн и соответствующих им классических частиц – поляритонов - можно объяснить тем, что поляритонная волна по своей природе является “гибридной”, т.е. и электромагнитной и механической. Уменьшение групповой скорости распространения такой волны соответствует тому, что доля механических колебаний в ней возрастает, а электромагнитных - соответственно падает.

3. Поляритонные волны в поглощающей и усиливающей среде

В обычной (поглощающей) материальной среде колебания лорентцевых осцилляторов являются затухающими. Это связано с тем, что часть энергии в процессе колебаний лорентцева осциллятора переходит в тепло, т.е. в другие степени свободы материальной среды. Кроме того, трение возникает из-за самого процесса излучения (так называемое “радиационное трение”). Учет силы трения можно осуществить феноменологическим образом (не раскрывая механизма силы трения): путем введения в уравнение (2.1) дополнительного слагаемого:

$$\mathrm{m}\frac{d^2\mathbf{u}(l)}{dt^2} = -\gamma_0\mathbf{u}(l) - 2\delta\frac{d\mathbf{u}(l)}{dt} + e\sqrt{\mathrm{F}}\mathbf{E}. \tag{3.1}$$

Соответственно, вместо уравнения (2.2.8), мы приходим к следующему соотношению для амплитуд отклонения $\mathbf{u}_0$ и напряженности $\mathbf{E}_0$ электрического поля:

$$-\omega^2\,\mathbf{u}_0 = -\omega_0^2\,\mathbf{u}_0 + \mathrm{i}\gamma\omega\mathbf{u}_0 + \mathrm{e}\sqrt{\mathrm{F}}\,\mathbf{E}_0. \tag{3.2}$$

Здесь введено обозначение $\gamma = 2\delta/m$ для величины, характеризующей затухание лорентцева осциллятора. Тогда для амплитуды вектора поляризации $\mathbf{P}_0$ имеет место:

$$\mathbf{P}_0 = \frac{e\sqrt{F}\mathbf{U}_0}{V} = \frac{e^2 F\mathbf{E}_0}{mV\left[\omega_0^2 - \omega^2 - i\gamma\omega\right]}. \tag{3.3}$$

Соответственно для диэлектрической проницаемости $\varepsilon(\omega)$, учитывая (2.11), получаем:

$$\varepsilon(\omega) = 1 + \frac{e^2 F}{mV\varepsilon_0\left[\omega_0^2 - \omega^2 - i\gamma\omega\right]}. \tag{3.4}$$

Таким образом, учет затухания лорентцева осциллятора приводит к тому, что диэлектрическая проницаемость $\varepsilon(\omega)$ становится комплексной величиной:

$$\varepsilon(\omega) = \varepsilon'(\omega) + i\varepsilon''(\omega), \tag{3.5}$$

где

$$\varepsilon'(\omega) = 1 + \frac{\omega_p^2(\omega_0^2 - \omega^2)}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}, \tag{3.6}$$

$$\varepsilon''(\omega) = \frac{\omega_p^2\gamma\omega}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}. \tag{3.7}$$

В формулах (3.6) и (3.7) мы использовали определение плазменной частоты ω_p, задаваемое соотношением (2.14). Комплексный характер функции $\varepsilon(\omega)$ приводит к тому, что показатель преломления $n(\omega) = \sqrt{\varepsilon(\omega)}$ также становится величиной комплексной: $n = p + i\chi$. Для действительной и мнимой частей показателя преломления имеют место формулы ($n = \varepsilon' + i\varepsilon''$):

$$p^2 - \chi^2 = \varepsilon', \tag{3.8a}$$

$$2p\chi = \varepsilon''. \tag{3.8b}$$

Из вида функций $\varepsilon'(\omega)$ и $\varepsilon''(\omega)$ и соответствующих им функции $p(\omega)$ и $\chi(\omega)$ для действительной и мнимой частей показателя преломления следует, что вблизи резонансной частоты $\omega = \omega_0$ показатель преломления уменьшается с увеличением частоты (с уменьшением длины волны).

Таким образом, учет затухания в модели лорентцевых осцилляторов позволяет объяснить как нормальную, так и аномальную дисперсию в материальных средах.

Другой эффект, связанный с затуханием лорентцевых осцилляторов, состоит в поглощении электромагнитного излучения при его распространении в материальной среде. Экспериментально поглощение электромагнитного излучения описывается законом Бугера:

$$I = I_0\exp(-\alpha L). \tag{3.9}$$

Здесь I_0 - интенсивность излучения на входе в материальную среду, а I - соответствующая интенсивность на выходе плоскопараллельной пластины толщиной L, α - показатель поглощения. Обычно предполагается, что коэффициент α является положительным, т.е. при прохождении электромагнитной волны через материальную среду должно происходить её ослабление, так как энергия исходной волны при прохождении через среду должна уменьшаться. Это согласуется с законом сохранения энергии, если учесть, что к материальной среде дополнительная энергия не подводится, а часть попавшей в вещество электромагнитной энергии переходит в тепло. Материальные среды, для которых коэффициент α является положительным, называются пассивными. Отметим, что возможна также ситуация, когда коэффициент α является отрицательным. Это осуществляется лишь в том случае, когда к

материальной среде подводится энергия извне. При этом может происходить усиление электромагнитной волны после прохождения её через материальную среду. Среды, усиливающие электромагнитные волны, называются активными. В дальнейшем мы будем их изучать более подробно.

Зависимость (3.9) характеризует спектр пропускания электромагнитного излучения пассивной материальной средой. Установим связь между показателем поглощения и характеристиками лорентцевых осцилляторов. Интенсивность электромагнитного излучения, распространяющегося в заданном направлении через изотропную среду, равна плотности потока энергии, т.е. усредненному по времени модулю вектора Умова-Пойнтинга:

$$I = |\mathbf{S}| = |\mathbf{E}\times\mathbf{H}|. \tag{3.10}$$

Для модулей напряженностей **E** и **H**, выраженных в комплексном виде, имеет место соотношение: $\mathbf{E} = \mathbf{E}_0 e^{i(\mathbf{kz}-\omega t)}$; $\mathbf{H} = \mathbf{H}_0 e^{i(\mathbf{kz}-\omega t)}$. Из соотношения (2.7), определяющего закон дисперсии поляритонных волн, имеем в случае комплексного показателя преломления:

$$k = \frac{\omega}{c_0}\sqrt{\varepsilon} = \frac{\omega}{c_0}(p + i\chi). \tag{3.11}$$

Соответственно для электрического и магнитного полей **E** и **H** получим:

$$E = E_0 e^{-\frac{\omega}{c_0}\chi Z} \cdot e^{i\left(\frac{\omega}{c_0}pZ - \omega t\right)}, \tag{3.12}$$

$$H = H_0 e^{-\frac{\omega}{c_0}\chi Z} \cdot e^{i\left(\frac{\omega}{c_0}pZ - \omega t\right)}. \tag{3.13}$$

Таким образом, для интенсивности I(z) имеет место:

$$I(z) = E_0 H_0 e^{-2\frac{\omega}{c_0}\chi Z}\left|e^{2i\left(\frac{\omega}{c_0}pZ - \omega t\right)}\right|^2 = I_0 e^{-2\frac{\omega}{c_0}\chi Z}. \tag{3.14}$$

Из (3.13) мы получаем выражение для показателя поглощения в виде:

$$\alpha = 2\frac{\omega}{c_0}\chi. \tag{3.15}$$

Учитывая соотношение (3.8b), имеем:

$$\alpha(\omega) = \frac{\omega}{c_0}\frac{\varepsilon''}{p} = \frac{\omega^2}{c_0 p}\frac{\omega_p^2\gamma}{\left(\omega_0^2 - \omega^2\right)^2 + \gamma^2\omega^2}. \tag{3.16}$$

Как следует из зависимости показателя поглощения от частоты, присутствие лорентцевых осцилляторов с собственной частотой ω_0 приводит к резонансному поглощению вблизи этой частоты; максимум полосы поглощения близок к резонансной частоте ω_0, а полуширина этой полосы - к коэффициенту затухания γ. Добротность $Q = \frac{\sqrt{\omega_0^2 - \gamma^2}}{\gamma}$ лорентцева осциллятора характеризует “остроту” резонансной кривой поглощения.

Учитывая выражение для плазменной частоты ω_p показатель поглощения может быть представлен в виде:

$$\alpha(\omega) = N_0\sigma(\omega), \tag{3.17}$$

где N_0 - концентрация лорентцевых осцилляторов, а $\sigma(\omega)$ - так называемое эффективное сечение:

$$\sigma(\omega)=\frac{e^2F\omega^2\gamma}{pc_0m\varepsilon_0\left[\left(\omega_0^2-\omega^2\right)^2+\gamma^2\omega^2\right]}. \qquad (3.18)$$

Эффективное сечение $\sigma(\omega_0)=\dfrac{e^2F}{pc_0m\varepsilon_0\gamma}$ имеет физический смысл площади поперечного сечения атома или молекулы, поглощающих электромагнитное излучение с частотой ω_0 и по порядку величины составляет 10^{-20} м^2. Таким образом, для конденсированной среды ($N_0 = 10^{28}$ 1/м^3) в области резонансного поглощения ($\omega = \omega_0$) имеем $\alpha(\omega_0) = 10^8$ м$^{-1}$. Величина показателя поглощения существенно меньше в случае разреженных сред, концентрация N_0 в которых на много порядков меньше, чем в жидкостях и твердых телах. Если в материальной среде имеется несколько типов лорентцевых осцилляторов, то соответственно, в спектре поглощения должно быть несколько резонансных максимумов. При этом спектр пропускания материальной среды обусловлен показателем поглощения следующего вида:

$$\alpha(\omega) = \omega^2\sum_j\frac{a_j}{(\omega_{0j}^2-\omega^2)^2+\gamma_j^2\omega^2}, \qquad (3.19)$$

где коэффициенты a_j зависят от сил F_j и коэффициентов γ_j соответствующих электронных осцилляторов. Вид спектров поглощения для конкретных веществ получается на основе построения зависимости интенсивности $I(\omega)$ прошедшего через плоскопараллельную пластинку или пленку (если поглощение очень велико) электромагнитного излучения.

Рассмотрим особенности поляритонных волн при переходе от пассивной (поглощающей) к активной, т.е усиливающей среде. В качестве примера рассмотрим слабоконцентрированый молекулярный раствор в нейтральной среде, для которого реализована инверсная заселенность между двумя уровнями энергии молекул, связанными с процессами поглощения и излучения. Будем полагать, что нижний уровень соответствует основному (невозбужденному) состоянию молекулы. В случае инверсной заселенности в выражении для показателя поглощения $\alpha(\omega) = N_0\sigma(\omega)$, вместо N_0 следует подставить отрицательную величину ($- N_i$), характеризующую значение инверсной заселенности (разности концентраци молекул на на нижнем и верхнем уровнях). Соответственно при этом значение квадрата плазменной частоты $(\omega_p)^2$ становится отрицательным: $(\omega_p)^2<0$. Изменение знака квадрата плазменной частоты приводит к радикальному изменению закона дисперсии поляритонных кривых согласно соотношению (2.23). В частности, верхняя поляритонная кривая приобретает асимптоты: $\omega = \omega_0$ и $\omega = c_0k$ при неограниченном возрастании волнового вектора, а нижняя поляритонная кривая начинается с нуля и заканчивается значением $\omega = \omega_l<\omega_0$ на оси ординат.

4. Акустооптические аналогии. Кристаллическая модель физического вакуума

Известно, что между оптическими и акустическими явлениями существуют глубокие аналогии, обусловленные волновой природой соответствующих процессов. Фотонам, т.е. элементарным возбуждениям физического вакуума, можно поставить в соответствие акустические фононы изотропной однородной материальной среды. Так как поляризация фотонов в вакууме предполагется поперечной, то в качестве аналогов фотонов следует рассматривать поперечные акустические фононы. При этом скорость света приобретает смысл скорости поперечного звука в такого рода среде.

В качестве простейшей модели кристаллического вакуума можно использовать одномерную кристаллическую цепочку, расположенную в трехмерном пространстве. Для такой цепочки возможны поперечные акустические волны, характеризующиеся законом дисперсии:

$$\omega = 2(s/a)\sin(ka/2). \quad (4.1)$$

Здесь а-период кристаллической цепочки; $(s/a)^2=(к/m)$, m-масса частиц, составляющих кристаллическую цепочку; к- соответствующий коэффициент упругости с учетом взаимодействия лишь ближайших соседей. При малых волновых векторах(длинноволновое приближение), или при стремлении периода “а” к нулю(приближение сплошной среды соотношение (4.1) переходит принимает вид:

$$\omega = sk. \quad (4.2)$$

Таким образом, константа s в соотношениях (4.1) и (4.2) имеет смысл скорости поперечных звуковых волн, распространяющихся в рассматриваемой кристаллической цепочке. Соотношение (4.2) совпадает с законом дисперсии для электромагнитных волн в вакууме: $\omega = c_0 k$, если, вместо скорости света использовать скорость звука. С другой стороны, в рамках применимости рассматриваемой модели, из соотношения (4.1) следует, что групповая скорость u света должна изменяться с изменением волнового вектора и длины волны электромагнитного излучения по закону: $u = s \cos(ka/2)$, т.е. уменьшаться с увеличением частоты и устремиться к нулю при $\cos(ka/2)=0$.

Будем полагать, что между ближайшими соседями цепочки имеет место гравитационное притяжение в соответствии с законом всемирного тяготения: $F=G(m^2/r^2)$, где $G=6{,}67x10^{-11}$ ($м^3$/кг c^2). Таким образом, при малых отклонениях dr от положений равновесия возникает возвращающая сила: $df=G(m^2/r^3)dr$, т.е. для коэффициента к получаем: $к= G(m^2/a^3)$. Соответственно для параметра s имеет место:

$$s^2 = G(m/a). \quad (4.3)$$

Размеры частиц, составляющих рассматриваемую кристаллическую цепочку, по порядку величины близки к периоду цепочки “а”. Если рассматривать их в виде шаров, то соответствующий момент инерции можно полагать равным: $I=(ma^2/10)$ (а-диаметр шаров).

При условии свободного вращения таких шаров с угловой скоростью, равной (2s/ a), момент количества движения шара оказывается равным : (msa/5). Согласно постулату квантовой механики, этот момент должен быть равен постоянной Планка:

$$(msa/5)=h. \quad (4.4)$$

Сопоставляя соотношения (4.3) и (4.4), приходим выражению для периода а цепочки через фундаментальные постоянные h, G и $s=c_0$:

$$a^2=(5\, hG/ c_0^3). \quad (4.5)$$

Отсюда получаем оценку для периода кристаллической цепочки в рассматриваемой модели и соответствующую массу(из соотношения (4.4.)): $a=3{,}49x10^{-33}$см и $m=1{,}57x10^{-5}$г.

Если в кристаллической цепочке рассматриваемого типа реализуется фазовый переход с образованием модулированной фазы (сверхрешетки) за счет присутствия регулярных массивных дефектов, то такая структура моделируется цепочкой с дополнительными связями [9,10] (см. рис.1.1), с учетом взаимодействия между ближайшими соседями. Для такой цепочки закон дисперсии принимает вид

$$\omega^2 = \omega_0^2 +4(s^2/a^2)\sin^2(ka/2). \quad (4.6)$$

При малых импульсах, или в приближении континуума из (4.5) следует закон дисперсии:

$$\omega^2 = \omega_0^2 +s^2k^2. \quad (4.7)$$

При переходе от частот к энергиям и от волновых векторов к импульсам мы получаем известный в теории отсительности закон изменения энергии релятивистской частицы в зависиморсти от её импульса ($s=c_0$):

$$E^2 = E_0^2 +c_0^2\, p_0^2. \quad (4.8)$$

5. Заключение

Выполненный анализ свойств поляритонов в различных материальных средах показывает, что электромагнитные волны после проникновения в вещество могут существенным образом видоизменить свои свойства. Это связано с процессом сильного взаимодействия электромагнитных волн с заряженными осцилляторами, особенно в том случае , когда

собственные частоты осцилляторов материальной среды близки к частотам осцилляторов свободного электромагнитного поля. В результате этого взаимодействия фотон в материальной среде, т.е. поляритон, может двигаться с замедленной скоростью, а в предельном случае даже «остановиться». Следует отметить, что для замедленных фотонов в материальной среде должны резко возрасти вероятности различного рода неупругих процессов, т.е. наблюдаться эффект неупругой опалесценции. Аналогичный эффект должен иметь мест в микроканалах, заполненных диэлектриком, когда поперечный размер микроканала приближается к длине соответствующего электромагнитного излучения. Особые свойства обнаруживаются для фотонов, распространяющихся вблизи края запрещенной зоны в так называемых фотонных кристаллах, в частности, в искусственных опалах, синтезированных в последние годы. При этом также может происходить уменьшение групповой скорости распространения; кроме того, фотон в трехмерном фотонном кристалле может приобретать новые свойства как квазичастицы кристалла, связанные с величиной его эффективной массы.

В усиливающих (активных) средах фотоны приобретают новые, свойства, связанные с возможностью распространения волновых пакетов с групповой скоростью, превышающей скорость света в вакууме[11].

В условиях сильной перенормировки скорости распространения электромагнитных волн в веществе известные эффекты теории относительности должны стать предметом дальнейших экспериментальных и теоретических исследований.

Список литературы

[1] M.Born, K. Huang. Dynamical Theory Of Crystal Lattices. Oxford at the Clarendon Press. 1954, 488 p.

[2] М.Борн, Х.Кунь. Динамическая теория кристаллических решеток. Перевод с английского В.И. Когана. Под редакцией И.М. Лифшица. Издательство иностранной литературы, Москва, 1958, 488 стр.

[3] А.М. Косевич. Основы механики кристаллической решетки. Издательство “Наука”, Москва, 1972, 280 стр.

[4] А.М. Косевич. Физическая механика реальных кристаллов. Киев. “Наукова Думка”, 1981, 327 стр.

[5] В.С. Горелик. Комбинационное рассеяние света как метод исследования квазичастиц в кристаллах. В сб. «Современные проблемы спектроскопии комбинационного рассеяния света». Редактор- М.М. Сущинский. Изд. «Наука», 1978 г., стр.28-47.

[6] В.С. Горелик, Г.Г. Митин, Ю.Н. Поливанов. Поляритонные спектры рассеяния света в зоне диссоциированных состояний кристалла. Кристаллография, 1978, т. 23, стр.561-565.

[7] В.С. Горелик, О.П. Максимов, М. М. Сущинский. Исследование многочастичных процессов КР в кристаллах с помощью лазера на парах меди. Краткие сообщения по физике, 1977, №4, стр.19-23.

[8] V.S.Gorelik “Raman Scattering”. In “ Visible and Near Visible Light Scattering” Ed/. By Roy Pike and P. Sabatier. Academic Press. San Diego. san Fransisco, New York, Boston, London, Sydney, Tokyo. Vol.1, p.828-848, 2002.

[9] В.С. Горелик. Закон дисперсии колебательных экситонов для некоторыъх идеализированных моделей кристаллических решеиок. Препринт ФИАН, 1975, №93, 35 стр.

[10] V.S.Gorelik. Idealized models of Crystalline Lattices and the spectra of actual crystals. In “Inelastic Light Scattering In crystals”. Ed. By M.M. Sushchinskiy. Nova Science Publishers, Commack. Vol.180, p.100-142.

[11] В.А. Андреев, В.С. Горелик. О возможности существования сверхсветовых импульсов ВКР. Краткие сообщения по физике, 1999 г., №8, стр.3-10.

Идентификация интерпретирующих моделей В теории гравитации и космологии

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1. Введение

Эксперименты по уточнению физической теории в узком смысле – теории физической величины – основываются на предсказании доступных измерениям следствий или на интерпретации данных измерений, не нашедших объяснения в рамках других теорий. При этом под «другими» понимают теории, использующие «другие» интерпретирующие математические модели зависимости данной физической величины как выходной переменной от физических величин, входных переменных модели, характеризующих свойства объекта измерений. Соответствующие измерительные задачи по классификации [1] относят к задачам структурно-параметрической идентификации. Их решение для различных интерпретирующих моделей даже одними и теми же методами приводит в общем случае к неравноточным и, строго говоря, несопоставимым результатам. Особенностью многих измерительных задач гравитации и космологии является ограниченные возможности использования метода прямого измерения, малость эффектов в контролируемых условиях и неконтролируемость условий измерений в тех случаях, когда эти эффекты существенны [2].

Так, за пятьдесят лет со дня открытия космологического красного смещения, по мере увеличения объема учитываемых внегалактических объектов, оценки постоянной Хаббла H уменьшились на порядок [3]. Из четырех последних определений ньютоновской константы гравитации G три противоречат друг другу [4], причем по точности такие определения на два порядка выше относительных погрешностей определения других фундаментальных физических констант. Релятивистские эффекты получают все еще как разность между астрометрическими данными и расчетным значением ньютоновской теории [5]. Интерферометрические радиоизмерения изменения угла между квазарами 3C279 и 3C273 непосредственно до и после затмения 3C279 Солнцем хотя и подтверждают наличие гравитационного отклонения радиосигналов, но достигнутой точности измерений еще недостаточно для выбора между теориями Эйнштейна и Бранса-Дикке [6].

Ряд важных измерительных задач по проверке теории относительности (см. [6] и [7]), определению ньютоновской константы гравитации [8], исследованию шкалы космологических расстояний [9] и т.п. связан с необходимостью анализа условий применимости статистических методов не столько параметрической, сколько структурной идентификации интерпретирующих математических моделей физических величин. Именно в этом плане решение этих задач нуждается в корректном метрологическом сопровождении [10], в учете погрешностей используемых средств и методов измерений, а также погрешностей неадекватности за счет применяемых методов выбора структуры и параметров интерпретирующих моделей.

2. Идентификация интерпретирующих моделей

Согласно [1] интерпретирующими называют такие математические модели физических величин, структуру и параметры которых определяют в ходе решения измерительной задачи.

Теория идентификации интерпретирующих математических моделей физических величин [11] рассматривает эти модели не отдельно, а в рамках класса моделей, определяемого т.н. моделью максимальной сложности, причем статистическая проверка гипотез о структуре конкурирующих моделей основана на следующих принципах [12]:

- Дедекинда-Кантора-Вейерштрасса аксиоматического определения физических величин при метризации свойств объектов измерений (неопределенность шкал физических величин определяется полнотой и точностью соблюдения условий метризации);
- относительности истинного значения физической величины (оно является расчетным в строгой физической теории, константы которой определены по данным измерений наивысшей точности в соответствующей поверочной схеме, или вводится по определению);
- единства результатов воспроизведения, измерения и вычисления значений физических величин (их действительное значение – результат измерения эталоном такого уровня поверочной схемы или результат такого определения расчетным путем, что его отличием от истинного значения в данной измерительной задаче можно пренебречь);
- совместного измерения всех переменных модели эталонами (погрешность неадекватности интерпретирующей математической модели объекта измерений – разность расчетного значения выходной переменной по данным совместных измерений входных переменных и результата ее измерения в соответствующих расчету условиях);
- максимума воспроизводимости в рамках схемы перекрестного наблюдения погрешности неадекватности интерпретирующей математической модели как погрешности экстраполяции

$$æ_2 \equiv \int_{-\infty}^{\infty} \inf_f \left\{ \hat{f}_{\Pi}(\xi), \hat{f}_{\text{К}}(\xi) \right\} d\xi ,$$

где $\hat{f}_{\Pi}(\xi)$ и $\hat{f}_{\text{К}}(\xi)$ – оценки плотности распределения вероятностей $f_{\Xi}(\xi)$ случайной составляющей погрешности неадекватности на пробной и контрольной частях данных совместных измерений переменных;

- линейности параметрических критериев точности

$$\theta_{2*} \equiv \mathbf{M}\left|\text{Y} - \theta_{1*}\right| \equiv \mathbf{M}\{\text{Y} - \theta_{1*}\} + 2\int_{-\infty}^{\theta_{1*}} F_*(\xi)\, d\xi ,$$

где $F_*(\xi)$ – функция распределения случайной составляющей выходной переменной вида * (R – равномерное, K – Коши, G – Гаусса, L – Лапласа и т.д.) с параметром положения θ_{1*} и параметром рассеяния θ_{2*};

- независимости случайной и неисключенной систематической (распределения вида R в пределах $\pm\theta_0$) составляющих погрешности измерений

$$f_{R*}(\varepsilon) = (2\theta_0)^{-1} \cdot \left[F_*(\varepsilon + \theta_0) - F_*(\varepsilon - \theta_0)\right].$$

Алгоритмы структурной идентификации интерпретирующих моделей на основе перечисленных принципов получили название метода максимума компактности (ММК), который в комбинации с традиционными алгоритмами параметрической идентификации «ММК+метод параметрической идентификации»[1] [1] реализован в системах метрологического сопровождения классами моделей «ММК-стат М» (многомерные степенные ряды), «ММК-спектр» (полигармонические модели) и «ММК-дин» (дифференциальные уравнения).

Примеры применения этих систем для решения типовых измерительных задач структурной идентификации интерпретирующих моделей приведены в [1], [11] и [13]. Анализ полу-

[1] ММКМНК – наименьших квадратов, ММКМНМ – наименьших модулей, ММКМЕДС – медианная интерполяция, ММКМП – максимального правдоподобия и т.п.

ченных результатов показывает, что во многих случаях более точное описание объекта измерений дают модели, в которых ряд структурных элементов отсутствует, а при наличии выделяющихся результатов совместных измерений – предпочтение отдается алгоритму медианной интерполяции ММКМЕДС.

3. Неравноточные измерения в гравитационных экспериментах

В 1998 г. CODATA рекомендовано новое значение ньютоновской константы гравитации: $6{,}673(10)\cdot 10^{-11}\ м^3\cdot с^{-2}\cdot кг^{-1}$ [14]. Это – «шаг назад» по сравнению с 1986 г. [15]: $6{,}67259(85)\cdot 10^{-11}\ м^3\cdot с^{-2}\cdot кг^{-1}$. Это значение было принято по данным опытов 1982 г. (см. Таблицу 1).

Определения ньютоновской константы гравитации G [15-19] **Таблица 1**

Год	Источник	Оценка, $м^3\cdot с^{-2}\cdot кг^{-1}$	Год	Источник	Оценка, $м^3\cdot с^{-2}\cdot кг^{-1}$
1930	Heyl P.R.	$6{,}6721(73)\cdot 10^{-11}$	1972	Facy, Pontikis	$6{,}6714(6)\cdot 10^{-11}$
1942	Heyl P.R., Chrzanowski P.	$6{,}6720(49)\cdot 10^{-11}$	1979	Sagitov et al.	$6{,}6745(8)\cdot 10^{-11}$
1969	Rose R.D., Beams J.W. et al.	$6{,}674(3)\cdot 10^{-11}$	1982	Luther, Towler	$6{,}6726(5)\cdot 10^{-11}$
1972	Pontikis	$6{,}67145(10)\cdot 10^{-11}$	1988	Karagioz	$6{,}6731(4)\cdot 10^{-11}$

Числа в скобках, средние квадратичные отклонения (СКО) средних (арифметических или взвешенных) – оценок константы, интерпретируют так:

$$6{,}6714(6)\cdot 10^{-11} = 6{,}67(14\pm 6)\cdot 10^{-11} = 6{,}67(08\ldots 20)\cdot 10^{-11}\ [1972],$$
$$6{,}6726(5)\cdot 10^{-11} = 6{,}67(26\pm 5)\cdot 10^{-11} = 6{,}67(21\ldots 31)\cdot 10^{-11}\ [1982],$$
$$6{,}6745(8)\cdot 10^{-11} = 6{,}67(45\pm 8)\cdot 10^{-11} = 6{,}67(37\ldots 53)\cdot 10^{-11}\ [1979].$$

При такой интерпретации вывод о противоречии оценок неизбежен.

Известно, что точность среднего <u>арифметического</u> $\overline{G}$ и <u>среднего взвешенного</u> $\overline{\overline{G}}$ полученных в эксперименте значений константы гравитации по данным соответственно равноточных или неравноточных измерений как статистических оценок должна с ростом объема выборки увеличиваться, если выполняется условие статистической однородности. Более того, полагают, что среднее взвешенное дает оценку даже более точную, чем наиболее точный из компонентов смеси неравноточных данных.

Так, пусть при неравноточных определениях константы G получены Q статистических рядов, содержащих $N_q = N_1$ членов и подчиняющихся распределениям Гаусса, параметры которых характеризуются средними арифметическими $\overline{G}_q$ и СКО $\hat{s}_q$, $q = \overline{1,Q}$. Рассмотрим оценку

$$\overline{\overline{G}} = \sum_{q=1}^{Q} p_q \cdot \overline{G}_q\,,\ p_q = \frac{1}{\hat{s}_q^2}\cdot\left(\sum_{k=1}^{Q}\frac{1}{\hat{s}_k^2}\right)^{-1}, \tag{1}$$

с весовыми коэффициентами p_q, обеспечивающими минимум ее дисперсии.

Для интерпретации оценок воспользуемся последовательностями независимых случайных величин, подчиняющихся распределению Гаусса с равными математическими ожиданиями и различными дисперсиями

$$\mathbf{G} = \{G_q : \mathbf{M}(G_q) = \mathrm{m}, \mathbf{D}(G_q) = \sigma_q^2,\ \sigma_1^2 \le \sigma_2^2 \le \ldots \le \sigma_q^2 \le \ldots \le \sigma_Q^2,\ q = \overline{1,Q}\}.$$

Согласно [20] оценка

$$\overline{\overline{G}}_{опт} = \left(\sum_{\nu=1}^{Q} 1/\sigma_\nu^2 \right)^{-1} \cdot \sum_{q=1}^{Q} \overline{G}_q / \sigma_q^2 .$$

Легко показать, что в этом случае дисперсия среднего взвешенного

$$\mathbf{D}\{\overline{\overline{G}}_{опт}\} = \frac{1}{N_1} \cdot \left(\sum_{q=1}^{Q} 1/\sigma_q^2 \right)^{-1} .$$

Использование неравенства средних [21] дает

$$\frac{\sigma_1^2}{N_1 \cdot Q} \le \mathbf{D}\{\overline{\overline{G}}_{опт}\} \le \frac{1}{N_1 \cdot Q^2} \sum_{q=1}^{Q} 1/\sigma_q^2 \le \frac{\sigma_Q^2}{N_1 \cdot Q}.$$

Пусть теперь $\sigma_1^2 << \sigma_2^2 = ... = \sigma_Q^2$. Тогда $\mathbf{D}\{\overline{\overline{G}}_{опт}\} = \left[N_1 \cdot \left(1/\sigma_1^2 + (Q-1)/\sigma_2^2 \right) \right]^{-1}$. Следовательно, $1/\mathbf{D}\{\overline{\overline{G}}_{опт}\} = N_1 \cdot \left(1/\sigma_1^2 + (Q-1)/\sigma_2^2 \right)$ или $1/\mathbf{D}\{\overline{\overline{G}}_{опт}\} > N_1/\sigma_1^2$. Отсюда немедленно следует, что $\mathbf{D}\{\overline{\overline{G}}_{опт}\} < \sigma_1^2 / N_1$ [22]!

Но, как известно (см. например, [23] и [24]), теория вероятностей и математическая статистика применимы соответственно к статистически устойчивым явлениям и статистически однородным данным. Поэтому рассмотрим этот парадокс в рамках метода максимального правдоподобия.

Еще в 20-е гг. прошлого века между А. Эддингтоном и Р. Фишером разгорелась дискуссия об оценке параметра рассеяния: Фишер отстаивал СКО, а Эддингтон (в те времена это было вызовом «нормальной» теории) – среднее абсолютное отклонение (САО). В 30-е гг. с появлением аксиоматики А.Н. Колмогорова отступления от «нормальной» теории усилились, появились непараметрическая статистика, в конце 40-х гг. – схема перекрестного наблюдения погрешностей, а в начале 50-х гг. – робастная статистика. И уже в 1960 г. в схеме «ε – загрязнения» для смеси двух распределений

$$f_2(x) = \frac{1-\varepsilon}{\sqrt{2\pi\sigma^2}} \cdot e^{-\frac{(x-\mathrm{m})^2}{2\sigma^2}} + \frac{\varepsilon}{\sqrt{2\pi(\beta\sigma)^2}} \cdot e^{-\frac{(x-\mathrm{m})^2}{2(\beta\sigma)^2}}, \quad 0{,}0018 \le \varepsilon \le 0{,}9982 ,$$

Дж. Тьюки показал, что рекомендации Эддингтона более реалистичны [25].

Другими словами, метод максимального правдоподобия предполагает интерпретацию данных в серии измерений как последовательность одинаково распределенных независимых случайных величин, и представление функции правдоподобия смесью распределений, которая в гауссовом случае имеет вид

$$\prod_{n=1}^{N} f_Q(\hat{G}_n) = \prod_{n=1}^{N} \sum_{q=1}^{Q} \left\{ \frac{\rho_q}{\sigma_q \sqrt{2\pi}} \cdot \exp\left[-\frac{(\hat{G}_n - m_q)^2}{2\sigma_q^2} \right] \right\}, \ \sum_{q=1}^{Q} \rho_q = 1, \ \rho_q = N_q / N, \ \sum_{q=1}^{Q} N_q = N, \quad (1)$$

где ρ_q, $q = \overline{1,Q}$, – вероятность появления случайной величины, подчиняющейся распределению Гаусса с параметрами m_q и σ_q[2]. Интегрирование (1) дает

$$\mathbf{M}\{\mathbf{G}\} = \sum_{q=1}^{Q} \rho_q \cdot m_q \quad \text{и} \quad \mathbf{D}\{\mathbf{G}\} = \sum_{q=1}^{Q} \rho_q \cdot [\sigma_q^2 + (m_q - \sum_{k=1}^{Q} \rho_k \cdot m_k)^2], \quad (2)$$

причем в корректно поставленном эксперименте по определению ньютоновской гравитационной константы $m_q = G_0$, и оценки (2) должны принимать вид

[2] Равноточным данным соответствует $N_q = 1$, $m_q = G_0$ и $\sigma_q = \sigma$.

$$\mathbf{M}\{\mathbf{G}\} = G_0 \text{ и } \mathbf{D}\{\mathbf{G}\} = \sum_{q=1}^{Q} \rho_q \cdot \sigma_q^2 .$$

Таким образом, правильная вероятностная интерпретация данных определений ньютоновской константы гравитации указывает на нее как на неопределенную величину, параметр положения распределения возможных значений которой и принимают за согласованное значение. В то же время точность определения согласованного значения характеризует не точность оценивания параметра положения, а параметр рассеяния распределения в целом.

Это значит, что «реальное» СКО в $\sqrt{N}$ раз больше!

Так, в экспериментах 1930-1972 гг. число частных определений ньютоновской константы гравитации колебалось в пределах от 10 до 40, т.е. СКО случайной составляющей оценки занижено не менее чем в 3-6 раз!

Это значит, что проблемы противоречия вероятностно-статистической интерпретации данных экспериментальных исследований оценок ньютоновской гравитационной константы носят параметрический характер.

Обратимся теперь к данным работы [26] определения константы гравитации (Таблица 2). Авторы этой работы в рамках теории Лесажа-Майораны интерпретируют близость оценки $\overline{\mathrm{G}}_{\text{лм}} = 6{,}74 \cdot 10^{-8}\ \text{см}^3 \cdot \text{г}^{-1} \cdot \text{с}^{-2}$ к среднему взвешенному значению $\overline{\overline{\mathrm{G}}} = 6{,}739 \cdot 10^{-8}\ \text{см}^3 \cdot \text{г}^{-1} \cdot \text{с}^{-2}$ по 19 источникам и существенность ее отличия от т.н. лабораторного значения $\mathrm{G}_{\text{л}} = 6{,}6725 \cdot 10^{-8}\ \text{см}^3 \cdot \text{г}^{-1} \cdot \text{с}^{-2}$ как аргумент в пользу гипотезы о гравитационном экранном эффекте.

Приведем табличные данные к нулевой глубине и воспользуемся рекомендациями [1] математической теории измерительных задач: проверим методом максимума компактности (ММК) гипотезу о наличии зависимости между глубинами парных измерений и оценками гравитационной постоянной.

С наименьшей погрешностью неадекватности в классе многомерных степенных рядов все данные интерпретирует ММКМНК–модель

$$\mathrm{G}(\mathrm{h}_1, \mathrm{h}_2) = (6{,}743 + 3{,}241 \cdot 10^{-5}\,\mathrm{h}_1 - 1{,}366 \cdot 10^{-5}\,\mathrm{h}_2 \pm 0{,}026) \cdot 10^{-8}\ \text{см}^3 \cdot \text{г}^{-1} \cdot \text{с}^{-2},$$

причем для ММК–моделей указывается не СКО, а средний модуль погрешности (САО) неадекватности, приведенный к СКО умножением на $\sqrt{\pi/2}$, как суммарная характеристика правильности и сходимости расчетных значений модели по отношению к результатам измерений.

Результаты геофизических экспериментов [27-29] **Таблица 2**

№	Объект	h_1, м	h_2, м	$\mathrm{G}, 10^8\ \text{см}^3 \cdot \text{г}^{-1} \cdot \text{с}^{-2}$	№	Объект	h_1, м	h_2, м	$\mathrm{G}, 10^8\ \text{см}^3 \cdot \text{г}^{-1} \cdot \text{с}^{-2}$
1	Шахта 1	96	587	6,795 ± 0,021	11	Море	0	379	6,810 ± 0.020
2	Шахта 2	0	685	6,733 ± 0,004	12	Скважина в ледовом щите Гренландии	213	396	6,725 ± 0,057
3		58	685	6,739 ± 0,003	13		396	597	6,725 ± 0.054
4		58	209	6,724 ± 0,014	14		597	762	6,738 ± 0.048
5		233	389	6,726 ± 0,012	15		762	945	6.738 ± 0,044
6		418	685	6,749 ± 0,013	16		945	1309	6,758 ± 0,038
7	Шахта 3	251	590	6,705 ± 0,016	17		1309	1491	6,780 ± 0.032
8	Шахта 4	0	1000	6,720 ± 0,024	18		1491	1673	6,777 ± 0,045
9	Скважина 1	3712	3962	6,810 ± 0,070	19	Море	100	5000	6.677 ± 0,013
10	Скважина 2	0	2000	6,704 ± 0,057					

Идентификация же модели для разностей глубин измерений Δh в качестве оптимальной по критерию минимума погрешности неадекватности дает ММКМЕДС–модель $G(\Delta h) = (6{,}737 \pm 0{,}034) \cdot 10^{-8}\ см^3 \cdot г^{-1} \cdot с^{-2}$.

Этот результат с точки зрения существования экранного эффекта не требует комментариев и представляет структурный аспект проблемы вероятностно-статистической интерпретации данных экспериментальных исследований в области определения ньютоновской константы гравитации.

Вместе с тем оба аспекта проблемы и интерпретация фундаментальной константы теории гравитации как неопределенной величины, распределением вероятностей возможных значений которой нельзя пренебречь всего лишь указывают на отсутствие достаточно полно контролируемых условий проведения гравитационных экспериментов в динамическом смысле и на отсутствие состоятельной теории физического механизма явления гравитации.

4. Интерпретирующие модели диаграммы Хаббла

«Красное смещение» в z спектрах внегалактических объектов было обнаружено В. Слайфером [30], а Э. Хаббл указал на его линейную зависимость от расстояния до галактик, определяемого по светимости ярчайших звезд или ярчайших галактик в скоплениях [31]. Полярные точки зрения, т.н. «допплеровская» и «недопплеровская», на природу внегалактического красного смещения представлены в обзорах Г. Тамманна [32] и Х. Арпа [33]. Но в обоих случаях одним из аргументов этих точек зрения являются модели зависимостей между наблюдаемыми физическими величинами.

Так, например, в работе [34] проведено сопоставление регрессионных зависимостей видимой светимости $m(z)$, поверхностной яркости $\mu(z)$ и углового размера $\lg \theta(z)$ для 12000 галактик и 4000 квазаров, в результате чего определена квадратичная зависимость красного смещения от расстояния, противоречащая, по мнению автора, квазилинейной модели.

Первоначально диаграмма Хаббла представляла собой зависимость «лучевая скорость – расстояние», полученную по 19 внегалактическим объектам. Бросающийся в глаза линейный характер этой зависимости стал причиной ее использования в качестве шкалы космологических расстояний. Проблема заключалась в «градуировке» этой шкалы путем определения коэффициента наклона – постоянной Хаббла. Сам Э. Хаббл оценил ее как 500 км·с$^{-1}$·Мпк$^{-1}$, но впоследствии ее значение пересматривали несколько раз по мере увеличения объема данных и совершенствования методов определения расстояний до внегалактических объектов и довели до 55 км·с$^{-1}$·Мпк$^{-1}$.

Для экзотических объектов, радиогалактик и квазаров, диаграмма Хаббла представляет собой их распределение в логарифмических координатах «видимая звездная величина – красное смещение спектральных длин волн». При этом наклон диаграммы для квазаров (+0,10) никак не удавалось согласовать со стандартным ее наклоном для обычных галактик (+0,20). Различие для выборки из 169 квазаров доходило до 200 %. Но стоило удалить из выборки всего лишь один квазар 3C 280.1, как наклон диаграммы достиг +0,12. После удаления второго «выделяющегося» квазара (3C 273.0) наклон возрос до +0,18! А для 172 радиогалактик получилось +0,19!! Однако на тщательно отобранной выборке квазаров со спектрами без особенностей и надежно определенными угловыми размерами повторение процедуры извлечения привело к «перегибу» от +0,18 до +0,28 [3]!!!

Ценный опыт в этом отношении дало дальнейшее статистическое исследование диаграммы Хаббла. Как оказалось, алгоритмы структурно-параметрической идентификации ММКМЕДС и ММКМНК дают практически совпадающие оценки наклона: +0,29 [36]. Однако анализ правильности постановки измерительной задачи и условий применимости пере-

численных алгоритмов структурно-параметрической идентификации показал, что при традиционном представлении диаграммы Хаббла нарушено условие неконфлюэнтности, а именно: в регрессионных моделях аргументы функциональных зависимостей должны быть достаточно точно измеряемыми величинами. Этому условию удовлетворял показатель красного смещения – логарифм произведения скорости света на относительное смещение длины волны, т.е. функция в диаграмме, а аргумент – видимая звездная величина объектов – имеет существенный статистический разброс на множестве объектов одного типа.

Вместе с тем основным в измерительных задачах идентификации интерпретирующих моделей для диаграммы Хаббла [35] остается структурный аспект. Поэтому будем полагать модели различной структуры конкурирующими статистическими гипотезами, для проверки которых по критерию наибольшего правдоподобия воспользуемся алгоритмами ММК-идентификации [1]. При этом в качестве иллюстрации рассмотрим данные [9] о красных смещениях z и визуальных величинах V для 63 радиогалактик и 103 квазаров при непрерывной интерпретирующей модели максимальной сложности в виде степенного ряда со старшей степенью M=8 (число параметров – не более 9):

$$\lg(cz) = \psi(V) = \theta_{V0} + \sum_{m=1}^{M} \theta_{Vm} \cdot (10 \cdot V)^m \tag{3}$$

или

$$V(z) = \theta_{Z0} + \sum_{m=1}^{M} \theta_{Zm} \cdot [100 \cdot \lg(cz)]^m , \tag{4}$$

где c – скорость света в вакууме.

Использование «прямой» и «обратной» зависимостей связано с тем, что регрессионный анализ применим для моделей, у которых значения входных переменных точно известны. Точность выполнения этого условия проверяют сравнением этих зависимостей, что особенно просто для линейных моделей.

В противном случае применяют конфлюэнтный анализ.

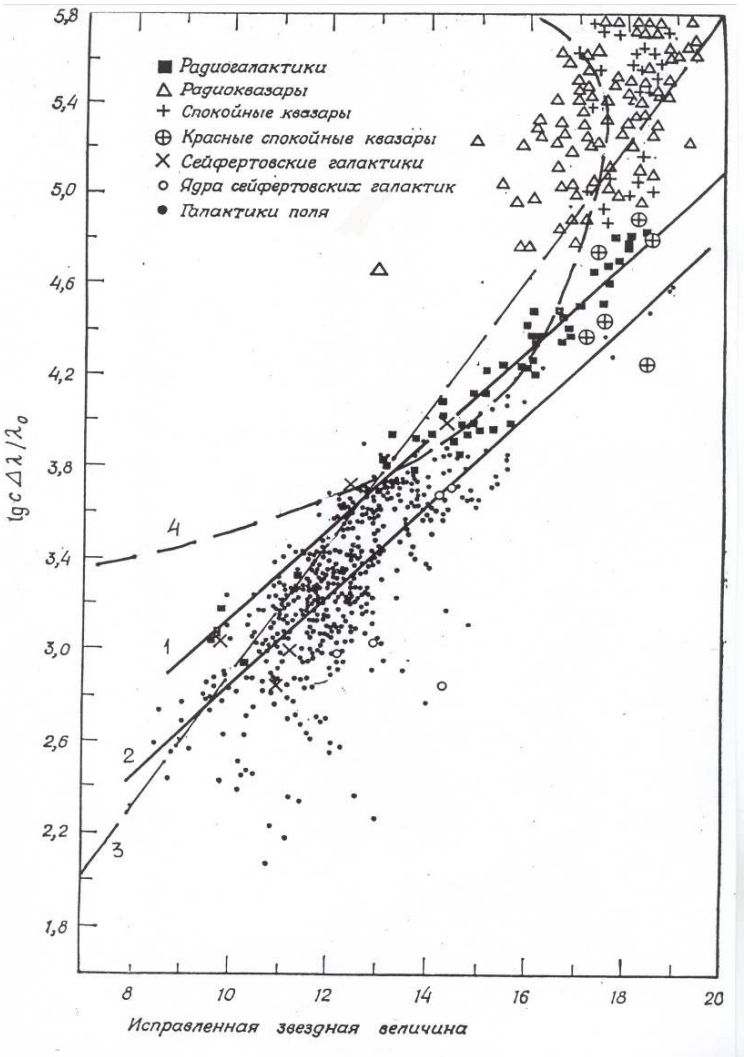
Идентификация модели (3) в данном случае подтвердила, что минимум погрешности неадекватности в диапазоне светимостей внегалактических объектов $12{,}80^{\mathrm{m}}\ldots19{,}00^{\mathrm{m}}$ достигается чисто линейной (с кодом структуры $\vartheta' = 01$) ММКМЕДС-моделью $\lg(cz) = \psi(V) = 0{,}2904564 \cdot V \pm 0{,}328$, но лучшей среди моделей вида (4) оказалась ММКМНК-модель (код структуры $\vartheta' = 1110011$) со средним модулем погрешности неадекватности 1,213 (см. рисунок).

Данные этого примера не претендуют на роль каких-либо оценок. Они всего лишь показывают, что существенной проблемой интерпретации диаграммы Хаббла является статистическая неоднородность данных.

5. Заключение

Многие проблемы и противоречия интерпретации данных измерительных экспериментов в теории гравитации и космологии порождены недооценкой роли метрологического обеспечения измерений и вычислений в задачах идентификации физических величин и зависимостей между ними. Причины такого положения дел связаны в первую очередь с неполнотой постановки и нарушением условий применимости методов решения измерительных задач. Сюда же следует отнести и еще ряд причин, в том числе: 1) несогласованное с аксиоматикой теории измерений определение понятия погрешности неадекватности математической модели объекта, 2) путаницу между размерностной и структурно-параметрической идентификацией, 3) использование шкал физических величин с реперными точками вне государственных поверочных схем и внесистемными единицами, 4) ориентацию на гауссово распределение погрешностей, 5) использование различных концепций вероятности при интерпретации по-

грешности и неопределенности результатов измерений, 6) некорректность обработки данных неравноточных измерений, 7) нарушение условия неконфлюэнтности для регрессионных моделей, 8) неполноту расчета точности результатов решения измерительных задач, полученных различными методами.



Зависимость красное смещение – звездная величина для 474 галактик поля, а также радиогалактик, квазизвездных объектов и сейфертовских галактик [9].

Звездная величина исправлена за эффект апертуры, К-поправку и поглощение света в нашей Галактике.

1 – данные Сэндейджа для ярчайших галактик в скоплениях

2 – данные по галактикам поля

3 – модель типа (3)

4 – модель типа (4)

Сводка результатов наиболее точных определений гравитационной постоянной G [9]

$м^3 \cdot с^{-2} \cdot кг^{-1}$

Метод	Материал	Частное значение	N	Общее среднее значение	СКО общего среднего	Относит. СКО
Крутильные весы (осцилляции), шарики из различных материалов [10]	Золото Платина Стекло	6,6782(16) 6,6640(13) 6,6740(12)	5 5 5	$6{,}6721(73)\cdot 10^{-11}$	$0{,}0073\cdot 10^{-11}$	0,001090
То же, платиновые шарики и вольфрамовые нити различного приготовления [11]	Отожженная Холоднотянутая	6,67554(51) 6,66854(83)	5 5	$6{,}6720(49)\cdot 10^{-11}$	$0{,}0049\cdot 10^{-11}$	0,000740

Ускоряющийся столик [12]				$6{,}674(3)\cdot 10^{-11}$	$0{,}0030\cdot 10^{-11}$	0,000450
Крутильные весы (резонанс), шарики из различного материала [13]	Серебро Медь Бронза Свинец	6,67162(17) 6,67157(17) 6,67122(21) 6,67126(22)	10 10 10 10	средн.взвешенное 6,67145(10) $\cdot 10^{-11}$	$0{,}0001\cdot 10^{-11}$	0,000015

Литература

1. Рекомендации по метрологии Р 50.2.004-2000 ГСИ. Определение характеристик математических моделей зависимостей между физическими величинами при решении измерительных задач. Основные положения. – М.: Издательство стандартов, 2000.
2. Левин С.Ф. Метрологическая аттестация математических моделей в измерительных задачах гравитации и космологии//Сб. тезисов докладов X РГК «Теоретические и экспериментальные проблемы общей теории относительности и гравитации», Владимир, 20-27 июня 1999 г. – М.: РГО, 1999. – С. 245.
3. Левин С.Ф. Оптимальная интерполяционная фильтрация статистических характеристик случайных функций в детерминированной версии метода Монте-Карло и закон красного смещения. – М.: НСК АН СССР, 1980.
4. Бронников К.А., Константинов М.Ю., Мельников В.Н. Сравнительный анализ проектов космических экспериментов по определению гравитационной постоянной. – Измерительная техника. – 1996. – № 5. – С. 3.
5. Synge J.L. Relativity: The General Theory. –Amsterdam: North-Holland Publishing Company, 1960 (Русский перевод под ред. А.З. Петрова: Синг Дж. Общая теория относительности. – М.: ИЛ, 1963).
6. Weinberg S. Gravitation and Cosmology. – N.Y.: John Wiley and Sons Inc., 1972 (Русский перевод под ред. Я.А. Смородинского: Вейнберг С. Гравитация и космология. – Волгоград: Издательство «Платон», 2000).
7. Eddington A.S. The mathematical theory of relativity. – Cambridge: University Press, 1924 (Русский перевод под ред. Д.Д. Иваненко: Эддингтон А.С. Теория относительности. – Л.: ОНТИ, 1934).
8. Сб. «Квантовая метрология и фундаментальные константы». – М.: Мир, 1981.
9. Lang K.R. Astrophysical formulae. – Berlin, Heidelberg, New York: Springer-Verlag, 1974 (Русский перевод под ред. Д.К. Надежина и Л.М. Озерного: Ленг К. Астрофизические формулы. – М.: Мир, 1978).
10. Levin S.F. Metrological support of measuring problems of identification for gravitational and optical measurement signals//«Gravitation, Cosmology and Relativistic astrophysics». – Kharkov: University of Kharkov, 2000. – P. 72.
11. Левин С.Ф. Теория измерительных задач идентификации. – Измерительная техника. – 2001. – № 7. – С. 8.
12. Levin S.F. Measurement problems of structural identification of mathematical models of physical objects//«Mathematics, statistics and computation to support measurement quality». – Sanct-Peterburg: VNIIM, 2002. – P. 29.
13. Левин С.Ф. Метод максимума компактности и комплексные измерительные задачи.– Измерительная техника.– 1995.– № 7. – С. 15.
14. Сб. «Российская метрологическая энциклопедия». – СПб.: «Лики России», 2001.
15. Melnikov V.N. Fundamental physical constants and multidimensional black holes. – Gravitation & Cosmology. – 2000. – Vol. 6. – No. 22. – P. 81.
16. Heyl P.R. Journ. Res. Nat. Bur. Stand. – 1930. – V.5. – P. 1243.
17. Heyl P.R., Chrzanowski P. Journ. Res. Nat. Bur. Stand. – 1941. – V.29. – P. 1.

18. Rose R.D., Parker H.M., Lowry R.A., Kuhlthau A.R., Beams J.W. – Phys. Rev. Lett. – 1969. – V.23. – P. 655.
19. Pontikis C. Compt. Rend. – 1972. – V.274B. – P. 437.
20. Бурдун Г.Д., Марков Б.Н. Основы метрологии. – М.: Изд-во стандартов, 1985.
21. Математическая энциклопедия. Т. 1-5. – М.: Изд-во «Сов. энциклопедия», 1977-1985.
22. Кузнецов В.А., Ялунина Г.В. Общая метрология. – М.: ИПК «Издательство стандартов», 2001.
23. Гнеденко Б.В. Курс теории вероятностей. – М.: Наука, 1965.
24. Гурский Е.И. Теория вероятностей с элементами математической статистики. – М.: Высшая школа, 1971.
25. Tukey J.W. A survey of sampling from contaminated distributions. – In: Contributions to Probability and Statistics (Ed. by Olkin I.). – Stanford: Stanford Univ. Press, 1960. – P.448.
26. Дедов В.П., Филимонов Б.П. Гравитационный экранный эффект в геофизических G-экспериментах//Труды 2-й Сибирской РНПК «Актуальные проблемы метрологии, сертификации и стандартизации». – Новосибирск: СНИИМ, 1999.
27. Stacey F. D. e. a. // Rev. Modern Physics. – 1987. – V.59. – № 1.
28. Ander M.E. e. a. // Phys. Rev. Lett. – 1989. – V.62. – № 9.
29. Zumberge M. A. e. a. // Phys. Rev. Lett. – 1991. – V.67. – № 22.
30. Slipher V.M. The detection of nebular rotation. – Lowell Obs. Bull. – 1914. – No. 62.
31. Hubble E.P. A relation between distance and radial velocity among extragalactic nebulae. – Proc. Nat. Acad. Sci. (Wash.) – 1929. – V.15. – P. 168.
32. Тамманн Г.А. Постоянная Хаббла и параметр ускорения//Космология. Теории и наблюдения. – М.: Мир, 1978. – С. 70.
33. Арп Х. Свидетельства существования красных смещений, не связанных с движением//Космология. Теории и наблюдения. – М.: Мир, 1978. – С. 89.
34. Троицкий В.С. Экспериментальная проверка релятивистской космологии указывает на гравитационную природу красного смещения. – Гравитация. – 1995. – Т. 1. – Вып. 1. – С. 71.
35. Humason M.L., Mayall N.U., Sandage A.R. Redshift and magnitudes of extragalactic nebulae. – Astron. J. – 1956. – V. 61. – P. 97.
36. Levin S.F. Estimating an accuracy of the solutions to measuring problems in gravity and cosmology under uncertainty conditions//V international conference on gravitation and astrophysics of Asian-pacific countries. – Moscow: PFUR, 2001. – P. 120.

The phenomenon of gravitational self-lensing

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The complex of the phenomena caused by a curvature of beams of an observably star which is caused by a gravitational field of same star is described.

1. Physical essence of the phenomenon

Let's name gravitational self-lensing a complex of the phenomena caused by a curvature of beams of a star which is caused by a gravitational field of same star.

The basic difference of the phenomenon of gravitational self-lensing close to it in physical essence of the phenomena, such as the deflection of lights, gravitational lensing and gravitational microlensing, is, that as a minimum two objects participate in realization of the last - a source of lights and a body bending them, for example, a gravitational lens. Emission and a curvature of light at self-lensing is carried out by the same physical object.

Let, for example, object letting out lights is the star having properties of the Sun - his geometrical R and gravitational r_g radiuses. Let also radius R of such object at removal from is observed on distance X in case of rectilinear distribution of lights would be seen under a corner ψ at constant brightness of object along his seen disk (figure 1).

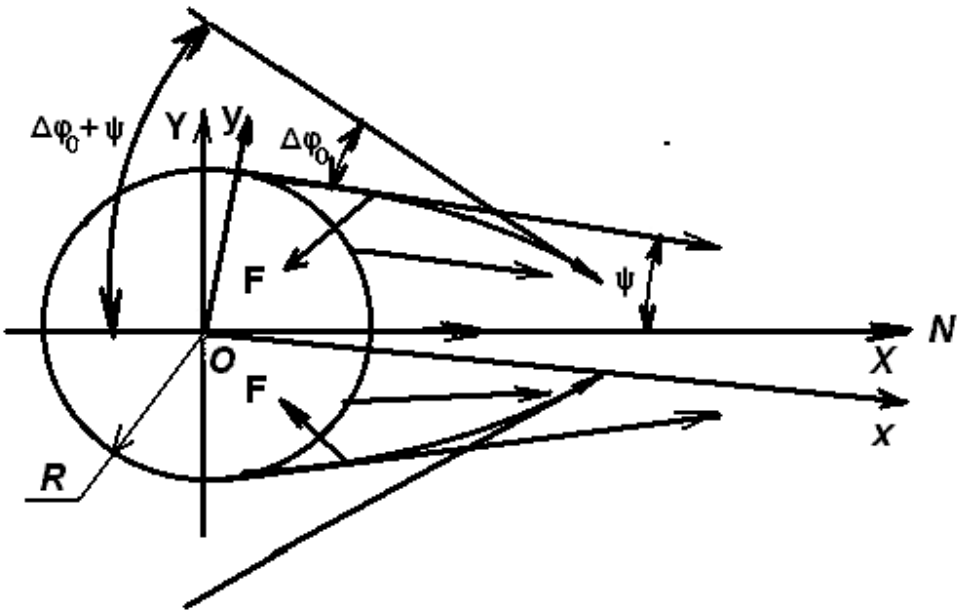

Figure 1

However, own gravitation of object results in occurrence of a part of force **F** directed perpendicularly to distribution of lights influenced beams that causes a deviation of light. Thus radius R appears seen of point N of supervision under a corner $\psi + \Delta\varphi_0$ which is more than corner ψ as corner $\Delta\varphi_0$ is formed by a line, a tangent in point N to a beam which was radiated by edge of object.

The equation of the light radiated by seen edge of object, on Einstein [1] is:

$$y = r_g \cdot \left(1 - \sqrt{1 + \left(\frac{x}{R} \right)^2} \right) + R \ , \tag{1}$$

where x - distance from a point of supervision up to the centre of the object, counted in coordinate system xOy which beginning is in the centre of weights of gravitating object, and the axis of ordinates is directed aside points of the approach of a light nearest to object which is removed from point O for length R called in aim distance.

The angle of an inclination of a tangent to a light, appropriate (1), thus is equal:

$$\varphi = -\frac{r_g}{R} \cdot \frac{1}{\sqrt{1 + \left(\frac{R}{x} \right)^2}} \ , \tag{2}$$

where the mark "minus" testifies to a deflection of a beam aside the centre of weights of object. The deflection of the light concerning object and acting to indefinitely removed observer, on absolute value appears equal to half of Eimstein's effect:

$$\Delta\varphi_0 = \frac{r_g}{R} \cdot \tag{3}$$

It is simple to notice, that already at $x > 10R$ the size φ (2) on absolute value differs from $\Delta\varphi_0$ (3) only on 0,5 %. Therefore a deflection of the light radiated by edge of object, from his valid angular direction at the self-lensing observably on distances from a source of beams about astronomical unit and more ($x > 200\ R$), it is necessary to count constant and equal $\Delta\varphi_0$ (3).

The corner ψ under which light is radiated in point N of supervision changes differently. It decreases in inverse proportion to distance X which is counted in system of readout XOY with the beginning in the centre of weights of gravitating object, an axis of abscissas which passes through point N of supervision:

$$\psi = \frac{R}{X} \cdot \tag{4}$$

Owing to (4), at growth X contribution of the sizes of object to formation of his image decreases, and the contribution of gravitation is increased. In case of a weak gravitational field, i.e. at small $\Delta\varphi_0$, for big X it is possible to write down $x=X$, that corresponds to the relation of angular sizes $\Delta\varphi_0$ (3) and ψ (4), equal:

$$\frac{\Delta\varphi_0}{\psi} = \frac{r_g}{R^2} X \cdot \tag{5}$$

For example, at supervision of the Sun from the Earth, i.e. at value X in one astronomical unit the contribution of self-lensing $\Delta\varphi$=0,87 " in the visible image of the Sun $\psi+\Delta\varphi_0$=1891,34…1955,78 " is very small. At removal of Sun - like object on distance $X = R^2 / r_g$, i.e. for 0,018 *light years* which we shall call characteristic, the contribution of gravitation to the seen image would make 50 %.

Accordingly, at removal of Sun - like object on distance in 10 *parsec* it is received his seen image in which the contribution of the sizes to the common image makes already only 0,3 %.

Thus, the phenomenon of gravitational self-lensing results to that the observably sizes of Sun - like objects at their removal from the observer on 1 *parsec* and more appear more than on 99 % generated by a gravitational field of these objects on a background of very small contribution in formed image of the own sizes of objects. Thus the contribution of gravitation in formation of the image grows with removal of object from the observer (figure 2).

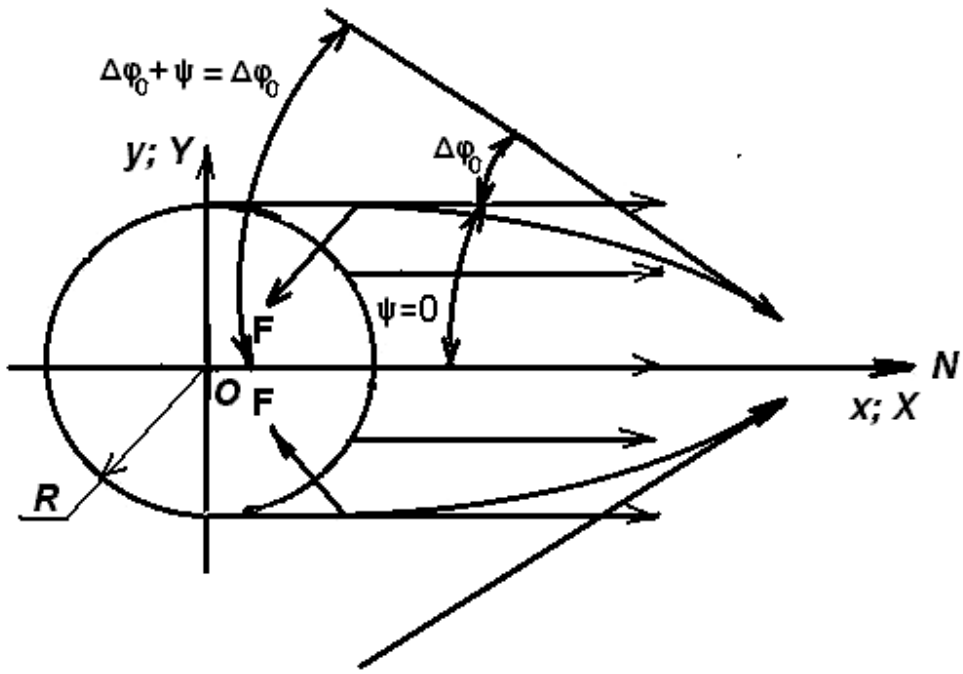

Figure 2

2. Effect of amplification of magnitude of stars at gravitational self-lensing

As the light exposure created by object, is determined by a solid angle with top in a point of the supervision, basing on the visible image of object, and the visible angular image of object due to the phenomenon of gravitational self-lensing is increased against his valid value, the phenomenon of gravitational self-lensing may be characterized by factor of amplification of light exposure. This factor is equal to the attitude of observably and valid values of the named solid angle, i.e. the attitude of squares observably $\psi+\Delta\varphi_0$ (3) - (4) and valid ψ (4) values of seen angular radius of object:

$$k_{mag} = \frac{\psi + \Delta\varphi_0}{\psi} = 1 + \left(\frac{r_g}{R^2 \cdot}\right)^2 X^2 = 1 + \rho_g^2 \xi^2 \quad , \tag{6}$$

where ρ_g and ξ - dimensionless values of gravitational radius r_g and distances X of observably object calculated in shares of radius of object:

$$\rho_g = \frac{r_g}{R \cdot} \;\; ; \quad (7) \qquad \xi = \frac{X}{R \cdot} \quad . \tag{8}$$

Thus the factor of amplification of light exposure grows with growth of distance X up to object.

According to (4) at removal of Sun-like object on characteristic distance the factor of amplification of light exposure would make 2 times.

As increase of luminosity (in magnitude) and factor of amplification of light exposure are connected by a formula

$$\Delta mag = -2{,}5 \lg\left(k_{mag}\right) \quad , \tag{9}$$

at removal of Sun-like object on characteristic distance the increase of luminosity makes 0,75 *mag.*

Thus, gravitational self-lensing increases observably luminosity of stars, and growth of luminosity is increased with growth of distance up to object. We shall notice, that growth of amplification of luminosity with increase of distance between object of supervision and the observer also was marked by A.Einstein in his work 1936 [2] devoted to a problem of gravitational microlensing.

3. The asymptotic description of distribution of light

At the analysis of the phenomena of a deflection of light, gravitational lensing and gravitational microlensing use asymptotic model in which true trajectories of distribution of light it is replaced with their asymptotes.

The einsteinian deflection of the light concerning gravitating object on any aim distance *Y*, makes:

$$\Delta\varphi = \frac{2\rho_g}{\eta} \quad , \tag{10}$$

where η - the aim distance expressed in radiuses *R* of object:

$$\eta = \frac{Y}{R} \quad . \tag{11}$$

According to the asymptotic approach believe, that lights of the removed star are distributed in parallel of axis *X* on distances η from it before crossing with axis *Y* without a curvature. Then lights are exposed to an instant deflection on corner $\Delta\varphi$ (10) which is kept along lights on all their way after crossing with axis *Y*.

Let's notice, that direct transferring of the formula (10) and the asymptotic representation caused by her on a case of gravitational self-lensing is impossible that this expression and diagrams of asymptotes do not correspond to distribution of a parallel bunch of the lights radiated by a surface of object in a direction of observer *N*.

Let's analyse a case of the removed observer when the parallel bunch of lights is distributed along axis *X*. Thus gravitational self-lensing results to that lights are bent, appearing laying on curves of a kind (1). These curves, being constructed in coordinate system *xOy*, appear revolved on some corner in system *XOY* so, that the corner of an inclination of lines, tangents to them in points of emission of lights and laying on a surface of radiating object, is equal to zero, and the top of a light is removed from a origin of coordinates on some aim distance $0 \le r \le R$ (fig. 3).

It is necessary to be stipulated, that such curves may pass as much as close to a origin of coordinates, and therefore in relation to some points of such curves the gravitational field of object cannot be counted weak. There of the inclination φ tangents to a curve strongly may change along all domain of definition. However, lights are distributed not along all length of curves (1), but along their limited part outside of object. In relation to this part the gravitational field still is necessary to count weak, and change $\Delta\varphi$ of a corner φ an inclination of a tangent to a curve along their these parts - small.

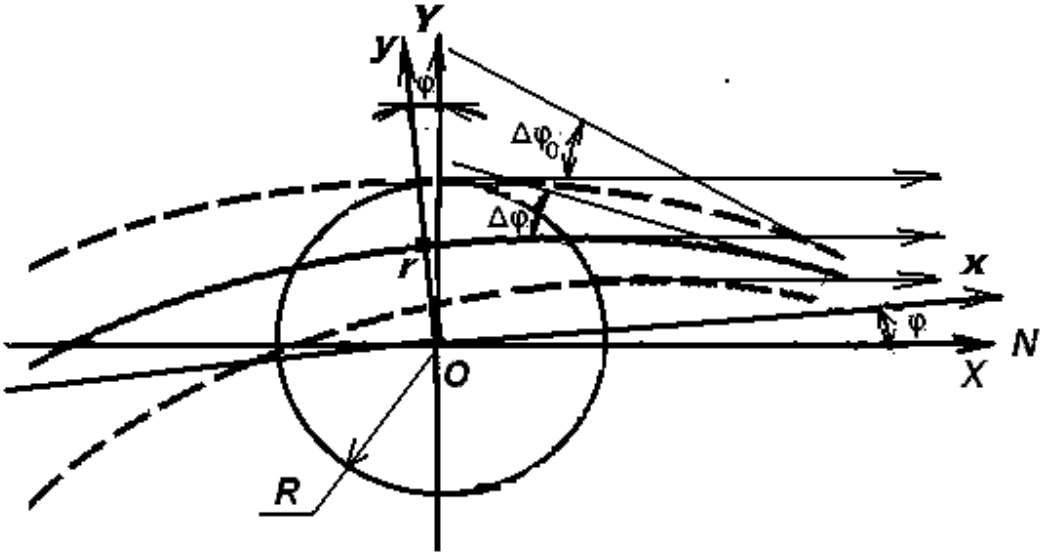

Figure 3

Let's calculate from (1) size r for the given coordinates x_B and y_B points of a surface of a radiating body. For this purpose we shall copy (1) as:

$$r^4 + 2r_g r^3 - 2y_B r^3 - 2y_B r_g r^2 + y_B^2 r^2 - x_B^2 r_g^2 = 0 \ . \tag{12}$$

Believing, that

$$r^4 >> 2r_g r^3 \ ; \quad (13) \qquad 2y_B r^3 >> y_B r_g r^2 \ , \tag{14}$$

let's simplify the equation (12):

$$r^4 - 2y_B r^3 + y_B^2 r^2 - x_B^2 r_g^2 = 0 \ , \tag{15}$$

Taking into account also, that

$$y_B^2 = R^2 - x_B^2 \ ; \quad (16) \qquad x_B^2 r^2 >> x_B^2 r_g^2 \ , \tag{17}$$

and, taking into account (16), we receive the further simplification the equation (15):

$$r^2 - 2y_B r + y_B^2 = 0 \ , \tag{18}$$

which decision is

$$r = y_B \ . \tag{19}$$

Thus, from the appointed condition of smalness of a gravitational field follows of equality of aim distance r of ordinate y_B points of emission in system xOy.

Further we shall determine a corner of an inclination of a tangent to a light in a point with coordinates (x_B; y_B), substituting (19) in (2):

$$\varphi_B = -\frac{r_g}{y_B} \cdot \frac{1}{\sqrt{1 + \left(\frac{y_B}{x_B}\right)^2}} \ , \tag{20}$$

with the account (16) is equal:

$$\varphi_B = -\frac{r_g}{R} \cdot \sqrt{\left(\frac{R}{y_B}\right)^2 - 1} \ . \tag{21}$$

Further in expressions (19) and (21) it is necessary to proceed from coordinates x and y systems in which the model of a light has a standard kind (1) - (2), in coordinates X and Y systems in which supervision is conducted. The listed coordinates are connected by the equations:

$$X = x\cos\varphi_B + y\sin\varphi_B \quad ; \quad (22) \qquad Y = -x\sin\varphi_B + y\cos\varphi_B \quad . \quad (23)$$

By virtue of the marked smallness of corners φ and consequently, and a corner φ_B, the equation (23) can be copied as:

$$y_B \approx Y_B + x_B\varphi_B \ . \tag{24}$$

Using (16) for definition of size x_B:

$$x_B = y_B\sqrt{\left(\frac{R}{y_B}\right)^2 - 1} \ , \tag{25}$$

and substituting in (23) sizes φ_B (21) and x_B (25), we receive

$$y_B^2\left(1 - \frac{r_g}{R}\right) - y_B Y_B + r_g R = 0 \ , \tag{26}$$

whence:

$$y_B = \frac{Y_B\left[1 \pm \sqrt{1 - 4\frac{r_g R}{Y_B^2}\left(1 - \frac{r_g}{R}\right)}\right]}{2\left(1 - \frac{r_g}{R}\right)} \ . \tag{27}$$

Thus in numerator (27) it is necessary to keep a mark "plus" as the negative mark corresponds to area in which geometrical radius R is comparable with gravitational radius r_g that contradicts the assumption of weakness of a field.

Neglecting the attitude of gravitational radius r_g to geometrical R in comparison with unit in parentheses, we have:

$$y_B = \frac{1}{2}\left[Y_B \pm \sqrt{Y_B^2 - 4r_g R}\right] \ . \tag{28}$$

Believing also, that radius R and ordinates Y_B may not accept the small values comparable to gravitational radius r_g, is finally received:

$$y_B = Y_B \ . \tag{29}$$

Thus, (21) it is possible to copy as

$$\varphi_B = -\frac{r_g}{Y_B} \cdot \frac{1}{\sqrt{1 + \left(\frac{Y_B}{x_B}\right)^2}} \ , \tag{30}$$

or, with the account

$$Y_B^2 = R^2 - X_B^2 \ , \tag{31}$$

as

$$\varphi_B = -\frac{r_g}{R} \cdot \sqrt{\left(\frac{R}{Y_B}\right)^2 - 1} \ , \tag{32}$$

and (19) - as

$$r = Y_B \ , \tag{33}$$

It is necessary to notice, that the attitude r_g / R for the Sun makes $4{,}84\times10^{-6}$ and consequently, for Sun-like object, even for values of attitude Y_B / R, included in (32), equal 10^{-4} the corner φ_B does not exceed $3\ ^0$, that testifies about enough good adequacy of the equations (13).– (14); (17); (24) parities between real physical sizes.

Thus, the light radiated in parallel of axis X from a point with ordinate Y_B, appears extending along a curve (1) with aim distance r up to the top, equal Y_B according to (33), and the radius - vector of top revolves to axis Y of ordinates on a corner φ_B (32).

The full deflection of the light radiated from top of a curve (1) on absolute value makes according to (3)

$$\Delta\varphi_0 = \frac{r_g}{Y_B} \ . \tag{34}$$

At emission of a light not from top of a curve (1), and from a point with coordinates X_B; Y_B, deflection of a light decreases against value $\Delta\varphi_0$ (34) on absolute value and makes:

$$\Delta\varphi = \frac{r_g}{Y_B} - \frac{r_g}{R} \cdot \sqrt{\left(\frac{R}{Y_B}\right)^2 - 1} \ . \tag{35}$$

Size Y_B varies in limits from zero up to R. It is simple to notice, that in the first case the size $\Delta\varphi$ (35) accepts zero value, and in the second - value

$$\Delta\varphi = \frac{r_g}{R} = \Delta\varphi_0 \ , \tag{36}$$

as that demands (3).

Normalizing values of size $\Delta\varphi$ (35) in shares of its limiting value φ_0 (36), we receive:

$$\delta\varphi = \frac{\Delta\varphi}{\Delta\varphi_0} = \frac{1}{\left(\frac{Y_B}{R}\right)} \left[1 - \sqrt{1 - \left(\frac{Y_B}{R}\right)^2} \right] , \tag{37}$$

where the size (37), thus, varies in limits from zero up to unit.

At last, replacing parameter Y_B of variable Y and expressing her in shares η of radius R (11) where the size η varies in limits from zero up to unit, we copy size $\delta\varphi$ (37) in a compact kind:

$$\delta\varphi = \frac{1}{\eta} \left[1 - \sqrt{1 - \eta^2} \right] \ . \tag{38}$$

To expression (38) there corresponds a curve in polar coordinates:

$$\eta = \sqrt{\frac{1}{1 + (\Delta\varphi\Delta\varphi_0)^2}} \ , \tag{39}$$

on which about enough good accuracy the top of a light goes at an inclination φ polar radius η to axis Y of the ordinates, making $0\ldots0{,}5\times10^{-2}$, i.e. at values η from a range about $10^{-4}\ldots1$.

Thus, a smallness of values of a corner φ, and also a smallness of radius R of Sun-like object in comparison with distance X from a origin of coordinates up to a point of supervision make possible asymptotic representation of a course of lights at gravitational self-lensing according to which lights are approximated by the direct lines which are starting with points of an axis of ordinates with coordinates η under corners:

$$\Delta\varphi = \delta\varphi\Delta\varphi_0 = \frac{\Delta\varphi_0}{\eta}\left[1-\sqrt{1-\eta^2}\right] \quad . \tag{38}$$

4. Effect of gravitational dimness of self-lensed images to edge

Let valid (at absence of self-lensing) brightness of points of a seen disk of a star is constant.
As the curvature of lights occurs non-uniformly along radius of a seen disk brightness of points of the image in conditions of gravitational self-lensing also appears non-uniform.

Let's execute the analysis of change of brightness along radius of object for infinitely removed observer.

Element of a seen disk of object $\eta d\eta d\psi$, where $d\psi$ - element of latitudial coordinates of object, at absence of self-lensing would be seen from big distance X under a solid angle:

$$d\Omega = \frac{\eta d\eta d\psi}{X^2} \quad . \tag{39}$$

Owing to self-lensing it is observed under a corner:

$$d\Omega_B = \Delta\varphi d(\Delta\varphi) cos(\Delta\varphi) \approx \Delta\varphi d(\Delta\varphi) \quad . \tag{40}$$

The coefficient of increase of a solid angle makes:

$$k_\Omega = \frac{\Omega_B}{\Omega} = \frac{\Delta\varphi}{\eta}\frac{d(\Delta\varphi)}{d\eta}X^2 \quad , \tag{41}$$

i.e.:

$$k_\Omega = (\Delta\varphi_0)^2 X^2 \left[\frac{1}{\eta^2}\left(\frac{1}{\sqrt{1-\eta^2}}-1\right)-\frac{\left(1-\sqrt{1-\eta^2}\right)^2}{\eta^4}\right] \quad . \tag{42}$$

With growth of coefficient of increase k_Ω brightness of points of the image removed from his centre on a share η of radius R falls on inversely proportional k_Ω dependences.

Normalizing a variable k_Ω (42) on its value at η=0 and calculating inversely value of this attitude, we receive the decreasing function (figure 4) appropriate to coefficient of dimness of the image to edge:

$$k = 0{,}25\left[\frac{1}{\eta^2}\left(\frac{1}{\sqrt{1-\eta^2}}-1\right)-\frac{\left(1-\sqrt{1-\eta^2}\right)^2}{\eta^4}\right]^{-1} \quad . \tag{43}$$

This dimness to edge we shall name gravitational dimness.

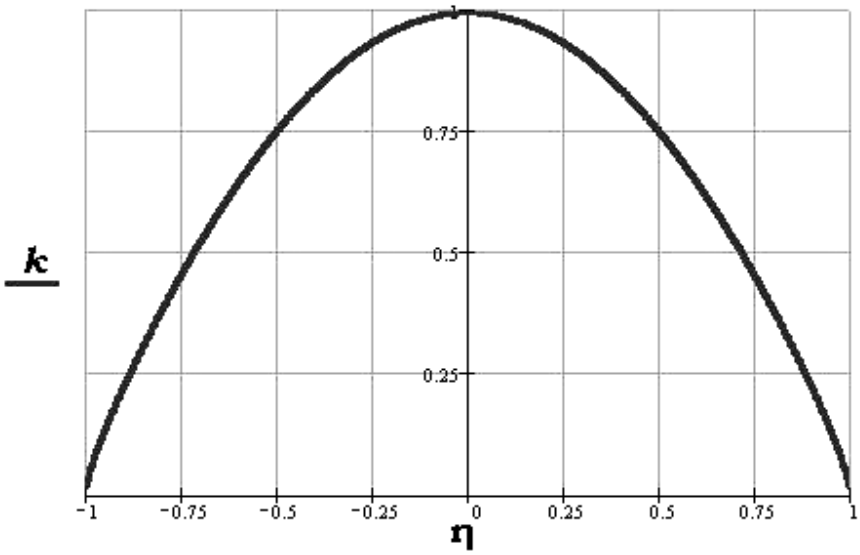

Figure 4

5. Conclusion

The phenomenon of gravitational self-lensing described above, and also accompanying it effects of amplification of shine and dimnesses of the image to edge may be taken into account during the decision of the following problems:

A) At an estimation of absolute star magnitudes of the astronomical objects especially removed. It is possible to assume, in particular, the essential contribution of the considered phenomenon to result of an estimation of luminosity of quasars with its overestimate;

B) At the analysis of double star systems, in particular, eclipsing-variables where the factor of dimness to edge is considered as empirical size;

C) At the analysis of optical thickness of space as Chwolson's - Einstein's corner of the removed objects appears close to their observably angular sizes;

D) At the analysis of gravitational fields of objects when their seen images owing to self-lensing can be considered as "prints" of the sizes and forms of a gravitational field;

E) By search of effects of strong gravitational fields, in particular, the essential contribution to formation of a field of the spin of object that results in a deflection of the form of a field and consequently, and the image self-lensed by him from the spherical symmetric form.

References

[1] A.Einstein, Ann.Phys., **49**, 769 – 822 (1916)
[2] A.Einstein, *Science*., **884**, 506 – 507 (1936)

Метод построения адаптивной модели определения движения космического объекта

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Анализируется метод построения адаптивной модели определения движения космического объекта реализуется с помощью использования компенсирующих поправок к моделям ошибок измерений и движения объекта наблюдения.

1. Ведение

Основными причинами возникновения ошибок определения движения космического объекта (КО) служат следующие ошибки:

- в определении начальных условий при решении задачи Коши [1]: $\frac{d\vec{r}}{dt} = \vec{f}(\vec{r}, R, t)$ с начальными условиями $\vec{r}_0 = \vec{r}\,(t_0)$,

  (1.1)

  где $\vec{r}$ - шестимерный вектор параметров движения КО, R=U+S+L+.. – потенциал сил, действующих на КО в полете, где основное влияние оказывает геопотенциал U, который может быть представлен в виде [1]:

  $$U = \frac{\mu}{r}\left\{1 + \sum_{n=2}^{\infty} J_n \left(\frac{r_a}{r}\right)^n P_n(\sin\varphi) + \sum_{n=2}^{\infty}\sum_{m=1}^{n}\left(\frac{r_a}{r}\right)^n P_n^{(m)}(\sin\varphi) \times (C_{nm}\cos m\lambda + D_{nm}\sin m\lambda)\right\},$$

  S и L – потенциалы сил притяжения Солнцем и Луной соответственно;

  $P_n(\sin\varphi)$ - полиномы, а $P_n^{(m)}(\sin\varphi)$ - присоединенные функции Лежандра соответственно;

  r, φ, λ - геоцентрические радиус, широта, долгота;

  J_n, C_{nm}, D_{nm} - коэффициенты разложения геопотенциала;

  μ - гравитационная постоянная Земли;

  r_a - средний экваториальный радиус Земли;

- ошибки в описании движения и расчетные ошибки.

Ошибки определения начальных условий обусловлены в свою очередь ошибками измерителя (погрешностей измерений и неточностью координатной привязки измерителя) и погрешностями при обработке результатов измерений. Последние в свою очередь можно разбить на ошибки за счет описания движения КО на интервале обработки измерений и расчетные ошибки.

Ошибки описания движения КО обусловлены как неточным знанием и учетом (известных) сил в потенциале R (1.1), так и наличием неизвестных и неучтенных в потенциале R сил, действующих на КО в полете.

Расчетные ошибки обусловлены как погрешностями при численном интегрировании системы дифференциальных уравнений (1.1), так и численными ошибками при вычислении оценки вектора состояния и при вычислении частных производных.

2. Метод построения адаптивной модели

Ошибку определения движения КО $\delta\vec{r}$ можно разложить на регулярную (систематическую) $M[\delta\vec{r}]$ и случайную $\delta\tilde{\vec{r}}$ составляющие, т.е. $\delta\vec{r} = M[\delta\vec{r}] + \delta\tilde{\vec{r}}$.

Возникновение регулярной составляющей ошибки прогноза обусловлено неточностью описания движения КА, погрешностями при численном интегрировании и систематическими (медленноменяющимися) ошибками траекторных измерений. Возникновение случайной составляющей ошибки прогноза обусловлено случайными ошибками измерений и теми силами, действующими на КО в полете, которые в некоторой степени можно считать случайными.

В качестве пути повышения точности определения положения КО может рассматриваться совместное оценивание начальных условий и (или) поправок ко всем (или к их части) параметрам, влияющим на точность прогноза, т.е. поправок к модели ошибок измерителя и к параметрам модели описания движения.

Предлагаемый метод как совокупность действий может быть представлен в виде 6 этапов. Этими этапами являются.

1. Апостериорная оценка точности.
2. Проведение анализа:
   - особенностей движения рассматриваемых КО;
   - наиболее вероятных причин (основных источников) возникновения ошибок прогнозирования при использовании существующей методики прогноза;
   - путей повышения точности прогноза положения рассматриваемых КО.
3. На основе результатов проведенного анализа и априорной информации принимается решение о возможных вариантах моделей описания движения КО и ошибок измерителя, т.е. параметры обеих моделей, поправки к которым могут быть использованы в качестве компенсирующих.
4. Исходный материал для проведения апостериорной оценки точности разбивается на три подвыборки: обучающую, проверочную и контрольную. На обучающей выборке(малого объема) производится вычисление поправок к параметрам, выбранным в пункте 3, для которых удовлетворяется правило включения по одиночке в расширяемый вектор состояния.
5. На проверочной выборке (которая может быть объединена с обучающей) строится (адекватная) модель прогноза положения КО методом пошаговой регрессии. Построение модели заканчивается, когда включение в модель оставшихся регрессоров, т.е. поправок, вычисленных в пункте 4, не приводит к существенному уменьшению функционала эмпирического риска.
6. На контрольной выборке проверяется статистическая устойчивость результатов апостериорной оценки точности прогноза.

Одним из наиболее важных этапов является третий – этап выбора кандидатов в компенсирующие поправки.

Задача поиска вектора компенсирующих поправок (как в поиске расширенного вектора состояния $\vec{y}_0 = \{\vec{r}_0, \vec{\alpha}\}$, где $\vec{\alpha}$ - вектор “мешающих” параметров [1], или уточнения только вектора начальных условий $\vec{r}_0$) может рассматриваться в качестве задачи нелинейного регрессионного анализа – восстановления зависимости [3]. Подобный подход к решению ис-

ходной задачи поиска компенсирующих поправок в составе расширенного вектора состояния (или отдельно) основан на следующем.

«Измеренные» составляющие вектора $\tilde{\vec{y}}_ф = \{\tilde{\vec{y}}_1, \tilde{\vec{y}}_2, ..\}$ (или $\tilde{\vec{r}}_ф = \{\tilde{\vec{r}}_1, \tilde{\vec{r}}_2, ..\}$) являются случайными величинами из-за наличия случайных ошибок в траекторных измерениях и случайных факторов в описании движения КО. Таким образом, вектор $\tilde{\vec{y}}_ф$ (или $\tilde{\vec{r}}_ф$) связан статистической стохастической зависимостью с расширенным вектором состояния $\vec{y}_0$ (или вектором только компенсирующих поправок), который может содержать как случайные, так и неслучайные составляющие. Математическое ожидание вектора $\tilde{\vec{y}}_ф$ (или $\tilde{\vec{r}}_ф$) считаем связанным с вектором $\vec{y}_0$ детерминированной зависимостью, не все параметры (поправки к вектору $\vec{y}_0$) которой известны, т.е. предполагаем, что известна регрессия $\tilde{\vec{y}}_ф$ (или $\tilde{\vec{r}}_ф$) на $\vec{y}_0$ с точностью до параметров.

Задача состоит в восстановлении регрессионной зависимости $\tilde{\tilde{\vec{y}}}_i$ от $\vec{y}_0$ по выборке $\tilde{\vec{y}}_i$ по выборке ограниченного объема (i=1,..,N).

Формальная постановка задачи состоит в следующем. Задан непрерывный оператор A, однозначно отражающий элементы $f(\vec{x},\vec{\alpha})$ метрического пространства $E^{(1)}$ в элементы $F(\vec{x},\vec{\alpha})$ метрического пространства $E^{(2)}$. Требуется найти решение операторного уравнения $Af(\vec{x},\vec{\alpha})= F(\vec{x},\vec{\alpha})$ в классе функций $f(\vec{x},\vec{\alpha})$, если функция $F(\vec{x},\vec{\alpha})$ не известна, но зато известны измерения $\vec{y} = \{y_1,...,y_N\}$ функции $F(\vec{x},\vec{\alpha})$ в точках $x_1,...,x_N$.

В рассматриваемом случае функция $F(\vec{x},\vec{\alpha})$ представляет собой векторную функцию орбитальных параметров $\vec{r}_i = \vec{r}(t_i)$ (или вектора $\vec{y}_i = \vec{y}(t_i)$). Ее «измеренными» значениями являются значения вектора $\tilde{\vec{r}}_i$ (или $\tilde{\vec{y}}_i$), полученные в результате обработки траекторных измерений в моменты времени t_i, (i = 1,...,N). Искомая функция $f(\vec{x},\vec{\alpha})$ представляет собой расширенный вектор состояния $\vec{y}_0$ (или при фиксированном векторе начальных условий $\vec{r}_0$, вектор компенсирующих поправок $\vec{\alpha}$), а оператор A представляет собой оператор интегрирования системы дифференциальных уравнений (1) с начальными условиями $\vec{r}_0 = \vec{r}(t_0)$. Решение системы дифференциальных уравнений (1) непрерывно зависит от начальных параметров $\vec{r}_0$ и параметров $\vec{\alpha}$, что говорит о непрерывности оператора A.

Задача восстановления зависимости сводится к минимизации функционала среднего риска [3]:

$$J(\vec{y}_0) = \int Q(\tilde{\vec{r}}, \vec{y}_0) P(\tilde{\vec{r}}) d\tilde{\vec{r}} \tag{1}$$

в условиях когда плотность распределения $P(\tilde{\vec{r}})$ обычно неизвестна, а функция потерь $Q(\tilde{\vec{r}}, \vec{y}_0)$ задана вместе со случайной выборкой $\tilde{\vec{r}}_1,..., \tilde{\vec{r}}_N$ объема N. Эта задача, называемой задачей минимизации среднего риска по эмпирическим данным, является достаточно общей [3].

Выберем в качестве функции потерь квадратичную функцию [4]:

$$Q(\tilde{\vec{r}}, \vec{y}_0) = \left[\tilde{\vec{r}} - \vec{r}(\vec{y}_0)\right]^T W_{\tilde{\vec{r}}} \left[\tilde{\vec{r}} - \vec{r}(\vec{y}_0)\right], \tag{2}$$

тогда по аналогии с [4] функционал эмпирического риска запишется как

$$J_э = \frac{1}{N}\sum_{i}^{N} Q(\tilde{\vec{r}}, \vec{y}_0). \qquad (3)$$

Если восстановление ведется в достаточно узком классе функций $\vec{r}(\vec{y}_0)$ по крайней мере в в одномерном случае отклонений вдоль орбиты $\Delta l(\vec{y}_0)$ и про функции потерь

$$Q(l_i, \Delta\vec{y}_0) = \Delta l_i^2(\vec{y}_0), \qquad (4)$$

Независимо от плотности $P(\tilde{\vec{r}})$ минимум функционала эмпирического риска (3) будет близок к минимуму функционала среднего риска (1) [3].

Как показывает практика, составляющая ошибки вдоль орбиты вносит основной вклад в ошибку прогнозирования движения КА, поэтому в дальнейшем будет рассматриваться минимизация только функционала эмпирического риска $J_э$ с функцией потерь (4).

Таким образом, задача поиска компенсирующих поправок (в составе или отдельно от от поправок к расширенному вектору состояния) сводится к использованию обобщенного метода наименьших квадратов [2]. Причем к методу наименьших квадратов, как одному из методов получения оценки расширенного вектора состояния $\vec{y}_0$, минимизирущей квадратичную форму невязок:

$$\eta(\vec{y}_0) = \text{const} \cdot \left[\tilde{\vec{r}}_ф - \vec{r}_ф(\vec{y}_0)\right]^T W_{\tilde{\vec{r}}_ф} \left[\tilde{\vec{r}}_ф - \vec{r}_ф(\vec{y}_0)\right], \qquad (5)$$

можно прийти, не используя классическую методику среднеквадратичной регрессии [2]. В этом случае схема поиска параметров $\vec{y}_0$ регрессии $\vec{r}_ф = \vec{r}_ф(\vec{y}_0)$, формально не совпадающая с методикой оценок вектора $\vec{y}_0$, входящего в детерминированную зависимость $\vec{r}_ф = \vec{r}_ф(\vec{y}_0)$, сводится к нелинейному оценивания методом наименьших квадратов, которое может производиться с помощью итерационной процедуры, реализующей метод Ньютона - Лекама [5].

В качестве критериев, позволяющих сделать выбор «наилучшей» (по определению Д.Химмельблау) модели из нескольких возможных или предполагаемых моделей, обычно используют по отдельности или в некоторой комбинации критерии, приведенные в работе [6]:

- ведется поиск наименьшего числа параметров регрессии, совместимого с разумной ошибкой;
- при выборе параметров регрессии используются разумные физические основания;
- выбор ведется по минимальной сумме квадратов отклонений между предсказанными и эмпирическими значениями.

Выбор модели в целом удовлетворителен, если отношение дисперсий S_r^2 / S_e^2 не превышает определенной величины, где S_r^2 - остаточная сумма квадратов $\underbrace{\min}_{\vec{y}_0}\eta(\vec{y}_0)$, деленная на число степеней свободы; S_e^2 - мера рассеяния ошибок прогноза, вызванного ошибками траекторных измерений. При этом предполагается [6], что модель приблизительно адекватно описывает экспериментальные данные.

Модель прогноза следует, видимо, подбирать, расширяя вектор состояния за счет минимального числа дополнительных параметров (или, используя только одни компенсирующие поправки), при введении которых норма регулярной составляющей вектора ощибок прогрнози-

рования движения КО уменьшается до уровня, существенно меньшего суммы дисперсий соответствующих случайных составляющих.

В модель определения движения следует подбирать, расширяя вектор состояния за счет минимального числа дополнительных параметров (или, используя только одни компенсирующие поправки), при введениии которых норма регулярной составляющей вектора ощибок прогрнозирования движения КО уменьшается до уровня, существенно меньшего суммы дисперсий соответствующих случайных составляющих.

При этом при формировании возможных моделей для выбора кандидатов в компенсирующие поправки (на обучающей выборке исходных данных) можно применять правило [2] последующего включения дополнительных параметров α_j ($j = 1,..,m$): $\dfrac{\left(\Delta\hat{\alpha}_j\right)^2}{\hat{\sigma}_0^2\sigma_{\alpha_j}^2} > F_{in}$

где $\Delta\hat{\alpha}_j$ - оценка поправки к параметру α_j; $\hat{\sigma}_0^2$ - оценка множителя, с точностью до которого известна априорная ковариационная матрица невязок прогноза; $\sigma_{\alpha_j}^2$ - расчетное значение дисперсии параметра α_j на основе априорной ковариационной матрицы траекторных измерений; в работе [2] F_{in} =2,5..3,9, (обычно выбирают $F_{0.05,1,\nu}$ - критическое значение распределения Фишера, где ν - число степеней своды при оценке σ_0^2).

Если какие-либо два регрессора сильно коррелированны друг с другом, то часто достаточно включения в модель одного из них, включение другого по предложенному критерию выбора последовательности дополнительных параметров оказывается нецелесообразным.

Для использования поправок к коэффициентам геопотенциала в качестве кандидатов в компенсирующие поправки может быть предложено следующее обоснование.

Как следует из исследований регулярная составляющая ошибки прогноза (вдоль орбиты), по-видимому, в основном обусловлена влиянием активных сил, действующих на КО в полете, а также недостаточно точным знанием и неучетом коэффициентов гармоник геопотенциала (в используемой «усеченной» модели, так как модели с более большим объемом учитываемых гармоник и нецелесообразно рассматривать, как из-за большего времени расчетов, так и из тех соображений, что геопотенциал является функцией времени, а практически ни одна из существующих моделей не отражает этот факт).

Задача повышения точности определения движения КО состоит из поиска компенсирующих поправок и дальнейшего их использования в модели описания движения КО (и/или в модели ошибок измерителя). Поиск производится на достаточно продолжительных интервалах (отдельно или совместно с поправками к начальным условиям), с таким расчетом, чтобы движение КО на рассматриваемых интервалах было бы «пассивным». При этом встает вопрос о сходимости итерационного процесса поиска поправок (в первую очередь к C_{nm} и D_{nm}). Для доказательства сходимости рассматриваемого итерационного процесса используем работы [7-9].

Для системы уравнений (1.1) в некоторой окрестности точки $\left(t_0, \vec{r}_0\right)$ семимерного эвклидова пространства теорема существования и единственности решения выполняется, что обеспечивается выбором вида функций, входящих в правые части рассматриваемой системы дифференциальных уравнений.

Единственное решение системы $\frac{d\vec{r}}{dt} = \vec{f}(\vec{r}, R, t)$ с начальными условиями можно продолжить на весь участок поиска поправок, к примеру δC_{nm} и δD_{nm}. Границы - m_1 m_2, за которые решение не может быть продолжено, где $m_1 < t < m_2$ в данном случае определяются границами участка «пассивного» полета.

Правые части рассматриваемых дифференциальных уравнений и их производных по параметрам r_i (i = 1,...,6) и C_{nm}, D_{nm} (в данном случае n = 2,...,6; m = 0,...,4) непрерывны в некотором открытом множестве Ψ шестимерного евклидова пространства $r_1,...,r_6$, но и в некотором открытом множестве Ω по переменным C_{nm} и D_{nm}. Таким образом правые части дифференциальных уравнений (1.1) и их производные по $\vec{y}_0 = \{\vec{r}_0, \vec{C}\}$ непрерывны в некотором открытом множестве $\widetilde{\Psi} = \Psi \oplus \Omega$ евклидова пространства E_{27}.

Тогда в силу теорем о непрерывной зависимости решения системы дифференциальных уравнений от начальных условий и параметров, непродолжаемое решение $\vec{r} = \vec{\varphi}(t, t_0, \vec{r}_0, \vec{C})$ с начальными значениями $t_0, \vec{r}_0$ определено как функция переменных $t, t_0, \vec{r}_0$ на некотором открытом множестве тех же переменных и непрерывно на нем (где $\vec{C} = \{C_{nm}, D_{nm}\}$).

В силу теорем о дифференцируемости решения по начальным условиям $\vec{r}_0$, и параметрам **C** непродолжаемое решение:

$$\vec{\varphi}(t, t_0, \vec{r}_0, \vec{C}) = \{\varphi_1(t, t_0, \vec{r}_0, \vec{C}), .., \varphi_6(t, t_0, \vec{r}_0, \vec{C})\}$$

системы уравнений (1.1) с начальными значениями $t_0, \vec{r}_0$ определены и непрерывны на некотором открытом множестве $\widetilde{\Psi}$ переменных $t_0, \vec{r}_0, \vec{C}$ вместе с частными производными:

$$\frac{\partial \varphi_i(t, t_0, \vec{r}_0, \vec{C})}{\partial r_{0j}}, \frac{\partial \varphi_i(t, t_0, \vec{r}_0, \vec{C})}{\partial C_k}$$

в рассматриваемом случае i,j = 1,...,6; k = 1,...,21).

Предлагаемая методика поиска компенсирующих поправок, как и поиск оценки вектора начальных условий $\mathbf{r}_0$ после двухпроходной обработки, основаны на минимизации некоторого функционала невязок:

$$\chi(\vec{y}_0) = \text{const}[\widetilde{\vec{y}}_ф - \vec{y}_ф(\vec{y}_0)] W_{\widetilde{\vec{y}}_ф} [\widetilde{\vec{y}}_ф - \vec{y}_ф(\vec{y}_0)] \text{ по } \vec{y}_0,$$

где под $\vec{y}_0$ может подразумеваться как вектор $\{t, \vec{C}^*\}$, так и вектор $\vec{C}^*$, а под вектором $\vec{y}_ф$ как вектор $\{\vec{r}_ф, \vec{C}^*\}$, так и вектор $\vec{r}_ф$;

$\vec{C}^*$ - вектор компенсирующих поправок.

Докажем сходимость итерационной процедуры поиска оценки при использовании модификации метода Ньютона-Лекама к (локальному) минимуму при некоторой доработке метода.

Следует отметить, что используемая в настоящий момент модификация метода Ньютона-Лекама, так как в ней не используются вторые частные производные от минимизируемого функционала по определяемым параметрам, одновременно является модификацией градиентного метода. Действительно:

$$\frac{\partial\chi(\vec{y}_0)}{\partial\vec{y}_0} = -\left(\frac{\partial\vec{y}_ф}{\partial\vec{y}_0}\right)^T W_{\tilde{\vec{y}}_ф}\left[\tilde{y}_ф - \vec{y}_ф(\vec{y}_0)\right]$$

$$\Delta\vec{y}_0^{(k+1)} = [B_ф^{(k)T} W_{\tilde{\vec{y}}_ф} B_ф^{(k)}]^{-1} B_ф^{(k)T} W_{\tilde{\vec{y}}_ф}[\tilde{\vec{y}}_ф - \vec{y}_ф(\vec{y}_0)] = -A^{(k)}\frac{\partial\chi(\vec{y}_0)}{\partial\vec{y}_0},$$

где $B_ф^{(k)} = \dfrac{\partial\vec{y}_ф(\vec{y}_0^{(k)})}{\partial\vec{y}_0}$ - матрица частных производных от вектора $\vec{y}_ф$ по вектору $\vec{y}_0$;

$W_{\tilde{\vec{y}}_ф}$ - весовая матрица вектора «измеряемых» параметров $\tilde{\vec{y}}_ф$;

$A^{(k)}$ - последовательность положительно определенных матриц.

Действительно из положительной определенности ковариационной матрицы $K_{\tilde{\vec{y}}_ф}$ следует положительная определенность матриц $W_{\tilde{\vec{y}}_ф} = K^{-1}_{\tilde{\vec{y}}_ф}$, $B_ф^{(k)T} W_{\tilde{\vec{y}}_ф} B_ф^{(k)}$, а следовательно и $A^{(k)} = [B_ф^{(k)T} W_{\tilde{\vec{y}}_ф} B_ф^{(k)}]^{-1}$.

Известно [8], что изложенная модификация градиентного метода приводит к сходимости итерационной процедуры, если провести следующую последовательность действий:

1)вычислим $\vec{y}_0 = \vec{y}_0^{(k)} - \dfrac{\partial\chi(\vec{y}_0)}{\partial\vec{y}_0}\Big|_{\vec{y}_0=\vec{y}_0^{(k)}}$;

2) вычислим $\chi(\vec{y}_0) = \chi\left(\vec{y}_0^{(k)} - \dfrac{\partial\chi(\vec{y}_0)}{\partial\vec{y}_0}\Big|_{\vec{y}_0=\vec{y}_0^{(k)}}\right)$;

3) проведем проверку неравенства для произвольного фиксированного η:

$$\chi(\vec{y}_0) - \chi(\vec{y}_0^{(k)}) \le \varepsilon\eta\left(\frac{\partial\chi(\vec{y}_0)}{\partial\vec{y}_0}\Big|_{\vec{y}_0=\vec{y}_0^{(k)}}, P^{(k)}\right),$$

где $P^{(k)} = -\dfrac{\partial\chi(\vec{y}_0)}{\partial\vec{y}_0}\Big|_{\vec{y}_0=\vec{y}_0^{(k)}}$,

$0<\varepsilon<1$ - произвольно выбранная константа, одна и та же для всех k=0,1,....;

4) если неравенство пункта(3) выполняется, то значение η берем в качестве искомого: $\eta_k = \eta$; если же неравенство не выполняется, то производим дробление η (путем умножения η на произвольное число $0<\lambda<1$) до тех пор, пока неравенство (пункта 3) не окажется справедливым.

Если при этом зависимость вектора $\vec{y}_ф = \vec{y}_ф(\vec{y}_0)$ является линейной функцией (а в случае использования в качестве компенсирующих поправок δC_{nm} и δD_{nm}, зависимость $\vec{r} = \vec{r}(t, t_0, \vec{r}_0, C_{nm}, D_{nm}, ...)$ от C_{nm} и D_{nm}, может быть аппроксимирована для рассмотренных интервалов прогнозирования линейной функцией от интервала прогноза), то функционал $\chi(\vec{y}_0)$ является сильно выпуклой функцией $\vec{y}_0$, так как матрица $\dfrac{d^2\chi(\vec{y}_0)}{d\vec{y}_0{}^2} = B_ф^T W_{\tilde{\vec{y}}_ф} B_ф$ является положительно определенной. Следовательно решение $\vec{y}_0^*$,

доставляющее минимум функционалу $\chi(\vec{y}_0)$, то есть решение $\chi(\vec{y}_0^*) = \underbrace{\min}_{\vec{y}_0} \eta(\vec{y}_0)$ будет единственным, так как сильно выпуклая функция на выпуклом компакте $\widetilde{\Psi}$ достигает минимума в единственной точке.

В заключение следует отметить, что использование предлагаемого метода для высокоэллиптических КО типа «Молния» позволило снизить регулярные составляющие ошибки прогнозирования до уровня, существенно меньшего сответствующих среднеквадратичных отклонений.

Литература

1. П.Е.Эльясберг. Определение движения по результатам измерений. М.: Наука. 1976.
2. Дж.Себер. Линейный регрессионный анализ. М.: Мир. 1980.
3. Алгоритмы и программы восстановления зависимостей. Под редакцией В.Н.Вапника. М.: Наука. 1984.
4. В.П.Вапник, С.С. Вербицкий, А.И.Михальский, Б.С.Ратнер, А.Н.Сергиевский, А.А.Сорокина.. Применение метода упорядоченной минимизации риска для нахождения сечений фотоядерных реакций. Краткие сообщения по физике. М.: ФИАН СССР. N9. 1975.
5. Ф.М. Гольцман. Физический эксперимент и статистические выводы. Ленинград. Издательство Ленинградского университета. 1982.
6. Д.Химмельблау. Анализ процессов статистическими методами. М.: Мир. 1973.
7. Курс обыкновенных дифференциальных уравнений. Киев, «Вища школа», 1974.
8. Б.Н.Пшеничный, Ю.Н.Данилин. Численные методы в экстремальных задачах. –М.: «Наука», 1975.
9. В.В.Степанов. Курс дифференциальных уравнений. Государственное издательство технико-теоретической литературы. -М.: 1950.

On the exact solutions construction in inflationary cosmology without restrictions on the scalar field potential of self-interaction

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The method of exact solutions construction in inflationary cosmology within the self-interacting scalar field theory is proposed. The main feature of the method lies in the fact that it needs no to impose any restrictions for the potential of self-interaction. This approach, in particular, leads for necessity of recalculations of the of the e-folds numbers in inflationary scenarios. The example for the scalar field with a logarithmic evolution in time is analyzed. The exact formula for e-folds number is compared to approximate one.

1. Introduction

Inflationary paradigm appeared with the aim to avoid the problems of standard Big bang cosmology. Nevertheless it was almost impossible to solve exactly the self-consistent system of equations for an inflationary model in the case of the scalar field potentials predicted by particle physics because the complexity of differential equations describing gravitating self-interacting scalar field. It was two well-known approximations in cosmology when the dynamic equation of the scalar field considered in the Friedmann-Robertson-Walker (FRW) Universe: the slow-roll approximation and the approximation of the rapid oscillations. It is two approximations above that give possibility to describe in a consistent way extremely fast expansion of the Universe being in a quasi-vacuum state and the particle creation in this 'inflated' space.

The relation between the approximate solutions in the slow-roll regime and the exact ones in the inflationary models was found out in the works [1–3]. In present contribution the inflationary model equations are solved exactly, without the approximations mentioned above. The example of the exact solution for a logarithmic evolution of the scalar field is analyzed.

2. Exact solutions of an inflationary model

The system of equations for self-interacting gravitating scalar field in spatially-flat homogeneous and isotropic Universe can be presented by the following equations (see, for example, [3]).

$$H^2 = \frac{æ}{3}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right), \tag{1}$$

$$\ddot{\phi} + 3H\dot{\phi} = -\frac{d}{d\phi}V(\phi), \tag{2}$$

where ϕ is a scalar field, $V(\phi)$ is a potential of self-interaction of the scalar field, $H(t) = \frac{d}{dt}\ln R(t)$, $R(t)$ is a scalar factor, æ is Einstein cosmological constant. Equations (1)-(2) can be transformed to the form [3]

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$$H^2 = \frac{æ}{3} W(\phi), \tag{3}$$

$$3H\dot{\phi} = -\frac{d}{d\phi} W(\phi). \tag{4}$$

Here the effective potential of self-interaction $W(\phi)$ is introduced by the relation

$$W(\phi) = V(\phi) + \frac{1}{2} U^2(\phi), \tag{5}$$

where $U(\phi) = \dot{\phi}$. The function $W(\phi)$ is the total energy as the function of the scalar field values.

The solution for the scalar factor $R(\phi)$ describes by the formula

$$R(\phi) = R_0 \exp\{-æ \int \frac{W}{W'} d\phi\}. \tag{6}$$

The relation between the functions $W(\phi)$ and $U(\phi)$ is

$$\sqrt{3æ} U W^{1/2} = -W'. \tag{7}$$

Thus only one from the two functions is an arbitrary.

From the other hand the initial system (1)- (2) can be presented in the form, which have been used in fine tuning of the potential method [4, 5]

$$V(t) = \frac{1}{\kappa} \left(\Lambda + \frac{\ddot{R}}{R} + \frac{2\dot{R}^2}{R^2} + \frac{2\epsilon}{R^2} \right), \tag{8}$$

$$\phi(t) = \pm \sqrt{\frac{2}{\kappa}} \int \left(\sqrt{-\frac{d^2 \ln R}{dt^2} + \frac{\epsilon}{R^2}} \right) dt. \tag{9}$$

The equation (9) was derived from the consequence of Einstein equation for the scalar field 'velosity' in terms of scalar factor

$$[\dot{\phi}(t)]^2 = \frac{2}{\kappa} \left(-\frac{\ddot{R}}{R} + \frac{\dot{R}^2}{R^2} + \frac{\epsilon}{R^2} \right) = -\frac{2}{\kappa} \left(\dot{H} + \frac{\epsilon}{R^2} \right). \tag{10}$$

By substitution the equations (8) and (10) into the expression for the total energy (5) one can obtain

$$W(\phi) = \frac{3}{\kappa} \left(\frac{\dot{R}^2}{R^2} + \frac{\epsilon}{R^2} \right). \tag{11}$$

In the case of spatially-flat Universe, $\epsilon = 0$, the equations (11) and (10) can be simplified to

$$W(\phi) = \frac{3}{\kappa} H^2 \quad [\dot{\phi}(t)]^2 = \frac{2}{\kappa} \dot{H}. \tag{12}$$

The derived formulas (3, 4, 12) correspond to those one which actively used in the slow-roll approximation. The last requires the strong restriction on the potential [7]:

$$|V''(\phi)| \leq 9H^2 = 24\pi V/m_{pl}^2, \quad |V'/V| \leq \sqrt{48\pi}/m_{pl}. \tag{13}$$

Presented in this section approach needs no any restrictions on the potential $V(\phi)$ of this kind. Thus the potential $V = V(\phi)$ may have large enough the first and the second derivatives

in respect to ϕ. That is the potential $V(\phi)$ can rise fast enough, included the case of inflationary regime in the very early Universe. Let us consider now the example with a logarithmic evolution at time of the scalar field.

3. Corrections to the number of Hubble times for the scalar field with a logarithmic evolution

Let us consider, using the definite example, how can be changed the number of Hubble times (e-folds). The Hubble times are defined for the period from some time t to fixed time t_e by the integral

$$\mathcal{N}(t) \equiv \int_t^{t_e} H(t)dt.$$

Let us apply this definition to the scalar field with a logarithmic evolution in time

$$\phi = \alpha \ln(t+1). \tag{14}$$

According to the exact formula (6) the number of e-folds in terms of the scalar field ϕ can be calculated and reads

$$\mathcal{N} = \frac{R(t)}{R_0} = \exp\{H_* t + \alpha_1^2 \ln(t+1)\}, \tag{15}$$

where H_* and α_1 are constants.

The potential $V(\phi)$ and the total energy $W(\phi)$ can be expressed in the explicit form:

$$V(\phi) = A_1 \exp\{-2\phi/\alpha\} + B_1 \exp\{-\phi/\alpha\} + C \tag{16}$$

$$W(\phi) = A \exp\{-2\phi/\alpha\} + B \exp\{-\phi/\alpha\} + C \tag{17}$$

$$\tag{18}$$

It is clear that the potential $V(\phi)$ and the total energy $W(\phi)$ have the only distinction in the constants A and A_1, B and B_1.

To define $\mathcal{N}$ using the formula (15) it needs to find the attitude W'/W, which can be reduced to

$$W'/W = -\alpha\{1 - \frac{A\exp\{-2\phi/\alpha\} - C}{2A\exp\{-2\phi/\alpha\} + B\exp\{-\phi/\alpha\}}\} \tag{19}$$

The last expression gives possibility to integrate the equation (6) by ϕ

$$\mathcal{N} = \alpha\kappa(\phi_e - \phi_0) + \frac{\alpha}{2}(\phi_e - \phi_0) - (\alpha\kappa H_*)^{-1}\{e^{\phi_e/\alpha} - e^{\phi_0/\alpha}\} - \frac{3}{4}\alpha^2 \ln\left|\frac{\alpha_1^2 + H_* e^{\phi_e/\alpha}}{\alpha_1^2 + H_* e^{\phi_0/\alpha}}\right|, \tag{20}$$

where ϕ_0 and ϕ_e are the initial value and the final values of a scalar field during the period of the inflation.

Thus if we compare the result (20) with the same obtained in the slow-roll approximation we conclude that only the first term in the expression (20) corresponds to the slow-roll regime with accepted restrictions. Consideration of the rest terms in (20) can give some changes for initial and final values for the scalar field to match the result with observational data. Also the

absence of the restrictions to the potential $V(\phi)$ may leads to new possible modes of cosmological perturbations from inflation.

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Список литературы

[1] S.V. Chervon, V.M. Zhuravlev CD-ROM (04/28+) of 19-th Texas Symposium "Relativistic Astrophysics and Cosmology",Paris, Dec.14-18, 1998, Nuclear Physics B (Proc.Suppl.) 80 (2000). ArXiv gr-qc/990757, 1999.

[2] S.V. Chervon, V.M. Zhuravlev, *Izv.Vyssh.Ucheb.Zaved. Fiz.* **N 11** 40 (2000).

[3] V.M. Zhuravlev, S.V. Chervon, *Zh. Exp. Teor. Fiz.* **118** 259 (2000).

[4] С.В. Червон, *Нелинейные поля в теории гравитации и космологии.* Изд. Средневолжского научного центра, Ульяновск (1997) 191 с.

[5] Chervon S.V., Zhuravlev V.M., Shchigolev V.K. Phys.Let. **B 398**, 269 (1997).

[6] A. H. Guth, Phys. Rev. D **23**, 347 (1981).

[7] Steinhardt P.J. and M.S.Turner, Phys.Rev.**D.29**, 2162 (1984).

[8] A.A. Starobinskiy, Phys. Letters, **918**, N1, 24 (1980)

[9] С.Вейнберг, *Гравитация и космология*// М.:Мир, 1975.- 696 с.

[10] А.Д.Линде, *Физика элементарных частиц и инфляционная космология.*, М.:Наука, (1990).

Singularity-free accelerating Universe with dilaton dark matter

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Matter with dilaton charge is proposed to consider as the dark matter within the framework of the gravitational theory with Weyl-Cartan geometrical structure [1]. The modified Friedman-Lemaître equation is obtained for the homogeneous and isotropic Universe with cosmological term and filled with the dilaton dark matter. From this equation the absence of the initial singularity in the cosmological solution follows. Also the existence of two points of inflection of the scale factor function is established, the first of which corresponds to the early stage of the Universe and the second one corresponds to the post–Friedmann era when the expansion with deceleration is replaced by the expansion with acceleration.

The inflation-like solution is obtained in case of the superrigid equation of state of the dark matter at the early stage of the Universe. Also the possible equations of state for the dilaton dark matter are found on the basis of the modern observational data. The theory predicts that in the modern era the dilaton dark matter has to be cold and self-interacting.

The hypothesis on the dilaton dark matter as the dark self-interacting (by means of the dilaton charge) matter explains, why the dark matter is detected only as a consequence of gravitational effects and cannot be discovered by means of the nongravitational interaction with particles for which the dilaton charge is equal to zero. This hypothesis also leads to the conclusion that the expansion with acceleration begins when the dark matter energy density becomes equal to order of magnitude to the vacuum energy density that occurs in the post –Friedmann era.

References

[1] O.V.Baburova, B.N.Frolov, *Class. Quant. Grav.*, **20**, 1423-1442 (2003)

Mechanical scenario for the reaction $n \to p^+ + e^- + \tilde{\nu}$

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Small perturbations of averaged ideal turbulence reproduce the electromagnetic field. A hollow cavity models the neutron. The cavity stabilized via a perturbation of the turbulence energy serves as a model of the proton. An isle of quiescent fluid models a localized electron. The antineutrino corresponds to a positive disturbance of the turbulence energy needed in order to compensate the difference in perturbations of the energy produced by the electron and proton.

The properties of physical space known as fields and particles can be modelled in terms of continuum mechanics. The concept of substratum for physics is used to this end. On a large scale the substratum can be approximated by a turbulent medium.

We consider the averaged turbulence of an ideal fluid. The governing is the linearized Reynolds equation[1],[2]:

$$\varsigma\partial_t\langle u_i\rangle + \varsigma\partial_k\langle u_i' u_k'\rangle + \partial_i\langle p\,\rangle = 0 \tag{1}$$

$$\partial_i\langle u_i\rangle = 0$$

where ς is the medium density, $\langle\mathbf{u}\rangle$ the averaged velocity of the fluid flow, $\langle p\,\rangle$ the averaged pressure and $\mathbf{u}'$ the turbulent fluctuation of the velocity, so that

$$\mathbf{u} = \langle\mathbf{u}\rangle + \mathbf{u}'$$

Here and further on we use denotations $\partial_t = \partial/\partial t$, $\partial_k = \partial/\partial x_k$. Summation over recurrent index is implied throughout.

We assume that in the unperturbed state the turbulence is homogeneous and isotropic, i.e.

$$\langle\mathbf{u}\rangle^{(0)} = 0$$

$$\langle p\,\rangle^{(0)} = p_{\,0} \tag{2}$$

$$\langle u_i u_k\rangle^{(0)} = c^2\delta_{ik}$$

where $p_{\,0}, c =$ const. Integrating equation (1) for an isotropic turbulence we get a kind of Bernoulli equation:

$$\varsigma\langle u_1' u_1'\rangle + \langle p\,\rangle = \varsigma c^2 + p_{\,0} \tag{3}$$

$$\langle u_i' u_k'\rangle = \langle u_1' u_1'\rangle\delta_{ik}$$

Formally, by (3), any distribution of the turbulence energy density $1/2\varsigma\,\langle u_i' u_i'\rangle$ may occur.

With the definitions

$$A_i = \kappa c\left(\langle u_i\rangle - \langle u_i\rangle^{(0)}\right)$$

$$\varsigma\varphi = \kappa\left(\langle p\,\rangle - \langle p\,\rangle^{(0)}\right) \tag{4}$$

$$E_i = \kappa\partial_k\left(\langle u_i u_k\rangle - \langle u_i u_k\rangle^{(0)}\right)$$

where κ is an arbitrary constant, (1) takes the form of the Maxwell's equation

$$\frac{1}{c}\,\partial_t A_i + E_i + \partial_i\varphi = 0$$

The next linearized term in the chain of Reynolds equations looks as follows

$$\partial_t \langle u_i' u_k' \rangle + c^2 \left(\partial_i \langle u_k \rangle + \partial_k \langle u_i \rangle \right) + h_{ik} = 0 \tag{5}$$

where

$$\varsigma h_{ik} = \langle u_i' \partial_k p' \rangle + \langle u_k' \partial_i p' \rangle + \varsigma \partial_j \left\langle u_i' u_j' u_k' \right\rangle$$

With the definitions (4) and

$$j_i = \frac{\kappa}{4\pi} \partial_k h_{ik}$$

the differentiation of (5) with respect to x_k appears[1] to give an equation, which is isomorphic to another Maxwell's equation

$$\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t} - \nabla \times (\nabla \times \mathbf{A}) + \frac{4\pi}{c} \mathbf{j} = 0$$

So, small perturbations of an ideal turbulence reproduce the electromagnetic field.

Discontinuities, or defects, of the medium model particles. Typically, a spherical cavity included into the medium represents a point-like discontinuity. In macroscopic fluids a hollow bubble will fill in by a vapor until the gas pressure becomes equal to the fluid pressure. However, an ideal fluid does not consist of corpuscles and hence does not form a gas phase. For, it is a true continuum. A turbulent fluid may adjust itself to boundary conditions via the perturbation of the averaged energy (Fig.1, left). A stable configuration is thus formed. The field of turbulence perturbation produced was shown in[2] to take the form $1/r$.

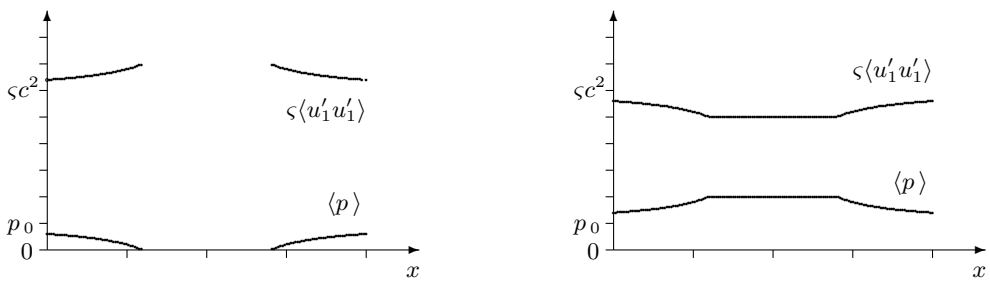

FIG. 1: The proton (left) and the antiproton (right).

We have for a stable cavity of the radius R located at $\mathbf{x}'$:

$$\langle p \rangle = p_0 - \frac{a}{|\mathbf{x} - \mathbf{x}'|} \tag{6}$$

$$a = p_0 R \tag{7}$$

It follows from (3) and (6) that

$$\varsigma \langle u_1' u_1' \rangle = \varsigma c^2 + \frac{a}{|\mathbf{x} - \mathbf{x}'|} \tag{8}$$

The form (8) implies an infinite quantity of the total energy perturbation

$$\frac{1}{2} \int_\Omega \varsigma \left(\langle u_i' u_i' \rangle - \langle u_i' u_i' \rangle^{(0)} \right) d^3 x$$

where the medium volume $\Omega \to \infty$. So, the positive deviation (8) from the background (2) should be compensated by a negative deviation of the turbulence energy of the same form, yet with the opposite sign of the coefficient (7):

$$a = -p_0 R \tag{9}$$

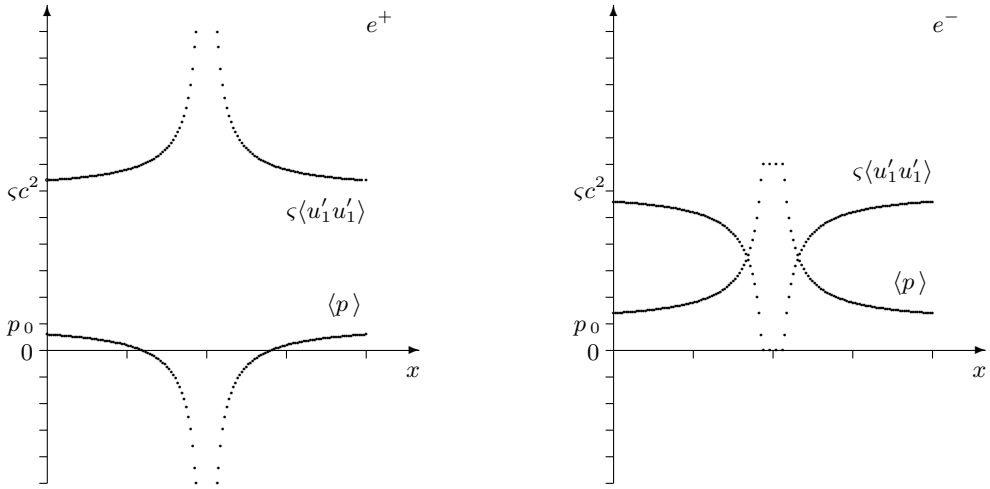

FIG. 2: The positron (left) and the electron (right). Dots show perturbations beyond the linear approximation.

Supposing that the energy attains the minimum value $\langle u_1' u_1'\rangle = 0$ at the core $|\mathbf{x} - \mathbf{x}'| = r_{\mathrm{e}}$ of the negative disturbance center, we find from (8):

$$a = -\varsigma c^2 r_{\mathrm{e}} \tag{10}$$

The cavity stabilized in the turbulent fluid models the proton (Fig.1, left). The isle of the quiescent fluid serves as a model of the localized electron (Fig.2, right).

Equating (9) and (10) we get a relation between the radii of the proton and localized electron :

$$\frac{r_{\mathrm{e}}}{R} = \frac{p_0}{\varsigma c^2} \tag{11}$$

Evaluating the masses of the electron and proton as

$$m_{\mathrm{e}} = \varsigma \frac{4}{3}\pi r_{\mathrm{e}}^3$$

and

$$M = \varsigma \frac{4}{3}\pi R^3$$

respectively, we find from (11) that aether should be modelled by the turbulence of the high energy and low pressure:

$$p_0 << \varsigma c^2$$

The algebraic sum of the energies of perturbations produced by negative (Fig.2, right) and positive (Fig.1, left) disturbance centers is nonzero. It is a negative quantity that corresponds to the region $r < R$ of Fig.2, right where $\varsigma\,\langle u_1' u_1'\rangle < c^2 - p_0$. This surplus of the energy is covered by a positive disturbance of the background turbulence (Fig.3, left, top) emitted. The latter corresponds to the antineutrino. Configurations shown in (Fig.3) wholly fall outside the linear approximation. Therefore, they can be considered only as a sketch or a prototype of some real structure. In the bounds of the linear model neutrinos should be treated properly as a kind of the shock wave. In this event, quasistatic configurations shown in (Fig.3) need to be corrected, above all by reducing perturbations of the pressure.

Thus, we constructed a linear mesoscopic mechanical scenario of the reaction

$$n \to p^+ + e^- + \tilde{\nu}$$

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[1] O.V.Troshkin, "On wave properties of an incompressible turbulent fluid", Physica A, 1990, **168**, No 2, 881-898.

[2] V.P.Dmitriyev, "Towards an exact mechanical analogy of particles and fields", Nuovo Cimento A, 1998, **111**, No 5, 501-511. `http://xxx.arXiv.org/abs/physics/9904029`

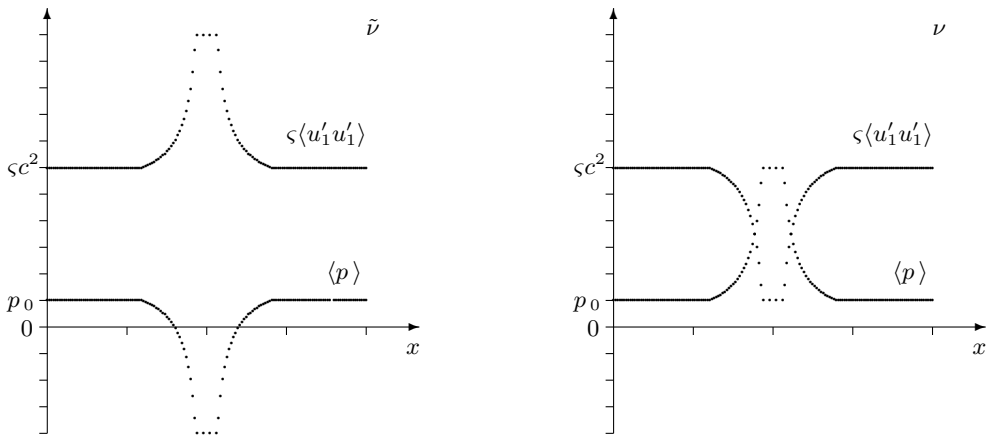

FIG. 3: Prototypes of the antineutrino (left) and neutrino (right). Dots show perturbations beyond the linear approximation.

Geometrized space-time & the world-ether

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The theories of science, subject to validation by experiment, are created in a Nominalist spirit, and they correlate observations and interpretations of objective reality. These theories, and their accompanying models, do not provide a final description of objective reality, because they evolve to incorporate new developments, and more accurate theoretical systems, and the accompanying conceptual apparatus, replace the less adequate systems of earlier times. During the evolution of more accurate theories and models, it usually happens that alternative interpretations of a particular set of accepted observations are proposed, using different concepts. As a general rule, the one which finds most widespread acceptance is the one which provides the most comprehensive, simple and accurate interpretation of observed phenomena, and which solves outstanding problems without introducing complex *ad hoc* conceptual or methodological devices. Mathematics defines the structure and operation of theories and models. The "First Interpretation" – the mathematical formal structure itself – is of primary importance. The "Second Interpretations" relate the mathematical formal structure to concepts ultimately derived by abstraction and idealization from physical experience. Within the context of relativity, geometrical second interpretations have enjoyed priority and are the norm, though a comprehensive mechanical interpretation, based on the Kelvin-Larmor ether, has been formulated and is now being advanced as an alternative to the usual Einstein-Minkowski exposition. This paper argues that far from being incompatible with each other, and representing mutually exclusive non-classical and classical interpretations of fundamental concepts, the geometrical and dynamical (ether) expositions can be shown to be directly related through a common formal structure. This paper briefly indicates how this can be done. History and philosophy of science are useful in describing how, when and why formal structures evolved, and how, when and why particular formal structures became associated with one dominant "second interpretation" when several alternatives were available.

Disclosing models play a vital role in scientific disciplines. Their function is to correlate observations of a particular phenomena; to inter-relate interpretations of a range of phenomena, and to extend consistency through a developing, comprehensive theory. The mathematical description of a disclosing model's action provides the relevant formal structure. There is two-way reaction between formal structure and disclosing model. Sometimes mathematical development of a formal structure suggests a new disclosing model; often a disclosing model suggests developments in the formal structure. Disclosing models (such as the Kelvin ether) may be abstracted from macroscopic observations of industrial mechanisms, scientific instruments, laboratory equipment or "philosophical machines". They may emerge from the rules of mathematical development. They may be mechanical, geometrical, or electro-biological as in the case of the neural state machine. They may be actual or imaginary; classical or non-classical. Preference is given to the simplest disclosing model which unifies a comprehensive set of phenomena. It is possible to inter-relate the geometrical disclosing models of space-time operations in Einstein-Minkowski relativity; the disclosing models of matter in the dynamical ether, and the idealized interferometer model used in

many derivations of "Poincare-Lorentz" relativity theory. Geometrized space-time can be interpreted by frame-space theories using rods and clocks (or equivalent idealized interferometers), and provided with a mechanical analogue – the modified vortex sponge, after Kelvin, Larmor, Hartley, Kelly, Dmitryev, Winterberg and others. Geometrizing the mechanical ether analogue gives the geometrized space time of the Einstein-Minkowski formulation. It makes sense to call this geometrized space-time a relativistic world ether, as Kostro shows in his study of Einstein and the ether.

Arguments concerning the respective standing of the Einstein-Minkowski and the Poincare-Lorentz formulation of the relativistic formal structure have persisted since 1910, the implication being that they are not physically equivalent, and that one is more accurate, methodologically superior, or free from inconsistencies compared to the other. This implication should be questioned. During the period 1900-1910, there were several expositions called "relativity", and a variety of names were associated with particular formulations: Lorentz-Einstein; Poincare-Lorentz; Lorentz-J.J. Thomson; Langevin. It was the establishment of General Relativity, between 1916 and 1920, which gave priority of place to the Einstein interpretation. An early form of special relativity, known by various names, emerged from vacuum tube experiments and studies of the electron, of a practical and theoretical nature, undertaken by Searle, Heaviside, Lenard, Lodge, J. J. Thomson, Kaufmann, Bucherer, Lorentz, Larmor and others. This resulted in the Lorentz Theory of Electrons, and originated the Lorentz Programme of Relativity. These developments were closely associated with the Electromagnetic World View, in which matter and its motions were regarded as electromagnetic phenomena. Matter was "reduced" into ether along with all measuring rods and clocks. The failure at the time to provide satisfactory interpretations of matter and ether in traditional mechanical terms, and the failure to detect the ether, did much to encourage the geometrization of physics. Geometrization had a long tradition in the 19th C, and the geometrizing of the Lorentz theory of electrons (Minkowski's own description of what he did) was virtually inevitable, following the failure of the Electromagnetic World View to resolve its difficulties. The Russian contribution to geometrical interpretations of science is particularly noteworthy. Geometrization resolved many difficulties, and the Einstein-Minkowski formulation of Special Relativity led to geometrized General Relativity, Geometrodynamics, and the geometrized interpretations of recent discoveries. Nevertheless, an alternative programme persists, which accepts the mathematical structure of relativity, but interprets the theory using a classical 3-d.space, Newtonian time, and ether. Simplifying a very complex situation, one can generalize as follows. The Lorentz Theory of Special Relativity, originally linked to particular theories of the electron, was given an expression in terms of measurements using rods and clocks in motion within an ether through which all action was transmitted, and in which energy and momentum were conserved. Historians have referred to Lorentz Theory-A and Lorentz Theory-B. The interpretation of relativity using an ether reference frame, and transported rods and clocks became the main alternative programme to the Einstein-Minkowski approach. It is usually called the Poincare-Lorentz formulation, or sometimes the FitzGerald-Larmor-Lorentz formulation. Definitions of its basic concepts, detailed derivations of its formal structure, and arguments in favour of its advantages over the Einstein theories are given in the papers of Broad, Ives, Builder, Prokhovnik, Selleri, Janossy, Podlaha, Mansouri & Sexl, d'Atkinson, Clube, Cornish, Levy, and many others. Between 1920 when the "Rod-Contraction, Clock-Retardation Ether Theory" emerged out of the Lorentz Theory-A (to use Erlichson's words), and the present, this theory has been developed to incorporate General Relativity and Cosmology. In much of this work, the ether serves as a reference frame, and it is not described as a complex "hidden mechanism". A great deal of work is being done to develop this programme today.

Since 1950, a successful "re-mechanization" of the world-view of physics has been achieved, following Hartley's development of the vortex-sponge ether analogue, which had been progressively evolved by Euler, Helmholtz, Kelvin and Larmor, and has since been developed further by Kelly, Winterberg, Dimityev and others. Hartley's key achievement was to show that the vortex-sponge could support spherical standing waves, representative of material particles. This enabled the "event-particles" of geometrized space-time to be interpreted in terms of activity in a dynamical ether. This ether analogue interprets the phenomena described by the Poincare-Lorentz group of theories, and has been used to interpret a very wide range of fundamental physical activity including symmetry breaking, zero point fluctuations, and quantum-mechanical behaviour. Donnelly has carried out laboratory experiments with super-cooled liquids which exhibit characteristics of the vortex sponge and exhibit "quantum-mechanical" behaviour. The dynamical interpretation of the event-particle is equivalent to the "idealized interferometer" used by Ives to develop a comprehensive Poincare-Lorentz exposition of relativity. Ives also used this concept to obtain the chronotopic interval within the context of the Poincare-Lorentz theory. The vortex-sponge ether provides a mechanical analogue or disclosing model. The model of matter as a standing wave system leads to the existence of minimum measurable intervals, or discontinuities in clock-time and as-measured space intervals on the very small scale. It may be possible to reconcile Dirac's understanding of the ether with the vortex-sponge. Berkson has argued that there are considerable similarities between the world view of Kelvin and Larmor, based on a dynamical ether as an energy bearing substance, and the world view of Einstein. This is only to be expected if the geometrized space-time continuum is the vortex-sponge geometrized. Kostro's valuable studies of Einstein's later understanding of "space with physical properties", and his discussion of the geometrical and dynamical interpretations emphasize the compatibility. Far from being antagonistic, they are aspects of the same thing. The dynamical description breaks the geometrical unity of Einstein-Minkowski space-time and deals with operations with rods and clocks in frames of reference. It deals with frame-space operations and theories. These theories can be given a classical or non-classical interpretation, depending on whether primacy in defining standards is given to matter geometry or light geometry during measuring operations. Ives stresses this in his work which remains one of the most complete and clear presentations of the "Rod-Contraction; Clock-Retardation Ether Theory". It is worth noting that between 1920 and 1960, a group of engineers inspired by Kron, Pestarini, Park, Alger and others showed that conventional engineering components and systems, such as electric motors and power networks, could be geometrized. The General Electric Company (USA) supported much of this work. In Russia there was an enduring tradition of geometrizing engineering theory, especially in the electro-mechanical research institutes. Kron, Banesh Hoffmann, and others published papers on electrical machines using geometrized classical and non-classical formulations analogous to Einstein-Minkowski relativity. The Japanese developed similar techniques. Kron developed topological interpretations of media and systems containing electro-magnetic and mechanical properties, originally with industrial power systems in mind, and it should not surprise anyone that geometrization of the vortex-sponge is perfectly possible.

The vortex-sponge has immense importance in engineering and other branches of science including meteorology, aerodynamics, cryogenics and biology. The need to solve problems in these areas is deepening insight into vortex-sponges and their behaviour, and physicists should be aware of these developments coming from other disciplines.

Enough has been done to demonstrate that the formal structure of relativity can be interpreted in terms of an ether theory accompanied by a mechanical analogue. Geometrized Space-Time can

fittingly be called World-Ether. At the frame-space level, the Poincare-Lorentz exposition of relativity is a legitimate second interpretation of the accepted formal structure, which can be given the more usual Einstein-Minkowski exposition.

There are important questions to be considered by those who advocate the ether interpretation of physics. They must try to reduce the number of ether theories and analogues and work out a unified theory and model. There is not enough linking up of one theory and ether model with another. Instead of concentrating on opportunities for co-operating with their colleagues, too many ether theorists spend their time in bitter rivalry over technical differences. They pursue narrow expositions and lose opportunities to add to a growing, comprehensive ether interpretation of modern physics. Because many physicists know no history, there is much duplication of theories, and the Ives version of Poincare-Lorentz relativity has been re-invented over and over again for at least 50 years. The development of the vortex-sponge analogue has, by contrast, been original and fruitful. Too many ether theorists are unreasonable in their hostility to the Einstein-Minkowski formulation. Why the persistent bias against Einstein's relativity and the hostility to geometrization? The geometrization of physics took place for very good reasons, and it solved outstanding problems. It has proved extraordinarily fruitful, and has been the norm for several generations of physicists and engineers. The exposition of relativity in terms of moving rods and clocks (after Ives) strikes them as ingenious but complex and possessing no obvious advantage. Many physicists find the vortex-sponge analogue a very difficult concept, requiring a way of thinking which was once current but which is no longer so. Why should geometrization be considered as inferior to this mechanical interpretation? These questions should be considered very carefully by all who are anxious to enhance the part played by the ether concept in physics. The author believes this hostility and implied incompatibility are the results of misconception. There is nothing wrong with Einstein-Minkowski relativity or the concept of geometrized space-time, which can be derived by geometrizing the Kelvin-Larmor vortex-sponge using the techniques of Kron, and which were originally got by geometrizing the Lorentz Theory of Electrons.

Description of electromagnetic radiation in complex motion media with account of the relativistic effects

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Results of theoretical and experimental investigation of new effects of moving media optics, arising in tasks of detection of electromagnetic and gravitational radiation from astrophysical objects were presented. On a basis of analysis of the obtained results the conclusion on influence of moving media optics effects on wide class of experiments even with non-relativistic velocities of medium motion was done.

1. Introduction

The advances of last decades in the field of laser physics, computering, and aeronautics led possibility to accomplish such complex scientific projects as ground and space gravitational wave (GW) observatories, the global satellite navigation systems.

The present metrological level, being realized in the projects, requires using all reserves for increasing sensitivity and protection from "noises" of measuring optical systems. The results of experimental investigations say about influences of such factors as: light pressure, light dragging in moving optical elements, frequency shift of refractive waves etc., which lead to initiation of new optical phenomena under special conditions, on the operation of the measuring optical systems. As the result a demand arises for theoretical describing and laboratory researching the optical experiments with availability of light pressure and also dependency of amplitude and phase characteristics for electromagnetic (EM) radiation on the velocity V and directions of motion of homogeneous and inhomogeneous media with account of the second order effects V^2/c^2.

The propagation of EM radiation in a moving medium has some specialties, which can be properly described within moving medium optics [1].

Because of complexity of experimental works in the field of optics, in main, the check was executed for normal velocity break [2].

Hence, experimental results, obtained from location of space crafts and by using laser interferometers for GW detection, point to necessity to take into account effects of moving medium optics, when one describes EM radiation in full-scale laser gravitational antenna, and in interferometers for geophysical researches.

In general case, a description of similar devices demands to create complex theoretical apparatus, allowing investigating interferometer operation in real-time regime with methods of dynamic modelling with account of the moving medium optics effects.

The problem of GW detection is associated with identification of the detected splashes and astrophysical sources; therefore we should analyze the spectral characteristics of astrophysical objects in optical region.

Hence, the known methods of determining the rotation velocity of astrophysical objects [3] don't allow defining the spatial orientation of a rotating object, that can make difficult the identification and calculating the GW source parameters. As the result, it arises a new task to develop a new approach for investigation of spectral characteristics of stars, which allows to find the orientation of rotation axis of an astrophysical object in space.

It is known that the essential influence on time of propagation of EM signals from remote astrophysical objects is exerted by an instellar medium [4]. Therefore, we should account instellar medium motion, calculating time of light signals propagation.

The additional increase of signal to noise relation is possible, when EM and GW signals from remote astrophysical sources are synchronously detected by a system of antenna and telescopes. Along this, as it's shown in [5], a description of satellite system operation should include accounting the relativistic effects.

Therefore, in general case, the process of the switched registration of astrophysical signals from system of remote detectors should be carried out, by accounting both relative motion of medium of all sources and receivers and motion of medium, in which a radiation propagates.

The analysis of the results of theoretical description and experimental investigation for the processes of EM radiation propagation in moving media attests that effects of moving medium optics have an influence on the wide class of experiments even for non-relativistic velocities of media motion. It leads to a new research direction, related with investigation of wave optical processes in moving media to be applied in the tasks of radiation detection from cosmic sources. Bellow, we present the results of a theoretical and experimental research of new effects of moving media optics.

2. Propagation of electromagnetic radiation in a medium with complex motion

On the basis of the solution of the dispersion equation of moving media optics, the theoretical investigation and then experimental that of spatial light dragging in a medium with complex motion were carried out. It was shown that propagation of a EM wave in a medium with complex motion may be described with the equation for the EM wave vector trajectory, which is a result of superposition of the primary and secondary EM waves, arising in the medium.

A trajectory of propagation of plane monochromatic EM wave in a rotating medium will place in the plane X, Z, and will described with the integral equation [6]

$$z(x) = \int_0^{x_{\max}(x,z)} \frac{k_{2z}(x,z)}{k_{2x}} dx, \tag{1}$$

$$x_{\max}(x,z) = \frac{1}{2}\sin 2\vartheta_2 \left[R_0 - ktg\vartheta_2 + \left(R_0^2 - 2R_0 ktg\vartheta_2 - k^2 \right)^{1/2} \right], \quad k = x - ztg\vartheta_2(x,z),$$

where R_0 is a radius of a surface, bounding a rotating medium, $R_0 >> \lambda_0$, $k_0 = 2\pi / \lambda_0$, λ_0 is wave length of radiation in an inertial reference frame (IRF) of an observer. The upper limit $x_{\max}(x,z)$ together with x, z is represented a drifting coordinate of the expected intersection of the trajectory of EM wave propagation with a cylindrical surface of a radius R_0; $\vartheta_2(x,z)$ is a refractive angle.

The wave vector projections k_{2x}, k_{2z} are found from the coordinate solution of the dispersion equation for refracted wave, by neglecting absorption and dispersion, for each local domain of a trajectory of a EM wave in a moving medium with the given refractive index n_2, and the incident angle ϑ_0, the angle velocity ω and the rotation law with the coordinates $x = 0$, $z = R_0$.

On the basis of a numerical solution of integral equations, describing kinematic characteristics of an EM wave propagation process, parameters of effects of the spatial light dragging and light beam distortion in a rotating medium were calculated.

The phenomenon of competition of longitudinal and transverse effects of light dragging in a rotating disk with increasing the incident angle of a beam onto cylindrical surface of the disk was found (fig. 1).

The numerical values for longitudinal and transverse light dragging effects are presented on the diagrams as dependences on ϑ_0 for the next parameters of an optical disk $k_0 = 10^{-7}\ \mathrm{m}^{-1}$, $n_2 = 1{,}5$, $R_0 = 0{,}1$ m, $\omega = 10^4$ Hz to compare.

The precise analytical solution for a trajectory of an EM wave vector in a medium with shift flow has been discovered. It was shown that it's needed to take into account the terms with β^2 for correct description of the spatial light dragging effects.

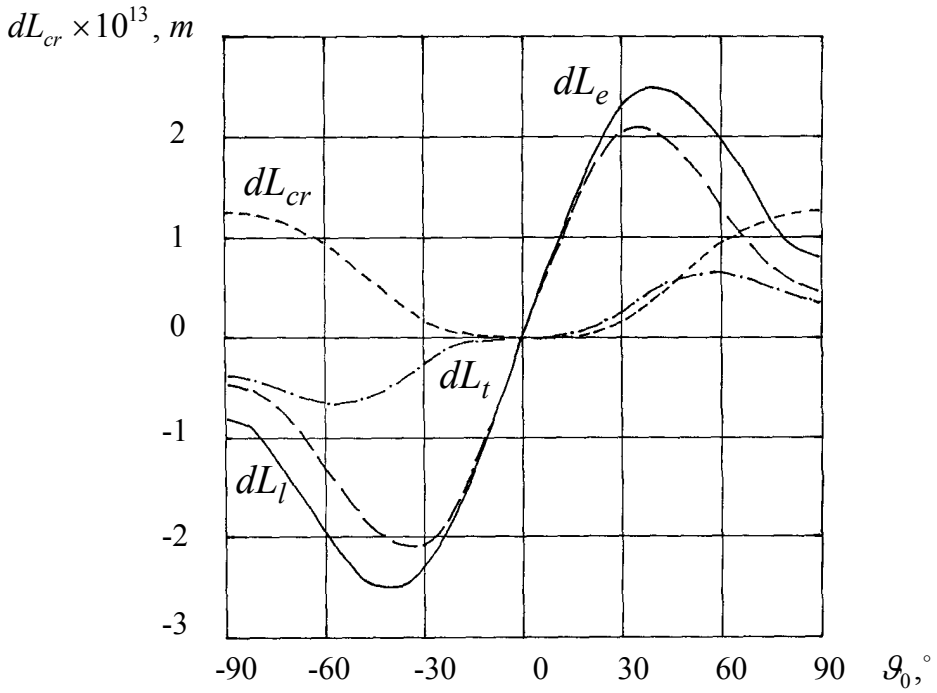

Fig. 1. The dependences of the equivalent path difference dL_e, the longitudinal component of path difference dL_t, the transverse component of path difference dL_l with light dragging effect for two EM waves, one of which propagated in a medium with $\omega = 0$, and another with $\omega \neq 0$, on the incident angle ϑ_0 onto a boundary of two media with account of shift of an EM radiation exit point on a surface of an optical disk. The dependence of the equivalent path difference due to bend of a trajectory without shift of an EM radiation exit point is presented as $dL_{cr}(\vartheta_0)$.

The interferometer methods of investigation of the spatial light dragging effect were suggested.

The disk interferometer for experimental research of the spatial light dragging effect in a rotating medium was developed and constructed (fig. 2).

A laser beam from the laser L is incident onto the light divider LD, then is split into two beams, which are reflected some times between flat surfaces of the optical disk OD, are reflected inside the prismatic reflector PR, exchanging its trajectories, go through OD again, then are mixed in LD, go through the lens O and make the interference pattern IP on the screen, where the photodetector PD is mounted.

For the first order the interference pattern shift magnitude due to the longitudinal light dragging effect is equal to

$$\Delta = \frac{16 l u_{2n}(n_2^2 - 1)}{\lambda c}, \tag{2}$$

where u_{2n} is linear velocity of a medium along a light beam trajectory; l and r define the geometry of light beam motion; λ is the wave length of radiation in vacuum in the observer's IRF. In actuality the IP shift in the interferometer may have more magnitude, because in the moving medium the refractive angle ϑ_2' differs from the calculated that, using Snell's law for the incident angle ϑ_2.

The resulting value of additional IP shift due to the effect I the carried out experiment with account all passages for two beams, all directions of disk rotation and two adjustments is equal to

$$\Delta_S = \frac{64 d n_2}{\lambda}\left(1 - \frac{1}{n_2}\right)\left(\frac{1}{\cos\vartheta_2'} - \frac{1}{\cos\vartheta_2}\right). \tag{3}$$

Here d is thickness of OD; n_2 is a refractive angle.

The theoretical estimation of total expected IP shift for the interferometer's parameters which were used in the experiment: $l = 0{,}089$ m, $u_{2n} = 0{,}6$ m/s, $\lambda = 0{,}632991$ mkm, $n_2 = 1{,}48$, $\vartheta_0 = 62^\circ$ was $\Delta_\Sigma = 0{,}0067$.

As the result of the carried out experiment for fiducial probability 0.9 the IP shift magnitude was obtained $\Delta_{\Sigma\acute{Y}} = 0{,}0076 \pm 0{,}0030$.

The results of the experiment verified the numerical calculations, received on the basis of integral equations, described the spatial light dragging effect in a rotating medium. As it's follows from the given theoretical and experimental research, the light dragging effect has essential magnitude under non-relativistic velocity of medium motion and should be accounted in the wide set of experiments.

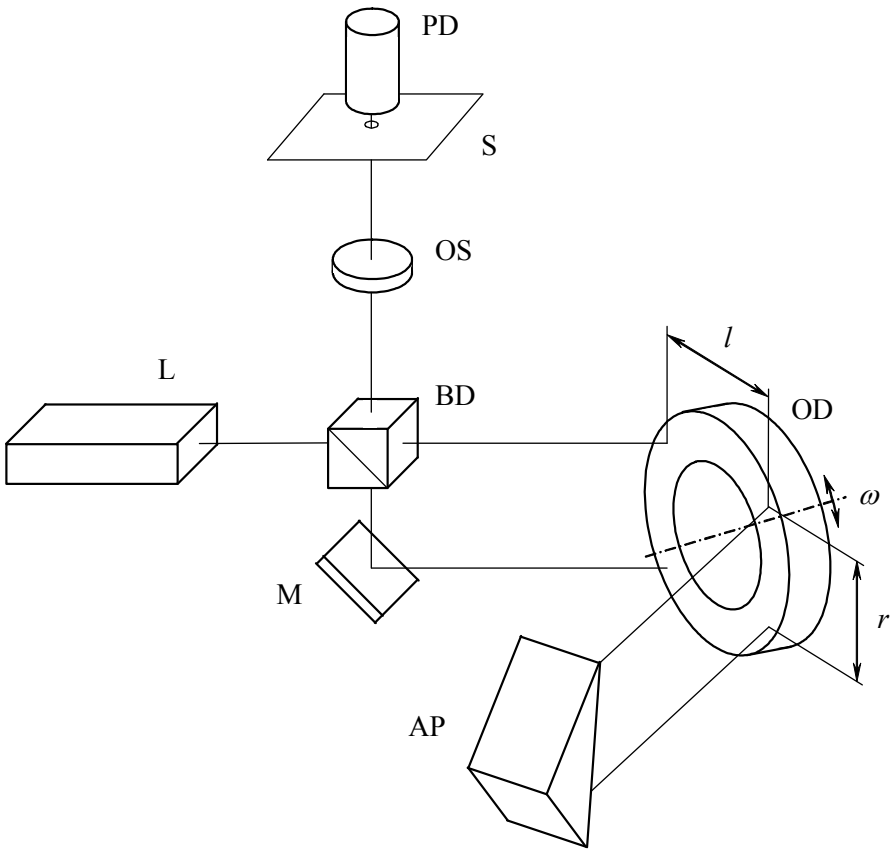

Fig. 2. The optical scheme of the interferometer with a rotating optical disk. The scheme realizes the method of IP shift detection by mea suring the time of IP moving on the photodetector.

3. Non-linear optical phenomenon in a multi-beam interferometer of Fabry-Perot type

On the basis of a solution of a equation system, describing EM field in Fabry-Perot resonator (FPR), analytical research of light pressure influence on interferometer operating

An equation system which describes EM field at the entry, at the exit and inside of FPR for arbitrary laws of optical pumping amplitude changing for initial phase EM wave for cavity phase adjustment, for mirror motion laws with account dependence of an EM wave frequency on mirror motion velocity.

Let's consider a case, when the phase adjustment δ satisfies the condition

$$\delta + 2\pi N = 2k_e L_0 - \alpha, \qquad N \in \mathrm{N},$$

where $\alpha = \alpha_1 + \alpha_2$ is resulting phase shift, when beams reflect from mirrors S_1 and S_2; L_0 is unperturbed optical length of FPR. By neglecting losses on mirrors that the condition is fulfilled

$$R_i^2 + T_i^2 + B_i^2 = 1, \qquad i = 1, 2.$$

Here T_i, R_i, B_i are amplitude coefficients of transmission, reflection and absorption for mirrors.

The equation for intensities has a view

$$I_{ij}(t) = \varepsilon_0 c a_{ij}^2 \mathrm{Y}_{ij}(t), \tag{4}$$

$$a_{ij} = \begin{pmatrix} 1 & 0 & T_1 T_2 \\ 0 & T_1 & T_1 \\ T_1^2 R_2 & T_1 R_2 & T_1 R_2 \end{pmatrix}, \qquad i, j = 0,1,2, \tag{5}$$

$$\mathrm{Y}_{ij}(t) = \left(\sum_{n=1}^{\infty} E_{nij}^0(t) \cos \Phi_{nij}^0(t) \right)^2 + \left(\sum_{n=1}^{\infty} E_{nij}^0(t) \sin \Phi_{nij}^0(t) \right)^2. \tag{6}$$

Here ε_0 is the electric constant; $\tilde{n}$ is the light velocity in vacuum; the functions $E_{nij}^0(t)$, $\Phi_{nij}^0(t)$ describe amplitude and phase components of EM field on the FPR mirrors with account of mirror shift, dependence of EM wave frequency on mirror motion velocity in the FPR, influence of GW signal and arbitrary laws of amplitude and phase changing for EM wave at the entry of FPR.

To present the complete theoretical description of FPR in the field of forces of light pressure, gravity and external perturbation, the self-consistent system of differential equations was obtained.

The equations of motion for test bodies with FPR mirrors which are suspended on the inextensible threads to the foundation in the Earth gravitational field, represents oscillator equations in the field of external forces

$$\hat{G}(\beta_i; \omega_{0i} \setminus x_i) = \frac{1}{M_i} \sum_{n=1}^{N} F_{ni}(t), \; i = 1,2, \tag{7}$$

where, for convenience, we introduce a linear operator, acting on the variable x_i

$$\hat{G}(\beta_i; \omega_{0i} \setminus x_i) = \frac{d^2 x_i}{dt^2} + 2\beta_i \frac{d^1 x_i}{dt^1} + \omega_{0i}^2 \frac{d^0 x_i}{dt^0}, \tag{8}$$

the indexes $i = 1,2$ correspond to equations for the mirrors S_1 and S_2; β_i are the coefficients of oscillation attenuation for mirrors; ω_{0i} – are fundamental frequencies of test bodies with mirrors S_i, which have equal aperture squares S; M_i are masses of test bodies; $F_{ni}(t)$ are the external forces; N is a natural number.

Influence of GW signal on FPR mirrors motion can be presented as

$$F_{11}(t) = -\frac{M_1 M_2}{M_1 + M_2} L_0 \frac{d^2 h}{dt^2} = -F_{12}(t). \tag{9}$$

The second derivative of disturbance of space metrics $h(t)$, masses of mirrors M_i and FPR length L_0 are contained in the formulae.

For the forces of light pressure it can be written:

$$F_{21}(t) = \frac{S}{c}\left[I_{00}(t) + R_1^2 I_{00}(t) + T_1^2 I_{22}(t) - I_{11}(t) - I_{21}(t)\right], \tag{9}$$

$$F_{22}(t) = \frac{S}{c}\left[I_{12}(t) + I_{21}(t) - T_2^2 I_{12}(t)\right], \tag{10}$$

where the intensities $I_{ij}(t)$ correspond to amplitudes $E_{ij}(t)$ on FPR mirrors.

As a result of analytical solution of integral equation which describes EM field inside the FPR parameters of low-frequency optical resonance (LFOR) in the FPR for wide region of interferometer's adjustments was calculated. The analysis of LFOR with account of additional terms, arising in the expressions for quality, frequency, and resonance damping coefficient was carried out. The consequence on possible distortion of parameters of FPR optical response on an external signal with availability of low frequency drift of phase adjustment, mirror coordinate or initial phase of EM wave was done [7].

By using Fokker-Planck's equation stationary solution, it was shown that in the free-mass multi-beam FPR the light pressure on interferometer mirrors has considerable influence on interferometer operating. Due to non-linear dependence of light pressure on the FPR mirror from mirror shift, spectral characteristics of mirror motion can be essential transformed.

4. Peculiarities of gravitational wave radiation with using the Fabry-Perot cavity

By using as a base the analysis of more possible sources of gravitational radiation, parameters and form of GW signal were defined, which must be used in calculations and optimization og gravitational antenna (GA) parameters. The calculations of FPR optical response are carried out for quasi-harmonical damping GW signals, arising in astrophysical objects collapses, and also in merging of twice stellar systems.

A GW classification on physical methods of GW detecting was offered. It was shown that, in whole, more perspective, on one side, and sufficiently technically provided, on other side, projects can be called those of wide-region laser interferometer GA, and the main element of the antenna is the multi-beam free-mass Fabry-Perot interferometer.

On the basis of the solution of the equation of FPR mirror motion in light pressure field and the analysis of spectral characteristics of the laser-free-mass FPR system the parameters of GA with FPR were defined, using heterodyne method of detecting the GW from astrophysical sources of radiation.

A phenomenon of transferring the low-frequency noise oscillations of FPR mirrors which are in the field of light pressure, in the high-frequency region of the spectrum was researched. The phenomenon may lead to decreasing the GA sensibility.

By using the received system of differential equations for FPR with a method of dynamic modelling in real time, research of multi-beam free-mass FPR in field of gravitational signal of quasi-harmonical type with account of light pressure on mirrors was carried out.

The LFOR characteristics in FPR were investigated. The LFOR was studied, by using optical pumping amplitude modulation. The optimal relationship among FPR parameters and GW signal, which allow to gain the optical response, was defined.

If the GW frequency if higher than the LFOR frequency, $\nu_{GW} > \nu_0$, the optical response of FPR has been modulated on high frequency, so the maximum of optical response can forestall the GW signal maximum with the magnitude Δt due to multi- beam interference nature (fig. 3).

In figure 3 the form of GW signal $h(t)$ of quasi-harmonical damping type and the form of FPR optical response in relative units $dP_T(t) = (P_T(t) - P_0)/P_0$ for frequency $\nu_{GW} = 3.5$ kHz are presented.

It was also shown in the work that LFOR in FPR may be used in cosmic experiment to detect GW.

For the parameters of the VIRGO project the quality is on level 10…2000 in region 10…2000 Hz. Decreasing L_0 the quality shifts in the region of high frequencies, hence, using the method of FPR optical pumping modulation can shift the quality to the region of low frequencies.

Large values of L_0, planned in cosmic experiments, allow theoretically to apply the LFOR in low frequency region, to which the high amplitudes of periodical GW from double stars correspond.

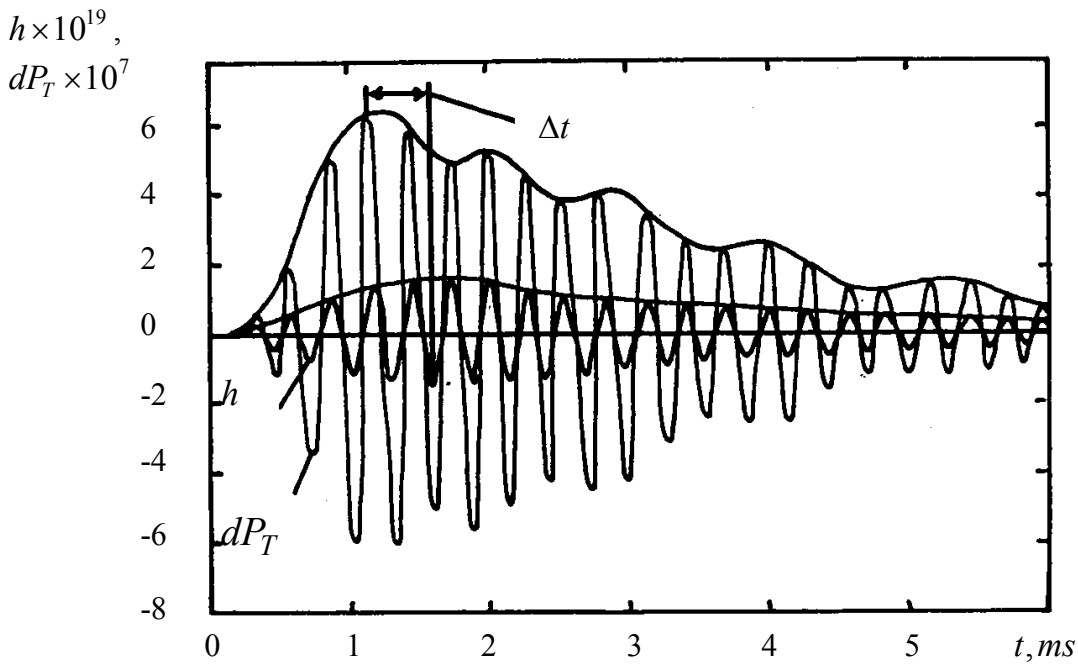

Fig. 3. Dependences of relative amplitude of FPR optical response and amplitude of GW signal on time with frequency of GW signal ν_{GW}, which is more than resonance frequency of LFOR ($\nu_0 = 2$ кHz, $\nu_{GW} = 3.5$ kHz).

At present the space antenna project LISA is at the stage of technical performance, in the project the space flying apparatuses create together the Mickelson's interferometer with shoulders 5 millions km. The planned accuracy of locating of test masses with interferometer mirrors, which don't have rigid bound with the flying apparatuses, stands at the level 5×10^{-11} m, which is sufficient to make some reflections in the interferometer and to detect astrophysical radiation, using the LFOR in FPR.

5. Determination of kinematic parameters from remote rotating astrophysical objects

On the basis of the solution of the integral equation which describes the profile of spectral line of rotating astrophysical objects, investigation of dependency of spectral line profile broadening on kinematic parameters from remote rotating stars was made. The equation system, connecting the

kinematic characteristics of astrophysical emitter, such as: equator velocity and rotatin axis inclination in space, with parallactic variations of spectral line width with account the relativistic law of frequency shift for rotating object was obtained.

The spatial orientation of equatorial velocity is characterized with the angles φ_1, φ_2 in planes which go through remote astrophysical object and spectral detectors $\theta_{1,3}$ and $\theta_{2,3}$. The spectrometer readings which are taken with detectors θ_{1-3}, differ from each other due to different angles of rotating axis inclination to observation ray i, i.e. due to some variations of angles φ_k, defined by expressions:

$$tg\Delta\varphi_1 = \frac{\rho_1 \cos\psi}{d - \rho_1 \sin\psi}, \tag{11}$$

$$tg\Delta\varphi_2 = \frac{\rho_2 \sqrt{1 - \cos^2\psi \sin^2\vartheta}}{d + \rho_2 \cos\psi \sin\vartheta}, \tag{12}$$

where ρ_1 and ρ_2 are distances between detectors θ_1, θ_3 and θ_2, θ_3; values ϑ, ψ and d are characterized the angle position and distance to astrophysical source, relative to system of spectrometric detectors, respectively.

It was shown that angles φ_k, orienting the equator plane of astrophysical object in space, can be found, if $\Delta\varphi_k$ are known, as:

$$\varphi_k = arcg\left\{ \frac{\Delta\lambda_{\text{ïð}\,k}}{\lambda_{\text{èñï}}} \frac{\tilde{n}}{V_R} \frac{1}{\Delta\varphi_k} \frac{\left(1 - V_R^2 / c^2 \sin^2 i\left(V_R, \Delta\varphi_{1,2}, \Delta\lambda_{\text{ïð}\,1,2}\right)\right)^{3/2}}{1 + V_R / c \sin^2 i\left(V_R, \Delta\varphi_{1,2}, \Delta\lambda_{\text{ïð}\,1,2}\right)} \right\}, \tag{13}$$

where $V_R = V_e \cos i$ is value of ray velocity of rotation at the equator which has been measured with middle detector θ_3; $\Delta\lambda_{\text{ïð}\,k}$ are differences of spectral line broadening magnitudes which are measured with spectral devices θ_1, θ_3 and θ_2, θ_3 for $k = 1,2$; $\lambda_{\text{èñï}}$ is wave length, according to the centre of a spectral line; c is the light velocity in vacuum.

The magnitude i, entering (13), can be defined from the six-order equation, connecting variations of spectral line profiles $\Delta\lambda_{\text{ïð}\,k}$ with geometric characteristics of the third detectors system.

The expressions for theoretical resolution, needed for defining the equator velocity and rotating axis inclination for astrophysical object in space were received.

The numerical calculations of necessary resolution of spatial devices for bright stars of upper part of main sequence on lines for ultra-violet and visible spectral regions were fulfilled. When optical base between detectors of order 40 a.u. that is planned magnitude in the research programs “Radioastron” and “SMILE”, the resolving power is on the level of achieved resolution of multi-beam interferometers of Fabry-Perot and Mickelson.

It was shown that for double-stars systems, if rotation axis inclination is known, we can define distance to astrophysical object, by measuring increments of spectral line edges.

The use of variations of spectral density of radiant emittance in spectral lines to determinate kinematic characteristics of rotating astrophysical objects was suggested. The system of equations, connecting the kinematic characteristics of astrophysical object with variations of spectral density of star radiant emittance was obtained.

The equation for time of propagation of EM signals in expending Universe in Robertson-Walker metric with account of instellar medium motion and radial motion of radiation source was found.

$$\int_{t_1}^{t_0} \frac{dt}{R(t)} = (1+\beta)\int_0^{r_1} \frac{(\beta + n(r))(1-\beta_n^2(r)) + \kappa(\beta - \beta_n(r))}{(1-\beta^2)(1-\beta_n^2(r)) - \kappa(\beta - \beta_n(r))^2} \frac{dr}{\sqrt{1-kr^2}}. \quad (14)$$

Here $R(t)$ is cosmological scale factor, k is the spatial curvature; t_1, t_0 are times of emittion and detection, counted off singular state, r_1 is dimensionless distance to space source of radiation in ground IRF; $\beta = -r_1 \frac{\dot{R}}{\tilde{n}R}$ is the velocity of removal of emitting astrophysical object in IRF observer; $n = n(r)$ is the refractive index of instellar medium or emitting field of astrophysical object; $\beta_n(r)$ is the instellar medium velocity along the trajectory of light propagation in considered part of space; r - is the current radial coordinate; κ is the parameter, characterizing dielectric and magnetic features of medium.

The solution of the equation (14) for constant refractive index of instellar medium and refractive index of hyperbolic type was obtained. It was shown that the EM wave propagation in expending Universe may be accompanied by the light dragging effect in the field, where instellar medium or medium of astrophysical object emitting region have near-light velocity. Therefore, by calculating distance to emitting object, the light dragging effect of EM wave by moving medium should be accounted with.

The discussed effect can lead to different times of propagation of EM and gravitational splashes, generated by cosmological astrophysical objects.

6. Synchronous detection of signals by detectors, moving in different initial frames of references

On the basis of Mellor's method the transformations of space-time, connecting the coordinates of arbitrary two synchronized IRF, moving in random directions with different velocities in observer IRF, were obtained. It was done the consequence on necessarity to account the non-invariant relationships for partial differentials of physical variables when synchronic registration of astrophysical signals by detectors, moving in different IRFs.

The transformations of spatial and time coordinates between two $IRF_{1,2}$, moving in observer IRF with different velocities $\vec{V}_1$, $\vec{V}_2$, have a view

$$x_1^\mu = D_2^{-1}(\hat{\eta}_\mu, \vec{r}_2) - \frac{\alpha_0 V_{0\mu}}{V_0^2}\hat{\tau}_\mu(\vec{r}_2, \vec{V}_0) + \gamma_0 \hat{\beta}_\mu V_{0\mu} x_2^4, \quad \mu = 1,2,3, \quad (15)$$

$$t_1 = \gamma_0 \lambda_1 t_2 + \lambda_2 \frac{\gamma_0}{c}(\vec{r}_2, \vec{\beta}_0), \quad (16)$$

Here $\lambda_1 = 1 + (\vec{\beta}_0, \vec{\beta}_1)$, $\lambda_2 = 1 + \frac{(\vec{\beta}_0, \vec{\beta}_1)}{(\vec{r}_2^{\,n}, \vec{\beta}_0)}$, $\alpha_0 = \gamma_0 - 1$, $\gamma_0^{-2} = 1 - \beta_0^2$, $\vec{\beta}_0 = \vec{V}_0 / c$, $\vec{V}_0$ is the relative velocity between IRF_1 and IRF_2, $V_{0\mu}$ is the projection of relative velocity of IRF_i on the coordinate x^μ; $\vec{\beta}_1 = \vec{V}_1 / c$, $\vec{V}_1$ is velocity of IRF_1 in observer IRF; coefficients $\hat{\eta}_\mu$, $\hat{\tau}_\mu$ are defined with vectors $\vec{V}_1$, $\vec{V}_0$ and parameters D_1, D_2; $D_{1,2}$ are parameters, featuring relative angle

positions $IRF_{1,2}$; the expression for $\hat{\beta}_\mu$ is found from condition of invariance of total differential dx_1^μ; $\vec{r}_2^{\,n} = \frac{\vec{r}_2}{|\vec{r}_2|}$.

In a general case for providing synchronous detecting of astrophysical signals the magnitudes $t_{1,2}$ should be calculated with account of delay time of EM signal propagation in moving medium.

On the basis of the obtained transformations, the task of calculation of eigen-interval of time for the clock, uniformly accelerated relative to the clock, immovable and movable with constant velocity in observer IRF was considered. In long-time measuring in the limit of near-light velocities the readings of uniformly accelerated clock with near-light velocity and stationary clock in observer IRF can differ. The transformations can be used under comparison of readings, which were obtained by detector system, moving in different IRFs, when synchronous detection of astrophysical objects is applied.

7. Conclusion

The effects of moving media optics have influence on propagation of electromagnetic radiation in a moving medium and when moving objects reflect or radiate. Usually ones consider that moving medium doesn't have essential effect on results of measurements. However, the influence can be remarkable for: high-speed motions, large index of refraction, great propagation path of electromagnetic radiation, precise measurements.

Therefore they must influence on results of measurements: when locating within earth atmosphere, when reflecting from prismatic reflectors, mounted on a moving spaceship, when GPS operates, when radiation propagates in moving interstellar medium.

In a case of the plane motions the propagation of electromagnetic radiation can be described with integral equations, which allow determining the trajectory of light beams for geometrical optics approximation, calculating the equivalent path length in a moving medium, and calculating the accumulation of beam path difference in rotating and stationary media due to longitudinal and transversal dragging effects, and estimating the trajectory curvature. More over, there is effect of shift of way-out point from the medium.

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References

[1] Болотовский Б.М., Столяров С.Н. Отражение света от движущегося зеркала и родственные задачи//УФН. 1989. Т.159. №1. С.155-180.

[2] Загороднов О.Г., Файнберг Я.Б., Егоров А.М. Умножение частоты с помощью «схлопывания» плазмы// ЖЭТФ. 1960. Т.38. №1. С.7-9.

[3] Hutchings J.B., Stoecly T.R. V and i rotating stars from Copernicus UV data// PASP. 1977. V.89. №527. P.19-22.

[4] Weinberg S. Gravitation and Cosmology. Wiley, New York. 1972.

[5] Ashby N., Spilker J.J. (Jr.) Introduction to Relativistic Effects on the Global Positioning System//Global Positioning System: Theory and Applications. Washington, 1997. P.623-697.

[6] Gladyshev V.O. Curvature of the trajectory traced out by a monochromatic plane electromagnetic wave in a medium with rotation// JETP Lett. 1993. V.58(8). P.569-572.

[7] Gladyshev V.O., Morozov A.N. The theory of a Fabry-Perot interferometer in a gravitational-wave experiment // J. Moscow Phys. Soc. 1996. V.6. P.1-13.

The state of the weakly interacting particles of the Universe and the Einstein gravitational equations

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On the basis of hypotheses in which the matter density of the Universe is superior much to its estimates obtained from the last astronomical data we offer to consider elemental particles as coherent frames in open systems characterized by quasi-group symmetry. It results in necessity to use the inhomogeneous (quasi-homogeneous) space-time endowed by the nontrivial geometrical structure. In particular we shall consider that the Riemannian space-time is the effective one, postulating the metric tensor on the base of the reduced density matrix of the gauge fields.

From last astronomical observations (see for example [1]) it follows that not more than 5% of the Universe matter has the baryon nature. Thereof it is convenient to divide the Universe matter on rapid and slow subsystems. We shall consider that all known particles (it is possible excluding only neutrinos) belong to the rapid subsystem. Let's consider the Boltzmann hypothesis of the Universe birth owing to a gigantic fluctuation not in an empty space but in a medium which consists of weakly interacting particles characterized by zero temperature and forming the Bose condensate. Certainly, if the particles are fermions they should be in the coupled state. For the description of such state of the Universe matter (this state we shall consider pure one) it is necessary to introduce an amplitude of probability B with components $B_a^b(\omega)$ (a, b, c, d, e, f, g, h = 1, 2..., r) dependent from points ω of a manifold M_r (not excepting a limiting case $r \to \infty$). In this case we can not define the metric but for its definition we need a density matrix $\rho(B)$ (the rank of which equals to 1 for a pure state) determining its standard mode $BB^+ = \rho\ tr(BB^+)$ ($tr\rho = 1$, $\rho^+ = \rho$, the top index + is the symbol of the Hermitian conjugation).

Let as a result of a fluctuation the disintegration of the Bose condensate will begin with formation of fermions (for their description we shall introduce an amplitude of probability Ψ) and with an increase of pressure in some local area of Universe (in addition some time the temperature of the Universe particles could remain equal or close to zero point - so-called the inflation period). In consequence the rank of the density matrix ρ will begin to grow that characterizes the appearance of mixed states. An inverse process of relaxation (characterized by the formation of the Bose condensate and by the pressure decline) should go with an energy release which will go on heating of the Fermi liquid with formation of excited states - of known charged fermions (quarks and leptons). From this moment it is possible to introduce the metric and use the results obtained for the hot model of Universe (with those by inflationary modifications which have appeared recently) interpreting the Universe evolution as the process characterized by the increase of the entropy $S = - tr(\rho \ln\rho)$. Now Universe is at that stage of an evolution when the dominating number of particles has returned to the Bose condensate state. They are also displayed only at a weak interaction with particles of a visible matter.

For the description of a matter it is convenient to use differentiable fields given in a differentiable manifold M_n which we shall call as a space-time and the points x of which one will have coordinates x^i (i, j, k, l... = 1, 2..., n). Probably the rank of the density matrix ρ equals n, but it is impossible to eliminate that the generally given equality is satisfied only approximately when some components of a density matrix can be neglected. In any case we shall consider that among fields B the

mixtures Π_a^i were formed with non-zero vacuum means h_a^i which determine differentiable vector fields $\xi_a^i(x)$ as:

$$\Pi_a^i = B_a^b \xi_b^i \tag{1}$$

for considered area $\Omega_n \subset M_n$ (fields $\xi_a^i(x)$ determine a differential of a projection $d\pi$ from $\Omega_r \subset M_r$ in Ω_n). It allows to define a space-time M_n as a Riemannian manifold, the basic tensor $g_{ij}(x)$ of which we shall introduce through a reduced density matrix $\rho'(x)$.

So let components ρ_i^j of a reduced density matrix $\rho'(x)$ are determined by the way:

$$\rho_i^j = \xi^{+a}_i \rho_a^b \xi_b^j / (\xi^{+c}_k \rho_c^d \xi_d^k) \tag{2}$$

and let fields

$$g^{ij} = \eta^{k(i} \rho_k^{j)} (g^{lm} \eta_{lm}) \tag{3}$$

are components of a tensor of a converse to the basic tensor of the space-time M_n. By this components $g_{ij}(x)$ of the basic tensor must be the solutions of following equations: $g_{ik} g^{ij} = \delta_k^j$. (Hereinafter η_{ij} are metric tensor components of a tangent space to M_n, and η^{ik} are determined as the solution of equations: $\eta_{ik} \eta^{ij} = \delta_k^j$).

The influencing of the macroscopic observer will display in an approximation of the transition operators T by the differential operators ∂_i. First of all we shall require that for fermion fields deviations $X_a(\Psi) = T_a(\Psi) - \xi_a^i \partial_i\Psi$ were minimum in the "mean" [2]. For this purpose we shall consider the following integral

$$A = \int_{\Omega_n} \Lambda(\Psi) d_n V = \int_{\Omega_n} \kappa \overline{X}^a(\Psi) \rho_a^b(x) X_b(\Psi) d_n V \tag{4}$$

(κ is a constant, $\Lambda(\Psi)$ is a Lagrangian, the bar means the Dirac conjugation that is to be the superposition of Hermitian conjugation and the spatial inversion) which is the action. Let the action A is quasi-invariant at infinitesimal substitutions $x^i \to x^i + \delta x^i = x^i + \delta\omega^a \xi_a^i(x)$, $\Psi \to \Psi + \delta\Psi = \Psi + \delta\omega^a T_a(\Psi)$ of Lie local loop G_r, the structural tensor components C_{ab}^c of which satisfy to Jacobi generalized identity [2]. The given requirement causes to introduce the full Lagrangian (instead of a Lagrangian $\Lambda(\Psi)$) recording it as

$$\Lambda_t = \Lambda(\Psi) + \kappa'_o F_{ab}^c F_{de}^f [t^{ad} (s_c^e s_f^b - \upsilon s_c^b s_f^e) + t^{be} (s_f^a s_c^d - \upsilon s_c^a s_f^d) + u_{cf} (t^{ad} t^{be} - \upsilon t^{ab} t^{de})]/4 \tag{5}$$

(κ'_o, υ are constants). In addition intensities $F_{ab}^c(B)$ of the boson (gauge) fields $B_a^c(x)$ will look like

$$F_{ab}^c = \Theta_d^c (\Pi_a^i \partial_i B_b^d - \Pi_b^i \partial_i B_a^d + \Xi_{ab}^d), \tag{6}$$

where

$$\Theta_b^c = \delta_b^c - (B_d^c - \beta_d^c) \Pi_i^d \xi_b^i, \qquad \Xi_{ad}^b = (B_a^c T_{cd}^e - B_d^c T_{ca}^e) B_e^b - B_a^c B_d^e C_{ce}^b. \tag{7}$$

Hereinafter a selection of fields $\Pi_i^a(x)$ and β_c^a are limited by the relations:

$$\Pi_j^a \Pi_a^i = \delta_j^i, \qquad \beta_c^a \xi_a^i = h_c^i \tag{8}$$

(δ_j^i are Kronecker deltas). If

$$s_a^b = \delta_a^b, \qquad t^{ab} = \eta^{ab}, \qquad u_{ab} = \eta_{ab} \tag{9}$$

(η_{ab} are metric tensor components of the flat space and η^{ab} are tensor components of a converse to basic one) then the given Lagrangian is most suitable one at the description of the hot stage of the Universe evolution because it is most symmetrical one concerning intensities of the gauge fields F_{ab}^c. What is more we shall require the realization of the correlations: $T_{ac}^b \eta^{cd} + T_{ac}^d \eta^{cb} = 0$, that the transition operators T_{ac}^b generate the symmetry, which follows from the made assumptions. In absence of fields $\Pi_a^i(x)$ and $\Psi(x)$ at earlier stage of the Universe evolution the Lagrangian (5) becomes even more symmetrical ($\Lambda_t \propto B^4$), so that the formation of fermions (the appearance of fields Ψ in a full Lagrangian Λ_t) from primary bosons is a necessary condition (though not a sufficient one) of the transition of Universe to the modern stage of its development with a spontaneous symmetry breaking. Only the formation of the Bose condensate from pairs of some class of fermions (the neutrinos of different flavors) has resulted in a noticeable growth of rest-masses of those

vector bosons (W^+, W^-, Z^o), which interacted with this class of fermions. In parallel there could be a growth of rest-masses and other fundamental particles, though and not all (photon, directly with a neutrino not interacting, has not a rest-mass).

Let's connect non-zero vacuum means $\beta_a{}^b$ of gauge fields $B_a{}^b$ with a spontaneous violation of a symmetry, which has taken place in the early Universe and which is a phase transition with a formation of Bose condensate from fermion pairs. The transition to the modern stage of the Universe evolution for which it is possible to suspect the presence of cluster states of weakly interacting particles will be expressed in following formula for tensors $s_a{}^b$, t^{ab}, u_{ab}, $h_i{}^a$:

$$s_a{}^b = s\, \xi_a{}^i\, h_i{}^b + \xi_a{}^{\underline{c}}\, \varepsilon_{\underline{c}}{}^b, \quad t^{ab} = t\, \varepsilon_{(i)}{}^a\, \varepsilon_{(j)}{}^b\, \eta^{(i)(j)} + \varepsilon_{\underline{c}}{}^a\, \varepsilon_{\underline{d}}{}^b\, \eta^{\underline{cd}},$$
$$u_{ab} = u\, \xi_a{}^i\, \xi_b{}^j\, h_i{}^c\, h_j{}^d\, \eta_{cd} + \xi_a{}^{\underline{c}}\, \xi_b{}^{\underline{d}}\, \eta_{\underline{cd}}, \quad h_i{}^a = h_i{}^{(j)}\, \varepsilon_{(j)}{}^a \tag{10}$$

((i),(j),(k),(l), ... = 1,2,...,n; $\underline{a},\underline{b},\underline{c},\underline{d},\underline{e}$ = n+1,n+2,...,n+$\underline{r}$; $\underline{r}/r \ll 1$), where fields $h_i{}^{(j)}(x)$, taking into account the relations (10), are uniquely determined from equations: $h_k{}^a\, h_a{}^i = \delta_k{}^i$. Similarly tensors $\eta^{(i)(j)}$, $\eta^{\underline{ab}}$ are determined from equations: $\eta^{(i)(k)}\, \eta_{(j)(k)} = \delta_{(j)}{}^{(i)}$, $\eta^{\underline{ab}}\, \eta_{\underline{cb}} = \delta_{\underline{c}}{}^{\underline{a}}$, while tensors $\eta_{(i)(j)}$, $\eta_{\underline{ab}}$ are determined as follows: $\eta_{(i)(k)} = \eta_{ab}\, \varepsilon_{(i)}{}^a\, \varepsilon_{(k)}{}^b$, $\eta_{\underline{ab}} = \eta_{cd}\, \varepsilon_{\underline{a}}{}^c\, \varepsilon_{\underline{b}}{}^d$. We shall connect constants $\varepsilon_{(i)}{}^a$, $\varepsilon_{\underline{b}}{}^a$ with a selection of the gauge fields $\Pi_i{}^a(x)$ recording them as

$$\Pi_i{}^a = \Phi_i{}^{(j)}\, \varepsilon_{(j)}{}^a + \Phi_i{}^{\underline{b}}\, \varepsilon_{\underline{b}}{}^a \tag{11}$$

and let $\varepsilon_{\underline{b}}{}^a = 0$. Besides we shall apply the decomposition of fields $B_b{}^a(x)$ in the form

$$B_c{}^a = \zeta_i{}^a\, \Pi_c{}^i + \zeta_{\underline{b}}{}^a\, A_c{}^{\underline{b}}, \tag{12}$$

where $A_b{}^{\underline{a}} = B_b{}^c\, \xi_c{}^{\underline{a}}$. In addition components of intermediate tensor fields $\xi_a{}^i(x)$, $\xi_a{}^{\underline{b}}(x)$, $\zeta_i{}^a(x)$, $\zeta_{\underline{b}}{}^a(x)$ should be connected by the relations: $\zeta_i{}^a\, \xi_a{}^j = \delta_i{}^j$, $\zeta_i{}^a\, \xi_a{}^{\underline{b}} = 0$, $\zeta_{\underline{b}}{}^a\, \xi_a{}^j = 0$, $\xi_a{}^{\underline{c}}\, \zeta_{\underline{b}}{}^a = \delta_{\underline{b}}{}^{\underline{c}}$.

Let $n = 4$, $\upsilon = 2$, $tu = s^2$, $T_{c(k)}{}^a = T_{cb}{}^a\, \varepsilon_{(k)}{}^b = T_{c(k)}{}^{(i)}\, \varepsilon_{(i)}{}^a$, $T_{i(j)}{}^{(k)} = \zeta_i{}^a\, T_{a(j)}{}^{(k)}$, $T_{\underline{b}(j)}{}^{(k)} = \zeta_{\underline{b}}{}^a\, T_{a(j)}{}^{(k)}$ and

$$T_{a(k)}{}^{(i)}\, \eta^{(k)(j)} + T_{a(k)}{}^{(j)}\, \eta^{(k)(i)} = 0, \tag{13}$$

so that the full Lagrangian (5) will be rewritten as follows

$$\Lambda_t = \eta^{(j)(m)}\, [\kappa_o\, E_{(i)(j)}{}^{\underline{a}}\, E_{(k)(m)}{}^{\underline{b}}\, \eta^{(i)(k)}\, \eta_{\underline{ab}} + \kappa_1\, (F_{(i)(j)}{}^{(k)}\, F_{(l)(m)}{}^{(n)}\, \eta^{(i)(l)}\, \eta_{(k)(n)} +$$
$$2\, F_{(i)(j)}{}^{(k)}\, F_{(k)(m)}{}^{(i)} - 4\, F_{(i)(j)}{}^{(i)}\, F_{(k)(m)}{}^{(k)})]\, /4 + \Lambda(\Psi, D\Psi), \tag{14}$$

where

$$\kappa_o = \kappa'_o\, t^2, \qquad \kappa_1 = \kappa'_o\, t\, s^2, \tag{15}$$

$$F_{(i)(j)}{}^{(k)} = F_{ab}{}^c\, \varepsilon_{(i)}{}^a\, \varepsilon_{(j)}{}^b\, h_c{}^l\, h_l{}^{(k)} = \Phi_l{}^{(k)}\, F_{mn}{}^l\, \Phi_{(i)}{}^m\, \Phi_{(j)}{}^n = \Phi_m{}^{(k)}\, (\Phi_{(i)}{}^l\, \nabla_l \Phi_{(j)}{}^m -$$
$$\Phi_{(j)}{}^l\, \nabla_l \Phi_{(i)}{}^m) + \Phi_{(i)}{}^l\, T_{l(j)}{}^{(k)} - \Phi_{(j)}{}^l\, T_{l(i)}{}^{(k)} + A_{(i)}{}^{\underline{a}}\, T_{\underline{a}(j)}{}^{(k)} - A_{(j)}{}^{\underline{a}}\, T_{\underline{a}(I)}{}^{(k)}, \tag{16}$$

$$E_{ij}{}^{\underline{a}} = E_{(k)(l)}{}^{\underline{a}}\, \Phi_i{}^{(k)}\, \Phi_j{}^{(l)} = F_{bc}{}^d\, \xi_d{}^{\underline{a}}\, \varepsilon_{(k)}{}^b\, \varepsilon_{(l)}{}^c\, \Phi_i{}^{(k)}\, \Phi_j{}^{(l)} = \nabla_i A_j{}^{\underline{a}} - \nabla_j A_i{}^{\underline{a}} + A_i{}^{\underline{b}}\, A_j{}^{\underline{c}}\, C_{\underline{bc}}{}^{\underline{a}} + C_{i\underline{b}}{}^{\underline{a}}\, A_j{}^{\underline{b}} - C_{j\underline{b}}{}^{\underline{a}}$$
$$A_i{}^{\underline{b}} + C_{ij}{}^{\underline{a}}, \tag{17}$$

$$\Phi_{(k)}{}^i = \Pi_a{}^i\, \varepsilon_{(k)}{}^a, \qquad A_i{}^{\underline{b}} = A_{(j)}{}^{\underline{b}}\, \Phi_i{}^{(j)}, \tag{18}$$

$$C_{\underline{ab}}{}^{\underline{c}} = \xi_g{}^{\underline{c}} C_{de}{}^g \zeta_{\underline{a}}{}^d \zeta_{\underline{b}}{}^e, \qquad C_{i\underline{a}}{}^{\underline{c}} = \xi_e{}^{\underline{c}} (C_{bd}{}^e \zeta_i{}^b \zeta_{\underline{a}}{}^d + \partial_i \zeta_{\underline{a}}{}^e), \qquad C_{ij}{}^{\underline{c}} = \xi_d{}^{\underline{c}} (\zeta_i{}^a \zeta_j{}^b C_{ab}{}^d + \nabla_i \zeta_j{}^d - \nabla_j \zeta_i{}^d). \tag{19}$$

As a result of an equation of fields $\Phi_{(j)}{}^i(x)$ may be received in a standard manner [3] as the Einstein gravitational equations

$$\kappa_1\, (2\, R_{jik}{}^j - g_{ik}\, g^{lm}\, R_{jlm}{}^j) = \kappa_o\, \eta_{\underline{ab}}\, g^{jl}\, (E_{ij}{}^{\underline{a}}\, E_{kl}{}^{\underline{b}} - g_{ik}\, g^{mn}\, E_{jm}{}^{\underline{a}}\, E_{ln}{}^{\underline{b}} / 4) + g_{jk}\, D_i \Psi\, (\partial \Lambda / \partial D_j \Psi) - g_{ik}$$
$$\Lambda(\Psi, D\Psi) \tag{20}$$

($R_{ijk}{}^l$ is the curvature tensor of the connection $\Gamma_{ij}{}^k$ of the Riemannian space-time M_4; $\kappa_o = 1/(4\pi)$, $\kappa_1 = 1/(4\pi\, G_N)$, G_N is the gravitational constant; hereinafter the system of units are used $h/(2\pi) = c = 1$, where h is the Planck constant, c is a speed of light). Naturally, that the Einstein equations express the modern physical state of the Universe matter. All this confirms a capability for interpretation of fields $\Phi_{(j)}{}^i(x)$ or fields $\Phi_i{}^{(j)}(x)$ as gravity potentials but taking into account their dependence from properties of medium (vacuum), and also the opinion to consider components $g_{ij}(x)$ of a metric tensor of the space-time by potentials of a gravitational field, it is meaningful to call $\Phi_{(j)}{}^i(x)$ and $\Phi_i{}^{(j)}(x)$ by polarization fields. It is necessary to note that the condensed description of a medium, consisting

of weakly interacting particles, by polarization (gravitational) fields $\Phi_i^{(k)}(x)$ allows to "hide" the Bose condensate with the help of a nontrivial geometrical structure, in particular, using a Riemannian space-time of General Relativity.

Let's study an approaching, in which the space-time is possible to consider as a Minkowski space, the fields $\Phi_i^{(k)}$, $\Phi_{(k)}{}^i$ are constants and let $\underline{r} = 1$, that assumes $C_{\underline{ab}}{}^{\underline{c}} = 0$. For obtaining equations of fields $A_i^{\underline{b}}(x)$ in Feynman perturbation theory the calibration should be fixed. For this we shall add the following addend:

$$\Lambda_q = \kappa_o\, q_{\underline{bb}}\, g^{ij}\, g^{kl}\, (\partial_i A_j^{\underline{b}} - q_o\, C_i\, A_j^{\underline{b}})\, (\partial_k A_l^{\underline{b}} - q_o\, C_k\, A_l^{\underline{b}})\, /2 \qquad (21)$$

to a Lagrangian (14), where $q_o = \eta_{\underline{bb}} / q_{\underline{bb}}$, $C_i = C_{i\underline{b}}{}^{\underline{b}}$. Besides let

$$T_{a(k)}{}^{(i)}\, \eta^{(j)(k)} + T_{a(k)}{}^{(j)}\, \eta^{(i)(k)} = \varepsilon_a{}^{\underline{b}}\, t_{\underline{b}}\, \eta^{(i)(j)}\,. \qquad (22)$$

As a result of this equations of a vector field $A_i^{\underline{b}}(x)$ will be written as:

$$G^{jk}\, [\partial_j \partial_k A_i^{\underline{a}} - (1 - 1/q_o)\, \partial_i \partial_j A_k^{\underline{a}} + (1 - q_o)\, C_i\, C_j\, A_k^{\underline{a}}] + m^2\, A_i^{\underline{a}} = I_i^{\underline{a}}/\kappa_o, \qquad (23)$$

where $I_i^{\underline{a}} = (g_{ij} / \eta_{\underline{aa}})\, (\partial\Lambda(\Psi) / \partial A_j^{\underline{a}})$ and

$$m^2 = (n - 1)\, (n\text{-}2)\, \kappa_1\, t_{\underline{a}}{}^2 / (2\, \kappa_o\, \eta_{\underline{aa}}) - g^{jk}\, C_j\, C_k\,. \qquad (24)$$

Notice, that owing to the vacuum polarization ($C_i \neq 0$) the propagator of a vector boson has the rather cumbersome view

$$D_{ij}(p) = \{(1 - q)\, [(p_i\, p_j - C_i\, C_j)(p^k\, p_k - q\, m^2) + (1 - q)\, p^k\, C_k\, (p_i\, C_j + C_i\, p_j)]$$
$$[(p^l\, p_l - q\, m^2)^2 + (1 - q)^2\, (p^l\, C_l)^2]^{-1} - g_{ij}\, \}\, (p^m\, p_m - m^2)^{-1}\,, \qquad (25)$$

which is simplified and receives the familiar form ($- g_{ij} / (p_k\, p^k - m^2)$, p^k is the 4-momentum, and m is the mass of the vector boson) only in the Feynman calibration ($q_o = 1$).

So, the transition to the hot state of Universe was connected with the destruction of the Bose condensate and an increase of a Fermi gas pressure accordingly. In addition some time a temperature of background particles of Universe could remain equal or close to zero (the stage of the inflation). As a result the rest-mass of W^+, W^-, Z^o bosons have decreased so, that the weak interaction has stopped to be weak and all (or nearly so all) particles from a ground (vacuum) state started to participate in an installation of a thermodynamic equilibrium. The given phenomenon also has become the cause of an apparent increase of a density of particles in the early Universe. Suggesting, that mean density n_o of particles in the Universe did not vary at the same time and the script of the hot model in general is correct, we come to its following estimation $n_o \sim m_\pi^3 \sim 10^{-3}$ GeV^3 (m_π is a mass of a π meson). This result allows to give explanation to the known ratio [4] $H_o/G_N \approx m_\pi^3$, if to consider, that the Hubble constant H_o gives an estimation $1/H_o$ to the length $l \sim 1/(n_o\sigma_\nu)$ of free run of particles in "vacuum" on the modern stages of the Universe evolution (σ_ν is a scattering cross-section of neutrinos on a charged particle) and to take into account the estimation given earlier [5] for the gravitational constant G_N ($G_N \sim \sigma_\nu$). Thus the gravitational constant G_N is inversely proportional to the time of a free run of a charged particle in the neutrinos medium characterizing the kinetic phase of relaxation process in the Universe.

References

[1]. B. Leibundgut and J. Sollerman, *Europhysics News,* **32,** № 4, 121-125 (2001).
[2]. V.M. Koryukin, *Gravitation and Cosmology,* **5,** № 4, 321 (1999).
[3]. H.-J. Treder, *Gravitationstheorie und Aquivalenzprinzip* (Akademie - Verlag, Berlin 1971).
[4]. S. Weinberg, *Gravitation and Cosmology* (J. Wiley & Sons, New York, 1972).
[5]. V.M. Koryukin, *Izvestiya vuzov, Fizika,* № 10, 119-120 (1996).

Причинные свойства некоторых классов моделей пространства-времени[1]

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Анализируются проблемы причинности в моделях пространства-времени, допускающих распространение сигналов со сверхсветовой скоростью. Показано, что если распространение сверхсветовых сигналов описывается общековариантными уравнениями и, следовательно, осуществляется вдоль инвариантных кривых, как это имеет место в ряде известных моделей, то нарушения причинности отсутствуют. Показано также, что распространение сигналов со сверхсветовой скоростью не приводит к каким-либо парадоксам.

1. Введение

В последнее десятилетие в литературе достаточно широко стали обсуждаться модели пространства-времени, допускающие передачу сигналов со сверхсветовой скоростью. К таким моделям относятся, в частности, модели с лоренцевыми кротовыми норами [1,2], проходимыми в обоих направлениях, модели космических струн [3] и некоторые другие, модели распространения фотонов и нейтрино в гравитационном поле с учетом квантовых поправок [4], приводящие, при определённых условиях, к возможности распространения сигналов со сверхсветовой скоростью, а также некоторые гипотетические модели, предложенные для решения некоторых астрофизических проблем [5].

Обычно считается, что возможность передачи сигнала со сверхсветовой скоростью неизбежно ведёт к возможности передачи сигнала в прошлое, т.е. к нарушению причинности (см, например, [6,7]). В рамках специальной теории относительности эта неизбежность аргументируется следующим образом [6,7]: "Вы посылаете сигнал из A в B со сверхсветовой скоростью. Затем этот сигнал посылается обратно также со сверхсветовой скоростью, но в другой лоренцевой системе отсчёта. В результате сигнал может вернуться в исходную точку раньше, чем он был испущен". Поскольку считается, что возможность передачи сигнала в собственное прошлое приводит к парадоксам, то модели, допускающие распространение сигналов со сверхсветовой скоростью, долгое время не исследовались, а отсутствие таких сигналов часто рассматривается как необходимое требование к любой физической теории.

В настоящем докладе рассматриваются следующие основные вопросы:

- действительно ли возможность передачи сигнала со сверхсветовой скоростью неизбежно приводит к нарушению причинности?
- необходимые условия нарушения причинности в различных классах моделей;
- существуют ли парадоксы в моделях с нарушением причинности?

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- некоторые теоретические проблемы, связанные с существованием моделей пространства-времени с возможностью передачи сигналов со сверхсветовой скоростью и нарушениями причинности.

Существующие модели пространства-времени, допускающие распространение сигналов со сверхсветовой скоростью, можно условно разделить на два класса:

1 "топологические"(лоренцевы кротовые норы [1], космические струны [3] и др.),

2 теоретико-полевые (распространение фотонов и нейтрино в гравитационном поле со сверхсветовой скоростью [4, 8, 9]).

В связи с тем, что свойства моделей разных классов существенно отличаются, обсуждение перечисленных выше вопросов (кроме последнего) будет проводится для каждого класса отдельно.

2. Причинность в "топологических" моделях (на примере лоренцевых кротовых нор)

Обсуждение проблемы причинности в "топологических" моделях будет проведено на примере Лоренцевых кротовых нор, проходимых в обоих направлениях.

Простейшая модель пространства-времени с Лоренцевой кротовой норой получается путём приклеивания к внешнему пространству $M^4_{ext} = T_{ext} \times M^3_{ext}$ топологической "ручки", т.е. многообразия $M^4_{int} = T_{int} \times M^3_{int}$, где внутреннее пространство M^3_{int} имеет топологию цилиндра $M^3_{int} = I \times S^2$, . Здесь T_{int} и T_{ext} - внутренняя и внешняя временные координаты, M^3_{ext} - внешнее пространство, $I = (-L_1, L_2)$ - "ось"кротовой норы и S^2 - двумерная сфера. Величина $L = L_1 + L_2$ называется координатной длиной кротовой норы. Значения $L = L_1$ и $L = L_2$ соответствуют "левому"и "правому"отверстиям кротовой норы.

Пусть $t \in T_{ext}$ и $\tau \in T_{int}$ - внешняя и внутренняя временные координаты. Тогда при фиксированном значении τ кротовая нора соединяет точки внешней пространственно-подобной гиперповерхности $(t_1(\tau), M^3_{ext1})$ с точками внешней пространственно-подобной гиперповерхности $(t_2(\tau), M^3_{ext2})$, где в общем случае $t_1 \neq t_2$.

Пусть $\{\tau, \xi^1, \xi^2, \xi^3\}$ - локальные координаты кротовой норы, причём $-\infty < \tau < \infty$ - временная координата, ξ^1 - координата вдоль оси кротовой норы, а ξ^2 и ξ^3 - координаты вдоль отверстий кротовой норы, являющихся в простейшем случае двумерными сферами S^2.

Предположим для простоты, что координаты внешнего пространства являются сопутствующими левому отверстию кротовой норы. Тогда вблизи левого отверстия $-(\sigma_1 + L_1) < \xi^1 < -L_1$ имеем

$$t_{left} = \tau, \quad x^i_{left} = x^i(\xi^1, \xi^2, \xi^3), \tag{1}$$

тогда как вблизи правого отверстия $L_2 < \xi^1 < L_2 + \sigma_2$,

$$t_{right} = t_r(\tau,), \quad x^i_{right} = x^i(\tau, \xi^1, \xi^2, \xi^3). \tag{2}$$

Уравнения (1)-(2), определяющие склейку кротовой норы с внешним пространством, задают топологическую структуру пространства-времени и индуцируют граничные условия для внешней и внутренней метрик пространства-времени и полей источников. В результате, модель пространства-времени, содержащая лоренцевы кротовые норы должна рассматриваться в рамках граничной или смешанной граничной задачи [10], а не задачи Коши, как

это пытаются делать некоторые авторы [11–13], причём граничные условия, индуцированные условиями склейки (1)-(2), в силу определения геометрических объектов на многообразии, являющихся функциями точки [14], обеспечивают выполнение так называемых условий "самосогласованности", обсуждавшихся в [11–13]. Необходимо отметить также независимость условий склейки (1)-(2) от уравнений поля и внешних полей.

2.1. Принцип относительности и "парадокс близнецов" для Лоренцевой кротовой норы

Рассмотри частный случай условий (1) - (2), когда внешнее пространство является плоским, а кротовая нора соединяет события, разделенные пространственно-подобным интервалом. В силу последнего условия, в некоторой системе отсчёта внешнего пространства $t_{left} = t_{right} = \tau$, т.е. $t = \tau$ является временной координатой во всём пространстве-времени и кротовая нора соединяет события на одной и той же пространственно-подобной гиперповерхности. Предположим также, что во внешнем пространстве центры отверстий лежат на оси $z = x^1$.

Из условий склейки (1) - (2) видно, что движение одного из отверстий (в данном случае правого) кротовой не выводит его из гиперповерхности $t = \tau = const$, поэтому, в отличие от пространства Минковского, где все инерциальные системы отсчёта равноправны, в рассматриваемой модели условия склейки (1) - (2) выделяют класс инерциальных систем отсчёта внешнего пространства, в которых выполнены условия $t_{left} = t_{right} = \tau$. Поэтому принцип относительности не применим к движению отверстий кротовой норы во внешнем пространстве [15, 16], а рассуждения работ [1, 11, 12, 17] о неизбежном или "абсурдно лёгком"превращении кротовой норы в машину времени теряют силу [15, 16, 18]. Сделанное заключение согласуется с известными теоремами о причинной структуре пространства-времени и задаче Коши [19–21].

2.2. Нарушения причинности в моделях с лоренцевыми кротовыми норами

Оценим условия, приводящие к нарушению причинности в моделях с лоренцевыми кротовыми норами. Не уменьшая общности можно предположить, что размеры отверстий кротовой норы значительно меньше расстояния между ними во внешнем пространстве, а ось x^1 соединяет центра отверстий. Пусть, кроме того, для некоторого $\tau_1 > \tau_0$ выполнено неравенство $t_r(\tau_1) > \tau_1$. Рассмотрим световой сигнал, посланный в момент $t_r(\tau_1)$ от правого отверстия к левому через кротовую нору и возвратившийся, после выхода из левого отверстия, в исходную точку. Время, прошедшее между отправкой и получением сигнала равно:

$$\Delta t = \delta t_1 + \delta t_2 - \delta t_3 \tag{3}$$

где δt_1 и δt_2 - соответственно времена прохождения сигнала через кротовую нору и внешнее пространство, а $\delta t_3 = t(\tau_1) - \tau_1$. Очевидно, что нарушение причинности (существование замкнутых времениподобных кривых) возможно, если $\Delta t \leq 0$ (подробные оценки см. [10]).

Таким образом, нарушения причинности в моделях с лоренцевыми кротовыми норами, допускающими возможность передачи сигналов со сверхсветовой скоростью, зависит от условий соединения кротовой норы с внешним пространством-временем (1)-(2). В силу общековариантности уравнений поля и уравнений движения, условия (1)-(2), а следовательно, и выполнение или нарушение условий причинности, не зависят от движения отверстий кротовой норы или каких-либо иных физических процессов во внешнем пространстве [10,15,16,18]. Вместе с тем, условия соединения кротовой норы с внешним пространством дают дополни-

тельные условия на решения уравнений поля, автоматически обеспечивающие отсутствие каких-либо парадоксов.

Использованная аргументация применима и к случаю моделей с космическими струнами, где возможность распространения сигналов со сверхсветовой скоростью также имеет топологическую природу.

3. Причинность в теоретико-полевых моделях

Возможность распространения фотонов в гравитационном поле со сверхсветовой скоростью следует из эффективного действия для фотонов в гравитационном поле в однопетлевом приближении [8, 9]

$$S = \int d^4x\sqrt{-g}\left(-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{\alpha}{m_e^2}\left(aRF_{\mu\nu}F^{\mu\nu} + bR_{\mu\nu}F^{\mu\lambda}F^{\nu}_{\lambda} + cR_{\alpha\beta\mu\nu}F^{\mu\nu}F^{\alpha\beta}\right)\right), \qquad (4)$$

где $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ - тензор электромагнитного поля, $\alpha = 1/137$ - постоянная тонкой структуры, m_e - масса электрона, a, b и c - некоторые не равные нулю коэффициенты, точные значения которых приведены в [4, 8, 9]. В приближении слабого поля действие (4) приводит к уравнениям, характеристический конус которых в общем случае не совпадает с обычным световым конусом метрики пространства-времени,

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu, \qquad (5)$$

т.е. скорость распространения фотона может не совпадать с обычной скоростью света c. Геометрически это означает, что фотоны распространяются вдоль интервала ds^2_{ph}, который может быть положительным, отрицательным или нулевым в зависимости от направления распространения, поляризации и т.д. [8, 9]. Значения интервала $ds^2_{ph} < 0$ соответствуют распространению фотонов со сверхсветовой скоростью[2].

В простейшем случае распространение сигналов со сверхсветовой скоростью может быть описано с помощью изотропных интервалов некоторой эффективной метрики

$$d\widetilde{s}^2 = \widetilde{g}_{\mu\nu}dx^\mu dx^\nu, \qquad (6)$$

где $\widetilde{g}_{\mu\nu}$ - эффективный метрический тензор. В общем случае такое описание невозможно. Например, скорость фотона в гравитационном поле может зависеть от поляризации [4, 8, 9, 22], поэтому введение эффективной метрики не всегда дает полное описание распространения сигнала. Существенно однако, что распространение фотонов определяется общековариантными уравнениями, вытекающими из инвариантного интеграла действия (4). Следовательно, фотоны распространяются вдоль инвариантных кривых, не зависящих от движения источника или приемника.

Рассмотрим для простоты модель распространения сверхсветового сигнала в плоском пространстве-времени. Предположим, что сверхсветовой сигнал распространяется вдоль изотропных направлений некоторой эффективной метрики, изотропный конус которой не совпадает с изотропным конусом обычной лоренцевой метрики. А именно, рассмотрим плоскость Минковского с координатами (t, x) и интервалом

$$ds^2 = dt^2 - dx^2 \qquad (7)$$

[2]Предполагается, что метрика пространства-времени имеет сигнатуру $(+, -, -, -)$.

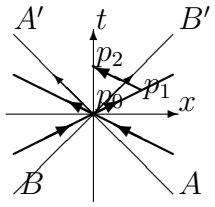

Рис. 1. Распространение сверхсветового сигнала в пространстве Минковского

и допустим существование сверхсветовых сигналов, имеющих инвариантный интервал

$$d\widetilde{s}^2 = u^2 dt^2 - dx^2 = 0 \tag{8}$$

где $u > 1$ - скорость распространения сигнала.

Пусть сверхсветовой сигнал испущен в точке $x_0 = 0$ в момент времени $t_0 = 0$ (событие p_0), получен в точке x_1 в момент t_1 и сразу же отправлен в обратном направлении (событие p_1) и, наконец, получен в исходной точке x_0 в момент времени t_2 (событие p_2) как показано на рис. 1, где тонкие линии AA' и BB' обозначают изотропный конус метрики Минковского (7), толстые линии обозначают изотропный конус эффективной метрики (8)), а стрелки показывают направления распространения сигналов.

Очевидно, что интервалы p_0p_1, p_1p_2 and p_0p_2 инвариантны относительно произвольных преобразований координат. Интервал (7) инвариантен относительно произвольных преобразований Лоренца

$$x' = \beta(x - Vt), \qquad t' = \beta(t - Vx) \tag{9}$$

где $\beta = 1/\sqrt{1 - V^2}$, тогда как интервал (8) примет вид

$$d\widetilde{s}^2 = \beta^2(adt'^2 - 2bdt'dx' - cdx'^2) \tag{10}$$

где $a = (u^2 - V^2)$, $b = (1 - u^2)V$ и $c = (1 - u^2V^2)$. Уравнение $d\widetilde{s}^2 = 0$ показывает, что в движущейся системе отсчёта сверхсветовой сигнал распространяется со скоростями

$$u'_+ = \frac{u - V}{1 - uV} \tag{11}$$

в положительном направлении оси x' (возможность $u'_+ < 0$ означает, что $t'_1 < t'_0$) и

$$u'_- = -\frac{u + V}{1 + uV} \tag{12}$$

в отрицательном направлении оси x'. Таким образом, в движущейся системе отсчёта скорость сигнала не только меняется по величине, но и становится пространственно асимметричной.

Уравнения (11) и (12) совпадают с известной формулой сложения скоростей. Следовательно, описание сверхсветовых сигналов с помощью изотропных кривых эффективной метрики (8) эквивалентно их описанию с помощью инвариантных пространственно-подобных кривых в обычной метрике Минковского (7).

В частности, для $V = 1/u$ уравнения (11), (12) дают $u'_+ = \infty$ и $u'_- = -(1+u^2)/2u$, события p_0 и p_1 в координатах (x', t') становятся одновременными ($t'_1 = t'_0$), но в обоих системах отсчёта (x, t) и (x', t') сигнал распространяется из p_0 в p_1 и из p_1 в p_2 но не наоборот (см. рис. 2). Пусть система отсчёта (x'', t'') движется относительно системы (x, t) со скоростью $V = -1/u$. Из (11) и (12) следует, что в системе отсчёта (x'', t'') одновременными будут

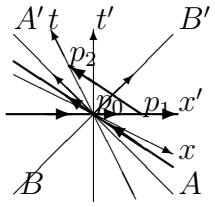

Рис. 2. Распространение сверхсветового сигнала в пространстве Минковского в движущейся системе отсчёта

события p_1 и p_2, причём сигнал по-прежнему распространяется из p_0 в p_1 и из p_1 в p_2 но не наоборот. Легко убедиться, что не существует системы отсчёта, в которой бы сверхсветовой сигнал распространялся из p_1 в p_0 или из p_2 в p_1. Это исключает возможность отправить сигнал в собственное прошлое.

Рассмотренный пример распространения сверхсветовых сигналов вдоль **инвариантных** изотропных интервалов некоторой эффективной метрики является **контрпримером** к известному утверждению, согласно которому распространение сигналов со сверхсветовой скоростью неизбежно ведет к нарушению причинности и возможности послать сигнал в собственное прошлое.

Обобщение рассмотренной модели на случай распространения сверхсветовых сигналов в гравитационном поле, в силу тривиальности, мы здесь рассматривать не будем.

Таким образом, в случае существования сверхсветовых сигналов, распространяющихся вдоль инвариантных пространственно-подобных интервалов (или вдоль изотропных интервалов эффективной метрики)

- существуют выделенные классы систем отсчёта;
- нарушается Лоренц-инвариантность;
- не происходит нарушений причинности.

"Стандартный" вывод о связи сверхсветовых сигналов с нарушениями причинности (см. например, [6, 7]) основан на предположении, что 3-скорость u сверхсветового сигнала относительно источника не зависит от движения источника. Это предположение, естественное в Ньютоновской механике, неприменимо к общерелятивистскому случаю распространения сверхсветовых сигналов, описываемых общековариантным уравнениями, в частности к рассматривавшемуся рядом авторов [4, 6, 8, 9, 22] распространению фотонов и нейтрино в гравитационном поле со сверхсветовой скоростью.

Отметим также, что модель распространения фотонов и нейтрино со сверхсветовой скоростью в гравитационном поле демонстрирует принципиальную возможность сосуществования в одном пространстве-времени нескольких метрических, точнее, причинных структур. Подобное предположение высказывалось ранее автором [23]. Некоторые космологические следствия этой гипотезы анализировались сравнительно недавно в [5], а одна из возможных геометрических моделей и некоторые ее следствия анализировались в [24–26].

Заключение

Итак, мы убедились, что, во-первых, наличие сигналов, распространяющихся со сверхсветовой скоростью, не обязательно приводит к нарушению причинности. Во-вторых, мы видели,

что, вопреки распространенному мнению, нарушение причинности (наличие в пространстве-времени замкнутых мировых линий) не приводит к каким-либо парадоксам.

В заключение отметим некоторые нерешённые теоретические проблемы, связанные с обсуждаемыми моделями.

В первую очередь возникает проблема корректности моделей, поскольку выше мы ограничились, как и авторы цитируемых работ, чисто геометрическим рассмотрением, не анализируя конкретные уравнения поля и уравнения движения. Поэтому неясно, имеют ли рассмотренные модели физический смысл.

В случае лоренцевых кротовых нор необходимо решать смешанную краевую задачу [10] и исследовать полученные решения на отсутствие или наличие сингулярностей. Такого рассмотрения до настоящего времени никто не проводил ввиду его крайней сложности. Кроме того, возникает вопрос о физическом смысле смешанной граничной задачи, возникающей в результате склейки кротовой норы с внешним пространством, поскольку граничные условия записываются вдоль времениподобных областей. Наконец, остается открытым вопрос о возможности прохождения через кротовую нору реального сигнала или физического объекта: как показывают некоторые работы [27, 28], в случае симметричных кротовых нор возмущения полей (физические сигналы) в некоторых моделях не проходят через через кротовую нору, даже если она геометрически проходима.

В случае фотонов и нейтрино вывод о возможности их распространения в гравитационном поле со сверхсветовой скоростью сделан в линейном приближении слабых возмущений, не оказывающих обратного влияния на метрику пространства-времени [4, 6, 8, 9, 22]. Корректность такого рассмотрения неочевидна, поскольку использовавшийся при этом метод ВКБ не применим к случаю существования нескольких характеристических конусов.

Список литературы

[1] M.S.Morris, K.S. Thorne, U. Yourtsever, *Phys. Rev. Lett.*, (1988), **61**, 1446.

[2] M.S. Morris, K.S. Thorne, *Amer. J. Phys.,* (1988), **59**, 395.

[3] J.R. Gott, *Phys. Rev. Lett.*, (1990), **66**, 1126.

[4] I.T. Drummond and S.J. Hathrell, Phys. Rev. D **22** (1980), 343.

[5] L. Gonzales-Mesters, Nucl. Phys. Proc. Supl., **48**, (1996), 131-136.

[6] A.D. Dolgov and I.D. Novikov, Phys. Lett. B (1998), **B442**, 82-89.

[7] S.W. Hawking, Phys. Rev. D., **D46**, 1992, 603.

[8] R.D. Daniels and G.M. Shore, Nucl. Phys. **B425** (1994), 634.

[9] G.M. Shore, Nucl. Phys. **B460** (1996), 379.

[10] М.Ю. Константинов. Гравитация и космология, (1997), т. 4, N 4, с. 299-304.

[11] I.D. Novikov, *JETF* (1989), **95**, 769 (in Russian).

[12] V.P. Frolov, I.D. Novikov, *Phys. Rev. D.*, (1990), **42**, 1057.

[13] A. Carlini, V.P. Frolov, M.B. Mensky, I.D. Novikov, H.H. Soleng, *Int. J. Mod. Phys. D.*, (1995), **4**, 557.

[14] S. Kobayashi, K. Nomizu, Foundation of differential geometry, v. 1-2, New-York - London, 1963.

[15] М.Ю. Константинов. Известия вузов. Физика. 1992, N 12, с. 83.

[16] M.Yu. Konstantinov. Int. J. Mod. Phys. D, (1995), v. 4, 247.

[17] M. Visser, *Phys. Rev. D*, (1997), **55**, 5212.

[18] M.Yu. Konstantinov. Causality in topologically nontrivial space-times Proceedings of the 8th Marsel Grossmann Meeting on General Relativity, Eds. Tsvi Piran and Remo Ruffini, World Sci., Singapore, 1999, 590-592.

[19] S. W. Hawking and G. F. R. Ellis, The large-scalle structure of space-time, Cambridge University Press, 1973.

[20] J.W. York - in: Sources of Gravitational Radiation, ed. L. Smarr, Cambridge, Cambridge U.P., 1979, p. 83.

[21] A. E. Fisher and J. E. Marsden, *Initial values problem and dynamical formulation of general relativity*, in: General Relativity. An Einstein Centenary survey, Eds. S.W. Hawking and W. Israel, Cambridge U.P., 1979.

[22] S. Mohanty, A.R. Prassanna, Nucl. Phys. **B526** (1998), 501-508.

[23] М.Ю. Константинов, в кн.: Проблемы теории гравитации и элементарных частиц, вып. 16, М., Енергоатом, 1985, с. 148-157.

[24] M.Yu. Konstantinov, Comments on the Hypothesis about Possible Class of Particles Able to Travel Faster than Light: Some Geometrical Models, Preprint RGS-VNIIMS-006/97 (preprint archive: physics/9705019)

[25] M.Yu. Konstantinov, Some Geometrical Models for the Possible Class of Particles Able to Travel Faster then Light, in: Proceedings of the 8th Marsel Grossmann Meeting on General Relativity, Eds. Tsvi Piran and Remo Ruffini, World Sci., Singapoure, 1999, 1435-1438.

[26] М.Ю. Константинов, в кн: "Труды 10-й Российской гравитационной конференции. Владимир, 20-27 июня 1999 г.", Гравитация и космология, (1999), т. 5, N 4 (приложение), с. 48-56.

[27] К.А. Бронников, Г.Н. Шикин, Модели самогравитирующих систем с классическими полями и их устойчивость. В сб.: Итоги науки и техники. Сер. Классическая теория поля и теория гравитации, т. 2, М., ВИНИТИ, 1991, сс. 4-55.

[28] N.D. Birrel and P.C.W. Davis, Nature, (1978), **272**, N 5648, 35-37.

String and loop quantum gravity theories unified in Platonic Ether With proof of Fermat´s Last Theorem and Beal´s Conjecture

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A significant tendency in current Western cosmology and theoretical physics is to "return to the Greek natural philosophy of mathematical beauty and perfect symmetry" (Winterberg [1998]). However, with the said and a few other notable exceptions, the inflated terminology and phraseology more and more extempore reverberated thereby are now assuming such a strangely wonderland ring that one might ask how deep the contiguity with the actual ancestors really is. None-the-less, even as a fairly capable worker in a closely adjoining branch of rational descriptive research with undeniable reproducible facts and findings, it is wise to join the latter-day Lingua Franca and let authoritative phenomenological as well as idiomatic citations from the most distinguished, nominally panepistemic organs of open scientific communication be the pass-words to the esotheric circles - of course not including the free *avantgarde* congregation here - that would otherwise accuse the oblique contributor of own charlatan parlance and exegesis.

A quite fresh and rich example that comprehensive quotations might also lead into the conceptual picture better than a secondary explication is the following excerpt from the leading *Science* magazine of an allegedly advanced *news focus* on "string and loop quantum gravity theories" (Cho [2002]), which also serves as an excellent introduction and allocation of this paper but would surely not be widely accepted had just I written it: "Space isn't smooth, and time does not flow…This foaminess has hamstrung physicists striving to explain how the universe sprang into existence and why it appears the way it does today…Even the leading candidate for a "theory of everything" - string theory - side-steps the sticky froth…but a few physicists have plowed headlong into the quantum foam. They've concocted a theory that precisely describes space-time on the smallest length and time scales. Loop quantum gravity, as it is called, directly reconciles the minutiae of quantum mechanics with Einstein´s general theory of relativity, which describes gravity as the warping of the very fabric of space-time. It also predicts that space comes in discrete chunks, so that there is a smallest possible area and a smallest possible volume. Just as matter is made of atoms and elementary particles, space consists of tiny indivisible bits…Loop quantum gravity builds the 'geometry of space-time' from scratch…String theory…which assumes that every fundamental particle is really a tiny loop known as a superstring…suffers from a fundamental weakness…the strings move in a space-time whose shape has been chosen from the beginning, as if they were actors on a previously constructed stage. A truly fundamental theory of gravity, everyone agrees, would build the stage itself…The solution describes slices of space each frozen at a fixed time. A single solution resembles a Cubist's dot-to-dot drawing….you can view the nodes as chunks of space and the links as paths that tell you which chunks talk to each other…As a result area can come only in discrete amounts, just as any sum of money can be counted in a whole number of pennies. Thus the spin networks tell theorists how to put space together, one little patch at a time.…both string theorists and loop quantum gravity researchers say they hope that their two approaches will merge someday…when light rattles through chunky space-time."

It is true that this was an extensive extract. But it covers a whole modernistic panorama and imagination of high pertinence here, and represents a crucial stage in the ongoing backwards-to-the-future reorientation to the classical roots; in the rather typical, almost surrealistic passage just cited

apparently groping from the monistic Big Bang ready-make via a slightly more evolved hand-in-glove settlement towards, if at all reaching target, the original dualistic tension and interplay "between the curved and the straight" at "the heart of Greek geometry and indeed of geometry in general" (Netz [2002]).

Where Plato stands forth as the generic figure most commonly implied - and occasionally spelt out. In the philosophical realm, for instance, Davies has reported on the "Platonist-Intuitionist debate about the nature of mathematics" where his specific project is "how to build an infinite machine within a continuous Newtonian Universe" [2001]. This essentially advances the Euclidean Universe and it is therefore of relevance to complement the Platonist-Intuitionist view with a 'Platonist-Institutionist' description of how then the natural progenitors truly conceived and used numbers as veritable building bricks in a productive synthesis by virtual self-assembly (Ikkala and ten Brinke, Kato, Whitesides and Grzybowski [all 2002]) of direct mathematical and physical space alike. Why differential calculus so entirely entered the "infinitely descending" (www [1997]) Zeno paradox retreat - or parenthesis - of the last few centuries poses a fascinating epistemology as such (Trell [2003]), but is here omitted since it has little to do with the prototype number practice (rather than number theory) which comprises the genuine heritage of the subject matter.

Since Davies and the type of infinite machine he promotes thus appear under Platonist label, it must be important as the true epistemology in kind to reintroduce what Plato and his contemporaries and disciples in fact thought and taught on the subject. In other words, Davie's explicitly intuitive position may need to be replaced by a "Scientific Realist" (Kukla [1998]) institution of the protagonists' actual posits on "the nature of mathematics" (Davies [2001]) as well as real realised space. Because they did not see substantial difference between matter and mathematics, between numbers and things. As Noel [1985] has expressed it: “the old Greek are famous for a completely brilliant idea, namely, to use spatial images to represent numbers”, where, notably, “Euclid’s mathematics was closely associated with his concept of the world, which in accordance with Aristotle was that the Universe was enclosed in a sphere, in the interior of which space and the bodies full-filled the properties of Euclidean Geometry."

In the current nanotechnological era there is as mentioned a strong Renaissance in that direction (Winterberg [2000]) but partially incoherent with its sources. When it is concluded that "Plato would have insisted that God created triangles, out of which the Universe is made" while "Platonists of the early 21st century may insist that what God created were mathematical objects, called superstrings, out of which the world is made" (Fraser [2001]) this is in vital respects a deviation from both the archetypes and the prototypes at hand. Superstrings are curved (Cho [2002]) but Plato (in *Timaios*) primarily reserved the spherical harmonies for the celestial rather than the terrestrial symmetries. For the latter he employed the (*per se* already well known) regular polyhedra "developed from the unit sphere" (Sutton [2001]) and in consequence lines up more with those today who argue, again, that in the dualistic interplay "between the curved and the straight" which is at "the heart of Greek geometry and indeed of geometry in general" (Netz [2002]), it is the rectilinear 'canvas' (Kamionkowski [2002]) that provides the flat screen (Bachall et al. [1999], Rees [2000]) of our physical realisation.

And the Platonic solid originally designated for this equally mathematical as material matrix was the cube, "completely filling the space with copies of itself" (Sutton [2002]). Triangles were engaged at many levels, but when it comes to their role as elementary spatial constituents, the involved "triangular part is a diagonally divided quadrate, four of which recreate the whole square, which then form cubes" (Sutton [2002]).

What Plato really insisted is therefore that what God created, or actually "folded from planar substrates" (Whitesides and Grzybowski [2002]), were uniform cubes, out of the atomic clone of which geometric Earth and Ether are made. And this was the general idea of the age since time im-

memorial, including the consequential numerical bearings. For instance, the geometry that Euclid learnt from his Ionian teachers "was originally based on watching how people built", and "the measurement of volume by the number of cubes with sides of standard length required to fill a solid space was probably first used by the Sumerians, who built with bricks" (Hogben [1937]).

How did the building proceed? There are at least two main continuous alternatives, one of which has been brought to the fore again both theoretically by e.g. Roger Penrose [1995] and in the recent nanotechnological "layer-by-layer" material self-aggregation and self-organization (Velikov et al [2002]). It can be described as a stepwise eccentric winding over the surface of the expanding box and has been used to literally underpin a previous proof of Fermat´s Last Theorem (FLT) (Trell [1997,1998a, 2002]).

The other, and most straightforward and practically manageable, is to first pave the floor, starting by a row from a corner along the side, after that turning for the next row, and so on till the ground square or rectangle is filled. Then, with unbroken succession in reverse order in the next tier, and so on, till the box is filled in a hence really analytical way, too, i.e. continuous, spacefilling and non-overcrossing. This mode would probably be closest at hand for Diophantos as well as for Pierre de Fermat, and will be focused upon in the continuation.

For it is important, that the comparative late Diophantos himself "stated the traditional definition of numbers to be a collection of units" when in his equations they "were simply put down without the use of a symbol" (Heath [1964], Zerhusen [1999]). The effective quantum leap in relation to modern linear functions is of course the integer instead of point nature of the numerical unit. And pointless, too, would be to make this a heuristic controversy since it is all about reality: reality for the founders, reality of means and ends; reality of the very facts and findings of the case, i.e., that when ancient mathematicians well up to Cardano calibrated numerical and physical space alike they used what during thousands of years between the Sumerian bricks and Roman *tessellas** was the most refined of manufactured self-assembling forms: the cube, the irreducible whole- number bit, **One**, a cubicle, kaba, 'cubit'™, "nanocube" (Murphy [2002]) of arbitrary unit side, providing the atomic set of a myriad literal dice not alone for God to throw but for themselves to stow by cumulative fulfilment (Noel [1985], Sutton [2002]) of their own, "nanobox" (Murphy [2002]) properties.

In order to reconstruct the original procedure, it may be reminded that gauging and calculations in those days were much like surveying (Noel [1985]). For the first degree, *positio* alignment, the unit number cells then automatically deliver the measuring-rod by longitudinal plus or minus stacking like in the contemporary *abacus* (and the Inca *Quipu* threads) over a single axis, here illustrated as the vertical (Fig. 1). However, the added, in a double sense manifold value of the direct spatial realisation of whole numbers does not become apparent until with Diophantos formalising their exponentiations and subsequent equations. The natural procedure that offers for a serial power expansion is a sideways instead of length- wise multiplication of the digit by itself, producing at the second degree stage a square tile, step-by-step like the Sumerians did till the quadrate or rectangle is continuously and non-overcrossingly tessellated (Fig. 2). Then, in the same fashion, next layer is filled, and next, and next, till the resulting first-order third degree 'hypercube' is also analytically completed (Fig. 2).

In turn, that 'hypercube of the first order' in same periodic progression re-multiplied by the base number yields a 4th power in the shape of a quasi-one-dimensional 'hyper-rod of the second order', which in forthcoming multiplications generates a 5th degree second order hypersquare, then 6th degree hypercube, then 7th degree hyperrod, 8th degree hypersquare etc. in an endless cyclical "self-assembly at all scales" (Whitesides and Grzybowski [2002]) that eventually contains all whole-number (and fractional) powers that there at all are (Fig. 2).

* Oxford Concise Etymological Dictionary of the English Language: *Tessella* is Latin for little cube, diminutive of *tessera* = a die (to play with), a small cube. *Tile, tiling* are derived from another Latin word, *tegula*.

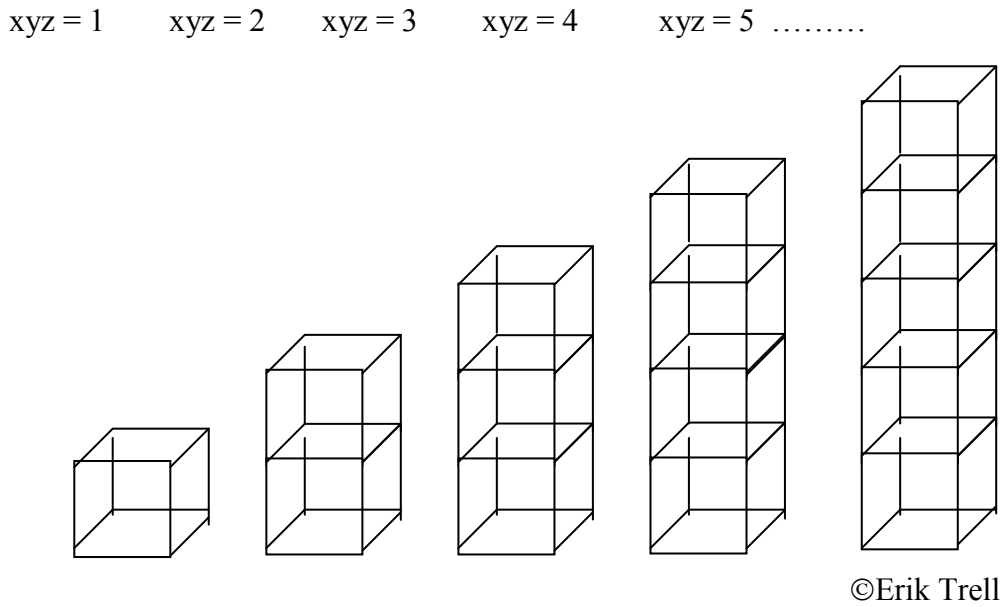

Fig. 1. Three-dimensional Diophantine whole-number cells (or, after Penrose [1995], polyominoes), one-dimensionally joined together in the arbitrary vertical direction to infinite series of integers of the first degree by the same discrete amount of the ground unit cubicle.

It is important to re-emphasise that the build is successive also within each sheet by the zigzag lining up of the individual tessellas so that they never clash. The entire Diophantine equation Block Universe is thus generated by a recursive, perpendicularly revolving algorithm in a maximum of three dimensions, thereby reproducing the hierarchically retarded, non-overcrossing, i.e. analytical space-filling of consecutively larger constellations, imaginable up to the size and twist of galaxies, no matter if taking place during actual time or an instantaneous phase transition in the sufficient ordinary Cartesian co-ordinate frame. A stepwise continuous "rod-coil-rod…self-assembly of phase-segregated crystal structures" (Kato [2002]) - which "in turn form assemblies or self-organize, possibly even forming hierarchies" (Ikkala and ten Brinke [2002]) - precipitates in a completely saturating, consecutively substrate-consuming way, displacing other stepwise cumulative syntheses (Fig. 2).

This is of utmost relevance, since, with bearing to and like Fermat´s Last Theorem (FLT), "far from being some unimportant curiosity in number theory it is in fact related to fundamental properties of space" (www [1996]) as well as of integers (www [1997]). And the geometrical uniformity, that all whole-number powers from n = 3 and infinitely onwards are realised in sufficiently three dimensions as saturated regular parallelepipeds which per primordial definition are composed by integer blocks alone, is of equal cardinal importance for the demonstrations *ad modum* Cardano to be exposed in the continuation.

That the (Western) situation was essentially the same up to the days of Cardano and hence also current for Fermat is namely another undeniable mathematical and philosophical fact, as most clearly demonstrated by the former in his Ars Magna [1545]. Quoted from Parshall [1988]; "For quadratic equations, Cardano, like his ancestors, built squares, but for third degree equations, he constructed cubes". He concluded "that only those problems which described some aspect of three-dimensional space were real and true. In his words: *"For as positio [the first power of the unknown] refers to a line, guadratum [the square of the unknown] to a surface, and cubum [the unknown cubed] to a solid body it would be very foolish for us to go beyond this point. Nature does not permit it"*" (Ib.).

That indeed Nature does not allow a truly analytic (that is, continuous, space-filling and non-overcrossing) simultaneous physical distribution over more than three linearly independent dimensions had been shown already by Aristotle, and so was the state of the art also for Fermat, when in the exclaimed (but unexplained) *demonstrationem mirabilem* in 1637 of his last theorem he ma-

nipulated plain *"cubos"* in equal *en bloc* manner without the use of algebraic symbols (www [1997]).

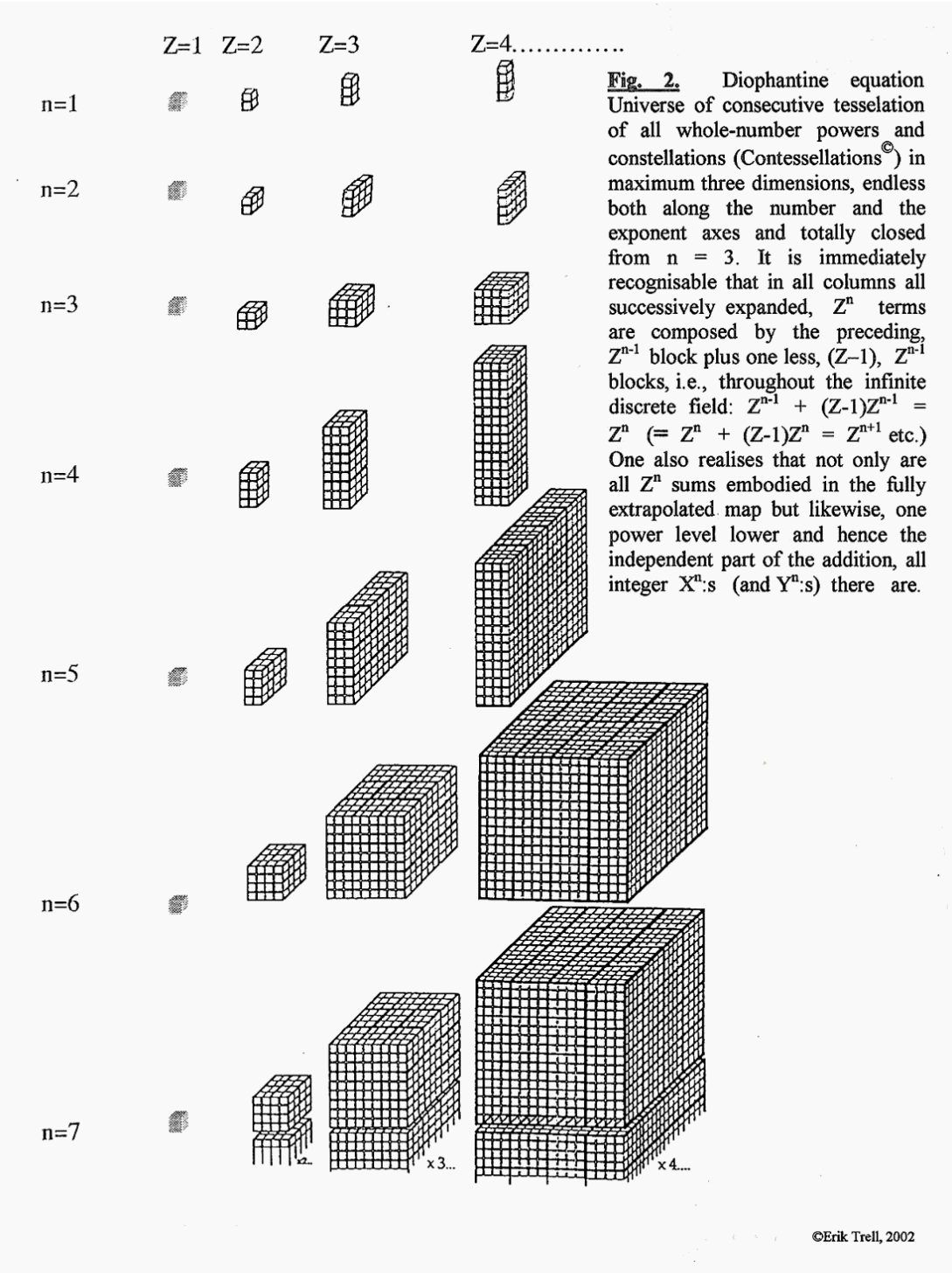

Fig. 2. Diophantine equation Universe of consecutive tesselation of all whole-number powers and constellations (Contessellations©) in maximum three dimensions, endless both along the number and the exponent axes and totally closed from n = 3. It is immediately recognisable that in all columns all successively expanded, Z^n terms are composed by the preceding, Z^{n-1} block plus one less, (Z–1), Z^{n-1} blocks, i.e., throughout the infinite discrete field: $Z^{n-1} + (Z-1)Z^{n-1} = Z^n$ (= $Z^n + (Z-1)Z^n = Z^{n+1}$ etc.) One also realises that not only are all Z^n sums embodied in the fully extrapolated map but likewise, one power level lower and hence the independent part of the addition, all integer X^n:s (and Y^n:s) there are.

But whereas Cardano "was unable to conceive of….a four-dimensional figure" geometrically (Parshall [1988]), this, and its continuation may well have been that instant flash of insight for the one century younger Fermat mind: just perpetuating the identified row-rectangle-octagon cycle to ensuing powers by the same undulating iteration and reiteration of the ground unit cube which comprised the genuine whole-number atom of the still prevailing protagonist era. The consequences would have been immediately recognised, too, for Fermat, but why he did not pass on the veritable blockbuster remains as an enigma. Perhaps he did not want to destroy future number theory fun, or it was just an act of that cryptic jeopardy game which seems to have been going on in the esoteric circles when mathematics was often a jealously protected secrecy.

While the previous proof of FLT (Trell [1997, 1998a]) follows the horizontal axis of Fig. 2, the present one engages the vertical. Thus considering the stepwise growth of each number for every new power, it is clearly an ascending differential function, too, and as such exhaustive, that is, filling and so occupying the whole space by its continuous iteration.

As demonstrated in Fig. 2, the second degree corresponds to a two-dimensional square in the arbitrary z direction by adding to the one-dimensional number column, X^1 (= X), one less further such columns: $X + (X-1)X = X^2$. The ensuing stage is equally straightforward. It is a periodical twisting, or unwinding of the space, where the third degree in like manner is entered along the x axis by the continued zigzag addition of $(X-1)\ X^2$ planes: $X^2 + (X-1)X^2 = X^3$.

And so it continues. Focusing on the stepwise growth of the exponents of all separate integers, FLT and the latter-day progeny called Beal's Conjecture (BC) can be proved, too, by this complementary "dynamical evolution of our toy model universe" (Penrose [1995]), which will here be performed in algebraic notation. Expressed in the forefather FLT designation, BC states that all possible whole-number power, $X^n + Y^m = Z^p$, additions must share an irreducible prime factor in all its terms (Mauldin [1997-], Mackenzie [1997]).

From what has been said earlier and by extrapolation from Fig. 2, it can be observed that all manifold blocks grow from the preceding one in the same column by adding upon this one less of the same than its base number:

$$X^n + (X-1)X^n = X^{n+1}$$

This borders to trivial but has profound bearings and consequences, notably in regard of the prevailing X = integer requisite. First, it is a universal relation; All X^n.s are represented, both by the first summand term and by the sum one step up (or successively higher by the relations $X^n + (X^2-1)X^n = X^{n+2}$ and, with non-integer roots of the multiplicative coefficient, $X^n + (X^3-1)X^n = X^{n+3}$, $X^n + (X^4-1)X^n = X^{n+4}$ etc. ad infinitum, according to the general formula, $X^n + (X^p-1)X^n = X^{n+p}$, where the specific case, p = n or multiples thereof, is excluded from integer solutions since when by definition X^n has a whole-number n:th root, (X^{n-1}) cannot have one).

It strikingly reminds of the actual world where three dimensions likewise are the most in which a continuous physical realisation can be simultaneously distributed in a non-overcrossing and space-filling, that is, analytical order. Already Aristotle deduced that with additional extensions the geodesics will get entangled by their equally higher-dimensional co-ordinate points no longer being able to avoid colliding with each other within one and the same static compartment. Also by observations on the own free mobility in experienced space but fixed transport in time he reached conclusions akin to modern expressions like that "invariant...orthogonal transformation of co-ordinates" can lastingly keep clear of obliterating themselves in a given neighbourhood over at the most three linearly independent axes so that when "in the theory of relativity, space and time co-ordinates appear on the same footing", the corresponding Lie algebra, or 4x4 matrix "inhomogeneous Lorentz transformations" must contain a "translational part" (Carmeli [1977]). The latter is here offered, too, as the perpetual way out from the final cubicle recess in a filled power box to the next. And of course, in such a sufficiently three-dimensional space there is a way out also for translation in purely relational time (Trell [1984]).

The principal condition is that all X^n.s are regenerated in the Z sum one power higher whereas the Y term is a full member only when its (X-1) or (X^p-1) multiplicator has an integer n:th root - and when not can still be retrieved and mobilised as a discrete factor subset within the sum block. Then, one starts to realise that $X^n + (X-1)X^n = X^{n+1}$ (etc.) is also the unique, i.e., the only possible non-overlapping or non-gapping binary n =3 manifold tessellation in the entire whole-number n >2 exponential space, which naturally verifies FLT by exclusion and the secondary BC by the inclusion in all terms of the common irreducible prime factor in X.

This is best mathematically expressed by the regular differential chain equation:

$$X^1 + (X-1)X^1 + (X-1)X^2 + (X-1)X^3 + (X-1)X^4 \ldots + (X-1)X^{n-1} = X^n$$

Which can be further generalised to

$$X^p + (X-1)X^p + (X-1)X^{p+1} + (X-1)X^{p+2} + (X-1)X^{p+3} \ldots + (X-1)X^{p+(n-1)} = X^{p+n}$$

hence providing a formal mathematical proof of the uniqueness of the ascending differential function by its "layer-by-layer…complete close-packed" (Velikov et al. [2002]) continuous iteration gradually sweeping over and so covering the entire Diophantine equation space. FLT and BC are demonstrated in the passing since all *integer*^{n} + *integer*^{n} additions in the exhaustive set yield *integer*$^{(\geq)n+1}$ sums, and the mutual X term obviously shares irreducible prime with itself.

Yet it may be of interest to illustrate the situation more expressively. The *sine qua non* of FLT and BC is the pure integer requirement of all the X, Y, Z base numbers, i.e. that the simultaneous 'external' coefficient of their X^n, Y^m and Z^p terms = 1. It is a binary splicing already at the outset absorbing all ligands in the solution by their mutual double-bonds and thus works like a global Eratosthenes' sieve (Noel [1985]), filtering the space from infinitely ascending 1^n, 2^n, 3^n, 4^n, 5^n…X exponential series so that the horizon for lower base number power inclusions is gradually pushed up precisely out of reach.

This goes over all magnitudes of n, even n = 1, because, for instance, 2 can only be combined with 4 to form 6. However, in that power it is an unbound relation since 2 can be combined with endlessly many other integers to form endlessly other integer sums, all members of the X^1 subset. When the power of the sum is 2, the situation is the same because it is formed by two basically first-degree terms; $X^1 + (X-1)X^1 = X^2$, and X^2 can thus be added together by other first-degree terms which might even be squares.

But from $X^2 + (X-1)X^2 = X^3$ and onwards the relation is locked in all its members; the first term X^2 piece exactly and exclusively determining also the unique missing second degree quantitative fraction delivered to the sum member of the common set which exactly and exclusively has to be filled by the missing puzzle piece of the addition. By such homogeneity of its algorithm, the totality of binary Diophantine additions comprised by the universal $X^n + (X-1)X^n = X^{n+1}$ (etc.) equation technically forms a folded but wholly even and dense X^n membrane, or 'n-brane', which, at all its points, by a mathematically equally constant, fixed and unbroken gear of itself lifts itself to the next level of itself. The totality elevates to the totality, in just one and the shortest rise, between one floor and the next, all monolayer shafts in the single interstice filled to the last unit corner, doubly obstructing other manoeuvres. In consequence, FLT and BC are proved by the effective displacement of other, necessarily higher solutions, by the gradual occupation from the bottom of all lowest solutions with the universal X^n as first term.

From the whole-number condition of the second term it is possible to regenerate <u>all</u> FLT and BC additions, most transparently by reformulating the equation to:

$$(X^n + 1)^n + X^n(X + 1)^n = (X^n + 1)^n + [(X^n)^{-n}(X^n + 1)]^n = (X^n + 1)^{n+1}$$

This is clearly in ground level exponential state as shown when posed as

$$1^x (X^n+1)^n + 1^x X^n(X^n+1)^n = 1^x (X^n+1)^{n+1},$$

and is accordingly unique already because of one rational solution alone to equations with all base terms of degree 2 and over. It is easy to exemplify for any X^n, e.g. $5^{13} = 1220703125$, when the coupled equation becomes:

$$(1220703126)^{13} + (1220703125) \times (1220703126)^{13} = (1220703126)^{14},$$

that is, $(1220703126)^{13} + [5(1220703126)]^{13} = (1220703126)^{14}$, and indeed for any magnitude, e.g., when $X = 12345^{6789}$, and n = 6789:

$$(12345^{6789}+1)^{6789} + (12345^{6789}) \times (12345^{6789} + 1)^{6789} = (12345^{6789}+1)^{6790} =$$
$$(12345^{6789}+1)^{6789} + [(12345)(12345^{6789} + 1)]^{6789} = (12345^{6789}+1)^{6790}$$

This extraction of all second terms can be systematised by Davies "brute force" variety of Eratosthenes' sieve [2001], viz. first let it (the infinite machine) "solve the problem for $n = 1$"; then it "passes to…. $n = 2$" and so on "down the chain": X = 1:

$$\text{for } 1^1 : (1+1)^1 + [(1^1)^{-1}(1^1+1)]^1 = (2)^1 + (1 \text{ x } 2)^1 = (2)^2 ;$$
$$\text{for } 1^2 : (1+1)^2 + [(1^2)^{-2}(1^2+1)]^2 = (2)^2 + (1 \text{ x } 2)^2 = (2)^3 ;$$
$$\text{for } 1^3 : (1+1)^3 + [(1^3)^{-3}(1^3+1)]^3 = (2)^3 + (1 \text{ x } 2)^3 = (2)^4 ;$$
$$\text{for } 1^4 : (1+1)^4 + [(1^4)^{-4}(1^4+1)]^4 = (2)^4 + (1 \text{ x } 2)^4 = (2)^5 ;$$
$$\text{for } 1^5 : (1+1)^5 + [(1^5)^{-5}(1^5+1)]^5 = (2)^5 + (1 \text{ x } 2)^5 = (2)^6 ;$$

etc. ad infinitum;

And X = 2

$$\text{for } 2^1 : (2+1)^1 + [(2^1)^{-1}(2+1)]^1 = (3)^1 + (2 \text{ x } 3)^1 = (3)^2 ;$$
$$\text{for } 2^2 : (4+1)^2 + [(2^2)^{-2}(4+1)]^2 = (5)^2 + (2 \text{ x } 5)^2 = (5)^3 ;$$
$$\text{for } 2^3 : (8+1)^3 + [(2^3)^{-3}(8+1)]^3 = (9)^3 + (2 \text{ x } 9)^3 = (9)^4 ;$$
$$\text{for } 2^4 : (16+1)^4 + [(2^4)^{-4}(16+1)]^4 = (17)^4 + (2 \text{ x } 17)^4 = (17)^5 ;$$
$$\text{for } 2^5 : (32+1)^5 + [(2^5)^{-5}(32+1)]^5 = (33)^5 + (2 \text{ x } 33)^5 = (33)^6 ;$$

etc. ad infinitum;

and X = 3;

$$\text{for } 3^1 : (3+1)^1 + [(3^1)^{-1}(3+1)]^1 = (4)^1 + (3 \text{ x } 4)^1 = (4)^2 ;$$
$$\text{for } 3^2 : (9+1)^2 + [(3^2)^{-2}(9+1)]^2 = (10)^2 + (3 \text{ x } 10)^2 = (10)^3 ;$$
$$\text{for } 3^3 : (27+1)^3 + [(3^3)^{-3}(27+1)]^3 = (28)^3 + (3 \text{ x } 28)^3 = (28)^4 ;$$
$$\text{for } 3^4 : (81+1)^4 + [(3^4)^{-4}(81+1)]^4 = (82)^4 + (3 \text{ x } 82)^4 = (82)^5 ;$$
$$\text{for } 3^5 : (243+1)^5 + [(3^5)^{-5}(243+1)]^5 = (244)^5 + (3 \text{ x } 244)^5 = (244)^6 ;$$

etc. ad infinitum;

And so it goes on, for every consecutive X and every consecutive n, and hence, for every whole-number X^n introjected in the second term there is but one pure FLT/BC equation where all terms are ground whole-number powers, i.e., in the irreducible form with all external coefficients = 1 [that, for instance, $(82)^4 + (3 \text{ x } 82)^4 = (82)^5$ can be expressed as e.g. $(6724)^2 + (60516)^2 = (37073984321)^1$ does not alter that], screening off other solutions.

Because the equation thus drains the whole space of binary additions of whole-number powers it also proves both FLT and BC since (here stated in most general form) $(X^n+1)^n + [(X^n)^{-n} (X^n+1)]^n = (X^n+1)^{n+1}$ excludes nth power sums (FLT), and the mutual (X^n+1) shares least prime factor (BC).

In conclusion, what has been quite extensively done here is a "brute force" (Davies [2001]) exposition that every discrete X, Y and Z power can be explicitly retrieved by a simple, but universal numerical formula. As so much brute force it is actually superfluous because contemplating that the entire whole-number Diophantine Equation Block Universe chart of Fig. 2 and its infinite extrapolation really comprises each and every separate element in its total space and that each and every of them likewise has a specific numeric formula of its solitary constitution, it is indeed almost a truism that there cannot be more formulas either so that both FLT and BC instantly follow. However, as "in the natural selection of ideas, the standard of rigorous, geometrical demonstration applied to algebraic fact which Cardano adopted represented a favourable variation in the theory of algebra" (Parshall [1988]), the proof can be given a more distinct infinite machine execution in sole geometrical terms, utilising the pivotal (re)discovery that all whole-number powers can be represented as regular parallelepipeds.

It is then possible to build right upon Cardano's solution of the third degree equation, which he performed by cubes, in which category the rectangular parallelepipeds were also included (Fig 3 a-e). There, like "Cardano asked his readers to complete the cubes formed on AB+BC, in just the same way the Arabs or Leonardo completed the square" (Parshall [1988]), individual parallelepipeds will be accordingly manufactured by a virtual 'manifold extruder', recreating what is here re-

garded as the at least fully plausible prior art. Recalling the vertical growth of power levels in Fig. 2, i.e. the geometrical "shafts" of manifold increment, what singular engine is it that Fig. 3 by direct filling out the shaded form from the living past outlines? In principle its infinite machine action is akin to the way Archimedes measured volume by displacement from the outset basin, and it could accordingly be called an one-stroke parallelepiped re-assembler, which beyond this slightly peculiar designation is precisely what it perpetually does. When an arbitrary whole-number parallelepiped is cornered from the origin within a larger arbitrary whole-number parallelepiped, the difference be

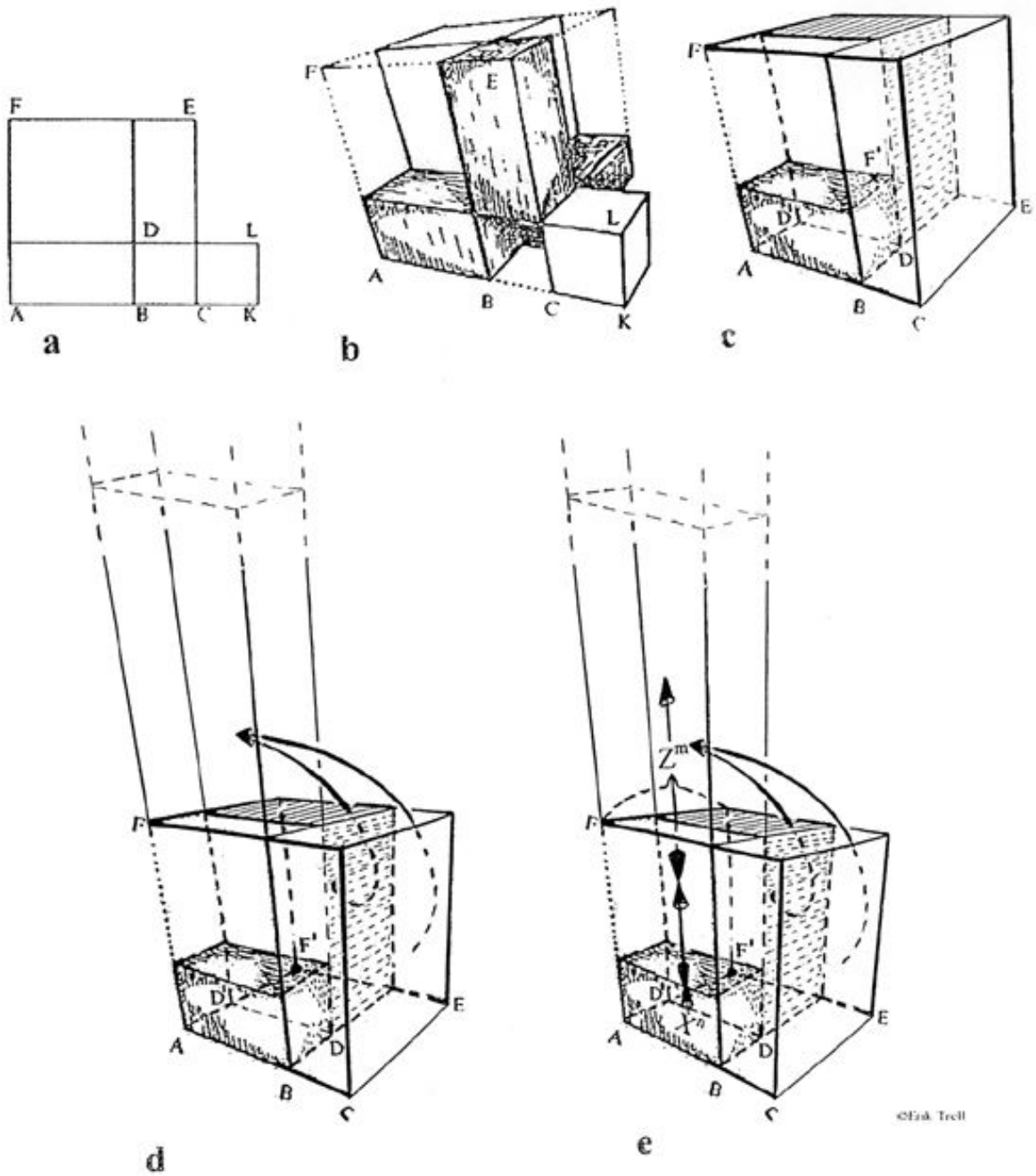

Fig. 3. **a)** "Bottom plate" of Cardano's cubic solution of the general third-degree equation. **b)** Cardanos graph of cubical completion of same. **c)** Present extraction cornered X^n inclusion in = ACEF' cube with identification of side, back and roof difference slabs amounting to Y^p in FLT/BC addition. **d)** Injection of side and back slabs on top of roof slab of same irreducible crossection area as X^n. **e)** This means that Y^p is a multiple of X^n, hence sharing prime factor. And since $X^n + Y^p$ is whole Z^m, this must be a multiple of X^n, too, hence sharing rime factor as well, but cannot be of n:th degree when $Y^{p(=n)}$ is, because the Y number is occupied at the n:th degree where X^n is too small to fill the gap to next higher, Z number to be lifted to n:th power.

tween the two forms one side and one back and one roof slab eccentrically enveloping the inclusion body. Only if they can be combined to a full whole-number parallelepiped, too, the basic requirements of a FLT/BC equation are fulfilled, the further, severely restricting provision of which is that all the parallelepipeds are whole-number powers.

However, we don´t need to worry about this at the moment, but analyse the general Archimedean communicating vessel premises established. It is obvious that any continuous permutation of net marginal volume, be it of the side or back slab alone or of both, around the combined inclusion body and its roof slab can be transferred as a likewise continuous multiple of and over the cross-section of the mutual column of these (Fig. 3c,d), where the occasional coincidence that all confluences thereby are whole numbers, let alone whole-number powers (Fig. 3e), are but rare and extremely rare special cases, respectively.

Since the specific whole-number coupling is established by the inclusion body fully defined as X^n and its cross-section therefore is a multiple of X, this applies to all sections of the Archimedean channel above it, so that already at this stage it can be determined that they must share least common prime denominator as well. Hence the proper genealogy of the BC baby comes out in the preliminaries as drained in the FLT bath-water.

In the further filtering of this also FLT follows from a mere consideration of its hypothetical stipulations, explicitly stating that the second term, Y, must likewise be of n:th degree. In practice, the potential second term candidates are therefore all n:th root whole-number multiples of X^n. Combined with X^n, this gives rise to the Z^m sum (Fig. 3e), which obviously is a multiple of X, too. However, since all consecutive whole-number multiples of X^n are already occupied by the $Y^{p(=n)}$ term, the Z^m sum must be pushed up to the nearest higher dignity if at all a whole-number power itself, thus proving FLT.

From this, the unique formula, $X^n + (X-1)X^n = X^{n+1}$, can be derived the other way around. The shared, X^{n-1} diameter of all three members in all satisfied FLT/BC additions means that the partaking parallelepipeds are directly dependent upon the one-dimensional X multiplicative factor and hence possible to derive strictly mathematically instead of hypothetically through a consecutive division by the said X^{n-1} constant. Thereby, the first term, X^n, is tested as the smallest (because when the second term, here called Y^p, is smaller it has already been screened off as the first one). In consequence, the first Y^p to be probed is the genuine whole-number power immediately larger than X^n, the next is the next larger, and so on.

When Y^p is not rationally divisible with X, division with X^{n-1} results in an additive interval which is an irrational excess larger than X along the mutual Archimedean tube. And when Y^p is rationally or evenly divisible with X, division with X^{n-1} result in an additive interval which is likewise the corresponding excess larger than X. In any case, this rest, conveniently expressed as an unending or ending decimal fraction of X, is thus transferred to the Z^m sum.

And since the so numerically defined remainder recurs only in steps of X over the mutual divisor X^{n-1}, this now mathematically determined equation means that the potential Z^m is always (numerically determined) X times $X^{n-1} = X^n$ larger than the serially matching member of the Y^p ladder. If Z^m really is a whole-number power its step from the linked Y^p whole-number power must be a genuine integral fraction, so that Z^m is a multiple of X^n and a power of X, and therefore, proving FLT, cannot be of n:th degree (as can also be worked out the infinitely ascending numerically testing way, starting with X = 2 etc.) but must be X^{n+1}. And Y^p, by definition a whole-number power, is X shorter than X^{n+1} and thus of size $(X-1)X^n$ so that only the ones where (X-1) has a whole-number n:th root match the conditions.

This can be worked out in an infinitely ascending Eratosthenes' sieve procedure, too, the mechanics of which is a stepwise filtration of all X^n parallelepipeds over rising bases of all consecutive X^{n-1}. Thereby, X = 1 is trivial. But X = 2 is already essential. The smallest bottom plate in the column is 2^2. The smallest power that can form with and over it is 2^3 by multiplication with the height

2. The next larger power, $n \geq 3$, that can be combined from it and the contribution from the rest of the sum block is 2^4, by the addition on top of 2^3 by another 2^3, all according to the $X^n + (X-1)X^n = X^{n+1}$, in this case $2^3 + (2-1)2^3 = 2^{3+1}$ formula. The next larger parallelepiped power is 3^3, and clearly the difference to this can be transferred on top of 2^2 as an irrational multiple of that base. However, the difference cannot be a whole-number parallelepiped because of the smaller ones already being consumed in their exclusive binary bindings.

And continuing the chain, when the sum block is the next larger, 2^5, this is formed by the deposition of three 2^3 blocks on top of 2^3 $[=2^3 + (2^2 - 1)2^3 = 2^{3+2}]$. This sum block can be distributed with and over a 3^3 corner as well, then forming an irrational multiple of this, and the difference again not possibly whole-number power because all the smaller ones are already sifted away. And so on. The 2^3 chain (like all others) is unending, forming new powers in the sequence $2^3 + (2^{3,4,5,6\ldots-1})2^3$ etc. and so hooked away from rational inclusion into any other bottom plate (except powers of multiples of same prime factor, like 4, 6, 8 etc. when this is 2). It is like a gigantic zipper over every rising X^n series; every member's excess being specifically clasped by every smaller member there by a whole-number multiple of at least the lowest member.

The ascending extraction acts infinitely and successively ties every second term parallelepiped specifically to the complementary first term, hence making also the sum sharing the mutual least prime factor. So, with BC in tow, the proper spelling-out of the FLT acronym should now righteously be Fermat's Last Triumph. Contrary to the assertion that "the problem may require a brand-new approach that would not only re-prove the Fermat theorem but a whole lot more" (Mackenzie [1997]), the brand-old directions yield even better.

It is therefore not possible to dismiss the reproducible findings here as some illegitimate Fosbury flop (when yet the outcome counts) of turning the absolute scientific method upside-down. Truth is the reverse: a genuine scholastic return, exchanging again today's intermittent infinite descent with the actual infinite ascent springing from the orthodox roots of original panepistemology. Facts, that is, the very scientific substrate and quintessence not frivolously to be rejected, are that the primordial Sumerian as well as Platonic as well as Diophantine as well as Cardano building blocks of real space, equivalently ethereal and material and mathematical, were atomic cubical monads, and that still in the Renaissance three dimensions for physical extensions and distribution were all that was needed.

Facts are as well each and everything else that has been briefly summarized, as much as possible by direct quotation, in the present account, including their striking coming back in the novel literature, whether coined nanocube self-assembly or quantum gravity dot-to-dot chunks of space etc. as earlier surveyed, or, most recently, "cell-like modules" for "decentralized architecture" of "expanding cube design for a lattice robot" with "morphing ability" and "locomotion" (Mackenzie [2003a,b]).

In ground state strikingly reminiscent of the arrays in Fig.1, such '*Telecube*' and '*Crystal*' robots (Ib.a) are topologically significant also because they show from a combinatorial angle what Fig. 2 structurally discloses, that it is perfectly feasible and appropriate to project endlessly many dimensions over the ordinary three-dimensional space (Ib.b). Further, they indicate the "path to reality" more "like living organisms" proceed (Ib.a), that lies in the broad span of continuous morphing likewise doable by "discrete translations" (Ib.b) between the polar mathematical idealizations of the eccentric layer-to-layer self-aggregation (Trell [1997,1998a,2002]) and the linear space-filling of Fig. 2.

From the prevailing self-similarity of physical structure, i.e. that things come out periodically much alike in the microscopic as in the macroscopic formats, the modular lattice robot designs provide a useful model of particulate organization at all scales. It is then important to consider that like their cubical building units are but vehicles for locomotion and carriage of their internally loaded equipment and instruments, the reproduced cubits of classical physical space analogously permeates the bifurcating decentralized architecture and distribution of the ordinary Cartesian co-ordinate

framework of the currently reconfirmed flat Universe "canvas" (Kamionkowski [2002]) corresponding to the initially cited "quantum foam…of the very fabric of spacetime" (Cho [2002]), by and in which the material events are catalyzed and arranged, even commutually molded, rather than immediately embodied.

However, what Cho claims that previous "theory of everything….sidesteps the sticky froth" (Ib.) is not right. The theoretical fundaments that "build the stage itself" (Ib.) of such and even more complex mathematics are extensively explored and clarified over a long series of years and works by Santilli and co-workers, as to date most comprehensively summarized in his latest Magnum Opus, *Foundations of hadronic chemistry with applications to new clean energies and fuels* [2001], and from which I have borrowed the term 'isounit' for the constituent cubit chunk of space that is inherent, indeed obligate when in its extensions space itself is just a cube composed of smaller and smaller cubes composed of this the smallest cube (which doubtless is the true and faithful iso-unit of the Diophantine equation space).

On the physical plane, such cubical modules at their single and aggregate orders of magnitude may hence be compared with cellular walls and scaffolding sharing in direction and shaping of the overall appearance; of which there are both inner and outer aspects. To start with the former on the elementary level, it is at this stage that Marius Sophus Lie enters the arena (Trell and Santilli [1998]).

Is it possible to envision a mutual interplay and intermorphing between the outlined straight Cartesian matrix of the Platonic "quantum foam" and the curved "superstring" geodesics of the interior and interstitial particulate symmetries by that SU(3) = SO(3) x O(5) group and algebra describing the elementary particle symmetries (Trell [1983, 1990, 1991, 1992, 1998b,c, 2000]) as well as Lie´s original, in his own formulation, "transformations by which surfaces that touch each other are turned into similar surfaces….between the (*straight Cartesian, my remark*) Plucker line geometry and a geometry whose elements are the space´s spheres" (Trell and Santilli [1998]).

It is a remarkable fact, that unit transitions over the central A_2 root space vectors spanned in the most compact, 2^3, hypercube, from a free side of any single of its constituent eight Cartesian space segment cubits against the surface of its inner inscribed sphere of radius 1 project the ground Proton-Neutron Baryon isodoublet (and against the pole the □□□Hyperon and further series); against the intermediary inscribed sphere the □□Hyperon (and from there the □ series); and against the outer circumscribed sphere the □ (ditto plus the N series), with their precise masses and channels. A veritable - and verifiable - 'eightfold eightfold way', (Trell [1998c, 2000]) is thus provided for full reproduction of the basic elementary particles in any separate + or - corner cell in any of the three actual space extensions of the smallest compound space module. It is a mutual ground order situation with suggestive bearings to necessary exclusion/annihilation of internal opponents and further to dark mass/energy and to co-ordination as automatic condition of (self-)assembly, first at the atomic level in single, then at the molecular scale in "stacks of cubical modules" (Mackenzie [2003a]). And so on, and so on in sequential magnitudes.

These are not loose speculations since apart from the truly canonical underpinning referred to, more sophisticated but still generically analogous iso-, geno- and hypermathematical formulas embracing and embodying such mutual systems by their commensurate equation units and operations have already described the shaping of seashells, the configuration of DNA etc. (Santilli [2001]). But the somewhat new contribution of the present observations are their direct real forms, providing at least a beginner's kit of material and computer animation of inorganic as well as organic structure. For the elementary particle spectroscopy the drawings already exist, but for the next stages much more needs to be explored.

Details are beyond the scope here, however, but in general, the "expanding-cube design" and algorithms for 'wiring', morphing and "navigating that world" (Mackenzie [2003a,b]) is a promis-

ing, similar, and not too distant approach and its continuous and space-filling modus operandi through individual topological "body plan" (Ib.b) adaptations has close counterparts in the symmetry co-ordination and cubical/rectangular parallelepiped and size modulations of the smaller relatives.

By the fundamental reciprocity to spherical geometry, the self-assembly periodicity will primarily come in frequency of the golden section. That there will be a tendency to aggregation is also natural, as by that to collective dextro- or levorotation from equal corner occupancy of the constituents, and hence to helicity and innumerable other modes of folding due to neighbors who "compete for the same space" (Ib.a,b]. When the turns are 90 degrees the resulting form will be cubical, too, like in common salt. But with more intricate and heterogeneous ingredients and neighbors one might anticipate phenomena like when "in a lattice robot….stacks of lattice modules can reshuffle themselves into a nearly limitless variety of shapes", inspiring "animal metaphors, such as snakes, spiders, and centipedes. But they can also attach end to end and form wheels" (Ib.a).

Hence the ring is closed. We are back to the outset. Only the cube fills the space densely and thus the immanent partition of space is the cube, which is the great wisdom of Plato´s cosmology - and yet the particles and the planets and the galaxies are round. Only cubes can propagate truly spacefillingly and continuously while points cannot, which already Aristotle noticed (McGinnis [2003]) - and yet in the core there are points. How can cubes roll? We must depart from the concrete to the principal and yet be very direct - it is an effective neural network phase motor with a block and a piston moiety and mode; it is indeed the infinite machine *per se*, working on Science´s finest essence, namely, Truth and Reality, also as reproducible objective data in the authoritative Wittgenstein understanding that “the world is the totality of facts” (Hossack [2000]). And there is ample additional legacy, one of the more famous of which is Hilbert's formalism, i.e., that "mathematics should be regarded as being, at heart, nothing other than a collection of formal games, each one played according to completely specified rules" (Devlin [2002]).

Another maxim stimulating to "playing the Euclidean geometry game in terms of those objects" (Ib.) is the Scientific Realism stance which holds that "theoretical posits are as real as the tables and chairs" (Kukla [1998]). What then about reproducible Diophantine equation boxes and block Universe, not the least as a reintroduction of Kant's heuristic *Teleologie als ob* when "the idea of the archetype is currently making something of a comeback" (Laubichler [2003])? They seem entirely compatible with what Davies perceives about the ideological confession of the Platonist “intuitionists….to what can actually be proved in the real world” [2001].

When he agrees with both Turing and Earman and Norton [1996] that an “infinity machine is defined to be a computer” he aligns with the recent informatics signals that “mathematical research as well as physics and many other fields would benefit from increased emphasis on development of deployable mathematical software and relatively less emphasis on abstract mathematical results” and that “such software can lower the barriers between those who think in ‘practical’ terms and those who think in ‘abstract’ terms” (Petti [1995]).

But he exclusively employs his “machine in a continuous Newtonian universe. By this we mean a universe obeying Newton´s laws in which matter may be subdivided more and more finely while retaining the same properties”. However, it is known that this “infinite descent” approach (which also Fermat fruitlessly entered instead of - perhaps for the suggested reasons - publishing his *mirabilem* demonstration) “would not be tractable in a fractal-like universe of clusters within clusters ad infinitum, such as Carl Charlier envisaged early in the 20th century” (Unruh [2002]).

The same thus applies to any machine along that direction trying to “carry out an infinite number of computations within a finite time”, either “by performing the individual computations faster and faster”, or “design the computer in such a way that the memory can be doubled indefinitely”

or/and that “the machine can indeed produce a scaled-down version of itself”; all “in the manner of the Zeno paradox” (Davies [2001]).

But he overlooks one quite relevant aspect of modern computer output which the alternative, ascending construction copes with, namely that of animation, i.e. that “the scientific content in a physical model might in the future be captured in simulation” (Petti [1995]). The omission is paradoxical also because in effect many operations and algorithms he suggests are in principle ascending, for instance the earlier mentioned version of Eratosthenes’ sieve.

Similarly he aptly predicts that “our machine depends upon advanced nanotechnology”. However, the needed re-consolidation of constitutionally omniscientific Philosophy by its otherwise dissociated Natural History strand is again indicated by the corresponding state of the art that “an essential part of nanotechnology is self-assembly” (Whitesides and Grzybowski [2002]), from linear nanotubes to surface “layer-by layer growth” including “formation of…superstructure…as a result of the templating effect” of the primary deposition (Velikov et al. [2002]).

The congenial scientist would positively respond to such collegial “learning from one another. Different fields of science take different roads to understanding, each bring something to self-assembly” (Whitesides and Grzybowski [2002]), too. When Davies summarizes that “neither our machine nor Turing machines can actually be built, because of fundamental properties of the real universe”, he is therefore right; and none-the-less the real infinity machine is also the real universe, and can be reproduced. In an updated cosmology where the world returns as the possible analytical phase transition substantiation, or ”inflation”, from some singular scintillation, or ”fluctuation”, of an elementary quantum against an already available, again essentially ”flat universe” (Bachall et al. [1999], Rees [2000]), the present results hint at an exponentially enlarging dispersion of the thereby re-instituted Cartesian co-ordinate frame, or ‘canvas’ (Kamionkowski [2002]), for a stratified realization of the spark and current between contrasting, yet infinitesimally approximating logical categories, or "branes" (Seife [2002]), such as (for the third time) "between the curved and the straight" which is at "the heart of Greek geometry and indeed of geometry in general" (Netz [2002]).

Once more there is double endorsement since “during the past few decades, it is the canvas itself that has increasingly become the focus of study” (Kamionkowski [2002]). In conclusion, the ancient plan still holds the draft, but of the block rather than the piston of the motor. This is the straight-to-round reversal in the retrograde excursion, which together with contemporary observations supports that the eventual GUT of the infinity machine is not a single trail but that its comprehensive organic formula must be a symbiosis of complementary topological forms with a gradient and tension between them, call them "spin networks" and "strings", respectively, if so be; in any case a system of more than one principal member, just like the cell needs a wall as well as a nucleus to work.

Such terms and parables are no more philosophically profane than "tables and chairs", no more jargon and verbose than many an ancient or recent text, no more outlandish than the Platonic solids or Eternal-Universe branes; and they have equally profound implications: backwards to "a bright bio-inspired future" (Douglas [2003]) when, anew, "Nature provides alternative fabrication strategies" where "crystals with exquisite micro-ornamentation directly develop within preorganized frameworks" (Aizenberg et al.[2003]). Reproducible data like these are just the kind of stuff that should deserve to be seriously picked up and refined by the self-appointed *cogniti* instead of such adverse reactions that may not rarely rise from their sectarian quarters. In order to "merge someday" (Cho [2002]) we must all go back to go.

References

1. Aizenberg, J., Muller, D.A., Grazul, J.L., Hamann, D.R. [2003]: ‘Direct Fabrication of Micro-patterned Single Crystals’, *Science,* **299**, pp1205-8.

2. Bahcall, N.A., Ostriker, J.P., Perlmutter, S., Steinhard, P.J. [1999]: 'The Cosmic Triangle: Revealing the State of the Universe', *Science,* **284**, pp1481-5.
3. Bishop, E. [1967]: *Foundations of Constructive Analysis,* New York: McGraw-Hill.
4. Cardano, G. [1545]: *Ars Magna.* Milan.
5. Carmeli, M. [1977]: *Group Theory and General Relativity.* New York, St. Louis, San Francisco…..: McGraw-Hill International.
6. Cho, A. [2002]: 'Constructing Spacetime - No Strings Attached', *Science,* **298**, pp 1166-7.
7. Davies, E.B. [2001]: 'Building Infinite Machines', *British Journal for the Philosophy of Science,* **52**, pp 671-8.
8. Devlin, K. [2002]: 'Kurt Gödel - Separating Truth from Proof in Mathematics', *Science*, **298**, pp 1899-1900.
9. Douglas, T. [2003]: 'A Bright Bio-inspired Future', *Science*, **299**, pp 1192-3.
10. Earman, J. and Norton, F.D. [1996]: 'Infinite Pains: The trouble with Supertasks' in: A. Morton and S.P. Stich (*eds*), 1996. *Benacerraf and his critics,* Cambridge MA: Blackwell, pp 231-61.
11. Eberhard, C.A. [1999]: 'Diophantine equations', *www.ms.uky.edu/~carl/ma330/projects/diophanfin1.html*
12. Fraser, J.T. [2001]: 'And take upon´s the mystery of time', *Kronoscope Journal for the Study of Time,* **1**, pp. 7-13.
13. Heath, T.L. [1964]: *Diophantus of Alexandria: A Study in the History of Greek Algebra.* New York, Dover Publications.
14. Hogben, L. [1937]: *Mathematics for the Million.* London: Georg Allen&Unwin.
15. Hossack, K. [2000]: 'Plurals and Complexes', *British Journal for the Philosophy of Science,* **51**, pp 411-43.
16. Ikkala, O. and ten Brinke, G. [2002]: 'Functional materials Based on Self-Assembly of Polymeric Supramolecules', *Science,* **295**, pp 2407-9.
17. Kamionkowski, M. [2002]: 'A Hawking-eye View of the Universe', *Science,* **296**, p 267.
18. Kato, T. [2002]: 'Self-Assembly of Phase-Segregated Liquid Crystal Structures', *Science,* **295**, pp 2414-8.
19. Kukla, A. [1998]: *Studies in Scientific Realism.* Oxford: Oxford University Press.
20. Laubichler, M.D. [2003]: 'A Premodern Synthesis', *Science*, **299**, pp 516-17.
21. Mackenzie, D. [1997]: 'Number Theorists Embark on a New Treasure Hunt', *Science,* **279**, p 139.
22. Mackenzie, D. [2003a]: 'Shape Shifters Tread a Daunting Path Toward Reality', *Science,* **301**, pp 754-6.
23. Mackenzie, D. [2003b]: 'Topologists and Roboticists Explore an 'Inchoate World'', *Science,* **301**, p 756.
24. Mauldin, R.D. [1997-2002]: 'The Beal Conjecture and Prize', www.math.unt.edu/mauldin/beal.html.
25. McGinnis, J. [2003]: 'For Every Time there is a Season: John Philoponus on Plato and Aristotle´s Conception of Time', *Kronoscope,* **3**, pp 83-111.
26. Murphy, C.J. [2002]: 'Nanocubes and Nanoboxes', *Science*, **298**, pp 2139-41.
27. Netz, R. [2002]: 'Proof, Amazement, and the Unexpected', *Science,* **298**, pp 967-8.
28. Noel, E. (Ed.) [1985]: *Le matin des mathématiciens – Entretiens sur l'histoire des mathématiques présentés par Émile Noel.* Paris: Belin – Radio France.
29. Parshall, K.H. [1988]: 'The Art of Algebra from Al-Khwarizmi to Viète: a Study in the Natural Selection of Ideas', *History of Science,* **26**, pp 129-64.
30. Penrose, R. [1995]: *Shadows of the Mind.* Oxford:Oxford University Press.

31. Petti, R. [1995]: 'Why Math-Software Development Counts', *Computers in Physics,* **8**, pp 623-5.
32. Rees, M.J. [2000]: 'Piecing Together the Biggest Puzzle of All', *Science,* **290**, pp 1919-25.
33. Santilli, R.M. [2001]: *Foundations of hadronic chemistry with applications to new clean energies and fuels.* Boston, Dordrecht, London: Kluwer Academic Publishers.
34. Seife, C. [2002]: 'Eternal-Universe Idea Coming Full Circle', *Science,* **296**, p 639.
35. Sutton, D. [2001]: *Platonic and Archimedean Solids.* Walkmill, Cascob, Presteigne, Powys, Wales: Wooden books.
36. Trell, E. [1983]: 'Representation of particle masses in Hadronic SU(3) diagram, *Acta Physica Austriaca,* **55**, pp 97-110.
37. Trell, E. [1984]: 'Scheme for a time antenna in three-dimensional Hausdorff space', *Speculations in Science and Technology,* **7**, pp 269-77.
38. Trell, E. [1990]: 'Geometrical Reproduction of (u,d,s) Baryon, Meson, and Lepton Transformation Symmetries, Mass Relations, and Channels', *Hadronic Journal,* **13**, pp 277-97.
39. Trell, E. [1991]: 'On Rotational Symmetry and Real Geometrical Represen-tations of the Elementary Particles With Special Reference to the N and □ series', *Physics Essays,* **4**, pp 272-83.
40. Trell, E. [1992]: 'Real Forms of the Elementary Particles with A Report of the □□□Resonances', *Physics Essays,* **5**, pp 362-73.
41. Trell, E. [1997]: 'An alternative solution to Fermat´s Last Theorem: Infinite ascent in isotopic geometry', *Hadronic Journal Supplement*, **12**, pp 217-240.
42. Trell, E. [1998a]: 'Isotopic proof and reproof of Fermat's Last Theorem verifying Beal's Conjecture', *Algebras Groups and Geometries*, **15**, pp 299-318.
43. Trell, E. and Santilli, R.M. [1998b]: 'Marius Sophus Lie's Doctoral Thesis Over en Classe Geometriske Transformationer', *Algebras Groups and Geometries*, **15**, pp 395-445.
44. Trell, E. [1998c]: 'The Eightfold Eightfold Way: Application of Lie's True Geometriske Transformationer to Elementary Particles', *Algebras Groups and Geometries*, **15**, pp 447-71.
45. Trell, E. [2000]: 'The Eightfold Eightfold Way. A Lateral View on the Standard Model'. *Physical Interpretations of Relativity Theory (11-14 September 1998), Late Papers, London: British Society for the Philosophy of Science*, pp. 263-84.
46. Trell, E. [2002]: Tessellation of Diophantine Equation Block Universe *Physical Interpretations of Relativity Theory VIII (6-9 September 2002), Proceedings, London: British Society for the Philosophy of Science.*
47. Trell, E. [2003]: 'Book Review: Foundations of Hadronic Chemistry with Applications to New Clean Energies and Fuels', *International Journal of Hydrogen Energy*, **28**, pp 251-3.
48. Unruh, W.G. [2002]: 'Time, Space, and Us', *Science*, **295**, pp 1649-50.
49. Velikov, K.P., Christova, C.G., Dullens, R.A. and van Blaaderen, A. [2002]: 'Layer-by-Layer Growth of Binary Colloid Crystals', *Science,* **296**, pp 106-9.
50. Whitesides, G.M. and Grzybowski, B. [2002]: 'Self-Assembly at all Scales', *Science,* **295**, pp 2418-21.
51. Winterberg, F. [2000]: 'Elementary Particle Physics: Science or Dogma?', *Physical Interpretations of Relativity Theory (11-14 September, 1998), Supplementary Papers,* London: British Society for the Philosophy of Science, pp. 217-28.
wwwgroups.dcs.stand.ac.uk/~history/HistTopics/Fermat's_last_theorem.html [1996].
www.coe.uncc.edu/cas/flt.html. [1997]: 'History of Fermat´s Last Theorem'.
52. Zerhusen, A. [1999]: 'Diophantine Equations'. www.ms.uky.edu/~ carl/ma330/ projects/diophantin1.html

Detection of Weak Gravitational Waves by Interferometric Methods and Problem of Inverse Calculations

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The fundamental features of the detection of non-stationary undulatory perturbations of metrics based on the interference effects are considered. The advantage of Aharonov-Bohm's effect in superconductors for these purposes in comparison with the ordinary optical interference is demonstrated. Some circuitries of the interferometric detectors in order to be used with SQUID are suggested. The possibilities of lowering the noise temperature of the ultraweak signals detectors based on the analogy between the processes of high-sensitive measurements and the reversible calculations are discussed.

The question about the detectability and the possibility of principle to carry out in practice the recording of gravitational waves (GW) had been brought up for the first time in the works of Bondi [1] and Weber [2]. The Bondi's mental experiment (friction of the beads that are "pushed away by the perturbation of metric" under the action of GW, a prototype of the modern laser-interferometric detectors [3, 4]), as well as the Weber's real experiments [5] with the massive aluminum antenna, equipped with piezo-sensors, have implied an energy transfer of GW to the mechanical system. Before these authors represented their own statements, it has been proposed a recording method of GW [6], assuming the conversion of the wave energy into the elastic-strain energy of a body with magnetostrictive properties (fig. 1). The possibility of recording a magnetic response, caused by the deformation of a magnetostrictive sample, of the superconductive quantum interferometer (SQUID) has brought the high efficiency to the method. On the present-day sensitivity the SQUIDs are capable to register $10^{-7}\Phi_0\, Hz^{-1/2}$, where $\Phi_0 = 2.07\times10^{-15} Wb$ is the flux quantum. The basic estimations on the basis of the actual parameters of magnetostrictive materials have allowed us to think of the possibility of enhancing the sensitivity of the proposed method in terms of GW metric tensor oscillation amplitude to the level of $\left|\delta g_{ij}\right| = 2.5\times10^{-23}\, Hz^{-1/2}$.

However, when considering the recording processes based on the energy conversion of a GW field into the measurable signals, it must be taken into account that one out of the profound problems of Einstein's general relativity theory is the problem of determining the energy of the gravitational field itself [7, 8]. In particular, this problem comes up in the calculation of total energy fluxes transported by GW from a source. The divergences, arising at the same time about the simplest symmetries of the problem, have forced some authors [9] to deduce that GW, being purely "geometrical objects" itselves, in general do not transfer the energy. It is possible that the problems with the energy in physics of the nonstationary gravitation themselves are essentially original failures of the experimental detection of the gravitational radiation by the traditional methods. If this is really the case, then GW should be searched as nonstationary variations of metric by the direct approaches without transforming them into the vibrational energy of probe elastic bodies [5, 10] or the oscillations of the optical interferometer's mirror [3, 4].

The direct measurement of the metric's variations in interferometric experiments is possible due to measuring changes of the optical path difference, caused by the space-time curvature under the action of undulatory gravitational perturbations, in the interferometer's arms. The arising phase difference periodically shifts the interference pattern, that leads to the change of a light intensity at the

recording photomultiplier's input, and can be estimated in the end from the formula $\delta\varphi = 2\pi L\left|\delta g_{ij}\right|/\lambda$, where L is the interferometer's base length, δg_{ij} is the metric tensor's variation, λ is the operating wavelength of the interferometer. It is clear that in such experiments one should use in the interferometer the highly monochromatic light, because the monochromaticity holds down the error of the phase measurement. The radiation of ultrastable lasers meets this requirement. Because so far the X-ray lasers are not yet created and ultraviolet ones are not yet ultrastable, then the operating wavelength is still restricted below in the visible and near IR ranges (e. g., the high-stable infrared line of He-Ne laser [3]).

However, it is possible to reduce the operating wavelength in order to increase the capacity of phase response of the system in the interferometric experiments by using the quantum interference effect in superconductors with a weak link. The effective wavelength of the Cooper pairs' condensate, corresponding to the quantum interference in the geometry of Aharonov-Bohm's effect, is represented by formula $\lambda_C = \pi\hbar/(eA)$. For comparison with optics it is possible to write down the numerical value of the coefficient that relates λ_C expressed in angstroms to the modulus of vector potential $\vec{A}$ expressed in $tesla \times metre$ (Tm): $\lambda_C\left[\mathring{A}\right] \approx 10^{-7}/A\left[Tm\right]$. Consequently, even the technically attainable weak fields (with the orders of $A = 10^{-6}\,Tm = 1Œ \times cm$, $\lambda_C \approx 0.1\,\mathring{A}$!) make the quantum interference of superconductive condensate (i.e. the Josephson's effect and the Aharonov-Bohm's effect) more preferred than optical interference by the terms on an operating wavelength in experiments with the detection of gravitational perturbations (when the comparable basis L is). The important factor, which limit the sensitivity of optical interference systems, is the necessity of using the light which has not just high monochromaticity and stability, but also the considerable power (about hundreds of Watts) at the entrance of the interferometer [4]. The last condition is dictated by the photoelectric multiplier, which will be otherwise unable accumulate the signal with the sufficient value of the signal/noise ratio even in the single-quantum regime of photon counting. Under the conditions of very superconductivity the stability of parameters is ensured comparatively simply by freezing the magnetic flux, and a higher value of the critical current is provided for here instead of the required greater optical power.

The principle of GW detection without converting the wave energy into the vibrational energy of experimental elastic bodies in some ways is connected with the issue on a possibility of the information transfer without transfering energy. On the one hand, in our case this problem in so acute form of course is not used, because the interferometric detection methods admit the indirect energy exchange. The GW "controls" the phase in the interferometric detector. Being as a matter of fact a nonlinear one, such an effect permits response energy which has exceeded the energy originating perturbation. On the other hand, however, and in the sharpest formulation, the problem about the information transfer without the energy transfer obviously admits the affirmative reply, which has the direct attitude to an interferometric detection. This reply should be searched in the theory of reversible calculations on a quantum computer. As it is known, the reversibility on a quantum level [11] is provided by the fact that in the course of calculation the states are all along transformed by, remaining proper relative to the initial Hamiltonian of a problem. In this case the states can be degenerated on the energy.

Thereby, the interferometric methods of nonstationary metrics variations detection appear to be associated in a sense with a theory of a quantum computer. However, more the close relation rather takes place here with reversible computations namely. Customary proposition that for processing of a single information bit is needed to diffuse out no less than kT of energy, refers actually only to irreversible calculations, in "reversible computations" the redundant entropy is not produced. In this case even in irreversible computations when energy conversions of the initial and final system states

generally appears to be equal (two equal energetic minimum, separated by potential barrier), but the dissipation and the entropy growth is caused by the transition inconvertibility from any initial in the specified final state. It is clear that for such a transition there is really impossible to describe the inverse process uniquely.

In fact, the simplest circuit [12] of reversible computations implies simply the conservation in the course of the computation of all initial data, which makes it possible to turn the transformation at any stage, but and the calculation reversibility allows to eliminate the dissipation. If to draw an analogy of computations with measurements, then the main source of an entropy, which is the necessities of the initial prearrangment of a computer in the specified initial state, corresponds to an absorption of an idle frequency in super low-noise parametric amplifiers [13]. It is the inconvertibility of absorptive process of idle frequency results in the dissipation in parametric systems and does not allow to obtain here the "absolute zero" noise temperature. The generalization of the concept of transformation reversibility of system states simulating the computational process to a quantum level by the strategy construction of calculation is given, excluding the reduction of a wave function . Such is possible under transformations brought to operator activity on its eigenstates. In this case the uncertainty of eigenvalues becomes zero. Latter allows to draw an analogy of calculations on a quantum computer with measurements in parametric systems with a quantum squeezing.

More generally, the computation reversibility is achieved by the construction of such algorithm, when the column vector readings of computation result $\vec{r}$ is received from the vector of input data $\vec{e}$ via the transforming matrix $\hat{M}$, tolerating the construction of the inverse matrix $\hat{M}^{-1}$, so $\vec{r} = \hat{M}\vec{e}$, and/or $\vec{e} = \hat{M}^{-1}\vec{r}$. The reversible algorithms allow to produce the information processing without the increase of an entropy as of information, as thermodynamic. This means that on processing of one information bit is not required to diffuse out the kT amount of energy. In this case it is possible to believe that the effective temperature of a computer is equal to zero.

It is apparent from the fluctuation-dissipative theorem, in the measurement process the final noise temperature reveals because of the nonzero imaginary part of ageneralized susceptibility of a system, that is due to the finite input resistance of a receiver.

Namely the active resistance is responsible for the dissipation of the energy in measurements. This value in the full sense characterizes the irreversibility of the measurement process. The parallel between the computations and the measurements points of view points here on an advance possibility of the zero noise temperature, in case of possiblities to construct the algorithm reversible measurements. Clearly, such measuring receiver must be designed from pure-jet nonlinear elements. The parametric amplifier or the anhysteretic HF-SQUID's are the best on this role. In figure 2 the flow-chart of the anhysteretic HF-SQUID with a quantum squeezing is given. The effect of which has been previously considered by authors of this message in the work [14]. In anhysteretic SQUID the role of nonlinear reactivity is played by the kinematic inductance of Josephson tunnel junction, under control of the outer magnetic flux, introduced in superconductive ring of SQUID. As an illustration of possible approaches on the creating of the technique reversible measurements let us consider an activity of the simplest three-frequency nondegenerate paramplifier with a nonlinear capacitance. The signal amplification here is achieved due to an insertion of a negative active component of an impedance on an input frequency. In this case the negative active impedance arises solely due to mixing of three frequencies (the input, the idle and the pumping) on a nonlinear capacitance (fig. 3). Consequently, negative active component is obtained as a result of the work of purely reactive elements. At first glance, this system is not to comprise active resistances and according to the fluctuation-dissipative theorem may be characterized by the zero noise temperature. However, the parametric amplifier is not yet intended as the absolutely reversible measuring device.

For its operation the absorbing load of the idler frequency is necessary. This resistive load makes different the noise temperature from zero.

Consequently, the designing problem of parametric amplifier with the zero noise temperature or close to one is brought to the problem of development of the reversible dissipationless load. Here are possible three routes of problem solving. First one is analogous to the approach to reversible computations. In order to turn the process of information processing it is necessary to all the time to retain results of intermediate calculations. The latest asks the sufficient volume of the main memory. In this case one can produce only until for the storage of intermediate data memory there is an open place. Analogously, during the limited time, while in high-quality resonator does not end the transition processes, the initial processing can be regarded as dissipationless (or, more precisely, low dissipative) loading, while considering the Universe as an infinite-mode resonator without any time restriction needed for establishing the balance. On the other hand, it is possible simply to believe that the Universe has the quite low noise temperature as well as the load has.

Indeed, until the forcing equilibrium has not been made, the energy absorbed by the resonator is mainly putting in the growth of an oscillation amplitude, and it is not just dissipates in the form of the loss compensation for an oscillation period. Rather than to absorb the energy of an idler frequency in a resonator in the nonstationary regime, it could be radiate in outer space also, considering the Universe as the resonator with neither than unlimited time of equilibrium setting. On the other hand it is possible to simply believe that the Universe is a load with the rather low noise temperature. The second path of the dissipationless absorption is possible due to the reversibility property of a parametric amplifier itself. According to Manly-Row formula, it is easy to income in the condition of attenuation of input signal instead of amplification by means of the selection in a proper way of the relation between the working, the idle and pump frequencies. In such way, the parametric amplifier will be converted into the parametric load. As a result the circuit of the idler frequency of the fundamental amplifier must be linked with the input circuit of a parametric load. Such load is not intended as an absolutely dissipationless, because there must be the absorber “of its” idle frequency in the reversed amplifier. However, the noise temperature of a parametric load can be done below the physical temperature of the absorber. For a rough estimation of possiblities to lower one it is necessary to take the ratio of an input impedance of the parametric load and the impedance of the proper absorber of the idle frequency load.

In this case the input impedance as a result of a corresponding circuit adjustment may be done however large. Really, one is a relation of an input voltage variation, corresponding to the increment of an input current, to the value of latter. The variation of an input voltage as the response of a nonlinear system on the current influence may be “made” the many times stronger than in linear case.

On the role of the “third route” pretends the possibility of an interferometric suppression of a signal of the idler frequency (here is possible the analogy as with principle of coated optics, and with quantum computations too). For implementing of interferometric supressing the input signal needs to give on two equal parametric amplifiers with the combined pumping generator and inductively coupled loopes of the idler frequency, which are geometrically located so that the magnetic fluxes in them became mutually antiphase.

It is obviously the use of principles of state squeezing in the quantum interferometer, recording the nonstationary variations of metrics, would allow us to increase considerably the receiver sensitivity of a GW signal. However this can be realized only after following upgrade of a measurement sensitivity on the basic principle of designing of the strategy of classical reversible measurements with zero (or vanishing) noise temperature. If to base on the finiteness of the energy flow rate across unit area by the transported GW and to use for its estimation the conventional

formula of general relativity $S_G = cW_G = \frac{c^3\omega^2}{16\pi\gamma}\cdot\left(h_{yy}^2 + h_{yz}^2\right)$, when it is clear the process of the energy transfer between a wave and the probe body of a classical receiver becomes "superunreversible". Indeed, it is easy to compare the energy density in GW and acoustical one $W_{Aq} = E\varepsilon^2$, when the variation amplitude of the metric tensor g_{ij} is equal to the amplitude of the specific elongation (strain tensor e_{ij}, Young's modulus $E \approx 5\,GPa$) of the elastic medium. At the frequency $1kHz$ the ratio of energy density for these waves (W_G/W_{Aq}) appears to be enormous, approximately 10^{34}. It turns out that the empty Euclidean space possesses the huge elasticity; the effective Young's modulus by the order of 34 is higher than at a "usual" matter.

Such jump of the elasticity must generate the gigantic wave reflection on the boundary of matter-vacuum. By the same reason, the process of the energy transfer from GW into mechanical oscillations is done much better than the inverse process. This very inconvertibility at the similar transformation (when others the less fundamental noise sources will be obviated) will not eventually allow us to bring nearer the noise temperature of the GW detector to absolute zero and makes meaningless the next step – the application of the quantum squeezing in the recording system evidently. The way out from such fundamental deadlock is the application of described above recording systems without direct energy conversion. In these ones the conversion irreversibility is lacking.

The direct conversion of a gravitational perturbation into a phase response that SQUID is able to trap may be carried out by a superconductive transformer of the magnetic flux (fig. 4). The transformer is a closed superconductive circuit, which is composed from a couple of coils, each of them has an inductance value which is very close to the other's one: the conversion coil (*b*, fig. 4) and the coupling coil (*c*, fig. 4). The *coupling* (or *loop*) coil leads changes of the magnetic flux, generated by GW, to the "sensitive" element of SQUID. The *conversion* coil is put in such a way as to arrange the plane of its own rings to be parallel to the wave vector of perturbed GW. In such configuration (fig. 5), according to Stockes' theorem, any variation of the metric tensor may cause some variation of the integral of scalar product of vector potential $\vec{A}$ on a infinitesimal length of the converting coil superconductive circuit (in the XOZ plane fig. 5):

$$\delta\Phi = \delta\left\{\oint g_{ij}A^i dr^j\right\} \approx \oint \delta g_{ij}A^i dr^j$$

Therefore, any perturbation of the metric causes some small increment of the magnetic flux, whose (approximately) half will fall into the transmission coil. It is well known that when passing through the "ordinary" three-dimensional space, a plane gravitational wave incidents so that the circle, represented in the plane XOZ (*a*, fig. 5), which is perpendicular to the wave vector, is periodically stretched in one direction and squeezed in another direction (as it were "breathing") while it leaves the area of the visible ellipse constant all the time. The stretching/shrinkage of the ellipse's axes, as well as of all linear dimensions inside the plane XOZ which is perpendicular to the wave propagation direction, leads to the change of the conversion coil's working area (while the wave vector is parallel to the planes of its rings), and this fact allows us to estimate the flux increment without need of calculating any circulation integral. The relative stretching/shrinkage of the ellipse's sizes is estimated approximately by the variation of the metric tensor. Consequently, the relative increments of the conversion coil's effective area and the magnetic flux are estimated by that one. Therefore, if in the transformer one freezes a magnetic flux $\Phi_C = 20\,mWb \approx 10^{13}\Phi_0$, where $\Phi_0 = 2.07\times10^{-15}\,Wb$ is the flux quantum, then a gravitational wave whose amplitude of the metric

tensor's oscillation is in the order of $\left|\delta g_{ij}\right| \approx 10^{-20}$ will give at the input of SQUID a flux increment of order $\Delta\Phi \approx \Phi_C \left|\delta g_{ij}\right| \approx 10^{-7}\Phi_0$, that complies with the limit resolution of a modern two-stage DC-SQUID, which is in the order of 0.1 *Hz* [15].

According to our estimations, the design of terra-hertzian anhysteresis UHF-SQUIDs using the quantum squeezing (fitting with Josephson's plasmic oscillations) of coherent states is promising to increase the sensitivity by more than three orders (i.e. with $\Delta\Phi \approx 10^{-10}\Phi_0$). At the same time it becomes available the detection of GW with the amplitude of order $\left|\delta g_{ij}\right| \approx 10^{-23}$ in the band of 1 *Hz*. It is necessary to note that in the quoted estimations we have not yet taken in to account the signal losses, unavoidable when adjusting the SQUID's input circuit with the transmission coil (these losses are estimated to be of order 1–2).

Based on the principle of the reversibility of linear electrodynamic systems, it is possible to offer a circuit for directly converting the variations of a gravitational field into phase responses, by means of utilizing an active superconductive transformer for the fluxes which includes the Josephson's tunnel transition. In that case flux and phase are dependent on each other, but the linearity is still ensured by the smallness of the variations. The functions of the conversion and connection coils in an active transformer are integrated on the principle "two in one", and the tunnel transition is put into the gap of the superconductive loop. The described structure is essentially a "superhysteresis" HF-SQUID, i.e. $LI_C \square \Phi_0$, where L is coil inductance, I_C is critical current intensity of Josephson's tunnel transmission. The wide range of the multivalued hysteresis branch ($\pm LI_C$) is in compliance with the small value of the derivative $\left(d\Phi_{int}/d\Phi_{ext}\right) \approx \Phi_0/\left(LI_C\right)$ outside the vicinities of the points with $\Phi_{int} = \left(n + \tfrac{1}{2}\right)\Phi_0/2$ (in the linear domain); here Φ_{int} is internal flux through a loop, Φ_{ext} is external magnetic flux, and n is an integer.

The internal flux Φ_{int} (represented by ordinate axis in fig. 5) is determined by the phase difference in Josephson's transmission, $\Phi_{int} = \Phi_0$. In response to the distortion of the metric in the closed circuit of an active transformer, it is brought about an increment of just the phase diference, that leads to the change of the internal flux [16]. The variation of the last, according to the principle of reversibility, has to result in an increment of the external flux which is proportional to $\left(d\Phi_{int}/d\Phi_{ext}\right)^{-1}$. Consequently,

$$\partial\Phi_{ext} \approx \left(LI_C/\Phi_0\right)\cdot\partial\Phi_{int} \approx \left(LI_C/\Phi_0\right)\cdot\Phi_{int}\cdot\left|\delta g_{ij}\right| \approx \left(LI_C/\Phi_0\right)\cdot LI_C\cdot\left|\delta g_{ij}\right|,$$

or else, in terms of the flux quantum, the response of the external magnetic field on the dynamical variation of the metric is given by the expression $\left(d\Phi_{ext}/\Phi_0\right) \approx \left(LI_C/\Phi_0\right)^2\cdot\left|\delta g_{ij}\right|$. In order to estimate the limit resolution of the set-up in terms of metric instead of $\left(\Delta\Phi_{ext}/\Phi_0\right)$, it is necessary to set the resolution by the flux through SQUID, which is registering the external field, generated by an active transformer in response to the space-time distortion $\left|\delta g_{ij}\right| \approx \left(\partial\Phi/\Phi_0\right)/\left(LI_C/\Phi_0\right)^2$. If by way of LI_C to take $20\,\mu Wb = 10^9\Phi_0$, then even at the sensitivity of SQUID $\Delta\Phi/\Phi_0 = 10^{-6}\,Hz^{-1/2}$ the set-up will be able to detect GW with the record-breaking small amplitude $\left|\delta g_{ij}\right| = 10^{-24}\,Hz^{-1/2}$.

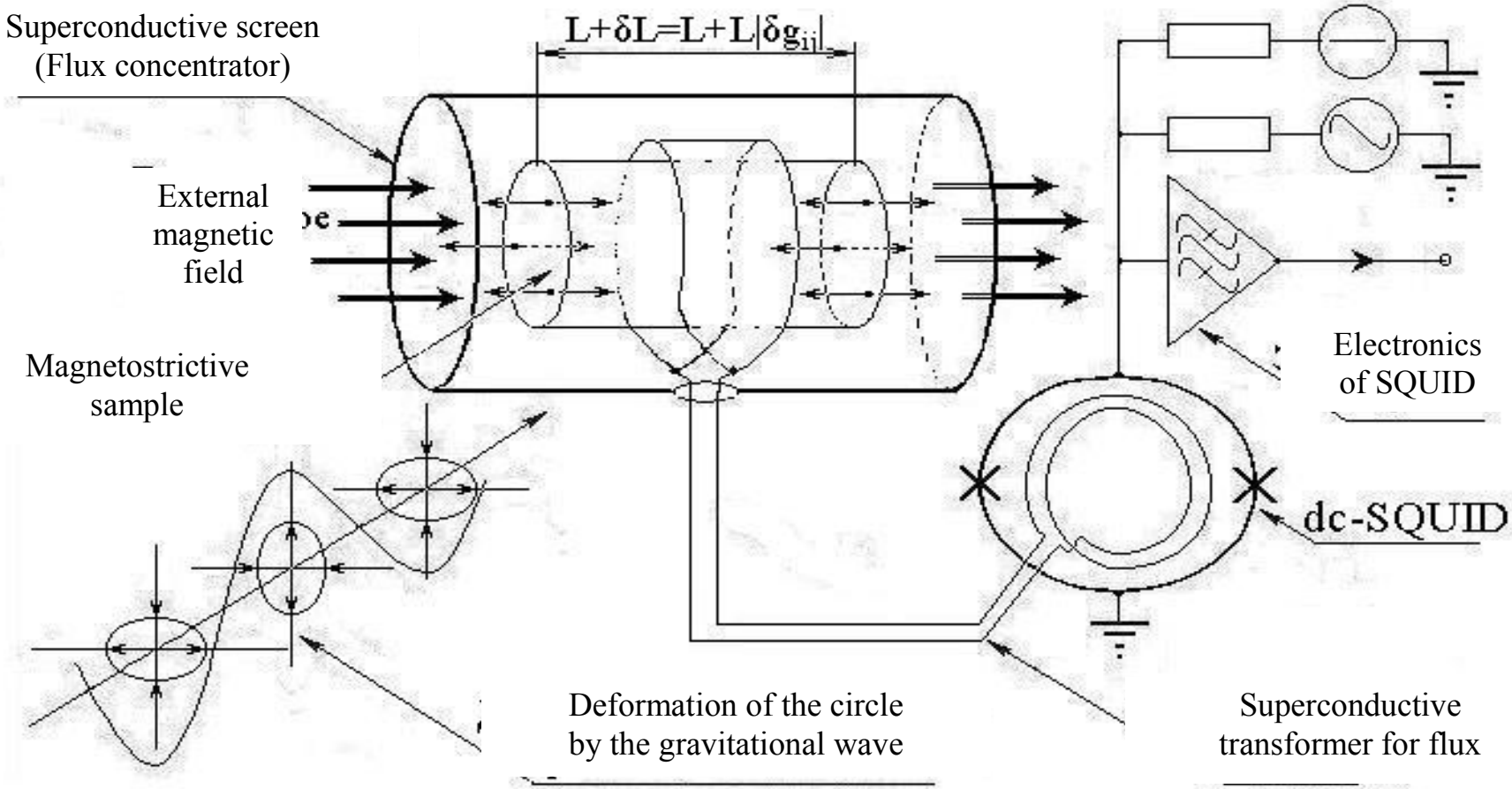

Fig. 1

The magnetostrictor's response by the flux: $\Delta\Phi = S\Lambda E(\Delta L/L)$, *where* $S = 200\,\mathrm{cm}^2 = 2\times10^{-2}\,\mathrm{m}^2$ *is area of the magnetostrictor's base,* $\Lambda \geq 2\times10^{-9}\,\mathrm{T/Pa}$ *is magnetostrictive sensitivity,* $E = 200\,\mathrm{GPa}$ *is Young's modulus. The resolution capacity of SQUID:* $\delta\Phi = 10^{-7}\Phi_0\,\mathrm{Hz}^{-1/2} = 2.07\times10^{-22}\,\mathrm{Wb}\cdot\mathrm{Hz}^{-1/2}$*; as* $\Delta\Phi = \delta\Phi \Rightarrow$ *the sensitivity in terms of the metric tensor's variational amplitude:* $|\delta g_{ij}| = (\Delta L/L) = 2.5\times10^{-23}\,\mathrm{Hz}^{-1/2}$.

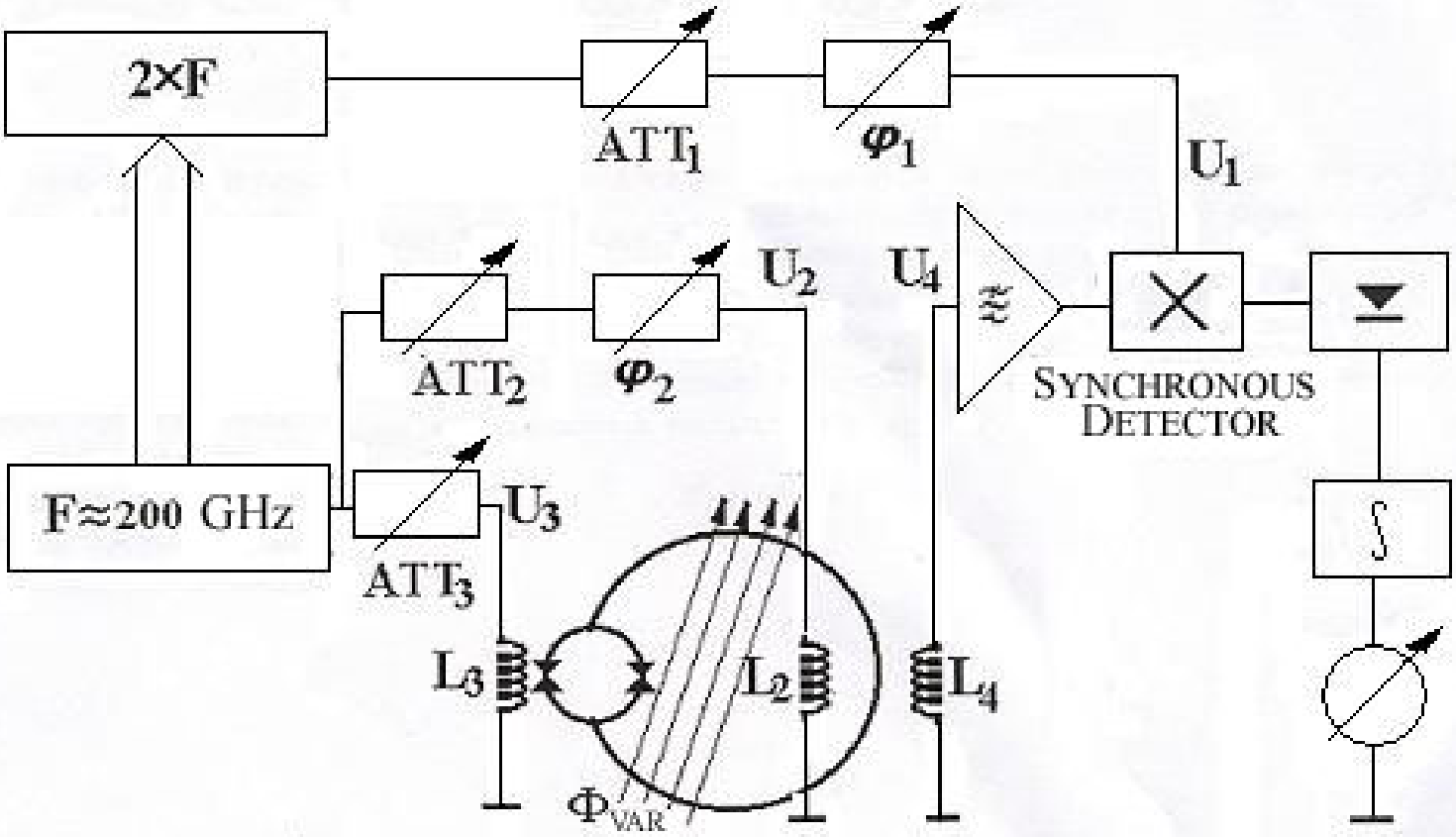

Fig. 2

The circuit diagram of a dynamic squeezing anhysteresis UHF-SQUID

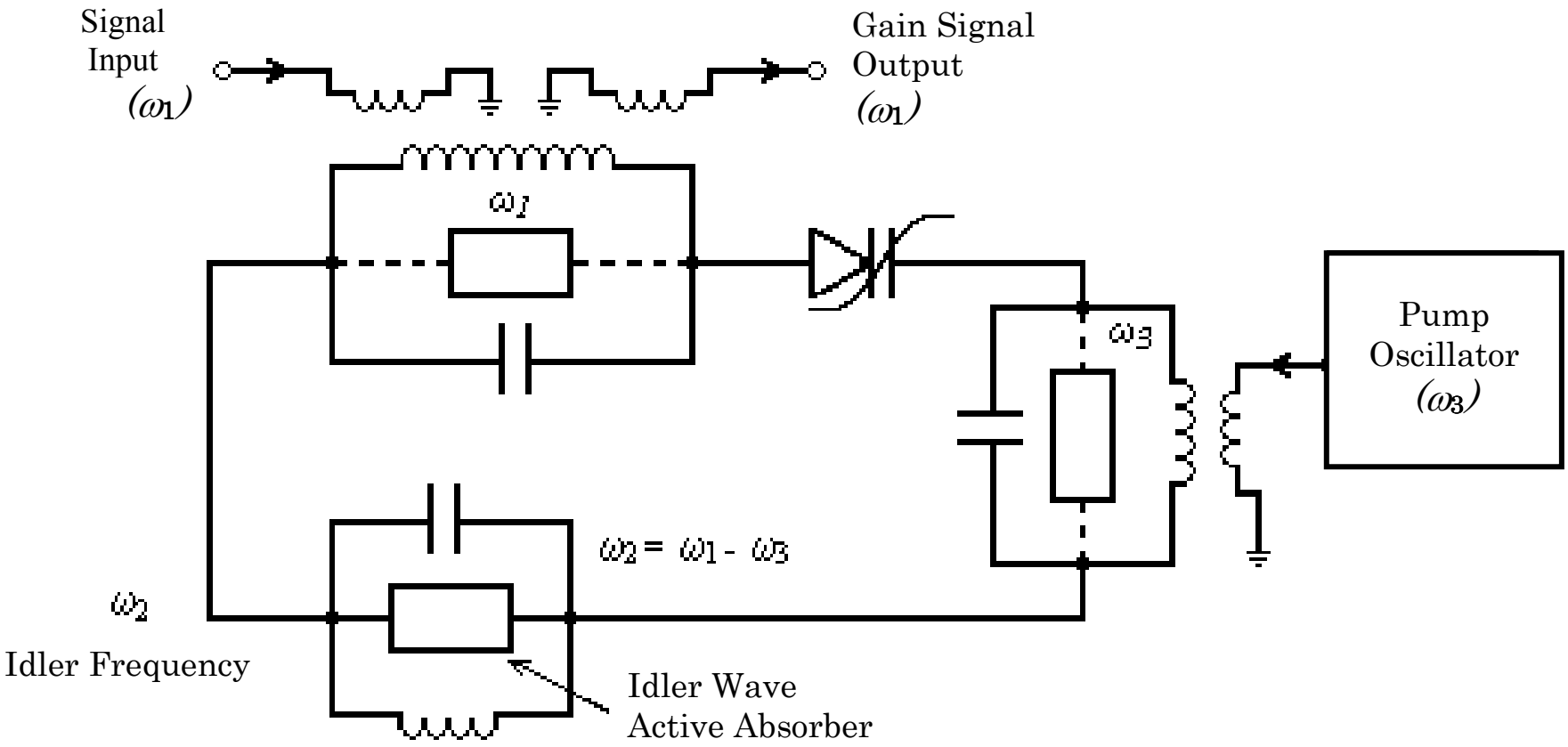
Signal
Input
(ω1)
Gain Signal
Output
(ω1)
ω1
ω3
Pump
Oscillator
(ω3)
ω2 = ω1 - ω3
ω2
Idler Frequency
Idler Wave
Active Absorber

Fig. 3

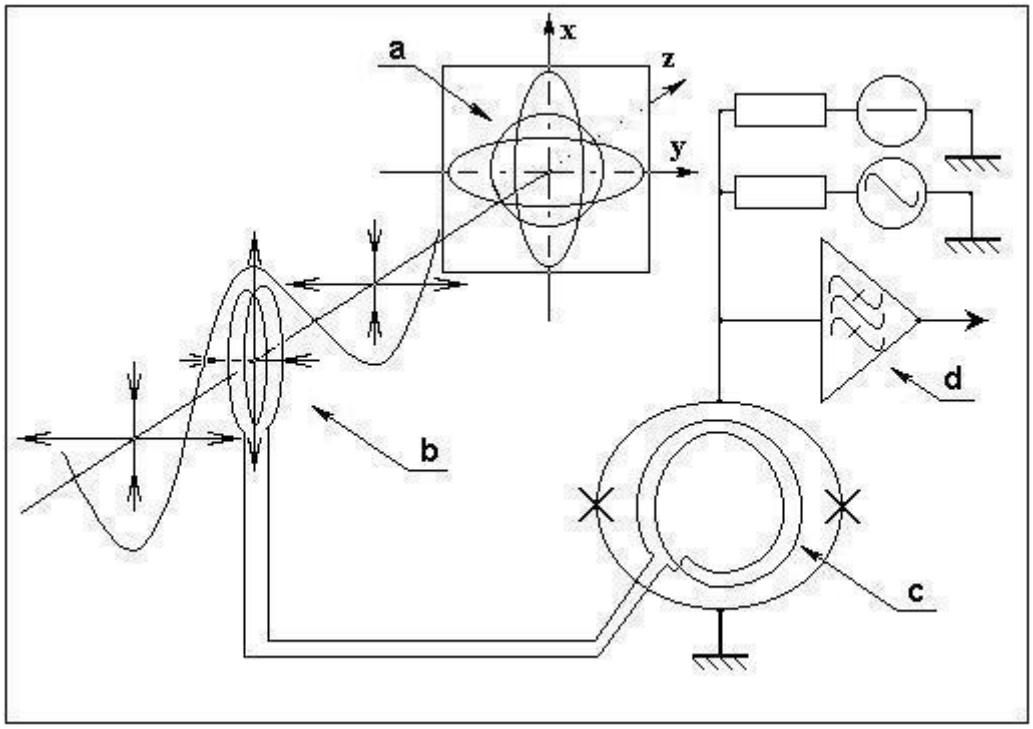

Fig. 4 – Representation of the schematic diagram of converting gravitational perturbations into the signals detected by SQUID: **a**) the distortion of a circle, depicted inside a plane, which is perpendicular to the wave vector (the "breathing" of the circle); **b**) the stretching of the effective area of the conversion coil in the superconductive flux transformer; **c**) the coupling coil that leads the magnetic signal in DC-SQUID; **d**) the input element from the electronic composition of DC-SQUID.

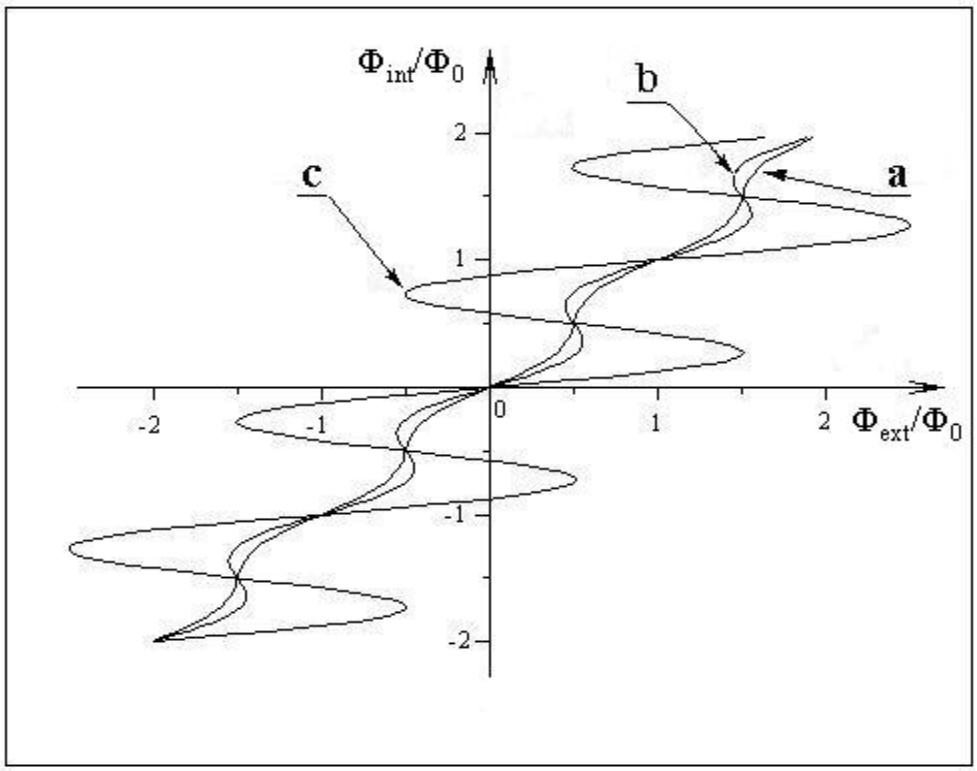

Fig. 5 – The plots representing the dependency of the internal magnetic flux (Φ_{int} as the ordinate axis) versus the external flux (Φ_{ext} as the abscissa axis) inside a superconductive loop, switching into the circuit

the Josephson's tunnel transmission (an active transformer): **a**) singlevalued anhysteresis loop; **b**) multiplevalued hysteresis loop; **c**) "ultrahysteresis" loop.

References

[1] H. Bondi. Nature, 179, 1072, (1957)
H.Bondi, F.A.E.Pirani, I.Robinson. Proceedings of the Royal Society, A251, 519, (1959)
[2] J.Weber. Phys. Rev., 117, 306 (1960)
[3] S.N. Bagaev. A.S.Dychkov, V.P.Chebotaev Pis'ma v JETP , 33-2, 79 (1981)
[4] M. Coles in: Proc. Int. Conf. on Gravitational Waves: Sources and Detectors eds. I. Ciufolini, F. Fedecaro World Scientific, Singapore (1997).
A. Giazotto in: Proc. Int. Conf. on Gravitational Waves: Sources and Detectors eds. I. Ciufolini, F. Fedecaro World Scientific, Singapore (1997)
S. Kawamura and N. Mio in: Proc. 2nd TAMA Internat. Workshop on Gravitational Wave Detection Tokio: Universal Academy, Tokio (1999)ACIGA http://www.anu.edu.au/ACIGA
[5] J.Sinsky, J.Weber. Phys. Rev. Lett. 18, 795 (1967)
[6] Письма в ЖЭТФ, т.60, вып.8, 1994, стр.595-599
[7] W.Pauli. Theory of Relativity 1983
[8] J.Weber General Relativivity and GravitationalWaves N.Y. 1962
[9] C. Moller. In: "Max-Plank Festschrift", 1958, Berlin, S.139
L. Infeld. Ann. Phys., 6, 341, 1959
A.E. Scheidegger. Rev. Mod. Phys., 25, 451, 1953
N. Rosen. In: "Jubilee of Relativity Theory", Basel, 1956
[10] ALLEGRO http:sam.phys.lsu.edu
AURIGA http://www.lnl.infn.it/
EXPLORER http://www.roma.infn.it/rog/explorer/explorer.html
NAUTILUS http://www.lnf.inf.it/esperimenty/rog/NAUTILUS/
NIOBE http://www.gravity.pd.uwa.edu.au/publ/niobe.html
[11] R. Feynman. Found. Phys., 16, 1986, 507-531
[12] C.H. Bennett, Logical Reversibility of Computation, IBM J.Res. Develop. G, 525-532, 1979
[13] F.N.H. Robinson (1974). Noise and Fluctuations in Electronic Devices and Circuits, Oxford University Press
[14] A. I. Golovashkin, G. V. Kuleshova, A. M. Tzhovrebov Proc. of MIFI Conference MIFI-2003, Moscow 20003 (in Russian)
[15] J. Clarke, IEEE Trans. Electron Dev. ED-27, 1896 (1980)
[16] Bryce S. DeWitt. Phys. Rev. Lett. 16, 1092 (1966)
G. Papini. Phys. Lett. 24A, 32 (1967)

Formation of physical fields and manifolds

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The mechanism of the evolutionary processes in material media is described. These processes lead to origination of physical structures from which the physical fields and manifolds are formed. It is shown that the physical fields are generated by material media (material systems). The role of the conservation laws in these processes is shown.

Such results have been obtained by means of the skew-symmetric differential forms. At present the theory of skew-symmetric (exterior) differential forms that possess invariant properties has been developed. As it is well known, the closed exterior forms discrete the conservation laws for **physical fields**. conservation laws correspond to physical structures. In the work the readers are introduced to the skew-symmetric differential forms, which possess the evolutionary properties and describe the conservation laws for **material media**. Transition from the evolutionary differential forms to the closed exterior differential forms discloses a mechanism of originating the physical structures and a process of forming physical fields and manifolds.

1. The exterior differential form

The exterior differential forms are skew-symmetrical differential forms defined on the manifolds with **closed metric forms**.

The exterior differential form of degree p (p-form) can be written as

$$\theta^p = \sum_{i_1 \cdots i_p} a_{i_1 \cdots i_p} dx^{i_1} \wedge dx^{i_2} \wedge \ldots \wedge dx^{i_p} \quad 0 \le p \le n \tag{1}$$

Here $a_{i_1 \cdots i_p}$ are functions of variables $x^{i_1}, x^{i_2}, \ldots, x^{i_p}$, n is the dimension of space, $\wedge$ is the operator of exterior multiplication, 1, dx^i, $dx^i \wedge dx^j$, $dx^i \wedge dx^j \wedge dx^k$, $\cdots$ is the local basis which satisfies the condition of exterior multiplication:

$$\begin{aligned} dx^i \wedge dx^i &= 0 \\ dx^i \wedge dx^j &= -dx^j \wedge dx^i \quad i \ne j \end{aligned} \tag{2}$$

[From here on the symbol $\sum$ will be omitted and it will be implied the summation over double indices. And besides, the symbol of exterior multiplication will be also omitted for the sake of convenience in account].

The differential of (exterior) form θ^p is expressed as

$$d\theta^p = \sum_{i_1 \cdots i_p} da_{i_1 \cdots i_p} dx^{i_1} dx^{i_2} \ldots dx^{i_p} \tag{3}$$

2. Closed exterior differential forms

A form is called the closed one if its differential is equal to zero:

$$d\theta^p = 0 \tag{4}$$

From the condition (4) one can see that the closed form is conserved quantity. This means that it may correspond to the conservation law, namely, to some conservative physical quantity.

If the form is closed only on pseudostructure, then the closure condition is written as

$$d_\pi \theta^p = 0 \tag{5}$$

and the pseudostructure π obeys the condition

$$d_\pi {}^*\theta^p = 0 \tag{6}$$

where ${}^*\theta^p$ is the dual form.

From conditions (5) and (6) one can see that the form closed on pseudostructure is a conservative object, namely, this quantity conserves on pseudostructure. This can also correspond to some conservation law, i.e. to conservative object.

Any closed form is a differential of the form of a lower degree: the total one $\theta^p = d\theta^{p-1}$ if the form is exact, or the interior one $\theta^p = d_\pi \theta^{p-1}$

on pseudostructure if the form is inexact. From this it follows that the form of lower degree may correspond to the potential, and the closed form by itself may correspond to the potential force.

The closed exterior diferential forms are invariant under invariant under all transforms that conserve the differential. The unitary transforms (0-form), the tangent and canonical transforms (1-form), the gradient and gauge transforms (2-form) and so on are examples. These are gauge transforms for spinor, scalar, vector, tensor (3-form) fields. It is to be pointed out that just such transforms are used in field theory.

3. Evolutionary differential forms

The evolutionary differential forms are skew-symmetrical differential forms defined on the manifolds with **unclosed metric forms**.

An evolutionary differential form of degree p (p-form) is written similarly to exterior differential form. But the evolutionary form differential cannot be written similarly to that presented for exterior differential forms (see formula 3). In the evolutionary form differential there appears an additional term connected with the fact that the basis of the form changes. For the differential forms defined on the manifold with unclosed metric form one has $d(dx^{\alpha_1} dx^{\alpha_2} \cdots dx^{\alpha_p}) \neq 0$. For this reason the differential of the evolutionary form ω can be written as

$$d\omega^p = da_{\alpha_1 \cdots \alpha_p} dx^{\alpha_1} dx^{\alpha_2} \cdots dx^{\alpha_p} + a_{\alpha_1 \cdots \alpha_p} d(dx^{\alpha_1} dx^{\alpha_2} \cdots dx^{\alpha_p}) \tag{7}$$

Every evolutionary form is unclosed form, since its commutator, and, conseguently, the differential of this form are nonzero (the evolutionary form commutator involves the commutator of unclosed metric form, which is nonzero).

For example, let us consider the first-degree form $\omega = a_\alpha dx^\alpha$. The differential of this form can be written as $d\omega = K_{\alpha\beta} dx^\alpha dx^\beta$, where $K_{\alpha\beta} = a_{\beta;\alpha} - a_{\alpha;\beta}$ are the components of the commutator of the form ω, and $a_{\beta;\alpha}, a_{\alpha;\beta}$ are the covariant derivatives. If we express the covariant derivatives in terms of the connectedness $\Gamma^\sigma_{\beta\alpha}$ (if it is possible), then they can be written $a_{\beta;\alpha} = \partial a_\beta / \partial x^\alpha + \Gamma^\sigma_{\beta\alpha} a_\sigma$, where the first term results from differentiating the form coefficients, and the second term results from differentiating the basis. If we substitute the expressions for covariant derivatives into the formula for the commutator components, then we obtain the following expression for the commutator components of the form ω

$$K_{\alpha\beta} = \left(\frac{\partial a_\beta}{\partial x^\alpha} - \frac{\partial a_\alpha}{\partial x^\beta} \right) + \left(\Gamma^\sigma_{\beta\alpha} - \Gamma^\sigma_{\alpha\beta} \right) a_\sigma \tag{8}$$

Here the expressions $\left(\Gamma^{\sigma}_{\beta\alpha} - \Gamma^{\sigma}_{\alpha\beta}\right)$ entered into the second term are just the components of commutator of the first-degree metric form. Since metric forms of the manifold are unclosed, the components of the metric form commutator $\left(\Gamma^{\sigma}_{\beta\alpha} - \Gamma^{\sigma}_{\alpha\beta}\right)$ are nonzero. Therefore, the second term of the differential form commutator is not equal to zero. For this reason, the differential form commutator proves to be nonzero. And this means that the differential form defined on the manifold with an unclosed metric form cannot be closed.

4. Conservation laws

It was noted above that the closed exterior differential forms describe the conservation laws. The form closed on pseudostructure and the dual form make up a conservative object, namely, a quantity, which is conservative on pseudostructure. Just such objects are the physical structures that form physical fields. Conditions (5) and (6) are the equations of the physical structure, and they are a mathematical description of the conservation law for physical fields. These conservation laws can be named **the exact conservation laws**.

The exact conservation laws are those that state the existence of conservative physical quantities or objects (the physical structures from which physical fields are formed). The exact conservation laws are related to physical fields. The closed exterior differential forms correspond to the exact conservation laws.

It appears that the evolutionary differential forms, which are unclosed, describe the conservation laws also. A difference consists in the fact that the closed exterior differential forms describe the conservation laws for physical fields, whereas the evolutionary differential forms describe the conservation laws for material media (material systems). As the conservation laws for material systems there serve the conservation laws for energy, linear momentum, angular momentum, and mass. These conservation laws, which can be called **the balance conservation laws**, are those that establish the balance between the variation of a physical quantity and the corresponding external action. From the equations, which describe the balance conservation laws, the evolutionary relation in differential forms is obtained.

The properties of the exterior and evolutionary differential forms, which correspond to the conservation laws, lie at the basis of the processes of forming physical fields and corresponding manifolds. To understand a mechanism of these processes one should look at the properties of the balance conservation laws.

5. Properties of the balance conservation laws

Let us analyze the equations that describe the balance conservation laws for energy and linear momentum.

We introduce two frames of reference: the first is an inertial one (this frame of reference is not connected with the material system), and the second is an accompanying one (this system is connected with the manifold built by the trajectories of the material system elements).

In the accompanying frame the energy equation is written in the form

$$\frac{\partial \psi}{\partial \xi^1} = A_1 \tag{9}$$

Here ξ^1 is the coordinate along the trajectory, A_1 is the quantity that depends on specific features of the system and on external energy actions onto the system.

In a similar manner, in the accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^{\nu}}=A_{\nu}, \qquad \nu=2,... \tag{10}$$

where ξ^{ν} are the coordinates in the direction normal to the trajectory, A_{ν} are the quantities that depend on the specific features of the system and external force actions.

Eqs. (9), (10) can be convoluted into the relation

$$d\psi = A_{\mu}d\xi^{\mu}, \qquad \mu = 1,\nu \tag{11}$$

Relation (2.11) can be written as

$$d\psi = \omega \tag{12}$$

here $\omega=A_{\mu}d\xi^{\mu}$ is the differential form of the first degree.

Since the balance conservation laws are evolutionary ones, the relation obtained is also an evolutionary relation.

The evolutionary relation is a nonidentical one as it involves an unclosed differential form.

Let us consider the commutator of the form $\omega=A_{\mu}d\xi^{\mu}$. The components of the commutator of such a form can be written as follows:

$$K_{\alpha\beta}=(\frac{\partial A_{\beta}}{\partial x^{\alpha}}-\frac{\partial A_{\alpha}}{\partial x^{\beta}}) \tag{13}$$

(here the term connected with the nondifferentiability of the manifold has not yet been taken into account). The coefficients of the form $\omega=A_{\mu}d\xi^{\mu}$ have been obtained either from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator constructed from the derivatives of such coefficients is nonzero. This means that the differential of the form is nonzero as well. Thus, the form ω proves to be unclosed. This means that the evolutionary relation cannot be an identical one. In the left-hand side of this relation it stands a differential, whereas in the right-hand side it stands an unclosed form that is not a differential.

Since the evolutionary relation is not identical, from this relation one cannot get the state differential $d\psi$ that may point to the equilibrium state of the material system. This means that the material system state is nonequilibrium. The nonequilibrium state of the material system induced by the action of internal forces leads to that the accompanying manifolds turns out to be a deforming manifold. The metric forms of such manifold cannot be closed. The metric form commutator, which describes the deformation of the manifold and is nonzero, enters into the commutator of the differential form $\omega=A_{\mu}d\xi^{\mu}$ defined on the accompanying manifold. That is, in formula (13) it will arise the second term connected with the metric form commutator with nonzero value. In this case the second term will correlate with the first term, and this tern cannot make the differential form commutator to be zero. That is, the differential form, which enters into the evolutionary equation, cannot become closed. And this means that the evolutionary relation cannot become the identical relation.

Relation (12) was obtained from the equation of the balance conservation laws for energy and linear momentum. In this relation the form ω is that of the first degree. If the equations of the balance conservation laws for angular momentum be added to the equations for energy and linear momentum, this

form in the evolutionary relation will be the form of the second degree. And in combination with the equation of the balance conservation law of mass this form will be the form of degree 3.

Thus, in the general case the evolutionary relation can be written as

$$d\psi = \omega^p, \qquad p = 0,1,2,3 \tag{14}$$

(The evolutionary relation for $p = 0$ is similar to that in the differential forms, and it was obtained from the interaction of energy and time). The differential forms ω^p, as well as the form ω, are unclosed forms, and hence evolutionary relation (14) is also nonidentical.

A role of the evolutionary relation obtained consists in the following. It enables one to describe the evolutionary processes in material medium, which lead to emergence of physical structures that form physical fields. From the nonidentical evolutionary relation the identical relations, which contain closed exterior forms, are obtained (under the degenerate transformations). The emergence of the closed exterior form points to the rise of the physical structure.

6. Obtaining an identical relation from a nonidentical one

Before it was shown that the evolutionary relation is nonidentical because it contains the evolutionary form ω^p, which is unclosed, namely, $d\omega^p \neq 0$.

If the transformation is degenerate, from the unclosed evolutionary form it can be obtained the differential form closed on pseudostructure. The differential of this form equals zero. That is, it is realized the transition

$$d\omega^p \neq 0 \;\rightarrow (\textit{degenerate transform}) \rightarrow \quad d_\pi \omega^p = 0, \;\; d_\pi{}^\bullet \omega^p = 0$$

Mathematically such a transition proceeds as a transition from the noninertial frame of reference to locally-inertial one. The evolutionary differential form ω^p is defined with respect to coordinate system that is connected with the accompanying manifold. This coordinate system is not an inertial or locally inertial system because the metric forms of the accompanying manifold are not closed ones. But the pseudostructure, on which the closed inexact form is defined, is connected with the locally-inertial system. The transition from the noninertial coordinate system to the locally inertial can proceeds only by means of the degenerate transformation, when the Jacobian of the transformation becomes zero.

The evolutionary relation on the pseudostructure π takes the form

$$d_\pi \psi = \omega_\pi^p \tag{15}$$

where the form ω_π^p is closed on the pseudostructure.

Since the form ω_π^p is a closed form, this form is a differential of differential of some differential form. In other words, this form can be written as $\omega_\pi^p = d_\pi \vartheta$. Evolutionary relation on pseudostructure is now written as

$$\omega_\pi^p = d_\pi \vartheta \tag{16}$$

There are differentials in the left-hand and right-hand sides of this relation. This means that under the degenerate transformation from the nonidentical evolutionary relation it follows the identical on pseudostructure relation.

7. Evolutionary process in material medium. Origination of the physical structures

The evolutionary relation is not identical and from this relation one cannot get the differential состояния $d\psi$. This means that the material system state is nonequilibrium. The nonequilibrium state means that there is an internal force in the material system. It is evident that the internal force originates at the expense of some quantity described by the evolutionary form commutator. (Everything that gives a contribution into the evolutionary form commutator leads to emergence of the internal force).

Transition from nonidentical relation (14) obtained from the balance conservation laws to identical relation (15) means the following. Firstly, an existence of the state differential $d_\pi\psi$ (left-hand side of relation (15)) points to a transition of the material system to the locally-equilibrium state. And, secondly, an emergence of the closed (on pseudostructure) inexact exterior form $d_\pi\psi$ (right-hand side of relation (15)) points to an origination of the physical structure, namely, the conservative object, this object is a conservative physical quantity (the form ω_π^p) on the pseudostructure (the dual form ${}^*\omega^p$, which defines the pseudostructure).

Transitions of the material system into the locally-equilibrium state, which are accompanied by origination of the physical structures, proceed in the evolutionary process under realization of the additional conditions connected with the degrees of freedom of the material system. This is described by a selfvariation of the evolutionary relation: the interaction between the evolutionary and dual (metric) forms, which is carried out by means of interactions between the terms of the evolutionary form commutator. Additional conditions connected with the degrees of freedom of the material system correspond to conditions of the degenerate transformation, which defines the pseudostructure. Thus the mathematical apparatus of the evolutionary differential forms elucidates a mechanism of the evolutionary process and of the emergence of physical structures.

8. Characteristics of physical structures

The emergence of physical structures in the evolutionary process proceeds spontaneously and is manifested as an emergence of certain observable formations. In this manner the causality of emerging various observable formations in material media is explained. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, creating massless particles and others.

The following correspondence between the characteristics of the formations (the physical structures) and characteristics of the evolutionary forms, of the evolutionary form commutators and of the material system is established: 1) an intensity of the formation (a potential force) - the value of the first term in the commutator of a nonintegrable form, 2) an analog of spin - the second term in the commutator that is connected with the metric form commutator,3) an absolute speed of propagation of the created formation (the speed in the inertial frame of reference) – additional conditions connected with degrees o f freedom of the material system,4) a speed of the formation propagation relative to the material system - additional conditions connected with degrees of freedom of the material system and the velocity of the local domain elements.

The physical structures are generated by numerous local domains of the material system and at numerous instants of realizing various degrees of freedom of the material system. It is evident that they can generate fields. In this manner physical fields are formatted. To obtain the physical structures that form a given physical field one has to examine the material system corresponding to this field and the appro-

priate evolutionary relation. In particular, to obtain the thermodynamic structures (fluctuations, phase transitions, etc) one has to analyze the evolutionary relation for the thermodynamic systems, to obtain the gas dynamic ones (waves, jumps, vortices, pulsations) one has to employ the evolutionary relation for gas dynamic systems, for the electromagnetic field one must employ a relation obtained from equations for charged particles.

9. Classification of physical structures

Closed forms that correspond to physical structures are generated by the evolutionary relation having the parameter p that defines a number of interacting balance conservation laws. Therefore, the physical structures can be classified by the parameter p. The other parameter is a degree of closed forms generated by the evolutionary relation. The evolutionary relation of degree p can generate the closed forms of degree $k = p, p\text{-}1, \ldots 0$. Therefore, physical structures can be classified by the parameter k as well.

10. Formation of pseudometric and metric spaces. (Integration of the nonidentical evolutionary relation)

It was shown above that the evolutionary relation of degree p can generate (in the presence of the degenerate transforms) closed forms of the degree $0, \ldots, p$. While generating closed forms of sequential degrees $k = p$, $k = p\text{-}1$, . . ., $k = 0$ the pseudostructures of dimensions $(n+1\text{-}k)$: 1, . . . , $n+1$ are obtained. As a result of transition to the exact closed form of zero degree the metric structure of the dimension $n+1$ is obtained. Under the influence of an external action (and in the presence of degrees of freedom) the material system can transfer the initial inertial space into the space of the dimension $n+1$.Sections of the cotangent bundles (Yang-Mills fields), cohomologies by de Rham, singular cohomologies, pseudo-Riemann and pseudo-Euclide spaces, and others are examples of the psedustructures and spaces that are formed by pseudostructures. Euclide and Riemann spaces are examples of metric manifolds that are obtained when going to the exact forms.

What can be said about the pseudo-Riemann manifold and Riemann space? The distinctive property of the Riemann manifold is an availability of the curvature. This means that the metric form commutator of the third degree is nonzero. Hence, it does not equal zero the evolutionary form commutator of the third degree $p = 3$, which involves into itself the metric form commutator. That is, the evolutionary form that enters into the evolutionary relation is unclosed, and the relation is nonidentical. When realizing pseugostructures of the dimensions 1, 2, 3, and 4 and obtaining the closed inexact forms of the degrees $k = 3$, $k = 2$, $k = 1$, $k = 0$ the pseudo-Riemann space is formed, and the transition to the exact form of zero degree corresponds to the transition to the Riemann space.

It is well known that while obtaining the Einstein equations it was suggested that there are fulfilled the conditions: The Bianchi identity is satisfied, the coefficients of connectedness are symmetric, the condition that the coefficients of connectedness are the Christoffel symbols, and an existence of the transformation, under which the coefficients of connectedness vanish. These conditions are the conditions of realization of the degenerate transformations for nonidentical relations obtained from the evolutionary relation of the degree $p = 3$ and after going to the exact relations. In this case to the Einstein equation there corresponts the identical equation of the first degree.

Квазирелятивистская и квазиквантовая кинематика зарядов (околозвуковые скорости) в классической электродинамике сплошных сред

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В классической электродинамике сплошных сред практически отсутствует теория движения зарядов со скоростями в окрестности скорости распространения собственных, так называемых решеточных, поляризационных волн среды. Хотя пример теории излучения Вавилова – Черенкова позволяет надеяться на физически интересные эффекты. Причиной, очевидно, является отсутствие каких-либо попыток экспериментального исследования таких систем вследствие невозможности создания условий для движения классических зарядов с такими скоростями. Ситуация принципиально изменилась после появления теории поляронов Ландау-Пекара, в которой обоснована возможность существования автолокализованных носителей заряда (поляронов) в средах.

Поляроны Ландау-Пекара взаимодействуют с поляризационными колебаниями среды так же, как классические заряды, хотя и носителя заряда в среде, и ее поляризационные колебания являются существенно квантовыми объектами. Причиной этих особенностей поведения поляронов Ландау-Пекара является то, что они образуются в результате сильного электрон-фононного взаимодействия. Квантовая теория когерентных состояний объясняет это явление тем, что носитель заряда в результате сильного электрон-фононного взаимодействия локализуется в области с радиусом r, а у гармоник поляризационных колебаний с длинами волн большими или примерно равными r возникают квантовые средние поляризации, которые часто называют деформацией вакуума этих гармоник. В формировании поляронов Ландау-Пекара эти деформации вакуума играют значительно более важную роль по сравнению с квантовыми флуктуациями поляризации в этих гармониках. Согласно теории когерентных состояний развитие во времени деформаций вакуума указанных гармоник описывается классическими уравнениями движения поляризации среды. Поэтому в теории Ландау-Пекара поляризация P среды описывается классическим полем, которое естественно нарушает трансляционную симметрию среды, хотя Гамильтониан среды с носителями заряда обладает трансляционной симметрией. Таким образом, поляроны Ландау-Пекара – хронологически, по-видимому, первый пример спонтанного нарушения симметрии в системе двух взаимодействующих квантованных полей.

В поляроне Ландау-Пекара движение носителя заряда значительно более быстрое, чем движение поляризации среды, и поэтому носитель поляризует среду как классический заряд, распределенный в пространстве по закону $\rho_0(\boldsymbol{r}) = e|\psi(\boldsymbol{r})|^2$, где $\psi(\boldsymbol{r})$ - волновая функция носителя в поляроне. Причем так как полярон Ландау-Пекара имеет радиус, значительно превосходящей постоянную решетки кристалла, то его движение можно рассматривать, считая среду континуумом. Все характерные черты поляризации среды при движении такого заряда можно найти, решив классическое уравнение движения поляризации $\boldsymbol{P}$, возбуждаемой соответствующим движением точечного заряда с δ-образным распределением плотности заряда (удобнее решать задачу о нахождении плотности $\rho = -div\boldsymbol{P}$ поляризационного заряда). Решение же для плотности ρ поляризационного заряда в поляроне может быть полу-

чено сверткой решения для случая свободного заряда с δ-образной плотностью (т.е. функции Грина) и плотности ρ_0 свободного заряда в поляроне.

Поскольку скорости теплового движения полярона даже при сравнительно низких температурах или при движении в несильном электрическом поле могут превосходить скорости u распространения поляризационных волн, то имеет смысл рассматривать функцию Грина при таких “околозвуковых” скоростях. Решение этой задачи описано в [1]. Для однородного и изотропного континуума оно сводится [2] к решению уравнения

$$\left[\frac{\partial^2}{\partial t^2}+\Omega^2-u^2\nabla^2\right]\rho(\boldsymbol{r},t)=-ec\Omega^2\delta(\boldsymbol{r},t), \tag{1}$$

где c имеет смысл константы взаимодействия свободного заряда с поляризацией среды.

Выбирая δ-функцию в правой части в виде δ(z-vt)δ(x)δ(y) получим при **v**, параллельном оси z, решение в виде

$$\rho(\boldsymbol{r},t)=\begin{cases}\dfrac{\exp\left\{-\Omega\left[(z-\upsilon t)^2/\beta^2+r^2\right]^{1/2}/u\right\}}{4\pi u^2\beta\left[(z-\upsilon t)^2/\beta^2+r^2\right]^{1/2}},\upsilon<u,\\ \dfrac{\cos\left\{\Omega\left[(z-\upsilon t)^2/\beta^2-r^2\right]^{1/2}/u\right\}}{2\pi u^2\beta\left[(z-\upsilon t)^2/\beta^2-r^2\right]^{1/2}},\upsilon>u,z-\upsilon t<0,r<\left|z-\upsilon t\right|/\beta,\\ 0,\upsilon>u,z-\upsilon t<0,r>\left|z-\upsilon t\right|/\beta,\\ 0,\upsilon>u,z-\upsilon t>0,\end{cases} \tag{2}$$

где $\beta^2=\left|1-v^2/u^2\right|, r^2=x^2+y^2$.

Таким образом, при равномерном прямолинейном движении свободного заряда с v<u распределенный вокруг него поляризационный заряд испытывает квазирелятивистское сжатие в направлении движения с хорошо известным в релятивистской механике коэффициентом типа $\sqrt{1-v^2/u^2}$.

Для поляронов Ландау-Пекара, имеющих конечный радиус, эта закономерность сохраняется. Вычислив плотность поляризационного заряда в поляроне как свертку $\int d\boldsymbol{r}'\rho_0(\boldsymbol{r}-\boldsymbol{r}',t)\left|\psi(\boldsymbol{r}')\right|^2$, где ψ(**r**) в соответствии с идеей вариационного метода расчета свойств полярона должна включать в себя вариационные параметры, можно достаточно точно, но ввиду сложности задачи только численно, найти зависимость энергии E полярона от его скорости. В работах [3-5] представлены результаты расчетов энергетической массы поляронов, определяемой соотношением $E=m_e u^2$, и тензора его инертной массы, определяемой соотношением $\frac{d\boldsymbol{p}}{dt}=\hat{m}\frac{d\boldsymbol{v}}{dt}$, где **p** – импульс полярона. С точностью до 10% они связаны с массой полярона m_0, рассчитанной Пекаром для случая v=u=0 и эффективной массой носителя m^* соотношениями

$$m_e-m^*=(m_0-m^*)\left(1+\frac{u^2}{R^2\Omega^2}\right)^{-1}\left(1-\frac{v^2}{R^2\Omega^2+u^2}\right)^{-1/2}$$

$$m_{||}-m^*=(m_0-m^*)\left(1+\frac{u^2}{R^2\Omega^2}\right)^{-1}\left(1-\frac{v^2}{R^2\Omega^2+u^2}\right)^{-3/2} \tag{3}$$

$$m_{\perp} - m^{*} = (m_0 - m^{*})\left(1 + \frac{u^2}{R^2\Omega^2}\right)^{-1}\left(1 - \frac{v^2}{R^2\Omega^2 + u^2}\right)^{-1/2},$$

где R – эффективный радиус полярона, который в 1.5 раза меньше значения параметра α^{-1}, входящего в волновую функцию носителя, использовавшуюся Пекаром [6], величины m_0 и R зависят от c и Ω. В зависимости (3) этих масс от скорости легко угадать квазирелятивистскую зависимость с поправкой на конечность радиуса полярона, обусловленную квантово-механической невозможностью локализации микрочастицы – носителя заряда в области с R=0. Интересно, что при $u \to 0 (v \le u)$ величины всех масс полярона, как и в релятивистском случае, сходятся к одной величине - к Пекаровской массе m_0.

Случай, аналогичный v>u в (2), в релятивистской механике не рассматривается. В физике твердого тела он является актуальным, так как носитель заряда в поляроне, образованном взаимодействием с фононами с максимальной групповой скоростью u, может взаимодействовать и с фононами, имеющими малую или даже равную нулю скорость u'. Тогда при u'<v<u в соответствии со случаем v>u в (2) полярон будет порождать осциллирующий поляризационный заряд. Распределение ρ'(**r**,t) этого заряда осциллирует внутри конуса, в вершине которого движется свободный заряд, а тангенс угла раствора конуса равен $r/|z| = 1/\beta$. Поверхностями постоянной фазы этого распределения будут гиперболоиды вращения, поверхность которых асимптотически приближается к поверхности корпуса. Вблизи вершины конуса поляризационный заряд ρ' имеет знак, противоположный знаку свободного заряда, и, следовательно, излучением когерентных "низкоскоростных" поляризационных колебаний свободный заряд тормозится. Движение его с постоянной скоростью v>u' возможно, например, в электрическом поле, действие которого на свободный заряд уравновешивает силу радиационного торможения.

При $\beta = \sqrt{v^2/u^2 - 1} \to \infty$ угол раствора конуса стремится к нулю. Этот предел при $u' \to 0$ проще получить решением уравнения (1) с u=0. Таким образом получим соотношение

$$\rho(\boldsymbol{r},t) = \frac{2l+1}{4R\Omega^2}\sin\left[\frac{2l+1}{2}\varphi - \Omega t\right]\delta(r-R)\delta(z) \qquad (4)$$

Его можно получить и интегрированием по r соответствующего выражения в правой части равенства (2) и затем переходом к пределу u=0. В этом приближении (u=0) Торнбер и Фейнман [7] вычисляли напряженность уравновешивающего электрического поля и получили величину порядка 10^7В/см. Значит, потери энергии на радиационное торможение поляронов могут достигать огромных для твердотельной электроники величин порядка 10^7эВ/см.

Как видно из выражения (4), при v>>u позади движущегося заряда можно найти такое место, что расположенный в нем второй такой же заряд своим полем излучения погашает поле излучения первого, за исключением первой полуволны ρ, расположенной между этими зарядами. В этом случае распределение заряда в системе будет подобно изображенному на рис.1. При такой конфигурации пара одинаковых зарядов может двигаться со скоростью v>>u без радиационного торможения. Легко подобрать константу c, обеспечивающую связанное существование этой пары.

В пределе u=0, который обычно используется в электродинамике сплошных сред, можно точно найти распределение ρ(**r**,t) для целого ряда интересных задач. Например, в случае подстановки в правую часть уравнения (1) вместо δ(z-vt)δ(x)δ(y) функции, в цилиндрической системе координат имеющей вид δ(φ-ωt)δ(z)δ(r-R)/R, возможны стационарные состояния вращения точечного заряда массы m^* при ω=2Ω/(2l+1), где l - положительное целое число,

включая ноль. Момент импульса частицы массы m^* при таком вращении оказывается пропорциональным полуцелому числу с коэффициентом пропорциональности, зависящим от заряда e, массы m^*, частоты Ω и величины c. По существу, параметрический резонанс движения частицы с собственными возбуждениями среды выступает причиной квантования стационарных состояний.

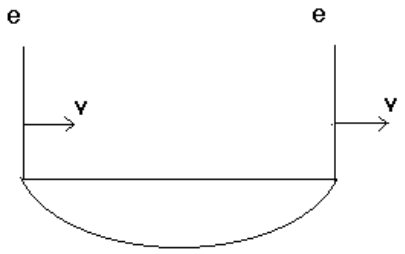

Рис.1

Интересно отметить, что возможны связанные состояния вращения пары частиц с одинаковыми зарядами и массами. Эти состояния можно рассчитать, подставляя в правую часть уравнения (1) $\delta(z)\delta(r-R)[\delta(\varphi-\omega t)+\delta(\varphi-\pi-\omega t)]/R$. При этом $\omega=\Omega/n$, где $n>0$ – целое число, а момент импульса этой пары частиц будет пропорционален n. Таким образом, эти состояния можно назвать моделью бозевских частиц, а состояния параметрического резонанса с полуцелыми “спинами” – моделью фермионов. Связанные резонансно пары одинаковых поляронов являются, естественно, бозевскими двухцентровыми биполяронами, которые в принципе могут образовывать сверхтекучий бозе-конденсат.

Связанные пары, соответствующие рис.1, проявляют свойства, аналогичным свойствам наблюдаемых адронов. В самом деле, при соударении с какой-либо частицей составляющие пары разлетаются с огромными потерями, аналогичными глубоко неупругим процессам с адронами. Конечно, в рамках классической электродинамики нельзя описать образование новых пар за счет возбуждения частиц из физического вакуума. Но эта аналогия позволяет предложить новый, не рассматривавшийся ранее механизм конфайнмента и глубоко неупругих процессов. Можно предположить, что кварки в адронах связаны взаимодействием с деформацией вакуума, соответствующей собственным его колебаниям с равной нулю скоростью распространения u. Такое связанное состояние может двигаться с любой скорость, меньшей скорости света в вакууме, без потерь. Но любое столкновение, например, с быстрым электроном, выбивая кварк из связанного состояния и сообщая ему большую скорость движения (меньшую c), приведет к возникновению тормозного когерентного излучения, сконцентрированного вдоль траектории кварка. Именно таким образом и протекают глубоко неупругие процессы в действительности.

Литература

1. A. E. Myasnikova, Phys. Rev. B **52**, 10457 (1995).
2. A. S. Davydov, V. Z. Enolskii, Phys. Status Solidi B **143**, 167 (1987).
3. А. Э. Мясникова, Э. Н. Мясников. ЖЭТФ **112,** 278 (1997).
4. A. E. Myasnikova and E. N. Myasnikov, Phys. Rev. B **56,** 5316 (1997).
5. А. Э. Мясникова, Э. Н. Мясников, ЖЭТФ **115,** 180 (1999).
6. С. И. Пекар, *Исследования по электронной теории кристаллов,* Гостехиздат, Москва (1951).
7. K. K. Thornber and R. P. Feynman, Phys. Rev. B **1**, 4099 (1970).

Gravitation as Mass Exchange

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In the framework of continuum mechanics and thermodynamics the possibility of the formulation of gravitation theory as the theory of mixture of exchange-reacting media is researched. An application to the acceleration of Universe expansion is given.

1. The well-known hydrodynamic fact of Newtonian attraction of like-sign sources (and repulsion of sources with opposite signs) serves as the basic argument. On any point source of variable mass $M(t)$ and power $\dot{M}(t)$, placed in an incompressible stream with the relative velocity $\mathbf{v}_0$ at a given point and the constant density ρ_0, the force $\mathbf{F} = \mathbf{v}_0 \dot{M}$ acts [1]. Interaction of two sources with the masses $M_1(t)$, $M_2(t)$ gives the force value

$$F = \frac{|\dot{M}_1 \dot{M}_2|}{4\pi\rho_0 r^2}$$

Let the ratio $\Omega = \dot{M}/M$ be a universal constant. Then one can write

$$F = \frac{\Omega^2}{4\pi\rho_0}\frac{M_1 M_2}{r^2} = \frac{G M_1 M_2}{r^2}, \qquad G = \frac{\Omega^2}{4\pi\rho_0} \tag{1}$$

This effect is based on the formation of the reactive pushing forces by interacting streams; this result cannot be obtained by regarding classical momentum exchange of single particles (in quantum mechanics, similar explication of gravitation interaction by neutrino background is given in [2]). The gravitational constant G can also be chosen proportionate to the ratio of the square of the relative source mass changing velocity to the density of background medium. In principle, the parameters Ω and ρ_0, appearing in G, can be arbitrarily small [3]. We also note that for $\rho_0 < 0$ the repulsion of like-sign sources and attraction of sources with opposite signs, as in electrostatics, are occurred.

This approach is like to the idea of K. Björknes where the balls, pulsing in phase in noncompressible fluid, are considered [4].

2. At first, in Newtonian mechanics we consider the theory of heterogeneous mixture of two ideal fluids, one of which (gravitational) has the constant density ρ_0 and the absolute temperature $T_0 = 0$. Let the relative velocity of mass exchange Ω be constant. We consider the adiabatic process when the total entropy $\int \rho S dv$ of the second fluid (gravitating) in the case of thermoisolated body is conserved. Then one can write a variation principle with the Lagrangian

$$\Lambda = \rho\left(\frac{v^2}{2} - U(\rho, S)\right) + \lambda\left(\frac{\partial\rho}{\partial t} + \nabla\cdot\rho\mathbf{v} - \Omega\rho\right) + \mu\left(\frac{\partial\rho S}{\partial t} + \nabla\cdot\rho S\mathbf{v}\right) +$$

$$+\rho_0\frac{v_1^2}{2} + \lambda_1\left(\rho_0\nabla\cdot\mathbf{v}_1 + \Omega\rho\right)$$

Here U is specific internal energy. The density ρ, the entropy S, the Lagrange multipliers λ, μ, ν and Lagrangian variables ξ^α, ξ_1^α are varied independently, indices $i, \alpha = 1, 2, 3$. The velocity and its variation have the form

$$v^i = -\frac{\partial x^i}{\partial\xi^\alpha}\frac{\partial\xi^\alpha}{\partial t}, \qquad \delta v^i = -\frac{\partial x^i}{\partial\xi^\alpha}\frac{d\,\delta\xi^\alpha}{dt}$$

The operator $d/dt = \partial/\partial t + \mathbf{v}\cdot\nabla$ is Galilei-invariant. By means of index mark 1 one denote the gravitational medium. Analogical forms have also the operators with index 1.

As a result we have the equations

$$\frac{d\rho}{dt} + \rho\,\nabla\cdot\mathbf{v} = \rho\Omega, \qquad \frac{d}{dt}\left[(v_i - S\nabla_i\mu - \nabla_i\lambda)\,\frac{\partial x^i}{\partial\xi^\alpha}\rho\det\left(\frac{\partial x}{\partial\xi}\right)\right] = 0 \qquad (2)$$

$$\rho_0\nabla\cdot\mathbf{v}_1 = -\rho\Omega, \qquad \frac{d}{dt_1}\left[(v_{1i} - \nabla_i\lambda_1)\frac{\partial x^i}{\partial\xi_1^\alpha}\rho_0\det\left(\frac{\partial x}{\partial\xi_1}\right)\right] = 0, \qquad (3)$$

$$\frac{d\lambda}{dt} = \frac{v^2}{2} - \Phi(p,T) + \Omega(\lambda_1 - \lambda), \qquad \frac{dS}{dt} + \Omega S = 0, \qquad \frac{d\mu}{dt} + T = 0,$$

$\Phi = U + p/\rho - TS$ is the Gibbs potential, $p = \rho^2\partial U/\partial\rho$ is the pressure, $T = \partial U/\partial S$ is the temperature of the gravitating medium.

These equations are Galilei-invariant (at corresponding transformation of λ, λ_1 as velocity potentials) and guarantee the conservation the total mass, momentum and energy for isolated body. Let

$$p_1 = \rho_0\left(\frac{v_1^2}{2} - \frac{d\lambda_1}{dt_1}\right)$$

Then in divergent form the equations of medium motion have the form

$$\frac{\partial}{\partial t}(\rho v_i) + \nabla_k(\rho v^k v_i + p\,\delta_i^k) = \rho\Omega\nabla_i\lambda_1,$$

$$\frac{\partial}{\partial t}(\rho_0 v_{1i}) + \nabla_k(\rho_0 v_1^k v_{1i} + p_1\delta_i^k) = -\rho\Omega\nabla_i\lambda_1$$

Thus, $\Omega\nabla\lambda_1 \equiv \nabla\varphi$ is the specific gravitational force.

The energy equations have the form

$$\frac{\partial}{\partial t}\left[\rho\left(\frac{v^2}{2} + U + \Omega(\lambda - \lambda_1)\right)\right] + \nabla\cdot\left[\rho\mathbf{v}\left(\frac{v^2}{2} + h + \Omega(\lambda - \lambda_1)\right)\right] = -\rho\Omega\frac{\partial\lambda_1}{\partial t},$$

$$\frac{\partial}{\partial t}\left(\rho_0\frac{v_1^2}{2}\right) + \nabla\cdot\left[\mathbf{v}_1\left(\rho_0\frac{v_1^2}{2} + p_1\right)\right] = \rho\Omega\frac{\partial\lambda_1}{\partial t}$$

Here $h = U + p/\rho$ is the enthalpy, $-\rho\Omega\partial\lambda_1/\partial t$ is the exchange influx of energy.

We multiply the first equation (3) by Ω, divide by ρ_0 and introduce G (1), then we obtain the equation

$$\nabla\cdot\mathbf{g} = -4\pi G\rho, \qquad \mathbf{g} = \mathbf{v}_1\Omega$$

which can be considered as the Poisson equation if $\mathbf{g}$ is gravitational field strength.

The equations of continuity and motion (2), (3) in corresponding Lagrangian variables are evidently integrated. From the equations of continuity we have

$$\rho\det\left(\frac{\partial x}{\partial\xi}\right) = \rho(\xi^\alpha, 0)\exp\Omega t, \qquad \det\left(\frac{\partial x}{\partial\xi_1}\right) = \exp\left(-\int_0^t\frac{4\pi G\rho}{\Omega}dt\right)$$

$$\mathbf{v} - S\nabla\mu - \nabla\lambda = \mathbf{f}(\xi^\alpha)\exp(-\Omega t), \qquad \mathbf{g} - \nabla\varphi = \mathbf{f}_1(\xi_1^\alpha)\exp\int_0^t\frac{4\pi G\rho}{\Omega}dt$$

Consequently, by virtue of observed potentiality of gravitational field the constant Ω must be negative and its value is sufficiently small. On the other hand, because of smallness of $|\Omega|$ the generalized vortex $1/2\,\mathrm{rot}(\mathbf{v} - S\nabla\mu)$ is practically freezed in the gravitating medium. Choosing $\mathbf{f}_1 = 0$ as initial data, one can assume $\mathbf{g} = \nabla\varphi$.

Hence, as $\Omega \to 0-$ the given theory fully coincides with Newtonian gravitation theory.

The gravitational pressure p_1 is defined by the Cauchy – Lagrange formula

$$p_1 = -\frac{1}{4\pi G}\left(\Omega\frac{\partial\varphi}{\partial t} + \frac{|\nabla\varphi|^2}{2}\right)$$

3. The approach of continuum mechanics permits also to extend the theory to relativistic mechanics in case of plane space-time. So, one can assume in the four-dimensional space-time

$$\Lambda = -\rho U(\rho, S) + \lambda\nabla_i(\rho u^i - \rho\Omega) + \mu\nabla_i(\rho S u^i) - \frac{\rho_1^2}{2\rho_0} + \lambda_1\nabla_i(\rho_1 u_1^i + \rho\Omega)$$

Here we assume for the gravitational cold medium the maximum hard state equation of the fluid with the sonic velocity being equal to the speed of light. The vectors $\mathbf{u}$ and $\mathbf{u}_1$ are the four-velocities, the metric g_{ij} has the signature $(+---)$, the value of the velocity of light is chosen to be equal to unity. The variable ρ, ρ_1, S, λ, μ, λ_1 and ξ^α, ξ_1^α, are varied independently, $\rho_0 = \mathrm{const}$. Here index $i = 0, 1, 2, 3$.

On account of $u^i\partial\xi^\alpha/\partial x^i = 0$ and $u_i u^i = 1$ the velocity variation

$$\delta u_i = \gamma_{\alpha\beta}\frac{\partial\xi^\alpha}{\partial x^i}u^k\nabla_k\delta\xi^\beta, \qquad \gamma^{\alpha\beta} = -g^{ij}\frac{\partial\xi^\alpha}{\partial x^i}\frac{\partial\xi^\beta}{\partial x^j}$$

$(\gamma_{\alpha\beta})^{-1} = (\gamma^{\alpha\beta})$.

Therefore we have

$$\nabla_i(\rho u^i) = \rho\Omega, \qquad \nabla_j T^{ij} = -\rho\Omega\nabla^i\lambda_1, \qquad u^i\nabla_i\lambda = \Omega(\lambda_1 - \lambda) - \Phi,$$

$$\nabla_i(\rho_1 u_1^i) = -\rho\Omega, \qquad \nabla_j T_1^{ij} = \rho\Omega\nabla^i\lambda_1, \qquad u_1^i\nabla_i\lambda_1 = -\frac{\rho_1}{\rho_0}, \qquad u^i\nabla_i\mu + T = 0, \tag{4}$$

$$T^{ij} = (\rho h + \rho\Omega(\lambda - \lambda_1))u^i u^j - pg^{ij}, \qquad T_1^{ij} = \frac{\rho_1^2}{\rho_0}u_1^i u_1^j - \frac{\rho_1^2}{2\rho_0}g^{ij}, \qquad \nabla_i(\rho S u^i) = 0$$

Here T^{ij} is the energy-momentum tensor.

Introducing the four-vector $\mathbf{g} = \rho_1\mathbf{u}_1\Omega/\rho_0$, one can represent equations for u_1^i (4) in the form

$$\nabla_i g^i = -4\pi G\rho, \qquad g^j(\nabla_j g_i - \nabla_i g_j) = 4\pi G\rho(g_i + \nabla_i\varphi) \tag{5}$$

Moreover, the relation $|\mathbf{g}|^2 + g^i\nabla_i\varphi = 0$ is a corollary of (5).

Equation (5) evidently has a class of potential solutions

$$g^i = -\nabla^i\varphi, \qquad \Box\varphi = 4\pi G\rho$$

As in Newtonian mechanics, it is possible to show that in general case gravitational vortices for $\rho > 0$ attenuate rapidly. Assuming $\mathbf{g} = -\nabla\varphi$, we have as $\Omega \to 0$ (but $G \neq 0$) the closed system of equations for the adiabatic process in fluid

$$\nabla(\rho u^i) = 0, \qquad \nabla_j(\rho(h - \varphi)u^i u^j - pg^{ij}) = -\rho\nabla^i\varphi,$$
$$\Box\varphi = 4\pi G\rho, \qquad u^i\nabla_i S = 0$$

In case of "vacuum", when $\rho = 0$ (but $\Omega \neq 0$) from (5) we have

$$\nabla_i g^i = 0, \qquad g^j(\nabla_j g_i - \nabla_i g_j) = 0 \tag{6}$$

The vortices of gravitational field are conserved.

4. In the cosmological problem the equations of the spherically-symmetrical homogeneous motion of cold dust ($p = 0$, $T = 0$) in framework of Newtonian mechanics are reduced to an equation for the scale factor $a(t)$ where ρ_* is some density constant

$$\rho = \frac{\rho_*}{a^3}e^{\Omega t}, \qquad \ddot{a} + \Omega\dot{a} + \frac{K}{a^2}e^{\Omega t} = 0, \qquad K = \frac{4\pi G\rho_*}{3} \tag{7}$$

Qualitative research of solutions of equation (7) reveals that in case of elliptical and parabolic trajectories the character of motion for all times is the same as for $\Omega = 0$. But in case of hyperbolic

motion the velocity of dust expansion increases as $\exp(-\Omega t)$. Comparison with lower estimation of Universe expansion [5] gives $\Omega \leq -10^{-18} 1/\mathrm{s}$ and $\rho_0 \geq \cdot 10^{-30} \mathrm{g/cm^3}$.

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References

1. Sedov L. I. On the ideal fluid flow around a body with counter-jet. // Dokl. AN SSSR, 1972, v. 206, No 1, p. 41–42 (in Russian).
2. Koryukin V. M. Kazimir effect, gravitation and relic neutrinos // Izv. vuzov. Physics, 1996, No 10, p. 119–120 (in Russian).
3. Golubiatnikov A. N. A continual approach to the exchange theory of gravitation // The Petrov School – 2000. Abstracts. Kazan, Russia, KSU–press, 2000, p. 27.
4. Whittaker E. A History of the Theories of Aether and Electricity. The Classical Theories. T. Nelson and Sons Ltd. London, 1910.
5. Riess A. G. et al. Observational evidence from supernova for an accelerating Universe and a cosmological constant // Astron. J., 1998, vol 116, p. 1009–1038.

About Singularity in General Relativity

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It was based earlier that the pole $r = r_g$ removes to a complex flat in light rays equation of Schwarzchild metrics. Hence Schwarzchild sphere is transparent in any direction. Therefore black holes ("gravitational grave" for light and substance) can't exist in the theory and they don't exist in nature. It was proved that high symmetry solutions are continued through the physics peculiarity of the type of infinite density. So the compression was followed by the contemporary expansion of the Universe. This point of view is proved by observations.

This point of view solves the central problem of modern physics. It was shown that the density is finite in general case. The physical singularity of the type of the infinite density is obtained under the spherically symmetrical gravitational collapse, which continues to the point r=0. The well-known example of the regular (with the finite density) changing of the motion to the center and the motion from the center is perihelion of a planet in which the trajectory is orthogonal radius. Due the position of the light cones it is possible only in R-region. Therefore the density can be simple if changing compression by expansion takes place in R-region. Under the gravitational radius inner R-region exists in vacuum for the charged sphere. This is the example of the solution without physical singularity. We discuss a more real case of electrically neutral substance when R-region appears inside the substance. For the lack of singularity the substance must be heterogeneous in space.

Wheeler J.A. considered the singularity the point where $\rho = \infty$, to be the central problem of the modern physics [5]. All the laws of physics are broken in the singularities. Hawking S.W. and Ellis G.F.R. [6; ch3] describe the possible crashing of the G.R. because of singularity. It will be shown that the appearance of the singularity of the type of infinite density is connected with the high symmetry of the solution and is not necessary in general case. This point of view was stated earlier [7, 8], but it was rejected. To prove this statement it is enough to give only an exact example, when under the gravitational radius compression follows expansion at finite density of the substance. This example was calculated [9-11] – it is the radial motion of the sphere from the charged dust [7]. Naturally, the question arises: is the conclusion about the possibility of the singularity lack true for the real celestial bodies, which consist of electrically neutral substance.

G.R equations are very complex nonlinear equations. Hence, nowadays only the simple cases can be considered, for example, when there is a spherical symmetry. About 40 years ago Novikov I.D. showed [7, 8] that in spherical symmetry there are regions of two types – R-regions where some segments of light trajectories and particles can be orthogonal radius r, and T-regions, where the motion orthogonal radius is impossible. The motion in T-region can be directed either to the center – compression or from the center – expansion. So we have Schwarzchild solution in vacuum:

$$\text{R-region: } r>r_g,\ r_g = 2Gm/c^2 \tag{1a}$$

$$\text{T-region: } 0<r<r_g, \tag{1b}$$

where r_g – the gravitational radius, G – the gravitational constant, m – the mass of the body. According to (1b) the motion to center in T-region can be finished only in the special point where r = 0.

There is the second inner R-region for the charged dust [7, 9-12].

R-region: $0<r<r_{g2}<r_g$ (2)

It is the only region where the compression is followed by expansion. The general space of Newton mechanic represents R-region where the changing compression by expansion often takes place. If a planet moves around the Sun along the elliptical orbit, the compression analog is its motion toward the Sun. At perigee trajectory of the planet is orthogonal radius. Then it starts moving from the Sun and it can be considered as expansion analog. The planet is moving in the Sun's gravitational field by inertia. The system doesn't need any additional forces to make 'recoil' – changing compression by expansion. To exchange compression by expansion it is necessary appearing R-region, but in electrically neutral substance outside the substance in vacuum there is no R-region [1b]. However it can appear inside the substance. We found it examining the example in our work [13]. Frankly speaking, we were not prepared for understanding this fact. The calculations are rather awkward and complex coordinates appear in intermediate calculations. The light rays equation $ds^2 = 0$ is quadratic by coordinate differentials and appearance of complex numbers is quite natural and provokes distrust. The problem of light spreading in metrics, depending on coordinates – gravitational field, is analogues the problem of waves spreading in heterogeneous medium [12]. It is obvious the final answer – observed quantities must be real, which was proved by calculations.

We discussed the complex map [13, 14] and the term "R-region" inside the substance wasn't used. These arguments were obtained under examining of the metric in vacuum outside the substance and so they don't depend on the equation of state of the substance. We remind that self-similar motion of the powder substance was examined [13]. According (1b) the fall in the vacuum reaches the point r = 0 and it turns out that at some moment t = 0 all the substance in Schwarzchild metric is situated in this point r = 0. The question arises: is it possible that in coordinate system connected with the substance the volume is distinguished from zero? Therefore the density will be finite.

We'll try to show that it is possible. The metrics inside the substance in the moment $t = t_0$ coincides with the metric of Fridman closed world [15, &108].

$$ds^2 = dt^2 - a^2(t-t_0)[d\chi^2 + \sin^2\chi(d\theta^2 + \sin^2\theta d\varphi^2)] \tag{3}$$

where χ - is the radial coordinate. At the substance border:

$$\chi = \chi_0(t) \tag{4}$$

The angular dependence must be the same as Schwarzchild metric has

$$ds^2 = g_{00}dt^2 - g_{11}dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{5}$$

It follows that

$$g_{22} = a^2(t-t_0)\sin^2\chi_0 = r^2 \tag{6}$$

For the filled with the substance volume being different from zero, it is required

$$a(t_0) \neq 0, a(t_0) - \min, \chi_0(t_0) = \pi \tag{7}$$

In the standard Fridman metric under homogeneous density inside the space minimum value of a is zero.

$$a(t) = |t - t_0|^{\mu}, 1/3 < \mu < 2/3 \tag{8}$$

As GR is not an analytical theory we write $|t - t_0|$. We noticed that in the book [15] it is admitted that if the theory was analytical, then in (8) imaginary quantity

v($t–t_0$) would appear, which forbids negative t.(There was nothing before Big Bang!) But the theory is not analytical, which follows from two aspects. Langrangian contains non-analytical multiplier $\sqrt{-g}$ and energy and entropy are not connected analytically – the formula includes exponent different from whole number.

It is very important that (8) contradicts (7) and so we have to depart from the model of homogeneous density $\rho \neq const$. Unfortunately, it is impossible to do it now because we meet fundamental difficulties. Mainly it is connected with the fact that in the GR as in ordinary gas dynamics under gravitational forces different zones of substance, moving toward each other, collides and so blast waves appear and therefore the energy is produced.

There is another specific difficulty in GR. Usually, it is very convenient to use contiguous coordinate of relict radiation. The heterogeneity is less than the number of the order 10^{-4}. The temperature is inversely proportional to the expansion time of the Universe.

The age of the Universe 10^{10} gives the dimension of the zone about 1 mln. light years. We think this examination gives very inflexible valuation, the real dimension can be bigger. We keep non-conservative point of view. The contemporary expansion followed compression – it withdraws many cosmological problems [8].

So far as the compression is unstable this point of view results to the considered above conclusion.

Every particle in this system has its own time which is shown by the fixed clock. Let the clock be synchronized at some moment of time. However, at the next moment clocks of different particles will show different time in general cases. If the particles if collide being at the same point of the space their clock will show different time. Therefore it is impossible to accept contiguous time as coordinate time in general case. But in a special case, when all the particles collide, singularity appears. It is an only case when we can have an analytical solution. To avoid this problem we won't go beyond Newton's quality examination.

If the density of the substance of the examined zone is ρ, time of collapse is equal

$$t - t_0 \approx \sqrt{\frac{1}{G\rho}} \tag{9}$$

Therefore densities zones collapse faster. We accept proper time of the particle as coordinate time. The particle must be fixed on the axis of the examined zone. We accept for the space coordinate the distance from the axis. The coordinate system is similar to one that takes place in Tolmen solution in coordinates r, t [1, 3, 10-14], where t – contiguous time, radial coordinate is determined under the condition that the square of the sphere is equal $4\pi r^2$. The dimensions of the high-density region are dependent of the angular moment of the particles. After passing the high-density region the expansion starts and expanding substance collides with compressing substance with less density, which does not have enough time to collapse. Blast waves appear at the collusion and their appearance increases entropy. Local dimensionless entropy, which is quotient of photons

number to androns number, is equal 10^8-10^9 nowadays. The explaining of such a big value is one of the astrophysics problem. Probably, that one of the reasons is producing of entropy of blast waves.

The points of the maximum density in different zones of the space don't appear simultaneously for non-homogeneous density. They are smeared all along the space and time. And this diminishes the maximum density! We present one more case in favor of the fact that Planck density is not reached. In the case of ideal symmetry the data of helium content are the evidence of the high-density substance at t = 1sec. This value of t corresponds the density, which is lower than Planck's one on 80 orders. Extrapolation to Planck density is not motivated.

Space and time dimensions of the region, where the point of the maximum density is situated, can be roughly evaluated by the temperature heterogeneity of relict radiation. The temperature heterogeneity is less number of the order 10^{-4}. The temperature is inverse proportional the expansion time of the Universe. The age of the Universe equal 10^{10} gives the dimension of the region about one 1 million light years. We think, this consideration gives very strict evaluation, real dimension can be bigger.

We are keeping non-conservative point of view: compression was followed by expansion and this withdraws many problems of cosmogony [2, 8, 16]. As far as the compression is unstable this point of view leads to the considered above picture, where the singularity of the type of infinite density might not exist.

References

1. Gertsenshtein M.E. Russian Physics "Uspekhi" 1999 №1 p.70
2. Gertsenshtein M.E. Russian Physics "Uspekhi" 1999 №4 p.65
3. Gertsenshtein M.E. Science and Technology in Russia 1999 №1 (31) p.8
4. Gertsenshtein M.E. Science and Technology in Russia 1999 №3 (33) p.1
5. Wheeler J.A. Collection Astrophysics and quantum "Mir" 1982, article 11 ASTROFIZIKA E COSMOLOGIA, GRAVITAZIONE QUANTIERELATIVITA FIRENZE (Флоренция) 1979
6. S.W. Hawking, G.F.R. Ellis. The large scale structure of space. Cambridge University press 1973.
7. Zeldovich Ya.B. Novikov I.D. The Gravity Theory and Evolution of Stars "Science" 1971
8. Zeldovich Ya.B. Novikov I.D. The Structure and Evolution of Universe " Science" 1975
9. Gertsenshtein M.E. Science and Technology Russia 2002, № 2 – 3 (46 - 47) p.23
10. Gertsenshtein M.E. XXIV International Workshop on the High Energy Physics and Field Theory. 27 – 29 June 2001 Protvino, Russia
11. Gertsenshtein M.E. Physics Letters A287 (2001) 59 – 61. August 2001.
12. Gertsenshtein M.E. Digest "Acoustics of heterogeneous substance". Year book 2001 Russia Acoustics Society 2001
13. Aivasyan U.M., Gertsenshtein M.E. JETPh 1969, v 59, №3, p.830
14. Gertsenshtein M.E. IZV. VUZ. "FIZIKA" 1984, №6 p.103
15. Landau L.D., Lifshits E.M. The Field Theory, "Science", 1967
16. Gabriele Veneziano. CERN Courier. March 1999, p.18.

The Lorentz non-invariance of the Faraday induction law

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It is known that, in general, the Faraday induction law does not issue from the Maxwell's equations, it represents an independent law of nature. Hence, the Lorentz-invariance of Maxwell's equations does not yet mean the Lorentz-invariance of Faraday induction law. In this connection the invariance properties of the Faraday law were investigated through integration over a closed mathematical line and over a conductor. It has been shown for mathematical line in space, that the mathematical expression for the Faraday induction law is not Lorentz-invariant. The principal difference of a conductor from a mathematical line is the appearance of internal electromagnetic fields induced by rearranged conduction electrons in external fields (polarization of conductor). In our analysis we distinguish two general cases: 1- the polarization effect contributes an induced e.m.f.; 2 – the polarization effect does not give such a contribution. Case 2 makes a conducting circuit similar to a mathematical line, where a formal violation of the Einstein relativity principle has been revealed. The paper finds a real realization of case 2, that makes possible an implementation of the experiments for qualitatively new test of special relativity.

1. Introduction

It is known that Maxwell's equations are Lorentz-invariant. Let us write the integral form of one of them:

$$\oint_{\Gamma} \vec{E} d\vec{l} = -\int_{S} \frac{\partial \vec{B}}{\partial t} d\vec{S}, \tag{1}$$

where Γ is the closed line enclosing the area S, and $d\vec{l}$ is the element of the circuit Γ. In case of a fixed line Γ and area S, Eq. (1) can be written as

$$\varepsilon = -\frac{\partial}{\partial t} \int_{S} \vec{B} d\vec{S}, \tag{2}$$

where

$$\varepsilon = \oint_{\Gamma} \vec{E} d\vec{l} \tag{3}$$

is the electromotive force (e.m.f.), and $\Phi = \int_{S} \vec{B} d\vec{S}$ is the magnetic flux across the area S. For fixed Γ and S Eq. (2) represents a direct inference of Eq. (1), and hence, it is Lorentz-invariant, too. Often Eq. (2) is considered to be closely related to the Faraday induction law. However, it is known that the Faraday law is valid not only for fixed Γ and S, but also for Γ, S, depending on time. In the general case (S=$S(t)$, Γ=Γ(t)), the experimentally established Faraday law

$$\varepsilon = -\frac{d}{dt} \int_{S(t)} \vec{B} d\vec{S}, \tag{4}$$

does not follow from Eq. (1), in particular, due to the inequality

$$\frac{d}{dt} \int_{S(t)} \vec{B} d\vec{S} \neq \int_{S(t)} \frac{d\vec{B}}{dt} d\vec{S} \tag{5}$$

for S variable with time. Historically the situation was just the opposite: the Faraday law (4) was discovered experimentally in the first half of 19th century, and it suggested to Maxwell his Eq. (1).

However, we see that, in general, Eqs. (1) and (4) are mathematically different relationships due to the inequality (5) for S=$S(t)$. Thus, the Lorentz-invariance of Maxwell's equations does not yet mean the Lorentz-invariance of the Faraday law (4). Hence, it should be tested separately.

2. Test of the Faraday induction law for the Lorentz-invariance

We carry out the test for the case where S=$S(t)$, Γ=$\Gamma(t)$, and, for simplicity, both the electric and magnetic fields are constant. In particular,

$$\vec{B}(\vec{r},t) = \vec{B}_0 \tag{6}$$

at least near and within the area S. In addition, we assume a flat area S, i.e., its normal $\vec{n}$ is a constant. Then

$$\varepsilon = -\frac{d}{dt}\int_{S(t)} \vec{B}d\vec{S} = -\vec{B}_0 \frac{d}{dt}\int_{S(t)} d\vec{S} = -\vec{B}_0\vec{n}\frac{dS(t)}{dt}. \tag{7}$$

For stationary current in a circuit the e.m.f. is transformed as

$$\varepsilon' = \varepsilon\sqrt{1-v^2/c^2}\ , \tag{8}$$

where $\vec{v}$ is a relative velocity between two inertial reference frames (here ε belongs to a resting (laboratory) frame). Two important inferences follow from transformation (8): 1 - the e.m.f. simultaneously vanishes in all inertial frames; 2- if $\varepsilon \neq 0$, its sign is the same for all observers. In fact, both these properties of e.m.f. are required by the causality principle. However, the *rhs* of Eq. (7) does not satisfy these requirements: under space-time transformations the magnetic field, in general, can appear in one inertial reference frame and disappear in another inertial frame, and it may change its sign under the same sign of dS/dt. It contradicts to the transformation law (8). Thus, the mathematical expression for the Faraday law of induction is not invariant with respect to the field transformations of SRT. This conclusion can be demonstrated with the particular physical problems, and one of them is presented in Fig. 1. It

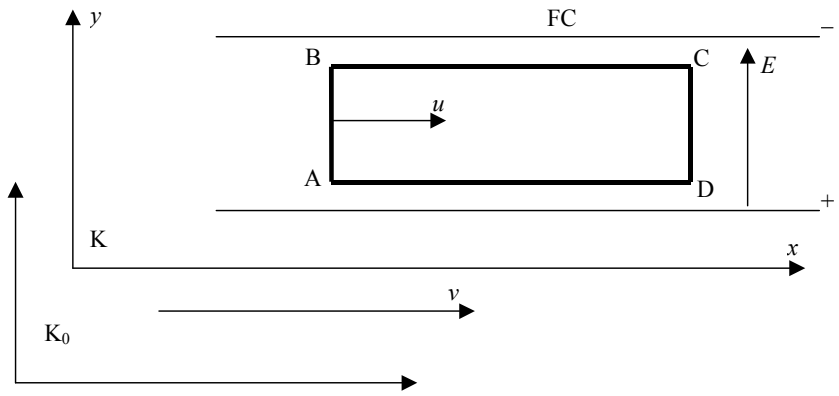

Fig. 1. The closed mathematical rectangular line A-B-C-D with moving side AB has been drawn inside the flat capacitor FC. The length of AB is l. The FC rests in the laboratory frame K, and the electric field in its inner volume is directed along the axis y and equal to E. The inertial frame K moves at the constant velocity v along the axis x of the external inertial frame K_0.

has been shown in ref. [1], that for a laboratory observer (the frame K) the e.m.f. is equal to zero, while in the frame K_0, wherein the frame K moves at the constant velocity v along the axis x, the e.m.f. is equal to

$$\varepsilon = \oint_{\Gamma} \left(\vec{v}(\vec{r}) \times \vec{B}\right) d\vec{l} = u_0 B_{0z} l - v B_{0z} l = \Delta u \frac{vEl}{c^2 \sqrt{1 - v^2/c^2}} = \frac{uvEl\sqrt{1 - v^2/c^2}}{c^2\left(1 + uv/c^2\right)}. \quad (9)$$

This result obviously contradicts to the Einstein relativity principle, but, as shown in Ref. [1], it agrees with the Faraday induction law in the frame K_0. At the same time, while we compute an e.m.f. along a mathematical line in space, we cannot refer the obtained results to physical reality. Therefore, in order to analyze a compatibility of the Faraday induction law and the Einstein relativity principle as physical laws, we have to consider an e.m.f. in closed conducting circuits.

3. The Faraday induction law in moving conductors

In this case we have to take into account the effect of rearrangement of conduction electrons in a conductor under a presence of external electromagnetic fields (polarization effect). The rearranged electrons create their own electric and magnetic fields, which can be negligible outside the conductor, but significant in its inner volume. It is clear that an influence of these fields cannot be analyzed in a general form due to their dependence on many factors (geometry of conductors, configuration of external fields, *etc*.). Nevertheless, in further consideration we may distinguish two different general cases: 1 – the electromagnetic fields, being created by such rearranged electrons, contribute the e.m.f. in a circuit; 2 – the electromagnetic fields from rearranged electrons give a negligible contribution to the e.m.f..

3.1. The Faraday induction law: polarization of conductor contributes an e.m.f. in a circuit

This case is simply realized under substituting of the mathematical line in Fig. 1 by conducting wire A-B-C-D, where an electrical connection between the moving side AB and the sides BC and AD is realized by means of sliding contacts.

First calculate e.m.f. in the loop A-B-C-D for a laboratory observer (inertial frame K). Conduction electrons in this loop are rearranged by such a way, so that to give a resultant vanishing electric field E_R inside the conductor:

$$\vec{E}_R = \vec{E}_{\text{ext}} + \vec{E}_{\text{int}} = 0,$$

where E_{ext} is the field of external source (FC), and E_{int} stands for the field, created by re-distributed conduction electrons. Since the external magnetic field is absent in the laboratory frame K, we obtain E_R, B_R=0 inside the loop. Due to a homogeneity of field transformations, the same equality $E_R', B_R' = 0$ remains valid for any other inertial observer. Hence, the e.m.f. in the circuit A-B-C-D is equal to zero for all inertial observers, including the frame K_0. It prevents a violation of the Einstein relativity principle for the problem under consideration. Simultaneously this result violates the Faraday induction law in the frame K_0. Indeed, the transformation of electromagnetic (EM) fields from K to K_0 has the form:

$$E_{0y} = \frac{E_y + vB_z}{\sqrt{1 - v^2/c^2}} = \frac{E}{\sqrt{1 - v^2/c^2}}, \quad (10)$$

$$B_{0z} = \frac{B_z + \left(v/c^2\right)E_y}{\sqrt{1 - v^2/c^2}} = \frac{vE}{c^2\sqrt{1 - v^2/c^2}}, \quad (11)$$

$E_{0x} = E_{0z} = 0$, $B_{0x} = B_{0y} = 0$.

Hence, the magnetic flux across the area ABCD is $\Phi = B_{0z} S_{\text{ABCD}} = \dfrac{S_{\text{ABCD}} vE}{c^2\sqrt{1 - v^2/c^2}}$,

(here we assume that the electric and magnetic field, being created by moving conduction electrons in the polarized side AB, is negligible in comparison with the fields of FC, i.e., the conducting wires are very thin). From there

$$\frac{d\Phi}{dt} = B_{0z}\frac{dS_{\mathrm{ABCD}}}{dt} = \frac{vE(u'-v)l}{c^2\sqrt{1-v^2/c^2}}, \tag{12}$$

where

$$u' = \frac{u+v}{1+uv/c^2} \tag{13}$$

is the velocity of segment AB in the frame K_0. Substituting Eq. (13) into Eq. (12), we obtain

$$\frac{d\Phi}{dt} = B_{0z}\frac{dS_{\mathrm{ABCD}}}{dt} = \frac{vE(u'-v)l}{c^2\sqrt{1-v^2/c^2}} = \frac{uvEl\sqrt{1-v^2/c^2}}{c^2\left(1+uv/c^2\right)}.$$

Thus, in the frame K_0 the e.m.f. in the circuit A-B-C-D is vanishing, while $d\Phi/dt \neq 0$. Such a violation of the Faraday induction law occurs due to a contribution of the polarization effect to the calculated e.m.f.

3.2. The Faraday induction law: polarization of conductor does not contribute an e.m.f. in a circuit

Such a situation can be realized due to a symmetry properties of a problem considered. One of such problems is presented in Fig. 2. There is a conducting rectangular loop A-B-W-U with the elongated segment AB inside a flat charged condenser FC. The segment AB is parallel to plates of FC. Thin vertical wires of the loop enter into the condenser via the tiny holes C and D in its lower plate, so that a distortion of the electric field $\vec{E}$ inside the condenser is negligible. The condenser moves along the axis y with the relative velocity u with respect to the loop. An inertial frame K_1 is attached to FC, while an inertial frame K_2 is attached to loop. There is some external inertial reference frame K_0, wherein the frame K_1 moves at the constant velocity v along the axis x. One requires to find an e.m.f. in the loop (indication of the voltmeter V) in the frames K_0 and K_2.

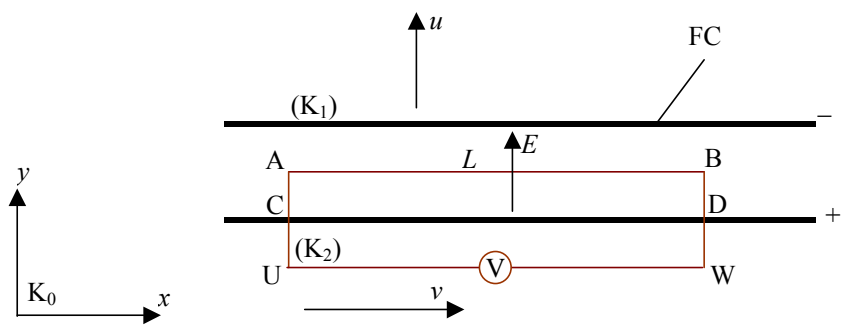

Fig. 2. The inertial frame K_1 is attached to the flat condenser FC, while the inertial frame K_2 is attached to rectangular conducting loop. The electric field in FC is equal to E. The upper lead AB of the loop lies inside the condenser, and its length is equal to L. The profile leads of loop pass across the tiny holes C and D in the lower plate of condenser.

One can see that the polarization effects in the sides of loop AC and BD compensates each other, while in the side AB the electric field of polarization is perpendicular to AB. Hence, the loop becomes equivalent to mathematical line with respect to the effects of polarization.

One can easily show that under a motion of FC along the axis y of the frame K_2 (frame of loop), the electric field does not change in K_2, and magnetic field continues to be equal to zero. Hence, the e.m.f. in A-B-W-U is equal to zero in the frame K_2.

In order to calculate an e.m.f. in the frame K_0, let us find the electric and magnetic fields inside of the FC in this frame according to transformations (10), (11):

$$E_{0y} = \frac{E}{\sqrt{1 - v^2/c^2}}, \tag{14}$$

$$B_{0z} = \frac{vE}{c^2\sqrt{1 - v^2/c^2}}, \tag{15}$$

$$E_{0x} = E_{0z} = 0, \; B_{0x} = B_{0y} = 0. \tag{16}$$

Now let us take into account a known result of relativitistic kinematics: in the frame K_0 the plates of FC are no longer planar to the xz plane. Due to a relativity of simultaneity of events, they constitute the angle $\gamma \approx uv/c^2$ with the plane xz (Fig. 3). Here it is interesting to notice

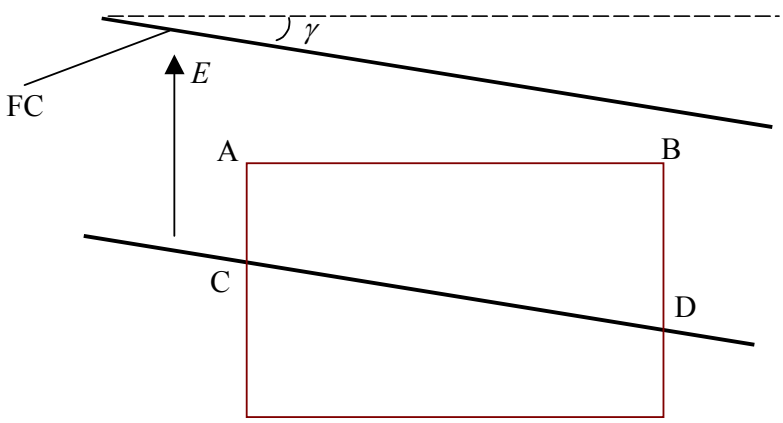

Fig. 3. The moving condenser FC, as it is seen in K_0.

that the vector $\vec{E}$ remains parallel to the axis y, see Eqs. (14) and (16), and it is no longer orthogonal to the plates of FC. Such a result is unusual for conventional electrostatics, but it is a solely possible for an observer in K_0. Indeed, in the frame K_0 the condenser has a component of its velocity v on the axis x, which induces an appearance of the magnetic field along the axis z (Eq. (15)), as well as the velocity component $u' = u\sqrt{1 - v^2/c^2}$ along the axis y, which is responsible for the appearance of magnetic Lorentz force $eu'B_{0z}$, acting to the conducting electrons of the plates FC along the axis x. (Here e is the electron's charge). Therefore, an equilibrium state of these electrons in the frame K_0 is only possible, when the proper electric field of FC is not orthogonal to its plates, and it has non-vanishing projection on the plates in order to compensate the force $eu'B_{0z}$. One can easily see that the equilibrium state of conduction electrons is realized just for the inclination angle $\gamma \approx uv/c^2$ of the vector $\vec{E}$ with respect to the normal of plates.

Now let us show that the non-orthogonality of electric field to the plates of FC in the frame K_0 leads to the appearance of an e.m.f. in the circuit A-B-W-U. Indeed, considering this circuit as mathematical line, we can write according to general expression for e.m.f. [2]

$$\varepsilon = \oint_{\Gamma} \left(\vec{E}(\vec{r},t) + \vec{v}(\vec{r}) \times \vec{B}(\vec{r},t)\right) d\vec{l} = El_{DB} + B_{0z}vl_{DB} - El_{AC} - B_{0z}vl_{AC} \approx E\left(l_{DB} - l_{AC}\right). \tag{17}$$

Here we adopt the accuracy of calculations to the order of magnitude c^{-2} and neglect by a contribution of the magnetic force in comparison with electric force. Taking into account that

$$l_{DB} - l_{AC} \approx \gamma L = \frac{uv}{c^2} L,$$

we obtain
$$\varepsilon = \frac{uv}{c^2} EL. \tag{18}$$

This result remains in force, when the mathematical line A-B-W-U is substituted by real conductor. Then the internal electric field in the segments AC and DB is equal to zero, and this condition is realized under different potentials between the points A and B due to the different lengths of AC and DB. Therefore, the potential difference U_{BA} determines the e.m.f. in the closed loop A-B-W-U. One can easily see that such an e.m.f. coincides with (18).

Further, proceeding from Eq. (15) and the equality $dS_{ABWU}/dt = -u'L$, one can show that the total time derivative of the magnetic flux across the area S_{ABCD} is equal to (18) with the opposite sign.

Thus, in the problem considered, where the effects of polarization of conductor do not influence the e.m.f. in a closed circuit, we obtain a validity of the Faraday induction law for different inertial observers and a violation of the Einstein relativity principle. It simultaneously signifies that this problem cannot be solved within special relativity. This statement does not mean any mathematical imperfection of relativity. It simply means that the empirically established Faraday induction law occurs to be non-invariant. Therefore, the problem in Fig. 2 can be solved only in case, if we stipulate, what inertial frame among involved is the absolute, and use this or that ether theory. In particular, if the frame K_0 in Fig. 2 is absolute, that the covariant ether theories [3] give the same solution (18), but now it is valid not only in K_0, but in all another inertial frames (including the laboratory frame K) in the adopted accuracy of calculations. Corresponding calculations are presented in [4]. The ref. [4] also shows that in practically more simple case (the profile sides of the loop lie far from the boundaries of the condenser, as depicted in Fig. 4), the e.m.f. increases two times in comparison with Eq. (18):

$$\varepsilon = \frac{2uv}{c^2} EL. \tag{19}$$

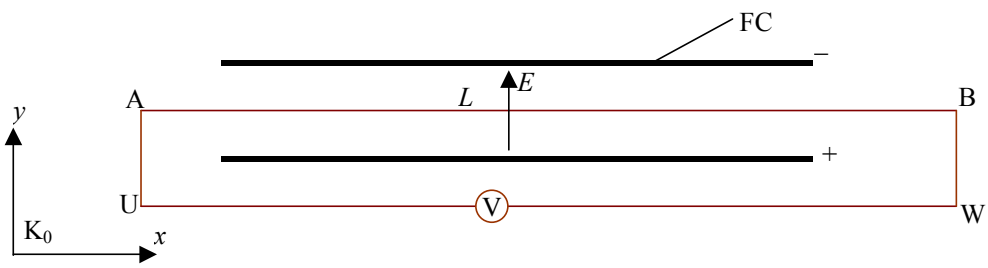

Fig. 4. In case, when the profile sides of the closed loop A-B-W-U lie outside the condenser FC, the e.m.f. increases two times in comparison with the problem in Fig. 2

4. About a qualitatively new test of special relativity in the induction experiments

Since the e.m.f. (19) should be detected in the laboratory frame, too, that its measurement can be used for revealing of an absolute velocity of the Earth. An attractive feature of the Faraday induction experiments is a possibility to multiply the e.m.f. by a number of turns n of the loop. An approximate scheme of such an experiment is depicted in Fig. 5.

Moving system MS, resting in a laboratory, provides an oscillating harmonic motion of the charged flat condenser at the angular frequency ω: $y = y_0 \sin \omega t$, where y_0 is the amplitude of oscillation. The velocity of oscillation is $u = y_0 \omega \sin \omega t$.

The side AB of the multi-turned conductive square loop, which rests in the laboratory, passes across the inner volume of the condenser. The high voltage, applied to the plates of the condenser, is equal to U. The distance between the plates of the condenser is l_0, and the length of the plates along the axis x is L. The output of the loop is connected with narrow-banded (near the frequency ω) amplifier A. The output of amplifier is connected with the oscilloscope to measure a possible e.m.f. in the loop. If the number of turns of loop is equal to n, that the e.m.f. in the circuit is defined as

$$\varepsilon = \frac{2uvE}{c^2} nL = \frac{2uv}{c^2} \frac{LUn}{l_0} . \tag{20}$$

Substituting into Eq. (11) the following numerical values: $u_{max} \approx 10$ cm/s, $U=2 \cdot 10^3$ V, $L=0.2$ m, $l_0=2$ mm, $n=100$ (the acceptable value for the laboratory conditions), $v \approx 10^{-3} c$ (typical velocities of Galax objects), we estimate the maximum value of e.m.f. as $\varepsilon \approx 10$ μV. For the amplifying coefficient of the amplifier A about 10^3, we will get the maximum voltage signal of e.m.f. $\varepsilon \approx 10$ mV, that can be measured by an oscilloscope. If our analysis is correct, that 24-hours and year variations of amplitude of e.m.f. should be detected due to rotation motion of Earth.

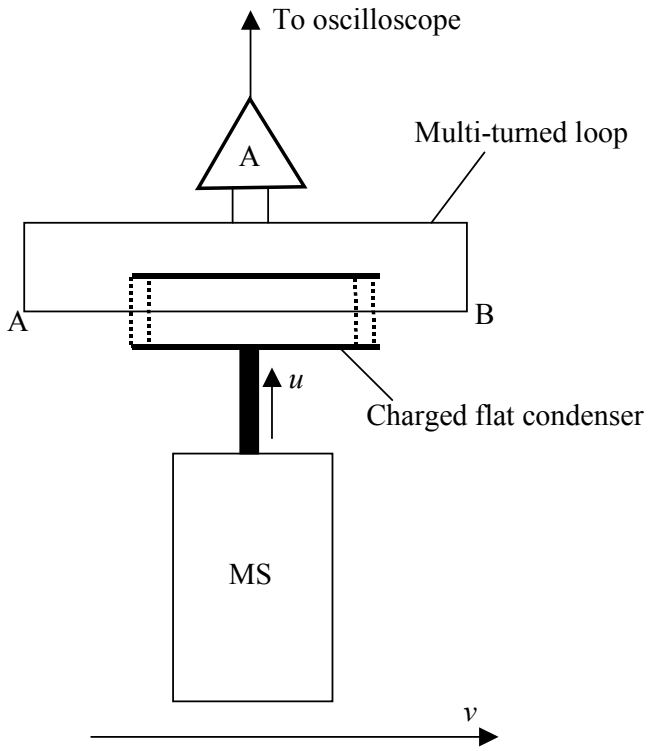

Fig. 5. Proposed scheme for a new test of SRT in the induction experiment

Now this experiment is prepared together with Drs. Oleg V. Missevitch and Victor A. Evdokimov. Omitting the particular technical details of the experiment, which will be described in a separate paper, we show in Fig. 5 one of the first experimental result, obtained for the numerical values, being closed to the presented above. It exhibits a certain change of e.m.f. during a day. A total detected change of e.m.f. was equal to 12 mV. At the same time, a big dispersion of the data obtained probably indicates a contribution of a systematic error to the results of measurements. It does not allow to make any definite conclusions. For further improvement of the experimental setup it is necessary either to exclude the systematic error, or to increase the value of eventual effect.

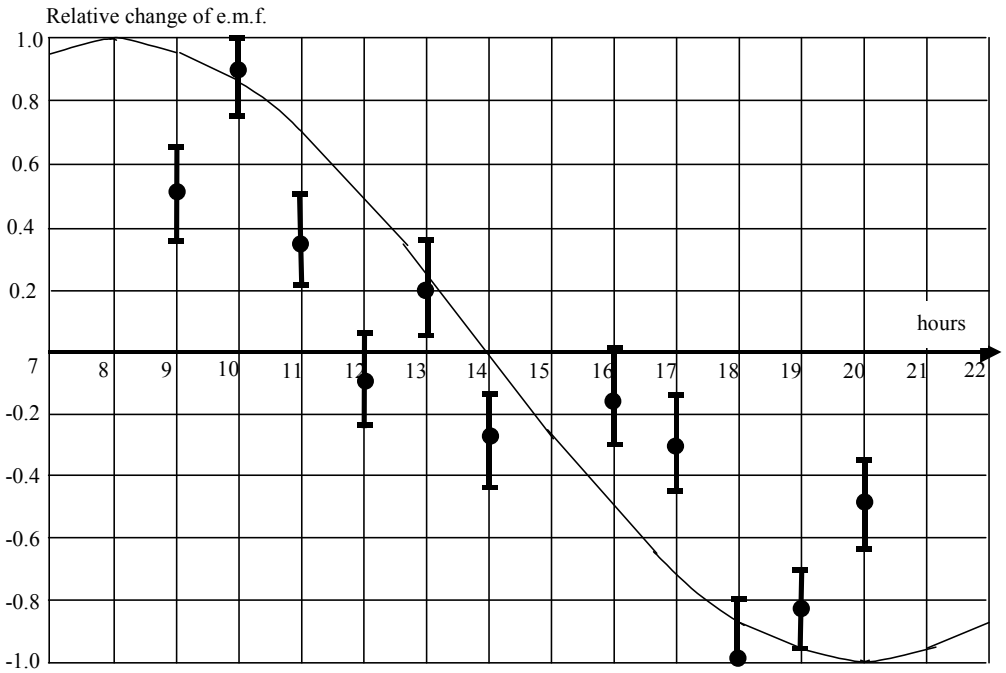

Fig. 6. Experimentally obtained dependence of e.m.f. (in relative units) on a local time in Minsk. The measurements were carried our on 23 June, 2003. The charged condenser moved along the direction West-East. The solid line shows an expected relative change of e.m.f. with time, if the absolute frame is attached to microwave relic radiation.

5. Conclusions

Thus, we have shown through integration over a mathematical line, that the Faraday induction law is not invariant with respect to the field transformations in special relativity. Under integration over a matter (conducting closed circuits) we have to additionally take into account the internal electromagnetic fields induced by rearranged conduction electrons (polarization effect). In the case where such internal fields contribute an e.m.f. in a conducting circuit, the Faraday induction law is violated, while the Einstein relativity principle remains valid. In these conditions it is especially interesting to analyze conducting circuits in which the effects of polarization do not contribute an e.m.f. due to symmetry properties of the problems considered. Then the circuit becomes similar to a mathematical line, where the Faraday induction law is true, while the Einstein relativity principle is violated. A physical problem of just this kind has been found, and the results of calculation of e.m.f. for different observers indeed disagree with relativistic conceptions. We stress that it does not reveal any mathematical "imperfection" of the relativity theory. It reflects a fact that the empirically discovered Faraday induction law is not Lorentz-invariant. This circumstance allows one a qualitatively new test of special relativity in the Faraday induction experiments.

References

[1]A.L. Kholmetskii, "On the non-invariance of the Faraday law of induction", *Apeiron,* **10,** (2003) 32.

[2]E. G. Cullwick, *Electromagnetism and Relativity*, London, Longmans, Green & Co. (1957).

[3]A.L. Kholmetskii, "Covariant ether theories and special relativity", *Physica Scripta,* **67,** (2003) 381.

[4]A.L. Kholmetskii, "The Faraday induction law and field transformations in special relativity", *Apeiron,* **10,** (2003) 118.

Landau-Lifshitz energy-momentum pseudotensor for metrics with spherical symmetry

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We have calculated the Landau-Lifshitz pseudotensor for various spherically symmetric systems as preparation for a later study for the case of rotation, which may be of interest in astrophysics. The considered systems are the static spherical star, the Schwarzschild geometry, the collapsing spherical dust ball of uniform density and the general pulsating or collapsing star.

1. Introduction

The point to point distribution of energy-momentum in the gravitational field is non-unique [1,2] in the theory of general relativity. This is inescapable because it is always possible to change coordinates to make the frame locally Lorentz at any chosen event. Gravitation must, however, make a contribution to the energy of a system since, for example, the mass of a star is less than the sum of the rest masses of its individual particles. In proving conservation laws of momentum and angular momentum for isolated systems, one can construct entities, which describe the energy-momentum content of the gravitational field. These entities are called energy-momentum pseudotensors [3-10]. The distribution of energy- momentum depends [1] on the choice of pseudotensor and on the choice of coordinates. Besides, the total momentum and angular momentum or the total energy radiated into the asymptotically flat space surrounding an isolated source also depends on the choice of pseudotensor or coordinates used in the calculation [11].

Einstein [12] was the first to introduce a pseudotensor, which is not symmetric and does not give a volume integral for the total angular momentum. In this work we have chosen the Landau-Lifshitz pseudotensor [2,13,14], which is symmetric and leads to volume integrals for momentum and angular momentum. Pseudotensors have been used [15,17] in studies of gravitational selfenergy.

2. Landau-Lifshitz energy-momentum pseudotensor

The Landau-Lifshitz (LL) pseudotensor $t_{LL}^{\mu\nu}$ is defined by writing the Einstein equations in the form [2,13,14]:

$$H_{LL}^{\mu\alpha\nu\beta}{}_{,\alpha\beta} = 16\pi(-g)\left(T^{\mu\nu} + t_{LL}^{\mu\nu}\right) = 16\pi\, T_{LL\ eff}^{\mu\nu} \tag{1}$$

where

$$H_{LL}^{\mu\alpha\nu\beta} = q^{\mu\nu}\, q^{\alpha\beta} - q^{\alpha\nu}\, q^{\mu\beta} \quad , \quad q^{\mu\nu} = \sqrt{-g}\, g^{\mu\nu} \tag{2}$$

and

$$16\pi(-g)t_{LL}^{\alpha\beta} = q^{\alpha\beta}{}_{,\lambda}\, q^{\lambda\mu}{}_{,\mu} - q^{\alpha\lambda}{}_{,\lambda}\, q^{\beta\mu}{}_{,\mu} + \frac{1}{2} g^{\alpha\beta} g_{\lambda\mu} \cdot q^{\lambda\nu}{}_{,\rho}\, q^{\rho\mu}{}_{,\nu} - g_{\mu\nu} q^{\mu\rho}{}_{,\lambda} \cdot$$
$$\cdot \left(g^{\alpha\lambda} q^{\beta\nu}{}_{,\rho} + g^{\beta\lambda} q^{\alpha\nu}{}_{,\rho}\right) + g_{\lambda\mu} g^{\nu\rho} q^{\alpha\lambda}{}_{,\nu}\, q^{\beta\mu}{}_{,\rho} + \frac{1}{8}\left(2g^{\alpha\lambda} g^{\beta\mu} - g^{\alpha\beta} g^{\lambda\mu}\right) \cdot \tag{3}$$
$$\cdot \left(2g_{\nu\rho} g_{\sigma\tau} - g_{\rho\sigma} g_{\nu\tau}\right) q^{\nu\tau}{}_{,\lambda}\, q^{\rho\sigma}{}_{,\mu}$$

The conserved momentum and angular momentum of an isolated system are given by:

$$P^{\mu} = \int T_{LL\ \ eff}^{\mu 0}\, d^3x\,,$$
$$J^{\mu\nu} = \int \left(x^{\mu} T_{LL\ \ eff}^{\nu 0} - x^{\nu} T_{LL\ \ eff}^{\mu 0}\right) d^3x\,, \tag{4}$$

with the conservation law $T_{LL\ \ eff}^{\mu\nu}{}_{,\nu} = 0$ In the expressions above, x^{μ} are asymptotically Minkowskian coordinates.

3. Some geometries with spherical symmetry

a).- The metric for a spherically symmetric star is given by:

$$ds^2 = -e^{2\Phi} dt^2 + \left(1 - \frac{2m(r)}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad . \tag{5}$$

We change coordinates t, r, θ, φ to t, x^1, x^2, x^3 where $r^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$. In the new asymptotically Minkowskian coordinates the metric takes the form:

$$ds^2 = -e^{2\Phi} dt^2 + g_{ij}\, dx^i\, dx^j \tag{6}$$

such that

$$g_{ij} = \frac{A}{r^2} x_i x_j + \delta_{ij} \quad , \quad A = \left(1 - \frac{2m(r)}{r}\right)^{-1} - 1 \tag{7}$$

A short calculation yields the following result for the effective energy density:

$$-g\left(T^{00} + t_{LL}^{00}\right) = \frac{1}{8\pi r^2} (rA)_{,r} \tag{8}$$

b).- We consider next the Schwarzschild geometry with metric:

$$ds^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \left(du^2 - dv^2\right) + r^2\, d\Omega^2 \tag{9}$$

in Kruskal-Szekeres coordinates [2,18,19]; *r(u,v)* is the Schwarszchild radial coordinate.

The coordinate transformation:

$$\bar{u} = 2M\, Ln\left[(u+a)^2 - v^2\right] \quad , \quad \bar{v} = 4M \tanh^{-1} \frac{v}{u+a} \tag{10}$$

which covers the region $(u+a) \rangle |v|$ where *a > 0,* puts the metric in the form

$$ds^2 = \frac{2M}{r} e^{-\frac{r}{2M}} \left[(u+a)^2 - v^2\right]\left(d\bar{u}^2 - d\bar{v}^2\right) + r^2\, d\Omega^2 \tag{11}$$

The additional coordinate transformation from $\bar{v}, \bar{u}, \theta, \varphi$ to asymptotically Minkowskian coordinates $\bar{v}, x^1, x^2, x^3$ where $(\bar{u})^2 = (x^1)^2 + (x^2)^2 + (x^3)^2$, gives

$$ds^2 = -\frac{2M}{r} e^{-\frac{r}{2M}} \left[(u+a)^2 - v^2\right] d\bar{v}^2 + \left(A\delta_{ij} + \frac{B}{\bar{u}^2} x_i x_j\right) dx^i dx^j \tag{12}$$

with $A = \frac{r^2}{\bar{u}^2}$ and $B = -g_{\bar{v}\bar{v}} - A$. The following expression is obtained for the LL energy density:

$$-g\, t_{LL}^{00} = \frac{1}{16\pi \bar{u}^2}\left[2(\bar{u}AB)_{,\bar{u}} - \left(\bar{u}\left(A^2\right)_{,\bar{u}}\right)_{,\bar{u}}\right] \tag{13}$$

The spacelike hypersurface $\bar{v} = constant$ includes the singularity at $r = 0$ when $\bar{v}$ is larger than a positive number which depends on a. The integral of $-g\, t_{LL}^{00}$ over any of these hypersurfaces gives *M,* as it should.

c.).- We now repeat the calculation for the case of the Schwarszchild metric in comoving coordinates.

This is appropriate to connect to an interior Friedman solution for the case of a collapsing ball of dust. In Novikov coordinates [20] the metric is written as:

$$ds^2 = -d\tau^2 + \left(\frac{R^{*2}+1}{R^{*2}}\right)\left(\frac{\partial r}{\partial R^*}\right)^2 dR^{*2} + r^2\, d\Omega^2 \tag{14}$$

and in terms of the new radial variable $R = 2M\left(R^{*2}+1\right)$:

$$ds^2 = -d\tau^2 + f(\tau, R)dR^2 + r^2(\tau, R)d\Omega^2 \tag{15}$$

The new coordinates τ, x^1, x^2, x^3 where $R^2 = \left(x^1\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2$ are asymtotically Minkowskian and thus (15) adopts the form:

$$ds^2 = -d\tau^2 + \left(A\delta_{ij} + \frac{B}{R^2}x_i x_j\right) dx^i\, dx^j \tag{16}$$

being $A = \frac{r^2}{R^2}$ and $B = f - A$. Then the *LL* energy density, is given by a relation similar to (13):

$$-g\, t_{LL}^{00} = \frac{1}{16\pi R^2}\left[2(RAB)_{,R} - \left(R\left(A^2\right)_{,R}\right)_{,R}\right]. \tag{17}$$

d).- The collapsing uniform density ball of dust has an interior Friedman solutions:

$$ds^2 = -d\tau^2 + a^2(\tau)\left[d\chi^2 + Sin^2\chi\, d\Omega^2\right], \tag{18}$$

with $a(\tau) = \frac{1}{2}a_m(1 - Cos\,\eta)$ and $\tau = \frac{1}{2}a_m(\eta + Sin\,\eta)$. This geometry connects at the surface χ_0 of the ball with the exterior Schwarzschild solution (15) using the radial coordinate $R = a_m Sin\,\chi$, then (18) implies:

$$ds^2 = -d\tau^2 + \left(\frac{a^2}{a_m^2 - R^2}\right) dR^2 + \frac{a^2 R^2}{a_m^2} d\Omega^2 \tag{19}$$

We further change the coordinates τ, R, θ, φ to τ, x^1, x^2, x^3 where $R^2 = \left(x^1\right)^2 + \left(x^2\right)^2 + \left(x^3\right)^2$, then the new coordinates connect to the exterior asymptotically Minkowskian coordinates which were used in (16). Thus the *LL* effective energy density $-g\left(T^{00} + t_{LL}^{00}\right)$ is given by (17) with:

$$A = \frac{a^2}{a_m^2} \quad , \qquad B = \frac{a^2}{a_m^2 - R^2} \ . \tag{20}$$

The interior contribution to the total energy *M* on a τ = constant hypersurface is

$$\frac{MR_0}{16(R_0 - 2M)}(1 + Cos\,\eta)^4 , \tag{21}$$

where R_0 is the radial (comoving) coordinate of the surface of the ball. This contribution decreases and becomes zero when the dust hits the singularity at $a(\tau)=0$ for $\tau=\frac{\pi}{2}a_m$. The exterior contribution to the total energy is M minus the value above for $\tau\langle\frac{\pi}{2}a_m$ and M for later times.

Finally, the *LL* effective energy density of any metric of the form (15) is given by expression (17) setting $A=\frac{r^2}{R^2}$ and $B=f-A$.

References

1. A. Trautman, Gravitation: an Introduction to Current Research, Ed. L. Witten, Wiley N.Y. (1962).
2. C.W. Misner, K.S. Thorne and J.A. Wheeler, Gravitation, W.H. Freeman, San Francisco (1973).
3. C. Moeller, Ann. Phys. **4** (1958) 347.
4. J. N. Goldberg, Phys. Rev. **111** (1958) 315.
5. P.A.M. Dirac, General theory of relativity, John Wiley, N.Y. (1975).
6. J. Stachel, Gen. Rel. Grav. **8** (1977) 705.
7. S. Persides, Gen. Rel. Grav. **10** (1979) 609.
8. T.N. Palmer Gen. Rel. Grav. **12** (1980) 149.
9. C.C. Chang, J. Nester and C.M. Chen, Phys. Rev. Lett. **83** (1999) 1897.
10. S.V. Babak, L.P. Grishchuk, Phys. Rev. **D61** (2000) 24.
11. G.T. Horowitz and P. Tod, Commun Math. Phys.**85** (1982) 429.
12. A. Einstein, Ann. Physik **49** (1916) 769.
13. L.D. Landau and E.M. Lifshitz, The Classical Theory of Fields, Addison-Wesley, Reading MA (1962).
14. J.L. Synge, Relativity: The general theory, North-Holland Pub., Amsterdam (1976).
15. A. Einstein, Berliner Berichte **349** (1919) 448.
16. A. Einstein and N. Rosen, Phys. Rev. **48** (1935) 73.
17. H.J. Treder and W. Yourgrau, Phys. Lett. **64** (1977) 25.
18. M. D. Kruskal, Phys. Rev. **119** (1960) 1743.
19. G. Szekeres, Publ. Mat. Debrecen **7** (1960) 285.
20. Ya. B. Zel´dovich and I.D. Novikov, Relativistic Astrophysics Vol.I: Stars and Relativity, University Chicago Press (1974).

The special relativity reinterpreted in view of the quantum mechanics

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The Newton theory of gravity describes gravitational interaction in the Euclidean space with the metric

$$l^2 = x^2 + y^2 + z^2 \tag{1}$$

and the universal time t. The gravitational field is static and is defined by the gravitational potential

$$\Phi = \frac{Gm}{r} \tag{2}$$

where G is the Newton constant, m is the mass of the source of gravity, r is the distance to the source. Equation (2) is invariant under the Galilei transformation

$$x' = x - vt \qquad y' = y \qquad z' = z \qquad t' = t \tag{3}$$

where coordinates of space x, y, z and time t describe the rest frame, coordinates of space x', y', z' and time t' describe the moving frame, v is the speed of the moving frame. Invariance of equations of Newton mechanics under the Galilei transformation is a manifestation of the principle of relativity in the Newton mechanics.

The Maxwell-Lorentz theory of electromagnetic interaction describes electromagnetic field as a wave with the scalar potential φ and the vector potential $\vec{A}$

$$\Delta\varphi - \frac{1}{c^2}\frac{\partial^2\varphi}{\partial t^2} = 0 \tag{4}$$

where Δ is the Laplacian, c is the speed of light. Equations (4,5) are not invariant under the Galilei transformation but are invariant under the Lorentz transformation

$$x' = \frac{x - vt}{(1 - v^2/c^2)^{1/2}} \qquad y' = y \qquad z' = z \qquad t' = \frac{t - xv/c^2}{(1 - v^2/c^2)^{1/2}}. \tag{6}$$

According to the Einstein theory of special relativity [1], invariance of equations of electrodynamics under the Lorentz transformation is a manifestation of the principle of relativity, with it is valid for all physics. Then the Lorentz transformation defines the space-time in the absence of the sources of gravity as the Minkowski space-time with the metric

$$x^2 + y^2 + z^2 - c^2t^2 = 0. \tag{7}$$

In the space-time defined by the Lorentz transformation, observers in the rest and in the moving frames describe events with the different intervals of length and time. An observer in his own frame measures the intervals of length and time as $\Delta x = x_2 - x_1$, $\Delta t = t_2 - t_1$ respectively. An observer in the other frame measures these intervals of length and time as $\Delta x' = \Delta x(1 - v^2/c^2)^{1/2}, \Delta t' = \Delta t/(1 - v^2/c^2)^{1/2}$ respectively. According to Einstein, an observer in his own frame measures the true (physical) intervals of length and time whereas an observer in the other frame do not. Then the sentences of an observer in some frame cannot be verified by observers in the other frames. Therefore the Einstein theory of special relativity contains non-verified sentences that is not acceptable. The theory has the physical meaning only if any event is described by observers in all inertial frames with the same intervals of length and time. Then the sentences of an observer in some frame can be verified by observers in the other frames.

In [2], the concept of space-time in the electrodynamics was proposed based on the dual particle-wave nature of the electromagnetic field. Within the framework of quantum mechanics, elec-

tromagnetic field is conceived as a quantum object, photon, which has the dual particle-wave nature. The space coordinate $\vec{r}$ and the momentum of photon $\vec{p}$ are bound by the Heisenberg uncertainty principle

$$pr \geq \hbar \tag{8}$$

where $\hbar$ is the Planck constant. Therefore one cannot simultaneously define the space coordinate and the momentum of photon. When considering classical electrodynamics, one should take into account the constraints imposed by the quantum mechanics. That is to consider electromagnetic field as a dual particle-wave object the space coordinate and the momentum of which are defined separately.

Consider electromagnetic field as a massless particle. Assume that the motion of the massless particle do not depend on the motion of the source of the electromagnetic field. Then it is described as

$$\Delta x - c\Delta t = 0. \tag{9}$$

Equation (9) is invariant under the Galilei transformation. Then the motion of the electromagnetic field as a massless particle defines the interval of the Euclidean space l and the interval of the universal time t. Hence it defines the scales of momentum (inverse wavelength) $p \propto 1/l$ and energy (frequency) $E \propto 1/t$. Invariance of equation for the motion of the electromagnetic field as a massless particle defines the background Euclidean space and the universal time. Then the principle of relativity for the electromagnetic field as a massless particle is restored in the form existed in the Newton mechanics.

Consider electromagnetic field as a wave described by eqs. (4,5). Recall that eqs. (4,5) are invariant under the Lorentz transformation. Assume that the Lorentz transformation defines dynamical space-time with the metric

$$\frac{1}{p_x^2} + \frac{1}{p_y^2} + \frac{1}{p_z^2} - \frac{c^2}{E^2} = 0. \tag{10}$$

Then the momentum and energy of the electromagnetic field are transformed as

$$p'_x = \frac{p_x(1 - v^2/c^2)^{1/2}}{1 - v/c} \qquad p'_y = p_y \qquad p'_z = p_z \qquad E' = \frac{E(1 - v^2/c^2)^{1/2}}{1 - v/c}. \tag{11}$$

With the use of the Lorentz transformation one can describe the shift of the frequency of the electromagnetic field in the moving frame (Doppler effect)

$$\omega' = \frac{\omega(1 - v^2/c^2)^{1/2}}{1 - v\cos\alpha/c} \tag{12}$$

where α is the angle (in the frame of the receiver) between the direction of the electromagnetic field and the direction of the emitter.

The first order term v/c is due to the inertial frame with the speed v. The second order term v^2/c^2 is due to the non-inertial frame with the potential of the inertial force v^2. Therefore it is reasonable to divide the Lorentz transformation into two parts. The first part, the modified Galilei transformation, describes transition between inertial frames

$$p'_x = \frac{p_x}{1 - v/c} \qquad p'_y = p_y \qquad p'_z = p_z \qquad E' = \frac{E}{1 - v/c}. \tag{13}$$

The modified Galilei transformation given by eq. (13) is a manifestation of the principle of relativity for the electromagnetic field as a wave. Then the first order effect is relative. That is the choice of the rest and the moving frame is arbitrary. An observer in his own frame determines the frequency of the electromagnetic field as ω whereas an observer in the other frame (moving frame) determines the frequency of the same electromagnetic field as $\omega' \propto (1 - v/c)^{-1}$, we neglect the second order effect. Unlike the Einstein special relativity where only an observer in his own frame

determines the true (physical) values, here both observers, in the rest and in the moving frame, determine the true (physical) but different values.

The second part, the Lorentz transformation neglecting the first order effect, describes transition between non-inertial frames

$$p'_x = p_x(1-v^2/c^2)^{1/2} \qquad p'_y = p_y \qquad p'_z = p_z \qquad E' = E(1-v^2/c^2)^{1/2}. \quad (14)$$

Then the second order effect is absolute. That is the choice of the rest and the moving frame is not arbitrary and is defined with respect to the inertial forces. More correctly in this case to specify the frames as the frame with zero potential (rest frame) and the frame with the potential v^2 (moving frame). Both observers, in the frame with zero potential and in the frame with the potential v^2, determine the frequency of the electromagnetic field in the frame with zero potential as ω and in the frame with the potential v^2 as $\omega'(1-v^2/c^2)^{1/2}$, we neglect the first order effect. Unlike the Einstein special relativity, the prime frequency of the electromagnetic field is the true (physical) one in the frame with the potential v^2 (moving frame).

So we arrive at the following interpretation of the special relativity. There exist the background Euclidean space and the universal time which define the intervals of length and time and hence the scales of momentum and energy. The dynamics of the electromagnetic interaction takes place in the Minkowski space-time. It is reasonable to define inertial mass (potential of the inertial force) and gravitational mass (gravitational potential) in the background Euclidean space and the universal time. That is we assume that the gravitational field is non-relativistic. The dynamics of electron gives an argument in favour of this assumption. Following [2] consider the dynamics of the electron with the charge e and the mass m in the electromagnetic field within the framework of the special relativity reinterpreted. In the frame moving with the speed v, the acceleration due to the electromagnetic force experienced by the electron is given by

$$w' = \frac{e}{m}\vec{E}(1-v^2/c^2)^{1/2} \quad (15)$$

where $\vec{E}$ is the strength of the electromagnetic field. From eq. (15) one can conclude that if the electromagnetic energy (charge) of the electron follows the Lorentz transformation then the gravitational energy (mass) of the electron do not.

Description of the electromagnetic field in the gravitational field may be given with the use of the principle of equivalence. While considering electromagnetic field as a massless particle it is reasonable to assume that its motion do not depend on the gravitational potential. So the gravitational potential do not bend the trajectory of the electromagnetic field as a massless particle. While considering electromagnetic field as a wave the gravitational potential affects the electromagnetic field. According to the principle of equivalence put the gravitational potential into correspondence with the potential of the inertial force

$$2\Phi = v^2. \quad (16)$$

Substituting eq. (16) into eq. (12) one obtains the shift of the frequency of the electromagnetic field in the gravitational field

$$\omega' = \omega(1-2\Phi/c^2)^{1/2}. \quad (17)$$

It should be stressed once again that the prime frequency is the true (physical) one in the gravitational field.

The phase of a plane electromagnetic wave is retarded in the gravitational field differently along the direction of the gravitational field. Hence a plane wave traveling transversely to the gravitational field changes its direction of propagation because the secondary fronts of the Huygens' construction advance faster where the gravitational potential is higher. It is necessary to take into account that according to the Lorentz transformation, in the laboratory frame, the energy of a plane

wave is two times higher in the direction of motion than in the transverse direction. Hence the wavelength is two times smaller in the direction of motion than in the transverse direction. Note that, in the Einstein special relativity, the true (physical) energy of the wave is determined in the system of reference of the wave. Hence the energy of a plane wave is the same in the direction of motion and in the transverse direction. In the special relativity under consideration, the energy of a plane wave (the first order effect) determined in the laboratory system of reference is also true (physical) but different from that in the system of reference of the wave. Then one should use the value determined in the system where the process is described, i.e. in the laboratory system of reference. From the above it follows that the deflection of a plane wave in the transverse gravitational field after a displacement Δx is given by

$$\phi = \frac{2\Delta x}{c^2}\frac{d\Phi}{dy}. \tag{18}$$

The deflection of a plane wave given by eq. (18) is two times higher than that predicted by the Einstein special relativity [1] and is consistent with the experimental results. Thus one can explain the bending of light (plane electromagnetic wave) in the gravitational field within the framework of the special relativity reinterpreted. Note that the deflection of the electromagnetic field as a wave is defined with respect to the trajectory of the electromagnetic field as a particle which is a straight line. That is the time of travel for the electromagnetic field is determined by the motion of the massless particle along the straight line from the emitter to the receiver with the speed c.

Einstein developed relativistic theory of gravity (general relativity) [1] in which the space-time is curved by the matter fields. There are two experimental tests [1] treated as evidences for the concept of the curved space-time, the bending of light in the gravitational field (one half of the effect is explained with the spatial curvature) and the anomalous shift of the perihelion of Mercury. In the special relativity reinterpreted, we have considered gravity as a static non-relativistic field (Newton potential) in the Euclidean space and the universal time. It was shown that the bending of light in the gravitational field may be explained within the framework of the special relativity reinterpreted without resorting to the concept of the curved space-time. Hence explanation of the anomalous shift of the perihelion of Mercury with the concept of the curved space-time is questioned.

Conclude the above consideration. The Einstein treatment of the special relativity introduces into the theory non-verified sentences hence it is questioned. The following interpretation of the special relativity is given based on the particle-wave concept of the electromagnetic field. Since electromagnetic field is a quantum object, its coordinate and momentum are determined separately. This is also valid when we consider electromagnetic field in the classical theory.

Electromagnetic field as a particle defines background Euclidean space and the universal time. Electromagnetic field as a wave defines momentum space-time with the Minkowski metric. Unlike the Einstein theory, the special relativity reinterpreted deals only with the true (physical) values. It is reasonable to treat gravity as a static non-relativistic field (Newton potential) in the Euclidean space and the universal time. The bending of light in the gravitational field may be explained within the framework of the special relativity reinterpreted. Explanation of the anomalous shift of the perihelion of Mercury should be given.

References

[1] W. PAULI, Theory of Relativity, Pergamon, New York (1958).

[2] D.L. KHOKHLOV, Space-time in the classical electrodynamics from the viewpoint of quantum mechanics, Spacetime & Substance, 4(14) (2002), 179-180.

[3] D.L. KHOKHLOV, Electromagnetic energy against gravitational energy, submitted to Spacetime & Substance.

Identification of structural systems and physical objects using radio wave control method

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Identification of structural systems and physical objects in order to increase quality and reliability of products is an important task. Manufacturing of high-reliable products needs it badly. Methods, which do not destruct objects and allow its following usage, play a major role when increasing the quality of production. The methods are called non-destructive methods. The methods allow discovering structural defects. As optimal physical-mechanical and heat-resistant properties necessary for effective usage are conditioned by formation of special structure and depend on degree of their correspondence with calculation results, the presence of structural defects can be considered as the key factor to define the performance.

The goal of the paper is the identification of structural systems and physical objects using radio wave method, which is the most effective to discover structural defects causing the decrease of performance of a physical object from the technological viewpoint.

Today, the radio wave control method can be classified (according to GOST 18353-79) in way of interaction with a physical object as: penetrating radiation, reflected radiation, scattered radiation and resonance radiation.

The schemes in accordance with the classification are active, i.e. an object undergoes radio wave radiation. The application of the schemes, however, is effective only to control non-metals [1].

The presented geometry of the radio wave method to control, which is also active, but generalized, i.e. joins features typical for all the four basic schemes of radio wave control method by using electric oscillation circuit, which includes the controlled object as its part (Fig. 1).

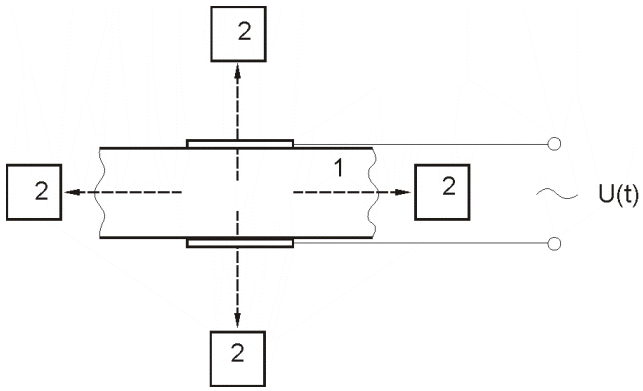


Fig. 1. Principal scheme of radio wave control method.
1 - structural scheme, 2 - receiver (the scheme shows some possible locations of the device).

The scheme is designed to allow controlling physical objects and structural systems made of both metal and non-metal materials.

Physical object is placed between plates of a capacitor and is a part of a complex electric oscillation circuit. In order to establish a quantitative relation between parameters of electro-magnetic wave passed through the interface and sought parameters of the physical object, the phenomenological approach based on Maxwell equation is used. Contact phenomena between systems are analyzed: conductor-conductor, conductor-dielectric, dielectric-dielectric.

The proposed scheme has been tested in laboratory conditions. Fig. 2 shows the test geometry, when the method is applied to discover defects for example of gluing failure.

The structural system consists of honeycomb panel made of coal-plastic shells and aluminum honeycombs, which is a part of a complex electric oscillation circuit. The circuit consists of a voltage source, capacitor, coil and resistor.

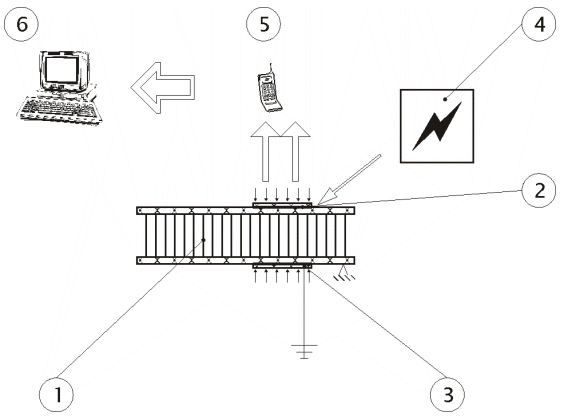


Fig. 2 Test geometry

1. structural object (honeycomb panel); 2. self-gumming foil (2 items); 3. metal ring (2 items); 4. source of voltage; 5. scanner; 6. data processing and visualizing device (PC).

When high-frequency alternating current from source 4 is applied to foil 2, the structural object starts radiating radio-magnetic waves over a wide range of frequencies, which are received by scanner and then transmitted to PC for following processing.

The scanning has been performed over radio-frequency range (i.e. a scanning receiver selects electro-magnetic waves (□=11 MHz and □□=±200 kHz).

The control results in:

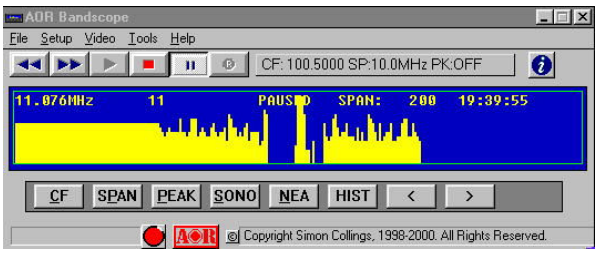
physical object has no defect regions;

physical object has one evident defect;

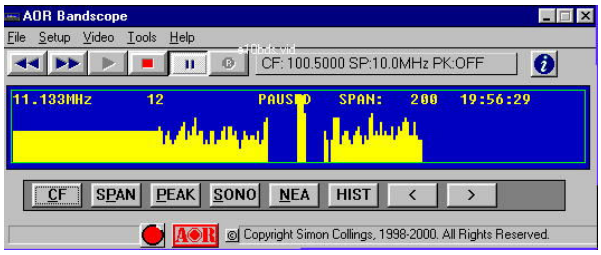
physical object has three evident defect.

The scanning receiver is directly connection with the PC, which has software to visualize secondary spectrum radiating by the object and process them.

Figs. 2 a, b, c show radio-wave spectrums of physical object with no defect regions, one defect region – gluing failure, and three defect regions – gluing failure. The method is based on comparison of spectrum for sample (no defect) physical object and identified object.



a



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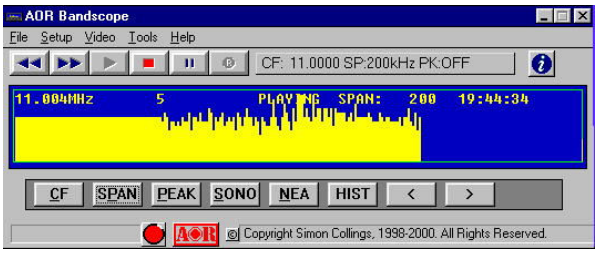

c.

Fig. 2 Spectrum of physical object with (a) no defect zones and regions, (b) one defect zone (evident defect – gluing failure), (c) three defect zones (evident defects – gluing failure).

The work results in the following conclusions:

- problem of identification of physical objects and structural systems can be considered as the problem of identification and comparison of spectral characteristics of a sample (calculation) object with tested one
- physical-mathematical model and principal scheme for identification of physical objects of structural systems has been developed
- the setup has been produced, test has been performed to prove to efficiency of the proposed method,
- the necessity to develop special numerical-analytical method to compare spectral characteristics for identification of physical objects of structural systems has been shown.

References

1. **Non-destructive control and testing.** Reference-book / V.V. Kluev, F.R. Sosnin, V.N. Filinov et al.; Edited by V.V. Kluev. M.: Mashinostroenie, 1995. - P. 488.

Нелинейное самогравитирующее спинорное поле: конфигурации типа космической струны

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Для классического спинорного поля с нелинейностью вида $F(S)$, где $S = \psi\overline{\psi}$, а F - произвольная функция, рассматривается вопрос о существовании регулярных самогравитирующих конфигураций, по определению, должны обладать регулярной осью симметрии и геометрией типа космической струны вдали от оси (т.е. метрика должна стремиться к метрике Минковского с некоторым угловым дефектом). Показано, что эти условия реализуются, если функция $F(S)$ стремится к конечному пределу $S \to \infty$ и убывает быстрее, чем S^2, при $S \to 0$. Получено общее точное решение уравнений поля, рассмотрены конкретные примеры.

Experimental evidence of nonlocal transaction in reverse time.

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Phenomenon of nonlocality allows superluminar transaction, and accordingly transaction in reverse time, without violation of relativity principles namely because of absence of any local carriers of interaction. Quantum nonlocality may persist at macro-level and manifest via nonlocal correlation of the dissipative processes. If a source-process is noncontrolled both retarded and advanced correlations are possible. The experiment on verification of this hypothesis has been performed. Nonlocal reaction of the probe-processes on large-scale geophysical and astrophysical processes was studied. As a result nonlocal transaction in reverse time has been reliably detected for the processes of solar, geomagnetic and synoptic activity. Due to diffusion propagation of the transaction value of advancement proved to be large.

1. Introduction

More than 30 years ago N.A. Kozyrev suggested concept of active time, developed in the framework of approach called causal mechanics [1]. Causal mechanics is originated from acceptance of fundamental asymmetry of time. Certainly, time asymmetry leads to violation of the energy conservation. Causal mechanics treated this violation as the energy of time emergence. Therefore, time has, except of the passive property of duration, some active ones (course, density), which involve superluminar interaction or, more exactly, transaction between any dissipative processes. Kozyrev performed extensive series of the experiments, which demonstrated unusual properties of this transaction, for example, availability of advanced correlation for non-controlled processes [1-3]. However his results met a contradictory reaction because of weak formalization of the theory and doubt about rigor of the experiments. But recently the basic statements of causal mechanics were strictly formulated [4-7] and the experiments at the modern level of rigor were performed [4-11]. This paper is devoted to results of these experiments.

The paper is ordered as follows. The next section will shortly review theoretical background and formulate the hypothesis. Section 3 will deal with setting up an experimental problem. Afterwards the experimental setup and data will be described in Sec 4. In Sec.5 description of the most interesting results will be presented. The conclusion will be in Sec. 6.

2. Macroscopic nonlocality

Generalization of Kozyrev′s results had shown similarity of the properties of transaction via active time with ones of quantum nonlocality [4-7, 11]. Nonlocality admits superluminar transaction, and accordingly transaction in reverse time, namely because of absence of any local carriers of interaction. It is generally believed that quantum nonlocality exists only at the micro-level. But during the last years theoretical considerations evolved of nonlocality persistence at the macroscopic

limit [12, 13], although a viable idea for experimental verification of such conclusion was not suggested (the standard blueprint of the interference experiment at the macro-limit is wittingly unfit). On the other hand a new way of entanglement formation via a common thermostat was suggested recently [14], and this way needs dissipativity of the quantum correlated processes. It means that dissipativity may not only lead to decoherence (namely this prevents manifestation of nonlocality at the macro-level), but on the contrary, it may play a constructive role. Our idea is to include dissipation by interpretation of quantum nonlocality within Weeler-Feynman action-at-a-distance electrodynamics [15], which we use in modern quantum treatment [16]. This theory considers direct particle field as superposition of retarded and advanced ones. The advanced field is unobservable and manifests only via radiation damping, which is the dissipative process.

But first of all let's take notice to likeness of axioms of causal mechanics and action-at-a-distance electrodynamics. In the electrodynamics transaction of the charges separated by finite distance $\delta\vec{x}$ and lapse δt (with zero interval)is postulated. Self-action of the charges is absent. Two from three Kozyrev′s axioms [1] assert the same, replacing only terms «charges» by «cause» and «effect» (third axiom postulates time asymmetry), while in Ref.[17] Kozyrev grounded that transaction occurs through zero interval. In Ref.[18] uncertainty of the terms «cause» and «effect» had been removed. Essentiality of the formalism is as follows. For any observables X and Y the independence functions i can be introduced:

$$i_{Y|X} = \frac{H(Y\,|\,X)}{H(Y)}, \quad i_{X|Y} = \frac{H(X\,|\,Y)}{H(X)}, \quad 0 \le i \le 1, \tag{2.1}$$

where H denote conditional and unconditional Shannon entropies. For example if Y is single-valued function of X then $i_{Y|X} = 0$, if Y does not depend on X, then $i_{Y|X} = 1$. Roughly saying, the independence functions behave inversely to module of correlation one.

Next, the causality function γ is considered:

$$\gamma = \frac{i_{Y|X}}{i_{X|Y}}, \quad 0 \le \gamma < \infty. \tag{2.2}$$

It can define that X is cause and Y is effect if $\gamma < 1$. And inversely: Y is cause and X is effect if $\gamma > 1$. The case $\gamma = 1$ means non-causal relation X and Y (they are related with some common cause).Theoretical and multiple of experimental examples have shown that such formal definition of causality does not contradict its intuitive understanding in obvious situation and can be used in non-obvious ones (e.g.[18-21]). Our definition allows formulating all three Kozyrev′s axioms in the form of one:

$$\begin{aligned} \gamma < 1 &\Rightarrow t_Y > t_X, \quad \vec{x}_Y \ne \vec{x}_X; \\ \gamma > 1 &\Rightarrow t_Y < t_X, \quad \vec{x}_Y \ne \vec{x}_X; \\ \gamma \to 1 &\Rightarrow t_Y \to t_X, \quad \vec{x}_Y \to \vec{x}_X. \end{aligned} \tag{2.3}$$

Statement (2.3) is very natural, but it is axiom of strong or local causality. For nonlocal transaction this statement might be invalid due to advanced correlations of the dissipative processes. Indeed any dissipative process is ultimately related to the radiation and therefore to the radiation damping. As a result it can be shown that advanced field connects the dissipative processes [4,5].

Time asymmetry is expressed as absorption asymmetry. While absorption of retarded field is perfect, absorption of advanced one must be imperfect. Having accepted that total field E is superposition

$$E = AE^{ret} + BE^{adv}, \tag{2.4}$$

Where A and B are constants and having denoted efficiency of absorption of retarded field *by a* ($a=1$ corresponds to perfect absorption, a=0 – to absence of absorption), advanced one by b, it is easily to obtain [16], that

$$A = \frac{1-b}{2-a-b}, \quad B = \frac{1-a}{2-a-b}, \tag{2.5}$$

Substitution to Eq. (2.4) A=1, B=0 corresponds to really observing situation, that is compatible with Eq. (2.5) only if a=1, $0 \leq b < 1$. It should be stressed wide a priori arbitrariness in value b, which may be close as to unit so to zero. Therefore the screening properties of the matter must be in one degree or another attenuated. The fact itself of imperfect absorption of the advanced field means a possibility of its separate detection.

From the operational consideration it is possible to formulate the following equation:

$$\dot{S} = \sigma \int \frac{\dot{s}}{x^2} \delta\left(t^2 - \frac{x^2}{v^2}\right) dV, \tag{2.6}$$

$$\sigma \sim \frac{\hbar^4}{m_e^2 c^4}, \quad v^2 \leq c^2,$$

where $\dot{S}$ is the entropy production in a probe process (that is detector), $\dot{s}$ is entropy production in the sources, σ is cross-section of transaction. The δ-function shows that transaction progresses with symmetrical retardation and advancement. In particular, if the transaction occurs through a medium by diffusion, then values of resulting retardation and advancement are large.

3. Setting up the experimental problem

The experiment task is to detect the entropy change of the environment according to Eq (2.6) under condition that all known classical local interactions are suppressed. Three types of detectors had been chosen by the criterion of maximal efficiency of a probe process: the first was based on the variations of self-potentials of weakly polarized electrodes in an electrolyte, the second – on the variations of dark current of the photomultiplier and the third – on the variations of ion mobility of an electrolyte. Consider briefly, for example, theory of the electrode detector.

Self-consistent solution for the potential u in the liquid phase is [22]:

$$u = \frac{2kT}{q} \ln \cos\left(x \arccos\left(\exp \frac{q\zeta}{2kT}\right)\right), \tag{3.1}$$

where q is charge of the main ion of the liquid phase, x is dimensionless length (x = 1 corresponds to half of the distance between the electrodes), ζ is full (electrokinetic) potential. The entropy S can be expressed in term of the normalized potential φ:

$$\varphi = \frac{u}{\int_0^1 u dx}, \tag{3.2}$$

$$S = -\int_0^1 \varphi \ln \varphi dx, \tag{3.3}$$

Substituting Eq. (3.1) to Eq. (3.2) and (3.3), one can obtain the expression for the entropy:

$$S \approx \ln \sigma - 2 \ln(\arccos(\exp w)), \tag{3.4}$$

where $w = \frac{q\zeta}{2kT} < 0, \quad |w| << 1$. Therefore, the entropy production is:

$$\dot{S} = \frac{\exp w}{\arccos(\exp w)\sqrt{1-\exp 2w}}\dot{w} \tag{3.5}$$

Prefactor of w is always positive, therefore from Eq. (3.4) and (3.5) it follows that S and ζ change in opposite phase. So far as variations of ζ are small in comparison with the averaged value, one can linearize Eq. (3.11) and obtain final simple expression:

$$\dot{S} \approx -\frac{1}{\sqrt{6}}\frac{|q|}{kT}\dot{\zeta} \tag{3.6}$$

All known local factors influencing on ζ: temperature, pressure, chemism, illumination, electric field, concentration and movement of the electrolyte must be excluded. In fact, only difference $U = \zeta_1 - \zeta_2$ of pair of the electrodes can be measured. Except external screening, influence of mentioned above noise-forming factors might be minimized by measuring U at minimal electrode separation. In this case variable U is:

$$U = g\left(\zeta_1^c - \zeta_2^c\right), \tag{3.7}$$

where ζ^c are constants, g is efficiency of the detector, the averaged measure of which is the variability coefficient.

For the photomultiplier detector analogue of ζ is the work function. Noise-forming factors influencing dark current to be excluded are: temperature, electric and magnetic field, illumination, moisture and feed voltage instability.

For the ion mobility detector entropy is linearly related with dispersion of the electric current in the small electrolyte volume. Noise-forming factor influencing the current to be excluded are: temperature, electromagnetic field and electrochemical reactions.

It is known that quantum nonlocality violates strong causality and persists weak one [15]. It means, that if a source-process is noncontrolled, we can observe both retarded and advanced correlations. But if an observer initiates a source-process, only retarded correlation is possible. That is why the most interesting source-process are large-scale natural ones. The experiment described below was devoted to study detectors reaction on various geophysical and astrophysical processes. The experiments with controlled lab artificial source-processes had also been conducted, though they had, of course, demonstrated only retarded correlation [9,10].

4. Experimental setups and data

The Geoelectromagnetic Research Institute (GEMRI) experimental setup included two types of detectors and apparatus for accompanying measurements (Fig.1).

The detector based on weakly polarized electrodes was constructed as follows. As the electrodes marine geophysical *C-Mn* ones were chosen. The electrodes were positioned in the glass vessel with the *NaCl* water solution; separation between contact windows measured 1.5 *cm*. The vessel was rigidly encapsulated so that evaporation as well as atmospheric pressure variations were fully eliminated. The vessel was positioned in the dewar, covered on the outside by the additional layers of heat insulation. For residual temperature variations control the sensor of temperature (allowing to measure it continuously accurate to 0.001 *K*) was positioned between internal wall of the dewar and the electrode vessel. Thus influence of all noise-forming factors, except temperature, was eliminated. Influence variation of the last was minimized and controlled. The quantity U was measured continuously accurate to 0.5 μV.

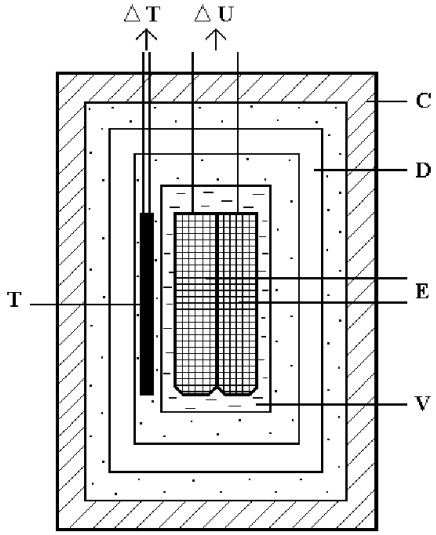

FIG.1 Simplified sketch of the electrode detector *C,* case (thickness of the walls is 20 *mm*); *D*, dewar; *V*, vessel with the electrolyte; *E*, electrodes (complicated internal design is not shown); *T*, temperature sensor. Materials: shade, caprolon; doubleshade, ebonite; dots, air; unshaded space, vacuum.

The second type detector was constructed on the base of photomultiplier with the *Cb-Cs* cathode of small area. The photomultiplier was positioned in the similar dewar with the temperature sensor and the additional external electric field screen. Possible magnetic field influence was controlled by quantum modulus magnetometer accurate to 10^{-5} *A/m.* The dark current *I* was measured continuously accurate to 0.05 *nA*. Magnetic field measurement served also as indicator of the most important for quantitative interpretation geophysical process – dissipation of ionospheric electric current.

Lastly, the overall air temperature in the lab was recorded continuously accurate to 0.1 *K*. Thus measurements on the setup included 2 major channels and 4 satellite ones. Accidentally during the part of period of the experiment with the setup described above and independently, similar measurements of *Ag-AgCl* marine electrodes self-potentials in other purposes were conducted by V.I. Nalivayko, kindly presented us his data. His setup did not provide measurements of the noise-forming factors and protection from them. Nonetheless, if a signal associated with the geophysical processes in *U* variations is sufficiently strong then, taking into account relatively small distance between the labs (300 *m*), it would have hoped on correlation of data.

The Center of Applied Physics (CAP) setup [23] included detector measuring fluctuation of conductivity (corresponding to fluctuation of ions mobility) in the electrolyte cell. These fluctuations were measured as high-frequency (5...15 *kHz*) voltage fluctuation on the cell under applied stabilized low-frequency (3 *Hz*) current. The steps taken to screening of detector against local influences were comparable with those of the GEMRI setup. Internal temperature of the detector was controlled as well as in the GEMRI detectors. External atmospherical temperature was continuously measured as index of the synoptic activity.

GEMRI setup measurements carried out in continuous regime during 366 days from 1996, December 10 to 1997, December 11. A data set included signal of the electrode detector *U*, its internal temperature T_U, signal of the dark current detector *I*, its internal temperature T_I, the external lab temperature T_e and modulus of geomagnetic field *F*. During the first month data sampling was chosen 5^m, then - 30^m, and in the last two days - 1^m. Simultaneous data spaced at 300*m* the setup of N.A. Nalivayko U_r and spaced at 40 *km* the CAP setup *b* (averaged over 5^m intervals disperse of the

voltage fluctuations on the electrolyte cell), were invoked for joint interpretation. The temperature of air in Moscow, the atmospheric pressure in Troitsk and standard international data on the geomagnetic and solar activity (radio wave flux at 9 standard frequencies within range 245 … 154000 *MHz)* were taken to study the large-scale processes. In addition, data on cosmic ray counting rates from the IZMIRAN neutron monitor, situated near (at distance 100 *m*) the GEMRI setup, were invoked as one more reasonable local factor of influence on the detectors.

Data were processed by the methods of causal, correlation, regression and spectral analysis.

5. Results

A. Relation of the signals of different detectors

Thus we had long-term measurements with 4 detectors of 3 types. Their signals proved to be rather high correlated. Mathematical exclusion of single possible common local factor not completely suppressed by screening, namely internal temperature leads to ase correlation increasing. Therefore there is not any local common cause of the signals. Level of correlation proved to be independent on type of detectors and only slightly dependent on their separation. Namely, correlation between nearby detectors r_{UI} = 0.78 ±0.01, spaced at 300 *m* r_{UUr} = 0.74± 0.01 and spaced at 40 *km* r_{Ub} =0.72±0.01. Such correlation can be explained by some large-scale common causes (geophysical or astrophysical processes), but their influence cannot be local.

Due to that correlation we shall consider in the following subsection mainly results with the electrode detector *U*.

B. Relation of the detector signal with the internal and external temperature

First of all, the environment temperature variations lead to its entropy changes. The problem is complicated by trivial local influence of small residual variations of internal temperature on the probe process, but we know that such influence can be only retarded.

Due to passive thermostating, dispersion of the internal temperature T_U in the dewar of the detector *U* is very small (it is decreased on two orders relative to one of the external lab temperature T_e). Indeed, there is small correlation pike r_{UT_U} = -0.33±0.02 (corresponding to the normal negative temperature coefficient of the electrodes -(141±9) $\mu V/K$ [24], which is accompanying by minimum of the independence function $i_{U|T_U} = 0.50^{+0.02}_{-0.01}$ at the time shift τ= -20.4^h (negative sign of τ corresponds to retardation *U* relative to T_U). But at the positive time shift τ = 12.8^h (where correlation must be classically damped out) there is unusual great correlation pike r_{UT_U} = 0.87±0.01 (anomaly positive sign) which is accompanying by minimum of $i_{U|T_U} = 0.43^{+0.01}_{-0.00}$.

Turn now to analysis of connection *U* with the external (lab) temperature T_e. As there are no heat sources inside of the dewar, where T_U is measured, then local connection of the potential variations with the temperature works along the causal chain $T_e \to T_U \to U$. It imposes the restrictions on independences [19]:

$$i_{U|T_e} \geq \max\left(i_{T_U|T_e}, i_{U|T_U}\right), \quad i_{T_e|U} \geq \max\left(i_{T_e|T_U}, i_{T_U|U}\right). \tag{5.1}$$

Violation of Ineq. (5.1) (which are macroscopic analogue of Bell inequalities) is sufficient evidence of nonlocality of transaction T_e and *U*. It has turned out that $i(\tau)$ has 3 almost symmetrical minima at τ=0 and ± 27.0^h. It corresponds qualitatively to results of known Kozyrev's astrophysical experiment [2,3]. Asymmetry amounts to stronger advanced signal as compared to retarded one: at τ = -27.0^h $i_{U|T_e} = 0.81^{+0.07}_{-0.00}$, $i_{T_e|U} = 0.77^{+0.10}_{-0.00}$; at $\tau = 0$ $i_{U|T_e} = 0.77^{+0.10}_{-0.00}$, $i_{T_e|U} = 0.72^{+0.13}_{-0.00}$; at τ= 27.0^h

$i_{U|T_e} = 0.75^{+0.11}_{-0.00}$, $i_{T_e|U} = 0.71^{+0.12}_{-0.00}$. Therewith independences of T_U and T_e have only single normal minimum: at τ= -11.5^h $i_{T_U|T_e} = 0.77^{+0.03}_{-0.00}$, $i_{T_e|T_U} = 0.84^{+0.05}_{-0.00}$, i.e. $T_e \rightarrow T_U$. Substituting these and mentioned above values of independences T_U and U, we concluded that there are two channels of connection T_e and U: classical local retarded and unusual nonlocal advanced. For the former at $\tau < 0$ left Ineq. (5.1) is asserted, for the last at $\tau > 0$ right Ineq. (5.1) is violated.

Availability of the synchronous channel of connection T_e with U is explained by interference of the retarded and advanced signals [15]. But symmetry by τ for T_e, U with asymmetry for T_U, U calls for analysis.

The space-time diagram of transaction of T_e, T_U, U is schematized in Fig. 2. τ^{ret} and τ^{adv} denote times of information passage along the retarded and advanced channel respectively, other notations are clear from the figure. T_e and T_U both are connected with U by pairs of the symmetrical retarded and advanced channels, while T_e connected with T_U only by retarded one, it follows from geometry of this scheme that $\delta^{adv}=\tau^{adv}(U|T_e)-\tau^{adv}(U|T_U)= \tau^{adv}(U|T_e)-[\tau^{ret}(U|T_U)- \tau^{ret}(T_U/T_e)]$. But $\tau^{ret}(T_U/T_e)= \tau^{ret}(U/T_e)-\tau^{ret}(U/T_U)=\delta\tau^{ret}$. Therefore $\delta^{adv} =2\delta^{ret}$. Substituting mentioned above values τ, one can be content with $\delta^{adv} =2\delta^{ret}$ accurate to 7%. Therefore the diagram of Fig. 2 quantitatively explains the positions of all pikes of independence functions.

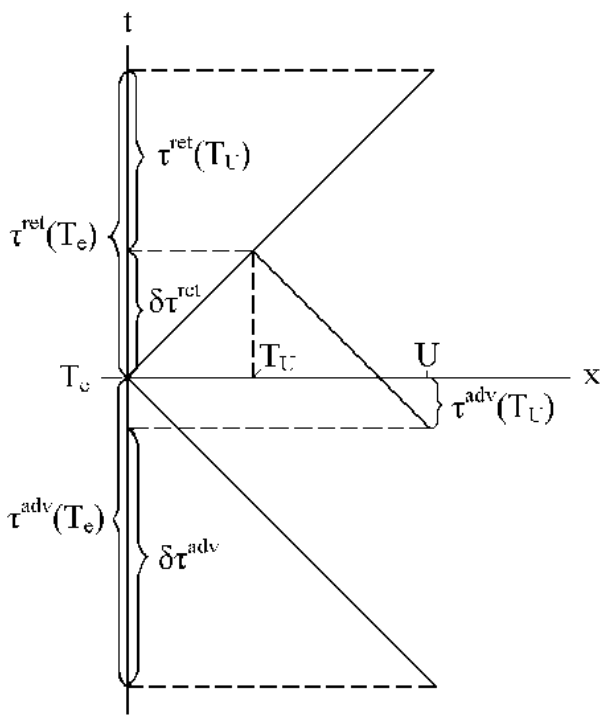

FIG. 2. Diagram of transaction of the external (lab) temperature T_e, internal temperature T_U and self potential difference of the electrodes U. Scales of distance x and time t are arbitrary. τ^{ret} and τ^{adv} are respectively retardation and advancement of the temperatures (T_U or T_e) relative to U. δ^{ret} and δ^{adv} are differences of respective τ^{ret} and τ^{adv}.

C. Relation of the detector signal with the synoptic activity

Consider now the variations of the atmospheric temperature T_a as an index of the synoptic activity. Taking into account passive thermostating, the local causal connection $T_a \rightarrow T_e \rightarrow T_U \rightarrow U$ must lead to weak correlation U with T_a at very long (many-days) retardation. Following usual geophysical practice of study of the large-scale processes, for exception of a possible influence of the small-scale

inhomogeneities compare measurements U and T_a at remote sites. Under a typical horizontal temperature scale (a few hundred km), distance between the GEMRI and CAP setups (40km) is quite optimal. That is why the measurements of T_a near CAP setup have been taken for comparison with U.

The most important feature of U,T_a dependence proved to be a dramatic exceeding of correlation at advancement U relatively T_a (τ>0) above weak (less than 0.4) retarded correlation (τ<0). Next there are five maxima r_{UT_a} at τ equal to -25, -13, 0, 13, 28 days. Symmetry relatively $\tau = 0$ is exactly analogously described above relation U with T_e. The greatest correlation is at $\tau = 13^d$: $r_{UT_a} = 0.725 \pm 0.005$. The causal analysis has shown corresponding minima of the independence function (at $\tau = 13^d$ $i_{U|T_a} = 0.72 \pm 0.01$, $\gamma = i_{T_a|U} / i_{U|T_a} = 1.02^{+0.02}_{-0.00}$, that is $T_a \rightarrow U$). Thus there is statistical reliable advanced connection of U to T_a.

More appropriate synoptic activity index is the atmospheric pressure, which scale a few thousand *km* . In Fig. 3 an example of causal analysis of the electrode detector signal and pressure is shown. At advancement 69 days (r_{UP} = - 0.78 ± 0.02) there is a deep independence function $i_{U|P}$ minimum and high causality function pike ($i_{U|P} \approx 0.30, \gamma = i_{P|U} / i_{U|P} \approx 2{,}3$) . The synoptic activity is a cause of the detector signal, but the progress in reverse time!

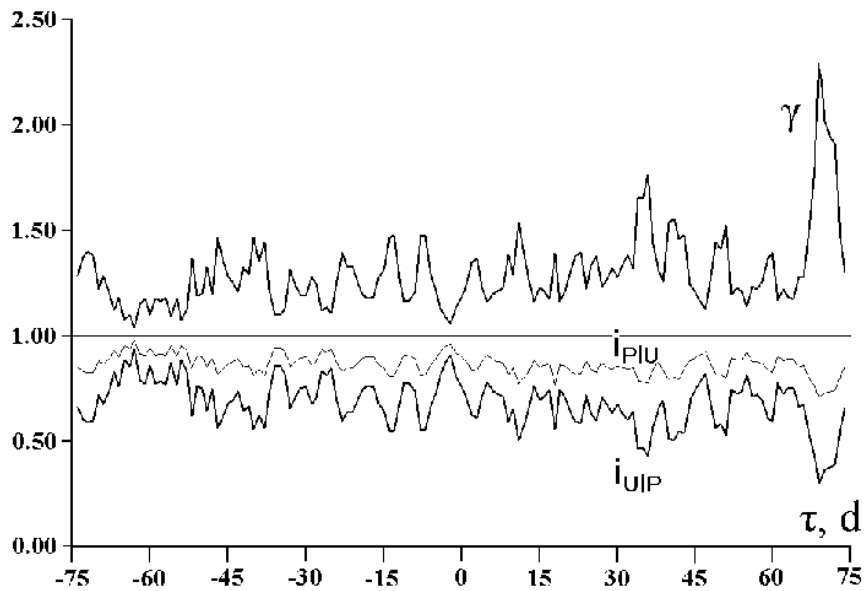

FIG.3. Independence i and causality γ functions of the electrode detector signal U (March – April, 1997) and the atmospheric pressure P. τ is time shift of P relative to U in days.

That result is independent on type of detector. In Fig. 4 the same example with the photomultiplier detector is shown. The picture is alike and advancement is almost the same, it equals 73 days (r_{IP} = - 0.86 ± 0.01).

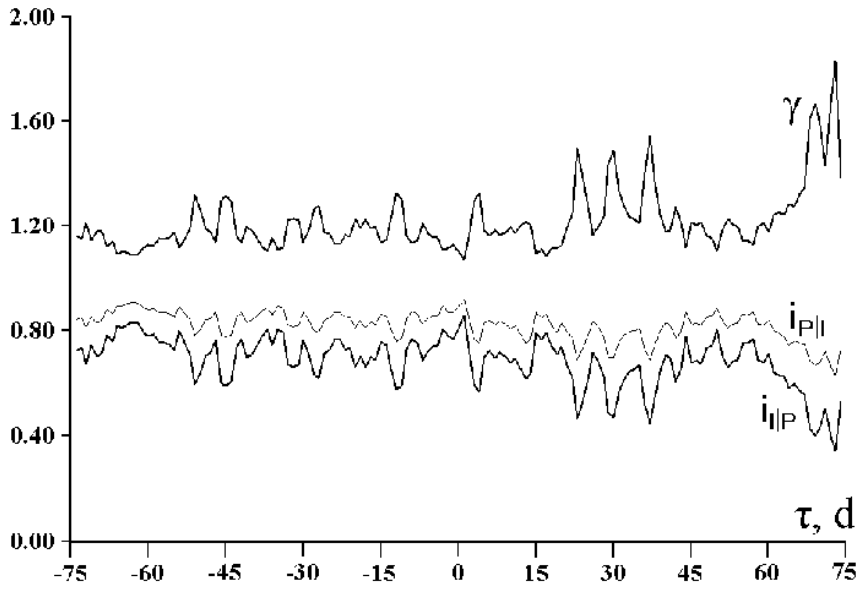

FIG.4. Independence i and causality γ functions of the photomultiplier detector signal I (March – April, 1997) and the atmospheric pressure P. τ is time shift of P relative to I in days.

It is even possible to give the simplest forecast. In Fig. 5 fore the same example time variation of pressure (passing a cyclone) and preceding on 73 days variation of the dark current are shown.

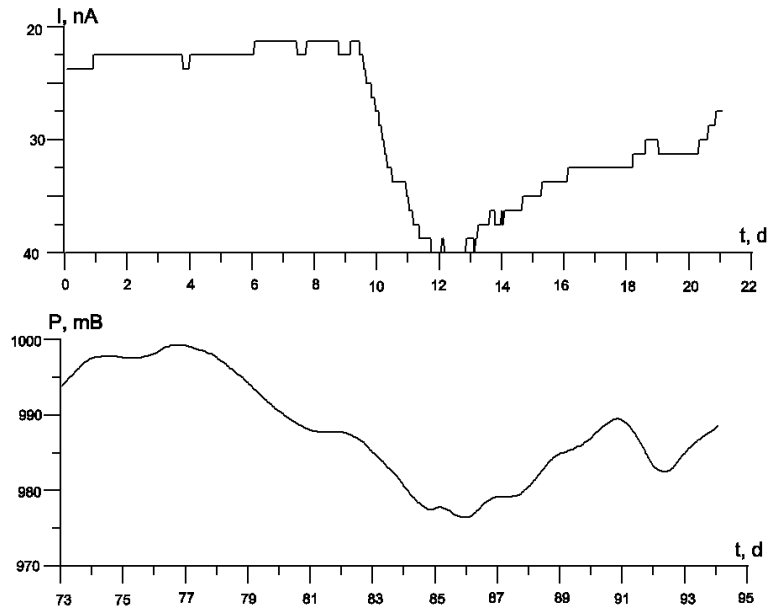

FIG.5. The variation of the detector signal I forecasting variation of atmospheric pressure P with advancement 73 days. The origin of time corresponds to March 24, 1997.

D. Relation of the detector signal with the geomagnetic activity.

It is beyond reason to consider U dependent on magnetic field F by any way. Therefore detection of relation of the potential with the Earth magnetic field variations would be a good test for the hypothesis (2.6), as these variations could be easily related with electric current dissipation in the

source (magnetosphere). Special experiments on influence on the detector of U by artificial magnetic field (up to 100 A / m) in frequency range from 0 to 1 Hz had confirmed absence of any reaction of U within sensitivity of the apparatus.

Analysis of long time series has shown existence of stable correlation r_{UF} = -0.56 ±0.01 only with advancement U relative to F (τ=48.0^h) . At this τ there is minimum $i_{F|U} = 0.79^{+0.02}_{-0.01}$. Thus relation U and F is statistically reliable, but both from prior reasons and from advancement U relative to F it can not be result of a direct influence F on U. Therefore F is indicator of some process interacting with U.

The spectral analysis in the range of maximal geomagnetic activity (10^{-6}…10^{-5}Hz) has shown that frequency dependence of amplitude ratio U/F is approximated by formula

$$U / F, \Omega m = 2.4 \cdot 10^{-5} / \sqrt{f}. \tag{5.2}$$

In terms of spectral densities $\hat{U}$ and $\hat{F}$ Eq. (5.2) turns into

$$\hat{U} / \hat{F} = \hat{\rho}_0 / f , \tag{5.3}$$

where $\hat{\rho}_0 = 8.5 \cdot 10^{-5} \Omega^2 m^2$, f is frequency. Eq. (5.3) describes flicker-noise, although we cannot localize the noising resistivity.

Whereas $U(f)/F(f)$ depends on frequency f, it has proved that $U(f)/F^2(f)$ does not depend on f: $U(f)/F^2(f)$=(1.7±0.2)$\cdot 10^{-5} \Omega\, m^2/A$. It is the most important result pointing to relation of U with the source entropy production.

For proof consider application of Eq. (2.6) to the concrete case. Magnetic field F is related with electric currents in the source – ionosphere, and also with induced currents in the Earth. For simplicity of the problem, neglect by the last and consider entropy production only in the source of F. It is easily to express the density of entropy production through electric field $E(f)$ (which in turn through impedance $Z(f)$ is related with $F(f)$), resistivity ρ and medium temperature T. ρ and $Z(f)$ consider for simplicity as scalar. Then:

$$\dot{s} = \frac{\langle E^2(f) \rangle}{\rho k T} = \frac{|Z(f)|^2 \langle F^2(f) \rangle}{\rho k T}. \tag{5.4}$$

Combining Eq. (2.6), (2.12), (2.13) we obtain explicit expression for $U(f)$, containing integral over (5.4) [11].The latter can be simplified using the known properties of electromagnetic field of the ionospheric source [25]. First, the field F is well approximated by the plane wave, therefore it is possible to factor out the F^2 from the integral. Second, using quasi-steady-state approximation of the plane wave impedance, substitute $|Z(f)|^2 = 2\pi f \mu_0 \rho$ in Eq.(5.4). As a result we have:

$$\frac{U(f)}{F^2(f)} = \frac{\sqrt{6}}{2} \frac{T_U g \sigma \mu_0}{|q|} \int \frac{dV}{T x^2} = \text{const}. \tag{5.5}$$

Thus the experimental fact $U(f)/F^2(f)$=const is explained within the Eq. (2.6).

To stress large scale of the process we can go from magnetic field measured by setup's magnetometer to indices of global geomagnetic activity. The most appropriate one proved to be *Dst*– index (which is calculated by data of equatorial geomagnetic observatories). The advancement of the detector signal relative to global activity turned about month. After appropriate filtration we also can give forecast of the geomagnetic activity. In Fig. 6 electrodes self-potential difference U forecasts the *Dst*–index of geomagnetic activity with advancement 33 days (data are filtered in the period window $1^d < T < 11^d$; at $\tau = 33^d$ there is minimum $i_{Dst|U} \approx 0.61$).

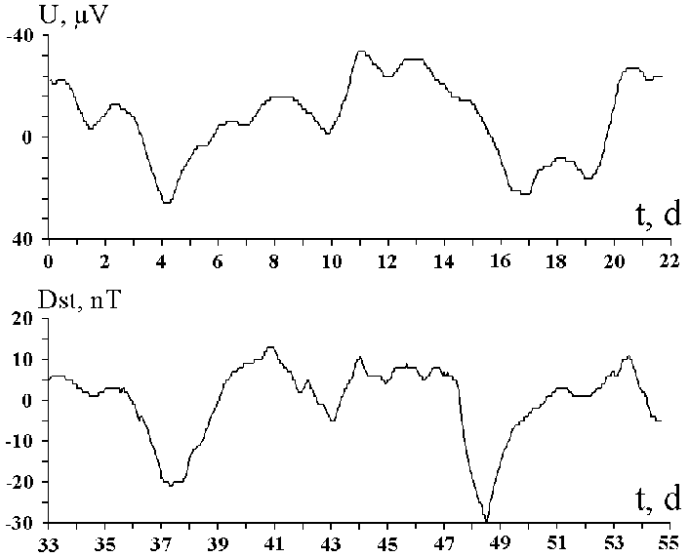

FIG.6. The variation of the detector signal *U* forecasting variation of the *Dst*-index of geomagnetic activity with advancement 33 days. Time origin corresponds to January 11, 1997.

E. Relation of the detector signal with solar activity

The spectral analysis has shown similarity of the signal and solar activity indices spectra. In particular there is maximum at period of solar rotation with amplitude of *U* about millivolt, and I about nanoamper.

Time domain analysis gives more interesting information. Since time averaged-terrestrial data always demonstrate stronger dependence our, data were processed with daily and monthly averaging. As the solar activity indices the solar radio wave flux at 9 standard frequencies within range 245…15400 *MHz* were taken. It should be stressed that detectors are not sensitive to the solar radio waves, their flux is only index of the source entropy production.

Analysis of daily averaged data has shown that both maximum of correlation and minimum of independence functions correspond to frequency 1425 *MHz*. This frequency corresponds radiation from the level of lower corona-upper chromosphere that is just from the level of maximal dissipation.

One can suggest the cosmic ray flux as possible mechanism of influence of the solar activity on detectors. Checking has shown that correlation of the detector signals with cosmic ray counting is statistically insignificant.

Maximum of dependence *U* on *R* (corresponding to min $i_{U|R}$) is at large advancement *U* relative to *R*. For the optimal frequency 1415 *MHz* $i_{U|R} = 0.59^{+0.01}_{-0.00}$ (and max $\gamma = i_{R|U} / i_{U|R} = 1.41 \pm 0.01$) is at $\tau = 39^{d}$.

Monthly averaged data have demonstrated this advancement connection even more brightly (though the causal analysis by depleted series is impossible). In Fig 7 such data of *U* and *R* shifted on 1 month are presented. The strong advanced correlation is evident At $\tau = 1^{M}$ $r_{UR} = 0.76 \pm 0.08$.

6. Conclusion

The experiment on the modern level of rigor has confirmed existence of Kozyrev′s transaction of the dissipative processes, which can be understood now as manifestation of macroscopic nonlocality. The most prominent property of this phenomenon is transaction in reverse time. Of

course, our theoretical approach was essentially heuristic. Therefore development of the theory at crossing of quantum nonlocality, action-a-distance electrodynamics and causal mechanics is burning.

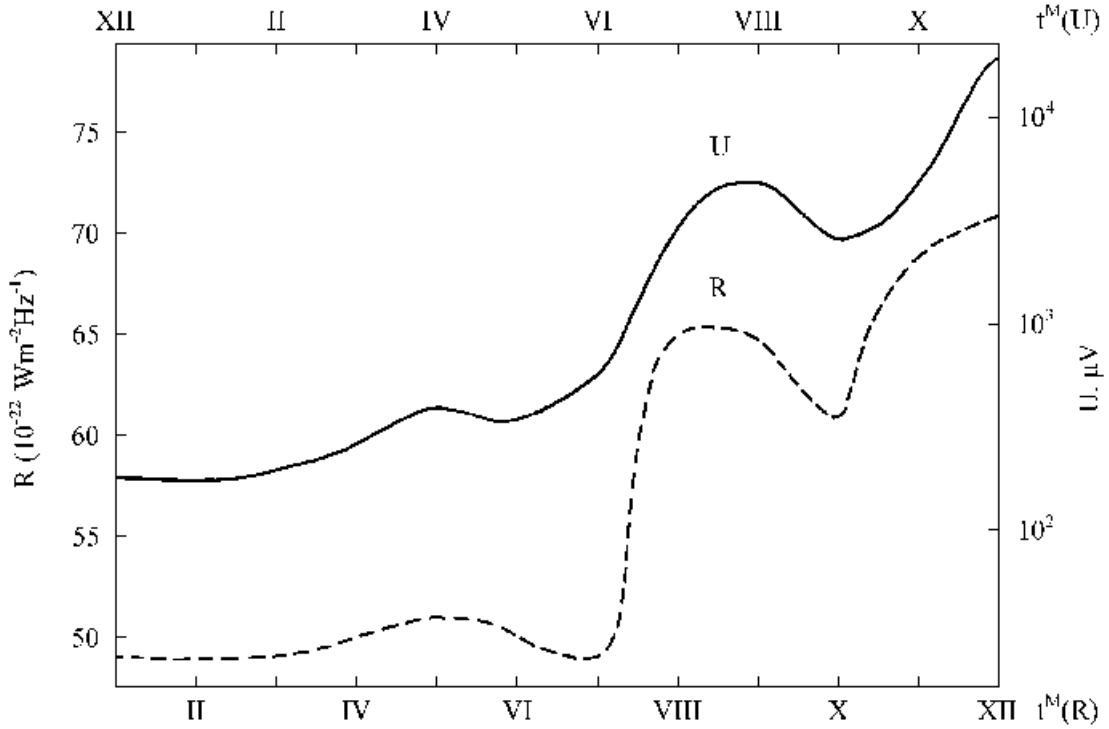

FIG.7. Time variation of monthly smoothed-out detector signal *U* forecasting variation of monthly smoothed-out solar radio wave flux *R* at frequency 1415 *MHz*. Time origin corresponds to December, 1996. The time interval spans the beginning of a regular solar activity cycle.

Acknowledgements

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References

[1] N. A. Kozyrev, in *Time is Science and Philosophy*, edited by J. Zeman, (Academia, Prague, 1971), pp.111-132.

[2] N. A. Kozyrev and V. V. Nasonov, in *Astrometry and Heavenly Mechanics,* edited by A. A. Efimov (VAGO Press, Moscow – Leningrad, 1978), p.168-179.

[3] N. A. Kozyrev and V. V. Nasonov, (in *Manifestation of Cosmic Factors on the Earth and Stars,* edited by A. A. Efimov (VAGO Press, Moscow-Leningrad, 1980), p.76-84.

[4] S. M. Korotaev, V. O. Serdyuk, M. O. Sorokin and J. M. Abramov, *Phys. Thought of Russia* **2**, 1-17 (1998).

[5] S. M. Korotaev, V.O. Serdyuk, M. O. Sorokin and J. M. Abramov, *Science & Technology in Russia* **1**, 16–19 (1999).

[6] S. M. Korotaev, V. O. Serdyuk, M. O. Sorokin and J. M. Abramov, *Physics and Chemistry of the Earth,* **24,** 735-740 **(**1999).

[7] S. M. Korotaev, V. O. Serdyuk and M. O. Sorokin, *Geomagnetism and Aeronomy* **40,** 323-730 **(**2000).

[8] S. K. Dvoruk, S. M. Korotaev, A. N. Morozov, A. L. Nazolin, V. O. Serdyuk, , A. V. Solovyov, M. O. Sorokin, S. E. Tabalin and G. V. Shishkin, *Applied Mechanics and Machine-Building Technologies* **1**, 61-66 (1998) .
[9] S. M. Korotaev, V. O. Serdyuk, M. O. Sorokin. and V. A**.** Machinin, *Physical Thought of Russia* **3**, 20-26
(2000).
[10] S.M. Korotaev, V.O. Serdyuk, M. O. Sorokin. and V. A**.** Machinin, *Herald of DGGGMS RAS* **3**, 100-106 (2000).
[11] S. M. Korotaev, A. N. Morozov, V. O. Serdyuk and M. O. Sorokin, *Russian Phys. J.* **5**, 3-14 (2002).
[12] N. D. Mermin, *Phys. Rev. Lett.* **65**, 1838-1840 (1990).
[13] D. Home and A. S. Majumdar, *Phys. Rev. A,* **52**, 4959-4962 (1995).
[14] A. M. Basharov , *J. Experimental and Theoretical Phys.* **121**, 1249-1260 (2002).
[15] J. S. Cramer, *Phys.Rev D,* **22** 362-376 (1980).
[16] F. Hoyle and J. V. Narlikar, *Rev. Mod. Phys.* **67**, 113-156 (1995).
[17] N. A. Kozyrev, in *Manifestation of Cosmic Factors on the Earth and Stars,* edited by A. A. Efimov (VAGO Press, Moscow-Leningrad 1980), p 85-93.
[18] S. M. Korotaev, *Geomagnetism and Aeronomy* **32**, 27-33 (1992).
[19] S. M. Korotaev, S. V. Shabelyansky and V. O. Serdyuk, *Izvestia Phys. of the Solid Earth* **6**, 77-86 (1992).
[20] S. M. Korotaev, O. A. Hachay and S. V. Shabelyansky, *Geomagnetism and Aeronomy* **33**, 128-133 (1993).
[21] S. M. Korotaev, *Geomagnetism and Aeronomy* **35,** 116-125 (1995).
[22] S. M. Korotaev, *Izvestia Phys. of the Solid Earth* **8,** 91-95 (1979).
[23] A. N. Morozov, *Irreversible Processes and Brownian Motion* (BMSTU-Press, Moscow, 1997).
[24] E. F. Zimin and M. M. Bogorodsky, *Izvestia Physics of the Solid Earth* **10,** 31-39 (1996).
[25] I. I. Rokityansky, *Geoelectromagnetic Investigation of the Earth's Crust and Mantle* (Springer-Verlag, Berlin1982).

On the physical field generated by rotating masses in Poincare-gauge theory of gravity

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It is shown that the gauge field in Poincare-gauge theory of gravity consists in two parts: the translational gauge field (t-field), which is generated by the energy-momentum current of external fields, and the rotational gauge field (r-field), which is generated by the sum of the angular and spin momentum currents of external fields. In connection with this the physical field generating by rotating masses should exist.

1. Introduction

The problem of a great interest consists in a possible existing of still unknown physical field that can be generated by rotating masses as a source. In particular, such possibility follows from the Theorem on the sources of the gauge fields [1–4], which states that the gauge field is generated by the Noether invariant corresponding to the Lie group that introduces this gauge field by the localization procedure. In this sense the gauge field in Poincare-gauge theory of gravity (PGTG) should be generated not only by the energy–momentum tensor, but also by the sum of angular and spin momentum currents as the source.

Here we shall discuss the following problems:

- What are the true gauge potentials in PGTG?
- What are the source currents of the field equations in PGTG?
- What fields can be generated by rotating masses in PGTG?

The corresponding field equations of PGTG were derived in [3,4] as the consequence of the general gauge field theory for the groups connecting with space-time transformations. We have shown [5] that under the localization procedure the Lorenz subgroup of the Poincare group introduces the *rotational* gauge field A_a^m (r-field), which is generated by the sum of the angular and spin momentum currents of external fields. The subgroup of translations introduces the *translational* gauge field A_a^k (t-field), which is generated by the energy-momentum current of external fields. These field equations are equivalent to the equations of PGGT in usual form, derived by the variation of the Lagrangian with respect to the tetrads $h^a{}_\mu$ and connections $A^m{}_\mu = A^m{}_a h^a{}_\mu$, the tetrads and also curvature and torsion being constructed with the help of both the t- and r-fields.

2. Noether theorem and the principle of local invariance

We start with the flat Minkowski space M_4 with the Cartesian coordinates x^a ($a=1,2,3,4$) and the metric $g_{ab}=\breve{g}(\vec{e}_a,\vec{e}_b)=\mathrm{diag}(1,1,1,\text{-}1)$ with the basis $\vec{e}_a=\partial_a=\partial/\partial x^a$. The fundamental group of M_4 is Poincare group P_4 (inhomogeneous Lorentz group),

$$\delta x^a = \omega^m I_m{}^a{}_b x^b + a^a = \omega^z X_z^a = -\omega^z \hat{M}_z x^k, \qquad \{\omega^z\} = \{\omega^m, a^k\},$$

$$\hat{M}_z = \{\hat{M}_m, \hat{M}_k\}, \qquad \hat{M}_m = -I_m{}^a{}_b x^b \frac{\partial}{\partial x^a}, \qquad \hat{M}_k = P_k = -\frac{\partial}{\partial x^k}.$$

Here we have introduced the abbreviations for the rotations and translations,

$$X_z^a = \{X_m^a, X_k^a\}, \qquad X_m^a = I_m{}^a{}_b x^b, \qquad X_k^a = \delta_k^a .$$

Let us introduce the curvilinear system of coordinate $x^\mu(x^a)$ on M_4:

$$dx^a = \hat{h}^a{}_\mu dx^\mu, \qquad \hat{h}^a{}_\mu = \vec{e}_\mu(x^a), \qquad \vec{e}_\mu = \partial_\mu = \partial/\partial x^\mu, \qquad ds^2 = g_{\mu\nu} dx^\mu dx^\nu,$$

$$\hat{g}_{\mu\nu} = g_{ab} \hat{h}^a{}_\mu \hat{h}^b{}_\nu, \qquad \hat{g} = \det(\hat{g}_{\mu\nu}) = \det(g_{ab})\hat{h}^2, \qquad \hat{h} = \det(\hat{h}^a{}_\mu) = \sqrt{|\hat{g}|},$$

Action integral is invariant under the Poincare group P_4 ,

$$J = \int_\Omega (dx)\sqrt{|\hat{g}|}\, L(Q^A, P_k Q^A), \qquad P_k = -\hat{h}^\mu{}_k \partial_\mu, \qquad \mathsf{L} = \sqrt{|\hat{g}|}\, L = \hat{h} L.$$

The first Noether theorem in a curvilinear system of coordinate yields the following,

$$0 = \int_\Omega (dx)\left[\frac{\delta \mathsf{L}}{\delta Q^A}\bar{\delta} Q^A + \partial_\mu\left(\mathsf{L}\, \hat{h}^\mu{}_k \delta x^k - \hat{h}^\mu{}_k \frac{\partial \mathsf{L}}{\partial P_k Q^A}\bar{\delta} Q^A\right)\right].$$

Here $\bar{\delta}$ denotes the variation of the form of the field. For example, for the field Q^A we have,

$$\bar{\delta} Q^A = \delta Q^A - \delta x^k \partial_\mu Q^A .$$

The field equation of the field Q^A is fulfilled,

$$0 = \frac{\delta \mathsf{L}}{\delta Q^A} = \frac{\partial \mathsf{L}}{\partial Q^A} + \partial_\mu\left(\hat{h}^\mu{}_k \frac{\partial \mathsf{L}}{\partial P_k Q^A}\right),$$

The result of Noether theorem can be represented as follows,

$$0 = \int_\Omega (dx)\hat{h}\hat{h}^\mu{}_k \partial_\mu\left(a^l t^k{}_l + \omega^m M^k{}_m\right),$$

where the following expressions for the energy-momentum $t^k{}_l$ and the full momentum $M^k{}_m$ (angular momentum plus spin momentum $J^k{}_m$) tensors are introduced,

$$t^k{}_l = L\delta_l^k - \frac{\partial L}{\partial P_k Q^A} P_l Q^A, \qquad M^k{}_m = J^k{}_m + I_m{}^l{}_b x^b t^k{}_l, \qquad J^k{}_m = -\frac{\partial L}{\partial P_k Q^A} I_m{}^A{}_B Q^A .$$

Noether Theorem yields the conservation laws for the energy-momentum and full momentum,

$$P_k t^k{}_l = 0, \qquad P_k M^k{}_m = 0.$$

Now we shall *localize* the Poincare group P_4 , the parameters of which become arbitrary functions of coordinates on M_4. The theory is based on four Postulates.

Postulate 1 *(Principle of local invariance).* The action integral

$$J = \int_\Omega (dx)\mathsf{L}(Q^A, P_k Q^A;\, A_a^R, P_k A_a^R), \tag{2.1}$$

where the Lagrangian density L describes the field Q^A, interaction of this field with the additional gauge field A_a^R and also the free gauge field A_a^R, is invariant under the action of the localized group $\mathrm{P}_4(x)$, the gauge field being transformed as follows,

$$\delta A_a^R = U_{z\ a}^R \omega^z + S_{z\ a}^{R\mu} \partial_\mu \omega^z , \tag{2.2}$$

where U and S are unknown matrices.

Postulate 2 *(Principle of the least action)*:

$$\frac{\delta \mathsf{L}}{\delta Q^A} = 0, \qquad \frac{\delta \mathsf{L}}{\delta A_a^R} = 0. \tag{2.3}$$

Postulate 3 *(Existence of the free gauge field)*. The full Lagrangian density L of the physical system has the following structure,

$$\mathsf{L} = \mathsf{L}_0 + \mathsf{L}_Q , \quad \mathsf{L}_0 = \mathsf{L}_0(A_a^R, P_k A_a^R), \quad \frac{\partial \mathsf{L}_0}{\partial Q^A} = 0, \quad \frac{\partial \mathsf{L}_0}{\partial P_k Q^A} = 0. \tag{2.4}$$

Postulate 4 *(Principle of the minimal gauge interaction)*:

$$\frac{\partial \mathsf{L}_Q}{\partial P_k A_a^R} = 0. \tag{2.5}$$

The second Noether theorem for the Lagrangian density (2.1) and the group $\mathrm{P}_4(x)$ yields the following,

$$\begin{aligned} 0 &= \int_\Omega (dx) \left[\frac{\delta \mathsf{L}}{\delta Q^A} \bar{\delta} Q^A + \frac{\delta \mathsf{L}}{\delta A_a^R} \bar{\delta} A_a^R \right] \\ &+ \int_\Omega (dx) \partial_\mu \left(\mathsf{L}\, \hat{h}^\mu{}_k\, \delta x^k - \hat{h}^\mu{}_k \frac{\partial \mathsf{L}}{\partial P_k Q^A} \bar{\delta} Q^A + \hat{h}^\mu{}_k \frac{\partial \mathsf{L}}{\partial P_k A_a^R} \bar{\delta} A_a^R \right). \end{aligned} \tag{2.6}$$

Here in δx^k, $\bar{\delta} Q^A$, $\bar{\delta} A_a^R$ we have arbitrary functions $\omega^z(x)$, $\partial_\mu \omega^z(x)$, $\partial_\mu \partial_\nu \omega^z(x)$, coefficients before them being equal to zero,

$$\begin{aligned} &\partial_\mu \left(\hat{h} \hat{h}^\mu{}_k\, \Theta_z^k \right) + \left(U_{za}^R + X_z^l P_k A_a^R \right) \frac{\delta \mathsf{L}}{\delta A_a^R} = 0, \\ &\hat{h} \hat{h}^\mu{}_k\, \Theta_z^k + \partial_\nu \mathsf{M}^{\nu\mu}{}_z + S_{za}^{R\mu} \frac{\delta \mathsf{L}}{\delta A_a^R} = 0, \qquad \mathsf{M}^{(\nu\mu)}{}_z = 0. \end{aligned} \tag{2.7}$$

where the following notations are introduced,

$$\begin{aligned} &\hat{h} \Theta_z^k = \mathsf{L}\, X_z^k - \left(I_m{}^A{}_B Q^B + X_z^l P_l Q^A \right) \frac{\partial \mathsf{L}}{\partial P_k Q^A} - \left(U_{za}^R + X_z^l P_k A_a^R \right) \frac{\partial \mathsf{L}}{\partial P_k A_a^R}, \\ &\mathsf{M}^{\nu\mu}{}_z = \hat{h}^\nu{}_k \frac{\partial \mathsf{L}}{\partial P_k A_a^R} S_{za}^{R\mu} . \end{aligned} \tag{2.8}$$

If the equations (2.3) for the gauge field are valid, then the equations (2.7) are simplified,

$$\partial_\mu \left(\hat{h} \hat{h}^\mu{}_k\, \Theta_z^k \right), \qquad \hat{h} \hat{h}^\mu{}_k\, \Theta_z^k + \partial_\nu \mathsf{M}^{\nu\mu}{}_z = 0, \qquad \mathsf{M}^{(\nu\mu)}{}_z = 0. \tag{2.9}$$

3. Structure of the Lagrangian densities L_Q and L_0

We introduce the differential operator M_R,

$$M_R = \{ M_m, \hat{M}_k \}, \qquad M_m = \hat{M}_m + I_m , \qquad \hat{M}_k = P_k . \tag{3.1}$$

and represent the gauge field in two components, $A_a^R = \{A_a^m, A_a^k\}$, where A_a^k describes the translational part of gauge field (t -field), and A_a^m describes the rotational part of gauge field (r -field).

Theorem 1 (B.N. Frolov, 1999, 2003). The gauge field A_a^R exists with transformational structure of Postulate 1 under the localized Poincare group $\mathrm{P}_4(x)$, and such the matrix functions Z , U and S of the gauge field exist that the Lagrangian density,

$$\mathsf{L}_Q = h L_Q(Q^A, D_a Q^A), \qquad h = Z\hat{h}, \tag{3.2}$$

satisfies to Postulate 1, L_Q being constructed from $L_Q(Q^A, P_a Q^A)$ by exchanging the operator P_a with the operator of the gauge derivative,

$$D_a = -A_a^R M_R \ . \tag{3.3}$$

Also the following representation for the gauge t -field is valid,

$$A_a^k = D_a x^k . \tag{3.4}$$

The ***proof*** of this Theorem has been performed in [3,4] and consists in proving three Prepositions.

Preposition 1.1. With the help of (3.1) the gauge derivative (3.3) can be represented as follows,

$$\begin{aligned} D_a Q^A &= h^\mu{}_a \partial_\mu Q^A - A_a^m I_m{}^A{}_B Q^B = h^\mu{}_a D_\mu Q^A , \\ D_\mu Q^A &= \partial_\mu Q^A - A^m{}_\mu I_m{}^A{}_B Q^B , \end{aligned} \tag{3.5}$$

where new quantities are introduced,

$$\begin{aligned} &Y_a^k = A_a^k + A_a^m X_m^k = A_a^R X_R^k , \qquad h^\mu{}_a = \hat{h}^\mu{}_k Y_a^k , \\ &h^a{}_\mu = Z_k^a \hat{h}^k{}_\mu , \qquad Z_k^a = (Y^{-1})_k^a , \qquad A^m{}_\mu = h^a{}_\mu A_a^m . \end{aligned} \tag{3.6}$$

It is easy to verify with the help of (3.5) and (3.6) that formula (3.4) is valid.

Preposition 1.2. We represent the Noether identities (2.7) as the system of differential equations for the unknown function L_Q , the Principle of the minimal gauge interaction (Postulate 4) being taken into account. The solvability conditions of the second system of these equations are satisfied if the unknown matrix functions Z and S have the form,

$$\begin{aligned} &S_{ma}^{n\mu} = \delta_m^n h^\mu{}_a , \qquad S_{ka}^{n\mu} = 0, \qquad S_{ma}^{l\mu} = 0, \qquad S_{ka}^{l\mu} = \delta_k^l h^\mu{}_a , \\ &Z = \det(Z_k^a), \qquad h = Z\hat{h} = \det(Z_k^a)\det(\hat{h}^a{}_\mu) = \det(h^a{}_\mu). \end{aligned} \tag{3.7}$$

Preposition 1.3.After substituting the results of Prepositions 1.1 and 1.2 into the first system of the equations (2.7) we shall see that this system of equations is satisfied identically by the Lagrangian density (3.2) provided that the unknown matrix function U has the form,

$$U_{ma}^n = c_{m\cdot q}^{\ \ n} A_a^q - I_{m\cdot a}^{\ \ b} A_b^n , \quad U_{ka}^n = 0, \quad U_{ma}^k = I_{m\cdot l}^{\ \ k} A_a^l - I_{m\cdot a}^{\ \ l} A_l^k , \quad U_{ka}^l = -A_a^n I_{n\cdot k}^{\ \ l} . \tag{3.8}$$

Corollary. After substituting (3.8) and the first line of (3.7) into (2.2), we get the transformational laws of the gauge fields $A_a^R = \{A_a^m, A_a^k\}$,

$$\delta A_a^m = \omega^n(x) c_{n\cdot q}^{\ \ m} A_a^q - \omega^n(x) I_{n\cdot a}^{\ \ b} A_b^m + h^\mu{}_a \partial_\mu \omega^m(x) = D_a \omega^m(x) - \omega^n(x) I_{n\cdot a}^{\ \ b} A_b^m ,$$
$$\delta A_a^k = \omega^m(x)(I_{m\cdot l}^{\ \ k} A_a^l - I_{m\cdot a}^{\ \ l} A_l^k) - A_a^m I_{m\cdot l}^{\ \ k} a^l \ h^\mu{}_a + h^\mu{}_a \partial_\mu a^k(x)$$
$$= D_a a^k(x) - \omega^m(x)(I_{m\cdot l}^{\ \ k} A_a^l - I_{m\cdot a}^{\ \ l} A_l^k), \tag{3.9}$$
$$\delta h^\mu{}_a = -\omega^m(x) I_{m\cdot b}^{\ \ a} h^\mu{}_b + h^\nu{}_a \partial_\nu \delta x^\mu , \qquad \delta h^a{}_\mu = \omega^m(x) I_{m\cdot b}^{\ \ a} h^b{}_\mu - h^a{}_\nu \partial_\mu \delta x^\nu ,$$
$$\delta A^m{}_\mu = D_\mu \omega^m - A^m{}_\nu \partial_\mu \delta x^\nu .$$

One of the main results of this corollary is that the tetrads $h^\mu{}_a$ and $h^a{}_\mu$ are not the true gauge potentials in contrast to the usually accepted opinion.

The structure of the gauge field Lagrangian density is established by the following theorem.

Theorem 2 (B.N. Frolov, 1999, 2003). The Lagrangian density

$$\mathsf{L}_0 = h L_0(F^m{}_{ab}, T^c{}_{ab}), \tag{3.10}$$

where

$$F^m{}_{ab} = 2h^\lambda{}_{[a} \partial_{|\lambda|} A_b^m + C^c{}_{ab} A_c^m - c_{n\cdot q}^{\ \ m} A_a^n A_b^q ,$$
$$T^c{}_{ab} = C^c{}_{ab} + 2I_{n\cdot[a}^{\ \ c} A_{b]}^n , \qquad C^c{}_{ab} = -2h^c{}_\tau h^\lambda{}_{[a} \partial_{|\lambda|} h^\tau{}_{b]} = 2h^\lambda{}_a h^\tau{}_b \partial_{[\lambda} h^c{}_{\tau]} , \tag{3.11}$$

satisfies to the Principle of the local invariance (Postulate 1).

4. Field equations of the gauge fields

The gauge field equations are the following,

$$\frac{\delta \mathsf{L}_0}{\delta A_a^k} = -\frac{\partial \mathsf{L}_Q}{\partial A_a^k}, \qquad \frac{\delta \mathsf{L}_0}{\delta A_a^m} = -\frac{\partial \mathsf{L}_Q}{\partial A_a^m}. \tag{4.1}$$

The right sides of these field equations can be represented in the form,

$$-\frac{\partial \mathsf{L}_Q}{\partial A_a^k} = Z_l^a \left(\mathsf{L}_Q \delta_k^l - \frac{\partial \mathsf{L}_Q}{\partial P_l Q^A} P_k Q^A \right) = h t^a{}_k ,$$
$$-\frac{\partial \mathsf{L}_Q}{\partial A_a^m} = I_m{}^l{}_b x^b \left(h t^k{}_l \right) + \frac{\partial \mathsf{L}_Q}{\partial D_a Q^A} I_m{}^A{}_B Q^A = h\left(\hat{M}^k{}_m + J^k{}_m \right). \tag{4.2}$$

The consequence of (4.1) and (4.2) is the theorem.

Theorem 3 (B.N. Frolov, 1963, 2003) (*Theorem on the source of the gauge field*). The source of the gauge field, introducing by the localized group $\Gamma(x)$, is the Noether current, corresponding to the non-localized group Γ.

5. Geometrical interpretation

In the geometrical interpretation of the theory the quantities $h^a{}_\mu$ and $A^m{}_\mu$ becomes tetrad fields and a Lorenz connection, and the quantities,

$$F^m{}_{\mu\nu} = F^m{}_{ab} h^a{}_\mu h^b{}_\nu = 2\partial_{[\mu} A^m{}_{\nu]} - c_{n\cdot q}^{\ \ m} A^n{}_\mu A^q{}_\nu ,$$
$$T^c{}_{\mu\nu} = T^c{}_{ab} h^a{}_\mu h^b{}_\nu = 2\partial_{[\mu} h^c{}_{\nu]} + 2I_{n\cdot a}^{\ \ c} h^a{}_{[\mu} A^n{}_{\nu]} , \tag{5.1}$$

become curvature and torsion tensors respectively.

The following theorem is valid.

Theorem 4 (B.N. Frolov, 1999, 2003). The system of the gauge field equations (4.1) derived by the variation with respect to the gauge fields $\{A^k{}_a,\ A^m{}_a\}$ is equivalent to the system of the field equations derived by the variation with respect to the fields $\{h^a{}_\mu,\ A^m{}_\mu\}$,

$$\frac{\delta \mathsf{L}_0}{\delta h^a{}_\mu} = -\frac{\delta \mathsf{L}_Q}{\delta h^a{}_\mu}, \qquad \frac{\delta \mathsf{L}_0}{\delta A^m{}_\mu} = -\frac{\delta \mathsf{L}_Q}{\delta A^m{}_\mu}. \tag{5.2}$$

The first of the gauge field equation (5.2) can be represented in the form,

$$\begin{gathered}\partial_\nu \frac{\partial \mathsf{L}_0}{\partial T^a{}_{\nu\mu}} = \frac{1}{2} h\left(t^\mu_{(0)a} + t^\mu_{(Q)a}\right), \qquad h t^\mu_{(Q)a} = \mathsf{L}_Q\, h^\mu{}_a - \frac{\partial \mathsf{L}_Q}{\partial D_\mu Q^A} D_a Q^A, \\ h t^\mu_{(0)a} = \mathsf{L}_0\, h^\mu{}_a - 2F^m{}_{a\nu} \frac{\partial \mathsf{L}_0}{\partial F^m{}_{\mu\nu}} - 2T^c{}_{a\nu} \frac{\partial \mathsf{L}_0}{\partial T^c{}_{\mu\nu}} + 2A^m{}_\nu\, I_{m\cdot a}{}^c \frac{\partial \mathsf{L}_0}{\partial T^c{}_{\mu\nu}},\end{gathered} \tag{5.3}$$

and the second one can be represented in the form,

$$\begin{gathered}\partial_\nu \frac{\partial \mathsf{L}_0}{\partial F^m{}_{\nu\mu}} = -\frac{1}{2} h\left(J^\mu_{(0)m} + J^\mu_{(Q)m}\right), \qquad h J^\mu_{(Q)m} = \frac{\partial \mathsf{L}_Q}{\partial A^m{}_\mu} = \frac{\partial \mathsf{L}_Q}{\partial D_\mu Q^A} I_m{}^A{}_B\, Q^B, \\ h J^\mu_{(0)m} = \frac{\partial \mathsf{L}_0}{\partial A^m{}_\mu} = 2\frac{\partial \mathsf{L}_0}{\partial F^n{}_{\mu\nu}} c_{m\cdot q}{}^n\, A^q{}_\nu + 2\frac{\partial \mathsf{L}_0}{\partial T^c{}_{\mu\nu}} I_{m\cdot a}{}^c\, h^a{}_\nu.\end{gathered} \tag{5.4}$$

The field equations (5.3) and (5.4) yield the conservational laws for the canonical energy-momentum tensor $t^\mu_{(Q)a}$ of the external field added by the energy-momentum tensor $t^\mu_{(0)a}$ of the free gauge field, and for the spin current $J^\mu_{(Q)m}$ of the external field added by the spin current $J^\mu_{(0)m}$ of the free gauge field,

$$\partial_\nu\left(h t^\mu_{(0)a} + h t^\mu_{(Q)a}\right) = 0, \qquad \partial_\nu\left(h J^\mu_{(0)m} + h J^\mu_{(Q)m}\right) = 0.$$

The field equations (5.3) and (5.4) can be also represented in geometrical form,

$$\begin{gathered}D_\nu \frac{\partial \mathsf{L}_0}{\partial T^a{}_{\nu\mu}} + F^m{}_{a\nu} \frac{\partial \mathsf{L}_0}{\partial F^m{}_{\mu\nu}} + 2T^c{}_{a\nu} \frac{\partial \mathsf{L}_0}{\partial T^c{}_{\mu\nu}} - \frac{1}{2}\mathsf{L}_0\, h^\mu{}_a = \frac{1}{2} h t^\mu_{(Q)a}, \\ D_\nu \frac{\partial \mathsf{L}_0}{\partial F^m{}_{\nu\mu}} = \frac{1}{2} h J^\mu_{(Q)m} + \frac{1}{2}\frac{\partial \mathsf{L}_0}{\partial T^b{}_{\nu\mu}} I_{m\cdot a}{}^b\, h^a{}_\nu.\end{gathered} \tag{5.5}$$

If we have $\mathsf{L}_0 = h\, L_0(F^m{}_{ab})$ instead of (3.10), then the first field equation (5.5) is simplified and generalizes the Hilbert–Einstein equation to arbitrary nonlinear Lagrangians,

$$F^m{}_{a\nu} \frac{\partial \mathsf{L}_0}{\partial F^m{}_{\mu\nu}} - \frac{1}{2}\mathsf{L}_0\, h^\mu{}_a = \frac{1}{2} h t^\mu_{(Q)a}.$$

7. Conclusions

The main result of the Theorem on the source of the gauge field (Theorem 4) is that the sources of PGTG are not only the energy-momentum and the spin momentum tensors as in the Einstein–Cartan theory [6,7], but also the angular momentum tensor. The gauge t- and r-fields are generated together by the energy-momentum, angular momentum and spin momentum tensors [5].

Therefore in PGTG rotating masses (for instant, galaxies, stars and planets), and also polarized medium should generate the r-field. In [8] the influence of the rotating Sun on the planets moving via the torsion generating has been investigated. Also a gyroscope on the Earth should change its weight subject to changing the direction of rotation because of the interaction with the rotating

Earth. There exist some experimental evidences of such effects. In [9–12] the change of the weight of rotating bodies or polarized medium have been observed that can be explained as the result of the interaction of these bodied and medium with rotating Earth. To this subject one also may refer the results of the N. Kozyrev's experiments with gyroscopes [13] and the mysterious $J-M$ relation between angular moments and masses of all material bodies in our Metagalaxy [14,15].

In Russia in Scientific Institute of Cosmic System (NIIKS of M.V. Khrunichev GKNP Center) some hopeful results have been received in experiments, in which the possibility has been demonstrated of using the decrease of the weight of rotating mass for constructing an engine that could move a body without any contacts with other bodies and without ejection of any reactive masses [16].

In case of General Relativity (GR) torsion vanishes and one gets only one field equation with the metric energy-momentum tensor as the source. But nevertheless the effects of GR depend on both of the t - and r - fields. In particular, the Lense–Thirring effect and the Kerr solution are induced by the r -field. The well-known problem of the constructing of the external source for the Kerr metric [17,18] may has its solution in considering the angular momentum of the external medium as the source.

In GR the coupling constants both of the t - and r - fields are equal to the Einstein gravitational constant. But in PGGT this choice is not determined by the theory and the coupling constants of the t-field and r-field have not to be equal to each other. In PGTG these constants can have different values, which should be estimated only on the basis of the experimental data.

References

[1] Frolov B.N. *Vestnik of Moscow University, Ser. Phys., Astron.* N6, 48–58 (1963) (in Russian).

[2] Frolov B.N. In: *Gravity, Particles and Space–time* (Ed. P. Pronin and G. Sardanashvily) World Scientific: Singapore (1996) 113–144.

[3] Frolov B.N. Problems of the theory of gravity with quadratic Lagrangians into spaces with torsion and nonmetricity. The Second PhD Thesis, Moscow: MSU (1999) 316 p. (in Russian).

[4] Frolov B.N. *Poincare–Gauged Theory of Gravity*, Moscow: "Prometey" Publishing House, MSPU (2003) 160 p. (in Russian).

[5] Frolov B.N. In *Abstracts of 11th Int. Conf.* "*Theoretical and Experimental Problems of General Relativity and Gravitation"* (1–7 July, Tomsk, Russia), Tomsk (2002) 44–45.

[6] Kibble T.W.B. *J. Math. Phys.* 2, 212–221 (1961).

[7] Trautman A. *Symposia Math*. 12, 139–162 (1973).

[8] Tucker R. In Proceedings of "*Symmetries and Gravity in Field Theory"* (Workshop in honor of Prof. J. A. de Azcarraga. June 9-11, 2003. Salamanca (Spain)). To be published.

[9] Hayasaka H. *Technology Report Tohoku Univ.* 43, N2 (1978).

[10] Hayasaka H., Takeuchi S. *Phys . Rev. Lett.* (1989).

[11] Rostschin V.V., Godin S.M. *JTF Letters*, 26, Iss. 24, 70–75 (2000) (in Russian).

[12] Volkov Ju.V. *Estestvennyje i Tehnicheskije nauki*, N1, 19 (2002) (in Russian).

[13] Kozyrev N.A. *Selective Works*, Leningrad Univ. Publishing House: Leningrad 1991.

[14] Muradian R.M. *Astrophys. Space Science* 69, 339 (1980).

[15] Sistero R.F. *Astroph. Lett*. 23, 235–237 (1983).

[16] Menshikov V.A., Akimov A.F., Kochekan A.A., Svetlichnyj V.A. *Engines without ejection of reactive masses: prerequisites and results*. Publishing House of NIIKS: Moscow (2003) 225 p. (in Russian).

[17] Lopez C.A. *Phys. Rev.* D30, 313 (1984).

[18] Burinskii A.Ya. *Gravit.&Cosm.* 8, N4(32), 261–271 (2002) (preprint hep–th 0008129).

Об инвариантности в релятивисткой термодинамике

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Несмотря на то, что прошло почти сто лет с момента создания специальной теории относительности (СТО) [1], в релятивисткой термодинамике до сих пор нет единого мнения по поводу вида преобразования основных термодинамических величин при переходе от неподвижной системы координат к системе, - движущейся со скоростью V.

Впервые этот вопрос был рассмотрен в [2] при следующих допущениях:
а) во всех инерциальных системах отсчета сохраняется вид первого и второго начал термодинамики

$$\Delta U = \Delta Q + \Delta A, \tag{1}$$

$$dS = \frac{\delta Q}{T} \geq 0, \tag{2}$$

здесь U – внутренняя энергия теромодинамической системы, Q – подведенное к ней количество теплоты, A – совершенная над системой работа, S и T - ее энтропия и термодинамическая температура соответственно.
б) процесс перевода термодинамической системы из неподвижного состояния в состояние движения со скоростью V (из неподвижной системы отсчета $\mathbf{K_0}$ в систему отсчета **К**, движущуюся со скоростью V) может быть осуществлен адиабатически и его критерием является постоянство энтропии:

$$S = S_0. \tag{3}$$

Далее в [2] было найдено выражение для работы, совершенной над термодинамической системой в движущейся системе отсчета **К** и определен вид преобразования для количества теплоты сообщенного термодинамической системе в неподвижной Q_0 и движущейся Q системах отсчета:

$$Q = Q_0\sqrt{1-\frac{\upsilon^2}{c^2}}\,. \tag{4}$$

С учетом (2) и (3) соотношение (4) автоматически приводит к следующему виду преобразования термодинамических температур в неподвижной T_0 и движущейся T системах отсчета:

$$T = T_0\sqrt{1-\frac{\upsilon^2}{c^2}}\,. \tag{5}$$

Сразу заметим, что при $\upsilon \to c$ соотношения (3) и (5) очень трудно, если не сказать большее, согласовать с теоремой Нернста (третьим началом термодинамики). Но, несмотря на это результаты работы [2] более полувека не вызывали сомнения. И лишь в начале 60-х годов появилась работа [3], в которой оспаривались соотношения (4) и (5).

В [3] автор по-прежнему придерживался справедливости допущений а) и б), но привел иной вывод для работы, совершенной над термодинамической системой в движущейся системе отсчета. В результате, вместо соотношений (4) и (5) в работе [3] получены следующие выражения:

$$Q = \frac{Q_0}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}, \tag{4a}$$

$$T = \frac{T_0}{\sqrt{1 - \frac{\upsilon^2}{c^2}}}. \tag{5a}$$

В последовавшей, после выхода работы [3], дискуссии было показано, что нельзя однозначно определить количество теплоты, переданное термодинамической системе в движущейся системе отсчета. Мнения разделились, хотя большинство согласилось с доводами, приведенными в работе [3]. Здесь следует упомянуть работу [4], в которой автор показывает, что в задачах релятивисткой кинематики справедливы соотношения (4) , (5), а в области релятивисткой динамики - (4а) и (5а). Сложившаяся ситуация, по мнению [5], не содержит противоречия, хотя и допускает существование альтернативных вариантов.

И в этом нет ничего удивительного. Ведь термодинамика - это область физической науки, обобщающей экспериментальные результаты определенного круга явлений. Релятивистская термодинамика, естественно, носит такой же характер и до появления «решающего эксперимента» и соотношения (4) , (5), и соотношения (4а) и (5а) имеют равное право на существование.

Все было бы так, если бы соотношения (5) и (5а) не противоречили специальному принципу относительности.

Поясним вышесказанное утверждение. Свойства вещества относительно пространственных и временных параметров являются однородными. Это означает, в частности, что характеристики вещества приводимые в справочной литературе носят удельный характер и со временем не меняются (если не учитывать все возрастающую точность последующих измерений их значений). Совсем по иному ведет себя вещество относительно температурных изменений. Например, диоксид водорода (H_2O) при температуре 300 К и нормальном давлении находится в жидком состоянии. Если, не меняя давления изменить температуру, то его состояние может кардинально измениться. Так при Т=200 К оно будет твердым (лед), а при Т=500 К – газообразным (пар).

Возьмем теперь, в качестве термодинамической системы, движущейся со скоростью 0,8с сосуд с водой. Пусть в сопутствующей системе отсчета (в системе отсчета, где сосуд покоится) его температура равна 300 К. Тогда, с учетом того, что давления в покоящейся и сопутствующей системах отсчета равны $p = p_0$, в неподвижной системе отсчета его температура согласно [2] будет равна 180 К, а в соответствии с [3] – 500 К. Очевидно, что имеется противоречие со специальным принципом относительности.

Вышесказанное основано на ***реальности*** физических величин, содержащих лоренцевский радикал. Но признание этой реальности отсутствовало в первые десятилетия 20-го столетия. Действительно, первым подтверждением следствий специальной теории относительности явился запуск в декабре 1942 года атомного реактора под руководством Э. Ферми. Тем самым, впервые, в лабораторных условиях, была экспериментально доказана взаимосвязь массы и энергии. Наблюдение в космических лучах мюонов, чье время жизни (в сопутствующей системе отсчета) составляет 2 мкс, экспериментальное подтверждение принципа автофазировки привело к тому, что уже к середине 50-х годов замедление времени и возрастание массы релятивистских объектов стало общепризнанным.

Поэтому, если мы признаем справедливость СТО, то, стоя на позициях [2] и [3], в покоящейся системе отсчета, в приведенном выше примере, мы будем наблюдать в сосуде лед или

пар, соответственно, несмотря на то, что в сопутствующей системе отсчета в нем «рыбки плавают».

Обращает на себя внимание и то, что в школьных и вузовских учебниках по физике пространство и время относительно (в соответствии с СТО), а термодинамическая температура – по-прежнему абсолютна. И это естественно, поскольку абсолютность пространства и времени, предполагавшаяся до появления СТО, была постулатом. И, как оказалось, постулатом неверным. Абсолютность же термодинамической температуры – это не постулат, а следствие физической теории. В этом смысле абсолютность термодинамической температуры носит тот же характер, что и абсолютность пространственно-временного интервала.

Следует упомянуть о работе [6], в которой предложен вариант релятивистской термодинамики, где $T = T_0$. Однако, в ней нет доказательства единственности предлагаемого решения, в силу чего она также носит альтернативный характер и не имеет широкого признания.

На наш взгляд, ошибка, приведшая к соотношениям (5) и (5а), содержится в допущении б). Условием адиабатичности процесса приобретения термодинамической системой скорости υ должно быть, по-видимому, более общее условие постоянства потока энтропии. Это условие, как показано в релятивистской гидродинамике [7], имеет вид:

$$\frac{\partial}{\partial x^i} S\upsilon^i = 0, \tag{6}$$

где υ^i – 4-скорость $\left(\upsilon^i = \left(\gamma, \frac{\vec{\upsilon}}{c}\gamma \right), \gamma = \left(1 - \frac{\upsilon^2}{c^2} \right)^{-\frac{1}{2}} \right)$.

Поскольку релятивистским объектом, в равной степени, являются как термодинамическая система, движущаяся с релятивистской скоростью, так и покоящаяся термодинамическая система, частицы которой движутся с релятивистскими скоростями, постольку, в равной степени, должны являться общими условия адиабатичности и в том, и в другом случае.

Мы ограничимся этим замечанием, поскольку вывод соотношений для вида энтропии в сопутствующей и покоящейся системах координат приведен в [6].

В заключение отметим, что релятивистким объектом, изучением свойств которого можно получить экспериментальные подтверждения теории, являются квазары. И хотя спектр излучения квазаров не является чисто тепловым, можно попытаться выделить его тепловую часть. В этом случае, рассматривая спектры похожих квазаров, отличающихся лишь скоростью, целесообразно провести их анализ с точки зрения законов теплового излучения.

Литература

1. Einstein A. – Ann. d. Phys., 1905, Bd 18, S. 639
2. Planck M. – Berl. Ber., 1907, S. 542
3. Ott H. Lorentz. – Lorentz-Transformation der Wärme und der Temperatur. Zeitscrift für Physik, 1963, 175, #1, S. 70 – 104
4. Shaffer J. – Kinematical and dynamical temperature transformations. – Nuovo cimento. B. - 1989. – 103, #3. – pp. 259 – 263
5. Физическая энциклопедия . Гл. ред. А. М. Прохоров. – М.: Большая Российская энциклопедия. т.4., с. 333-334., 1994
6. P. T. Landsberg, K. A. Johns. – The problem of moving thermometers. - Pros. Roy. Soc. A. 1968. – **306**, pp. 477 – 486
7. Ландау Л.Д., Лифшиц Е.М. Теоретическая физика. т.VI. Гидродинамика, гл.XV. – 3 изд. – М.: Наука,1986

On the Energy Problem in Gravity

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The most acknowledged definition of the total energy in general relativity comes back to Arnowitt-Deser-Misner who defined energy for an asymptotically flat space-time as the surface integral. This approach yields zero for closed Universes because there are no boundary terms on a formal ground. The open questions attracting interest up to now are:

1) Is it possible to express the surface integral as the volume integral of some reasonably defined energy density?
2) Is it possible to define energy for closed Universes?

Recently I considered the most general two-dimensional gravity coupled to a scalar field [1]. The model includes all known two-dimensional gravity models as well as spherically reduced general relativity coupled to a scalar field in four dimensions. Solving explicitly all constraints and gauge conditions unphysical modes are excluded leaving the scalar field alone as the only dynamical propagating degree of freedom. The effective Hamiltonian density yielding the effective equations of motion for a scalar field is found.

The total Hamiltonian in the original model equals to the linear combination of first class constraints and is zero on the constraint surface. The integral of the effective Hamiltonian density is shown to be equal to the boundary term. For spherically reduced general relativity it is precisely the term found by ADM. For closed Universes the energy density is also nontrivial. In fact, it is the same as for open Universes. The reason is as follows. There is no smooth solution for the constraints for closed spaces. Therefore to pose the variational problem correctly one must make a cut and pose the boundary conditions there. This boundary term on the constraint surface may be expressed as the volume integral of the effective Hamiltonian density which produces equations of motion for physical degrees of freedom. Hence the energy density can be defined in any case, and it does not depend on whether the space is open or closed.

[1] Katanaev M.O. "Effective Action For Scalar Fields in Two-dimensional Gravity". Ann.Phys. 296(2002) 1-50.

Quantum Cosmology revisited

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A homogenous isotropic cosmological model is considered in the framework of quantum geometrodynamic . Wheeler-DeWitt's equation is solved in the WKB approximation. The wave function phase determines a time dependence of the scale factor of the Universe born as a result of tunnelling. The probability of tunnelling is given by Gamow's formula. Closed universes prove to be less probable than flat and open ones. Possible generalizations of the model to universes' nonzero energies and angular momenta are discussed. The problems of time and an observer in quantum cosmology are also considered.

Nature of electric and magnetic Fields at the vicinity of the earth's surface

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Equations of electrodynamics describe interaction of charges with electric and magnetic fields as well as propagation of electromagnetic disturbances in free space. Electromagnetic fields possess momentum and energy and the latter could directly be experienced by our sense organs. It follows from these that electromagnetic fields are real physical entities.

Therefore, it is likely that electric and magnetic fields should be carried by the moving earth, just like all other physical objects are carried by this planet at the vicinity of its surface. We have deduced in this paper many electrodynamic equations (including auxiliary Lorentz Transformation Equations) for electric and magnetic bodies moving in the free space basing only on Maxwell's field equations. Therefore, if any electric and magnetic body move on the surface of the moving earth, electrodynamic equations should be affected by the motion of the earth. But surprisingly, all electrodynamic equations derived from Maxwell's field equations are seen to be unaffected by the motion of the earth, when experiments on earth are performed to verify them. This clearly implies that the earth really carries electric and magnetic fields along at the vicinity of its surface.

This simple consideration will explain (i) the Michelson-Morley type experiments (1881) and the Kennedy-Thorndike (1932) type experiments which confirm the constancy of two way velocity of terrestrial light in any medium that is stationary on the earth's surface, (ii) the Tomaschek and Miller's experiment (1924-25) which confirm the constancy of two way velocity of astral and solar light on the surface of the moving earth, (iii) two photon absorption experiment (Riis et al, 1989), NASA jet propulsion laboratory experiment (Krisher et al, 1990) which confirm one way velocity of light on the surface of the moving earth, (iv) the Trouton and Noble experiment (1903) which confirms the absence of magnetic field of the charges stationary on earth's surface where earth's motion should create magnetic fields for the charges. (v) Bradley's observation of aberration of star light when observed from the earth's surface (1728), (vi) Airy's observation of fixed aberration of star light when the aberration is observed from the surface of the earth with a telescope filled with water (1871), (vii) astronomers' observations that the laws of reflection, refraction diffraction and interference of terrestrial and astral light are independent of the motion of the earth, and (vii) the Sagnac effect (1913) where the velocity of light emitted from a source mounted on a rotating circular turntable and reflected by three mirrors placed on the circumference of the rotating turntable is seen to be dependent on the angular velocity of the turntable but independent of the motion of the earth.

More generally, this simple consideration will naturally explain why all electrodynamic phenomena on earth are independent of the motion of this planet and thereby, an alternative approach to special relativity in the Newtonian framework within the domain of electrodynamics emerges.

1. Itroduction

Oliver Heaviside (1850-1925), J.J. Thomson and their contemporaries based their electrodynamics from the consideration of Maxwell. In this communication, we have studied the electrodynamics of Heaviside and Thomson in free space as well as on the surface of the moving earth and have drawn from this study an interesting conclusion which will give a new direction to our understanding of nature , physics and special relativity.

2. Heaviside's electrodynamics

Maxwell (1831 – 1879) has elegantly explained the nature of propagation of electromagnetic disturbances in free space. Oliver Heaviside (1850 – 1925) and his followers proceeded to solve the potential

problems of a system of moving charges into Poisson's form by the elongation of the OX axis which was the direction of movement of the system of charges. They thus developed the way of solving dynamic potential problem in a static way through an unreal static auxiliary equation in the form of Poisson's potential equation which correlated between the static and dynamic states.

The E- field and the B-field of a system of charges and currents stationary in the free space are governed by Poisson's equations viz.,

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=-\frac{\rho}{e_0} \tag{1}$$

and

$$\frac{\partial^2 A}{\partial x^2}+\frac{\partial^2 A}{\partial y^2}+\frac{\partial^2 A}{\partial z^2}=-\frac{J}{e_0 c^2} \tag{2}$$

where Φ and ρ are the scalar potential and density of the system of charges and A and J are the vector potential and current density of the system of currents stationary in free space, e_o is permittivity of free space wherein 'c' is the velocity of light and the introduced Cartesian co-ordinate is in the free space.

From the above equations we have,

$$E_x=-\frac{\partial\Phi}{\partial x}, E_y=-\frac{\partial\Phi}{\partial y}, E_z=-\frac{\partial\Phi}{\partial z} \tag{3}$$

and

$$B=\nabla \times A \tag{4}$$

We may call this system of charges and currents stationary in free space as the stationary system S_o. But if this system of charges and currents moves in free space with constant velocity of translation, what will the E-field and the B-field in free space?

Now, we know that when this system moves in free space, the E field in free space will induce a B field (B*) which if varying will induce an electric field. Therefore, we may say qualitatively that when a system of charges moves, the E-field in free space changes its magnitude and direction and an induced B field (B*) emerges.

The same happens for the B-field. It will also change its magnitude and direction and an induced E field (E*) in free space emerges.

Let us now proceed to deduce the exact formulas for the E-field and the B-field while the system of charges and currents moves in free space with constant velocity of translation u, basing on Maxwell's equations, in a way first exemplified by Oliver Heaviside (1850-1925) and his followers.

When the system translates in free space in the OX direction with a constant velocity of translation u, the E-field and the B* field in free space as well known are governed by d'Alembert's equation, viz.,

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2\Phi}{\partial t^2}=-\frac{\rho}{e_0} \tag{5}$$

and

$$\frac{\partial^2 A^*}{\partial x^2}+\frac{\partial^2 A^*}{\partial y^2}+\frac{\partial^2 A^*}{\partial z^2}-\frac{1}{c^2}\frac{\partial^2 A^*}{\partial t^2}=-\frac{\rho u}{e_0 c^2} \tag{6}$$

Comparing (5) and (6) we have

$$A_x^*=\frac{u}{c^2}\Phi, A_y^*=0, A_z^*=0, (u_y \text{ and } u_z \text{ being } 0) \tag{7}$$

and

$$E_x=-\frac{\partial\Phi}{\partial x}-\frac{\partial A_x^*}{\partial t}, E_y=-\frac{\partial\Phi}{\partial y}-\frac{\partial A_y^*}{\partial t}=-\frac{\partial\Phi}{\partial y}, E_z=-\frac{\partial\Phi}{\partial z}-\frac{\partial A_z^*}{\partial t}=-\frac{\partial\Phi}{\partial z} \tag{8}$$

(A^*_y and A^*_z being 0)

and

$$B^*=\nabla \times A^* \tag{9}$$

We may call such a system, the dynamic system S where charges (and currents) move with the matter in free space with a constant velocity of translation u in the OX direction. Now, in such a situation, the potentials at the point (x, y, z) at the instant t and the potentials at the point (x + u dt, y, z) at the instant t+dt in free space will be the same. Therefore,

$$\Phi + \frac{\partial \Phi}{\partial t} dt + \frac{\partial \Phi}{\partial x}.udt = \Phi \tag{10}$$

$$\therefore \frac{\partial \Phi}{\partial t} = -u\frac{\partial \Phi}{\partial x} \quad \text{and} \quad \frac{\partial^2 \Phi}{\partial t^2} = +u^2 \frac{\partial^2 \Phi}{\partial x^2} \tag{11, 12}$$

Similarly, $$\frac{\partial A_x^*}{\partial t} = -u^2 \frac{\partial \Phi}{\partial x} / c^2 \quad \text{[using equations (7) and (11)]} \tag{13}$$

Now the equation (5) could be written as

$$\left(1 - \frac{u^2}{c^2}\right)\frac{\partial^2 \Phi}{\partial x^2} + \frac{\partial^2 \Phi}{\partial y^2} + \frac{\partial^2 \Phi}{\partial z^2} = -\frac{\rho}{e_0} \tag{14}$$

Now if we substitute

$$x^/ = \frac{x}{\sqrt{1 - \frac{u^2}{c^2}}},\ y^/ = y,\ z^/ = z \tag{15}$$

$$(\text{or } x/ = \gamma\,(x\text{-}ut),\ y/ = y,\ z/ = z \tag{16}$$

when $$\gamma = 1/k \text{ and } k = \sqrt{1 - \frac{u^2}{c^2}} \tag{17}$$

if electromagnetic action is considered after the time t of the instant, when the co-ordinate attached with the system coincide with the co-ordinate attached with the free space), the volume charge density in this transformation becomes

$$\rho/ = \rho k \tag{18}$$

and the equation (14) takes the form

$$\frac{\partial\ \Phi}{\partial x^{/2}} + \frac{\partial^2 \Phi}{\partial y^{/2}} + \frac{\partial^2 \Phi}{\partial z^{/2}} = -\frac{\rho}{e_0} \tag{19}$$

which is again a Poisson's equation.

Thus the value of Φ of the S system is connected with the potential $\Phi^/$ of a stationary auxiliary system $S^/$ in which all the co-ordinates parallel to OX have been changed in the ratio determined by the equation (15).

This transformed co-ordinate $S^/$ ($x^/$, $y^/$, $z^/$) system which is obviously imaginary will be used as a mathematical tool to correlate electromagnetic phenomena between static S_o and dynamic state S. [1,2,3]

This transformed auxiliary elongated co-ordinate system will be called as Auxiliary system $S^/$. Its role in electrodynamics was first exemplified by Oliver Heaviside who developed the way of solving dynamic potential problems in a static way through an unreal static auxiliary equation in the form of Poisson's equation which correlates between the static and dynamic states.

Now in the Auxiliary system $S^/$ we have,

$$\frac{\partial\ \Phi^/}{\partial x^{/2}} + \frac{\partial^2 \Phi^/}{\partial y^{/2}} + \frac{\partial^2 \Phi^/}{\partial z^{/2}} = -\frac{\rho^/}{e_0} = -\frac{\rho\sqrt{1 - \frac{u^2}{c^2}}}{e_0} \tag{20}$$

where $\Phi^/$ = electrostatic potential in Auxiliary system.

By comparison (19) and (20) we have, $\Phi^/ = \Phi k$

Or $$\Phi = \gamma \Phi/ \tag{20a}$$

Therefore, [using the equation (13) & (20a)]

$$E_x = -\frac{\partial \Phi}{\partial x} - \frac{\partial A_x^*}{\partial t} = -\frac{\partial \Phi}{\partial x} + \frac{u^2}{c^2}\frac{\partial \Phi}{\partial x} = -\frac{\partial \Phi^/}{\partial x^/} = E_x^/ \tag{21}$$

$$E_y = -\frac{\partial\Phi}{\partial y} - \frac{\partial A_y *}{\partial t} = -\frac{\partial\Phi}{\partial y}(Ay * being 0) = -\frac{\gamma\partial\Phi^{/}}{\partial y^{/}} = \gamma E_y^{/},$$

Similarly, $E_z = \gamma E_z^{/}$ From $B^* = \nabla \times A^*$, we have

$$B^*_x = 0,\ B^*_y = -\frac{u}{c^2}E_z = -\gamma\frac{u}{c^2}E_z^{/},\ B_z^* = \frac{u}{c^2}E_y = \gamma\frac{u}{c^2}E_y^{/} \tag{22}$$

The equations (21 & 22) are general and they may be used to determine the electric fields and the induced magnetic fields of moving charges of any shape and size.

For the independent magnetic field of the system of currents when the system is moving with a constant velocity of translation u, we have,

$$\frac{\partial^2 A_x}{\partial x^2} + \frac{\partial^2 A_x}{\partial y^2} + \frac{\partial A_x}{\partial z^2} - \frac{1}{c^2}\frac{\partial A_x}{\partial t^2} = -\frac{\rho V_x}{e_0 c^2} \tag{23}$$

and

$$\frac{\partial^2 A_y}{\partial x^2} + \frac{\partial^2 A_y}{\partial y^2} + \frac{\partial A_y}{\partial z^2} - \frac{1}{c^2}\frac{\partial A_y}{\partial t^2} = -\frac{\rho V_y}{e_0 c^2} \tag{24}$$

and the similar equation for the Z-component and which could be transformed (in a way previously shown) to the following equations viz.,

$$\frac{\partial^2 A_x}{\partial x^{/2}} + \frac{\partial^2 A_x}{\partial y^{/2}} + \frac{\partial A_x}{\partial z^{/2}} = -\frac{\rho V_x}{e_0 c^2} \tag{25}$$

$$\frac{\partial^2 A_y}{\partial x^{/2}} + \frac{\partial^2 A_y}{\partial y^{/2}} + \frac{\partial A_y}{\partial z^{/2}} = -\frac{\rho V_y}{e_0 c^2} \tag{26}$$

and the similar equation for the $Z^{/}$ component where ρV is the current density in S system.

In the auxiliary system $S^{/}$, for line currents flowing within the moving system in any arbitrary directions (i.e., when the magnetic field depends on the length of the current element but not on its cross-section), we have,

$$\frac{\partial^2 A^{/}{}_x}{\partial x^{/2}} + \frac{\partial^2 A^{/}{}_x}{\partial y^{/2}} + \frac{\partial A^{/}{}_x}{\partial z^{/2}} = -\frac{\rho^{/} V_x}{e_0 c^2} = -\frac{\rho\sqrt{1-\frac{u^2}{c^2}}.V_x}{e_0 c^2} \tag{27}$$

$$\frac{\partial^2 A^{/}{}_y}{\partial x^{/2}} + \frac{\partial^2 A^{/}{}_y}{\partial y^{/2}} + \frac{\partial A^{/}{}_z}{\partial z^{/2}} = -\frac{\rho V_y}{e_0 c^2} \tag{28}$$

and the similar equation for the Z'-component. By comparison of (25) and (27), (26) and (28), we have,

$$A_x = \gamma A^{/}{}_x,\ A_y = A^{/}{}_y,\ A_z = A^{/}{}_z \tag{29}$$

whence,

$$B_x = \left[\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial Z}\right] = \left[\frac{\partial A_z^{/}}{\partial y^{/}} - \frac{\partial A_y^{/}}{\partial Z^{/}}\right] = B_x^{/}$$

$$B_y = \left[\frac{\partial A_x}{\partial Z} - \frac{\partial A_z}{\partial x}\right] = \left[\frac{\gamma\partial A_x^{/}}{\partial Z^{/}} - \gamma\frac{\partial A_z^{/}}{\partial X^{/}}\right] = \gamma B_y^{/} \tag{30}$$

$$B_z = \left[\frac{\partial A_Y}{\partial X} - \frac{\partial A_X}{\partial Y}\right] = \left[\gamma\frac{\partial A_y^{/}}{\partial X^{/}} - \gamma\frac{\partial A_x^{/}}{\partial y^{/}}\right] = \gamma B_z^{/}$$

where $A_x^{/}$, $A_y^{/}$ and $A_z^{/}$ are the components of the Auxiliary magnetic potential in the Heavisidean imaginary elongated system $S^{/}$. For the induced vector, we have the relation $E^* = -u \times B$, from which we have,

$$E_x^* = 0,\quad E_y^* = uB_z = \gamma uB^{/}_z,\quad E^*_z = -uB_y = -\gamma uB^{/}_y \tag{31}$$

Now, if the sources of an independent electric field (originating from charges of any shape and size) and an independent magnetic field (originating from line currents flowing within the system in

any arbitrary directions) move with a constant velocity of translation u in free space with the system, then from the consideration of equations (21), (22), (30), and (31), we have the following auxiliary field equations from Maxwell :

$$Ex = Ex^{/} \qquad Bx = Bx^{/}$$

$$E_y = \gamma\left[E^{/}_y + uB^{/}_z\right] \qquad B_y = \gamma\left[B^{/}_y - \frac{u}{c^2}E^{/}_z\right]$$
$$E_z = \gamma\left[E^{/}_z - uB^{/}_y\right] \qquad B_z = \gamma\left[B^{/}_z + \frac{u}{c^2}E^{/}_y\right] \tag{32a}$$

Or

$$E^{/}_x = Ex \qquad B^{/}_x = Bx$$

$$E^{/}_y = \gamma\left[E_y - uB_z\right] \qquad B^{/}_y = \gamma\left[By + \frac{u}{c^2}E_z\right]$$
$$E^{/}_z = \gamma\left[E_z + uB_y\right] \qquad B^{/}_z = \gamma\left[By - \frac{u}{c^2}E_y\right] \tag{32b}$$

where E and B are the electric and the magnetic fields of the system of charges and currents having a constant velocity of translation u in free space, $E^{/}$ and $B^{/}$ are auxiliary quantities which relate E and B to static electric and magnetic fields of the system of charges stationary in free space. Those equations are also valid for induced electromagnetic fields when the inducted body moves with respect to the free space.

Corollary 1. In a stationary system S_o, if E_o be the source of B_o, and $E_o = vB_o$, where v is constant, then in the auxiliary system $S^{/}$, $E^{/}=vB^{/}$.

3. Application of Heaviside's electrodynamics in the free space

Corollary 2. It can easily be shown that Heaviside's fields obey Maxwell's equations just like Coulomb's fields do. Therefore, if a stationary dipole radiates in free space, it will also radiate while moving in free space at constant velocity of translation.

3.1. The electric field and the induced magnetic field of a point charge moving with a constant velocity of translation in free space.

Now following Heaviside, the Electric field E and the induced magnetic field B* of a point charge (Q) moving with a constant velocity of translation u in free space could be calculated at a point P (x, y, z) or (r,θ) in the free space (considering at a particular instant, the point charge as origin and OX is the direction of motion of the charge) in the following way :

$$E^{/}_x = \frac{Qx^{/}}{4\pi e_0 r^{/3}},\ E^{/}_y = \frac{Qy^{/}}{4\pi e_0 r^{/3}},\ E^{/}_z = \frac{Qz^{/}}{4\pi e_0 r^{/3}} \tag{33}$$

where $E_x^{/}$, $E_y^{/}$, and $E_z^{/}$ are the components of auxiliary electric field in the Heavisidean imaginary auxiliary system $S^{/}$ at $P^{/}$ ($x^{/}$, $y^{/}$, $z^{/}$), $r^{/} = \sqrt{x^{/2} + y^{/2} + z^{/2}}$ where $P^{/}$ ($x^{/}$, $y^{/}$, $z^{/}$) of the $S^{/}$ system is the corresponding point P (r, θ) of the S system whence, [E=$\left(E_x^2 + E_y^2 + E_z^2\right)^{1/2} = \left(E_x^{/2} + \gamma^2 E_y^{/2} + \gamma^2 E_z^{/2}\right)^{1/2}$ [using equation (21)]$= \frac{Q}{4\pi e_0 r^{/3}}\left(x^{/2} + \gamma^2 y^{/2} + \gamma^2 z^{/2}\right)^{1/2}$ [using equation (33)]$= \frac{Q}{4\pi e_0 r^{/3}}\gamma r$ [using equation (15)] $= \frac{Q}{4\pi e_0}\frac{\gamma r}{r^3\gamma^3\left[1 - \frac{u^2}{c^2}Sin^2\theta\right]^{3/2}}$

$$\because r^{/3} = \left(x^{/2} + y^{/2} + z^{/2}\right)^{3/2}$$
$$= \left(\gamma^2 x^2 + y^2 + z^2\right)^{3/2} \quad \text{[using equation (15)]}$$

$= \left(\gamma^2 r^2 \cos^2\theta + r^2 Sin^2\theta\right)^{3/2}$ (using the relation between Cartesian and polar co-ordinates)

$= \gamma^3 r^3 \left(1 - \frac{u^2}{c^2} Sin^2\theta\right)^{3/2}$]

$$\therefore E = \frac{Qk^2}{4\pi e_0 r^2 \left(1 - \frac{u^2}{c^2} Sin^2\theta\right)^{3/2}} \bar{r} \tag{34}$$

and from B* = ∇ x A*, we have,

$$B_x^* = 0,\ B_y^* = -\frac{u}{c^2} E_z = -\gamma \frac{u}{c^2} E_z^{/}$$
$$B_z^* = \frac{u}{c^2} E_y = \gamma \frac{u}{c^2} E_y^{/} \tag{35}$$

from which

$$B^* = \frac{u \times E}{c^2} \tag{36}$$

where B* is the induced magnetic field.

It could be shown with a little analysis that equation (34) and (36) are the same for a charged ellipsoid having its axis with ratios k:1:1 moving with a constant velocity of translation u in free space, k being in the direction of motion of the ellipsoid.

Thus Oliver Heaviside (1850 - 1925), the greatest electromagnetician after Maxwell (1831- 1879) has shown that a charged ellipsoid having its axis with ratios k:1:1 which while moving with a constant velocity u in free space produces the same external effect as that of a similarly moving point charge[4,5], k being in the direction of motion of the charge.

3.2. The electromagnetic momentum of a Heaviside's ellipsoid (with the axes δRk : δR : δR) which while moving rectilinearly with a velocity u in the OX direction in the free space

$$P_X = \int \left(D_y B^*_z - D_z B^*_y\right) dv = \frac{u}{c^2} e_0 \int \left(\gamma^2 E^{/}_y{}^2 + \gamma^2 E^{/}_z{}^2\right) k dv^{/} =$$

$$= \gamma \frac{u}{c^2} e_0 \int \left(E^{/}_y{}^2 + E^{/}_z{}^2\right) dv^{/} \left[\text{cf. equations (21)and (22)}\right]$$

$$= \frac{q^2 u}{6\pi e_0 c^2 \delta R k} = \mu u \tag{37}$$

(as $E^{/}_y$, $E^{/}_z$ and $dv^{/}$ are related to a sphere and so each integral is equal to $\frac{q^2}{12\pi e_0{}^2 \delta R}$) which is the electromagnetic momentum of a point charge moving rectilinearly in the free space with a velocity u, where μ_0 and μ are the electromagnetic masses of the point charge moving with the velocities 0 and u respectively in free space such that

$$\frac{q^2}{6\pi e_0 c^2 \delta R} = \mu_0 \text{and} \frac{\mu_0}{k} = \mu \tag{38}$$

Consult Hajra 1998[6] and Searle 1897 [7] for alternative deduction of electromagnetic momentum of a steadily moving point-charge

3.3. Electromagnetic force acting on a point charge moving in the free space

(a) at a direction parallel to the direction of the uniform electric field operating in the free space,

$$F_{II} = \frac{|dG|}{d|u|} a_{II} = \frac{\mu_0}{k^3} a_{II} \tag{39}$$

where a_{II} is the acceleration of the point charge in the direction parallel to the field.

And **(b) at a direction perpendicular to the direction of the uniform electric field operating in the free space**.

$$F_{\perp} = \frac{|G|}{|u|} a_{\perp} = \frac{\mu_0}{k} a_{\perp} \tag{40}$$

where $a_{\perp}$ is the acceleration of the point charge in the direction perpendicular to the field.

[A general treatment will show $F = \mu\, du/dt + u\,(F.u)/c^2$

3.4. Similarly, energy of a point charge having a constant velocity of translation in the free space

$$\mathcal{E} = \int \left(\frac{\partial G}{\partial t} \right) dx = \frac{\mu_0 c^2}{k} = \mu c^2 \tag{41}$$

3.5. Frequencies of light emitted from a source having a constant velocity of translation in the free space -

Let an electric force drive a point charge back and forth from one end to the other end of a radiating dipole stationary in the free space.

Then, $F_0 = -\mu_0 \omega_0^2 S,$ (42)

the velocity of oscillation being small, where μo is the electromagnetic mass of the charge in the stationary dipole, ωo is the radian frequency of oscillation of the charge, S is the separating distance of the dipole.

Now, if the dipole moves with a velocity u in the free space in any direction perpendicular to its direction of oscillations, the electric force and the magnetic force acting on the charge will be respectively from equations (34) and (36), [when θ= 900] γF_0 and $-\frac{u^2}{c^2}\gamma F_0$. Therefore, total electromagnetic force acting on the moving charge

$$\mathbf{F} = F_o k \tag{43}$$

Now, under the circumstances, when the dipole moves we have ,

$$F = -\mu\omega^2 S \tag{44}$$

where μ is the electromagnetic mass of the charge in the moving dipole, ω is the frequency of oscillation of the charge which is moving with a velocity u in the free space with the dipole and F is the electromagnetic force acting on the moving charge.

From equations (38), (42), (43) and (44) for the dipole moving with an uniform velocity in any direction perpendicular to its direction of motion we have,

$$\omega = \omega_0 k \tag{45}$$

Now, if that radiating dipole while moving with a velocity u towards a direction parallel to OX, is seen from the origin at any point P which makes an angle θ with OX axis at the origin, we have,

$$\omega\text{observed} = \frac{\omega_0 k}{1 + \frac{u}{c}\cos\theta} \quad \text{from Doppler} \tag{46}$$

whence $\omega\ \text{trans} = \omega_0 k$ (47)

i.e., the well known transverse Doppler effect, if the dipole moving in a direction parallel to OX and oscillating in a direction parallel to OZ is seen at a point (0,y,0) from the origin.

3.6. The period of oscillation (t)for a radiating dipole having a constant velocity of translation in the free space we have, $t_0 = \frac{2\pi}{\omega_0}$ (48)

where ω0 is the radian frequency of the radiating dipole stationary in the free space, and t0 is its period of oscillation. Now, if the same dipole moves with a velocity u and radiates in free space, we have,

$$t = \frac{2\pi}{\omega} \tag{49}$$

where t is the period of oscillation and ω is the radian frequency of the moving dipole.

Comparing equations (48) and (49) with the equation (45) we have

$$t=\gamma t_0 \tag{50}$$

or the period of oscillation of a moving dipole increases with its velocity in the free space.

3.7. Life spans of radioactive particles having a constant velocity of translation in the free space

a) Proton-proton Decay or Electron-electron Decay

Consider two similar point charges tied by some unknown forces. The repelling electric force is here tending to destroy the equilibrium whereas the unknown forces are keeping the charges tied together.

Therefore, spontaneous transformation of those particles will depend also on the repulsive electromagnetic force just like on time. Therefore, we may write for disintegration

$$N=N_0\, e\text{-}\lambda Ft \tag{51}$$

where N are the untransformed particles present at the time t from initial untransformed particle N_0 at t=0

Now, if N_o radioactive particles of similarly charged bodies are at rest in the free space, and if we have N untransformed particles after the time t0, then we have,

$$N=N_0 e\text{-}\lambda F_0 t_0 \tag{52}$$

where F0 is the repelling force acting on the charged particles at rest in the free space.

Now if the charged particles move with a velocity u in the free space in any direction perpendicular to their direction of oscillation, we will find N untransformed particles after a time t such that

$$N=N_0\, e^{-\lambda Ft} \tag{53}$$

Comparing the equations (52) and (53) with the equation (43), we have

$$t=\gamma to \tag{54}$$

b) Positive point charge - negative point charge decay

Consider two dissimilar point charges are separated by some unknown forces. The attracting electric force is trying to destroy the equilibrium, whereas the unknown forces are keeping the charges separated. By the similar arguments as in (a) we have t = γto

c) Similarly for negative or positive muon-type decay

we may consider that a muon is a point negative or positive charge tied with a point mass by the attractive electric force which is giving stability but some other unknown repelling forces responsible for decay are acting here to destabilise the equilibrium. So here,

$$N = N_0 e^{-\frac{\lambda t}{F}} \tag{55}$$

as N decreases with t but increases with F

Now, the decay equations of muons for stationary and moving states respectively could be written as follows :

$$N = N_0 e^{-\frac{\lambda t_0}{F_0}} \tag{56}$$

$$N = N_0 e^{-\frac{\lambda t}{F}} \tag{57}$$

The magnetic force is here 0, because the magnetic field is non-operative on moving mass, only the electric force is here $F = \gamma F_0$ (58)

Comparing the equations (56), (57) and (58), we may write for muon decay

$$t = \gamma t_o \tag{59}$$

Thus, we may conclude that the life-span of a charged electromagnetic radioactive particle increases with its velocity in the free space.

Now, if the source be stationary in the free space and the observer moves, from the consideration of Maxwell, there should be no transverse Doppler effect and no time increment. If transverse Doppler effect and time increment are confirmed experimentally in such cases, only then some special theories could be held superior in this regard.

3.8. Velocity of light in a medium moving in the free space

We know from Maxwell that the relation between the electric field (E0) and the magnetic field (B0) in a ray moving in OX direction inside a dielectric at rest in the free space is

$$\frac{(E_0)_y}{(B_0)_z} = \frac{c}{n} \tag{60}$$

where n is the refractive index of the medium.

Now, if the dielectric moves with a velocity u in the OX direction in the free space, Vx is the velocity of the ray in OX direction with respect to the free space and Ey and Bz are the electric field and the magnetic field of the ray in the moving dielectric, we have, $V_x = \frac{E_y}{B_z} = \frac{\gamma\left[E'_y + uB'_z\right]}{\gamma\left[B'_z + \frac{u}{c^2}E'_y\right]}$ [cf. Equation (32a)]. (In case when E_0 is the source of B_0)

$$= \frac{\frac{c}{n} + u}{1 + \frac{u}{nc}} \approx \frac{c}{n} + u\left(1 - \frac{1}{n^2}\right) \tag{61}$$

[using the Corollary 1 of the section 2, i.e., if $\frac{(E_0)_y}{(B_0)_z} = \frac{c}{n}$, then $\frac{E'_y}{B'_z} = \frac{c}{n}$]

3.9. Velocity of charges in a conductor moving in free space

Suppose that some charges are moving with a velocity υx in the OX direction over the surface of a conductor at rest in the free space where an electric field E0 and a magnetic field B0 are operating on the charges. Then from Lorentz,

$$(F_0)_y = q\left[(E_0)_y - \upsilon_x (B_0)_z\right] \tag{62}$$

where $(F_o)_y$ is the force acting on the charges in OY direction, which is equal to O. In such a situation,

$$\upsilon_x = \frac{(E_0)_y}{(B_0)_z} \tag{63}$$

Now, if the conductor moves with a velocity u in the OX direction in the free space, we have for the velocity (Vx) of charges with respect to the free space in the moving conductor.

$$V_x = \frac{E_y}{B_z} = \frac{\gamma\left[E'_y + uB'_z\right]}{\gamma\left[B'_z + \frac{u}{c^2}E'_y\right]} \quad \text{[cf. Equations (32a} \tag{64}$$

where Ey and Bz are electric and magnetic fields in the moving conductor

Now if $\frac{(E_0)_y}{(B_0)_z} = \upsilon_x$ and E_0 is the source of B_0 then from Corollary 1 of the Section 2 we have, $\frac{Ey'}{Bz'} = \upsilon_x$

Therefore,

$$V_x = \frac{u + \upsilon_x}{1 + \frac{u\upsilon_x}{c^2}} \tag{65}$$

In similar analyses, we can show with little manipulations

$$V_y = \frac{\upsilon_y k}{1 + \frac{u\upsilon_x}{c^2}} \quad \text{and} \quad V_z = \frac{\upsilon_z k}{1 + \frac{u\upsilon_x}{c^2}} \qquad (66, 67)$$

4. Derivation of Lorentz's auxiliary transformation equtions from Heaviside's auxiliary equations

Following Heaviside, Lorentz engaged himself in developing some transformation equations through which and their corollaries, he could solve optical problems of moving bodies as well as electrodynamic problems of charges moving with low and high velocities[8] in symmetric and easy form, reducing the equations of moving system to the form of ordinary formula that hold for a system at rest.

Heaviside's problem was to solve potential problem for charges having a constant velocity of translation in free space which he did by transforming the d' Alembert's equation in an invariant form with Poisson's in the auxiliary system.

Lorentz's problem was to solve the radiation problems of moving radiating bodies which he did by transforming the Maxwell's equation for the moving radiating body to the same form for the auxiliary system, which correlates between the static and dynamic radiation states.

By dint of Corollary 2 of the section 2, we have,

$$\frac{\partial^2 E}{\partial x^2} + \frac{\partial^2 E}{\partial y^2} + \frac{\partial^2 E}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0 \qquad (68)$$

(where E is the electric field of a radiating dipole moving with a constant velocity of translation u in the free space) from which we get $x^2 + y^2 + z^2 = c^2t^2$. (68a)

To solve radiation problems in an analogous way as shown in section 2, we are to keep the Maxwell's equation in the same form in the S/ system i.e., it is now required

$$\frac{\partial^2 E'}{\partial x'^2} + \frac{\partial^2 E'}{\partial y'^2} + \frac{\partial^2 E'}{\partial z'^2} - \frac{1}{c^2}\frac{\partial^2 E'}{\partial t'^2} = 0 \qquad (69)$$

where E' is the Heavisidean auxiliary electric field in the S' system.

i.e., $x'^2 + y'^2 + z'^2 = c^2t'^2$. (69a)

Now solving the equations $x^2 + y^2 + z^2 = c^2t^2$ and $x'^2 + y'^2 + z'^2 = c^2t'^2$, using the Heavisidean auxiliary equations i.e., $x' = \gamma(x-ut)$, $y' = y$, $z' = z$ (70)

(Which help us to determine electric fields and induced magnetic fields of moving charges of any shape and size, magnetic fields and induced electric fields of line currents flowing within the moving system in any arbitrary direction and similar fields of surface currents and volume currents flowing within the moving systems in the direction of movement of the system.)

We get $$t' = \gamma\left(t - \frac{ux}{c^2}\right) \qquad (71)$$

the famous auxiliary time equation of Lorentz.

An interesting fact about the equations is that the reverse of Lorentz Transformation equations (which could be deduced from Lorentz Transformation equations) have the same form as that of the Lorentz Transformation equations themselves i.e.,

If $x' = \gamma(x-ut)$, $y' = y$, $z' = z$, $t' = \gamma\left(t - \frac{ux}{c^2}\right)$ (72a, 72b, 72c, 72d)

Then $x = \gamma(x' + ut')$, $y = y'$, $z = z'$, $t = \gamma\left(t' + \frac{ux'}{c^2}\right)$ (73a, 72b,73c, 73d)

Therefore, Lorentz transformation equations could be reversely deduced from the dyads of equations (68a) & (69a) and (72a)& (73a) with the help of (72b)&72c) (which were later observed and used by A. Einstein in his theory).

It should be mentioned here that all the quantities $x^{/}$ $y^{/}$, $z^{/}$, $t^{/}$, $E^{/}$, $B^{/}$ etc. are auxiliary and unreal, so the auxiliary equation $\frac{\partial^2 E'}{\partial x'^2}+\frac{\partial^2 E'}{\partial y'^2}+\frac{\partial^2 E'}{\partial z'^2}-\frac{1}{c^2}\frac{\partial^2 E'}{\partial t'^2}=0$ is unreal.

Thus, from the stand point of classical electrodynamics, Lorentz Transformation Equations are tactical just like the Heaviside's tactical equations. These equations can be used to solve the electrodynamic problems of any moving charges, of any line currents and of surface currents and volume currents flowing within the system in the direction of the movement of the system.

Unfortunately, Lorentz transformation equations have no legitimacy to work for calculation of magnetic fields and induced electric fields due to surface current and volume current flowing within the system in any arbitrary direction.

Therefore, Lorentz transformation equations are neither general nor real, but very useful for point charge and line current electrodynamics (as has been shown in section 3).

To consider Lorentz transformation equations general is contrary to the principles of electrodynamics and to consider them real is oversimplification of electrodynamics and nature.

5. Discussion

Equations of electrodynamics describe interactions of charges with electric and magnetic fields and propagation of electromagnetic disturbances in free space. Electromagnetic fields possess momentum and energy which could be experienced by our sense organs. It follows from these that electromagnetic fields are real physical entities to the same extent as the charged bodies,(Kompaneyets 1961[9]).

There are numerous experiments to prove that the velocity of light is independent of the velocities of the small field-creating bodies (Alvager 1964[10], Brecher 1977[11]), which indicates that the electromagnetic fields are not carried with the body that creates it. In the same analogy, it is likely that the electromagnetic fields should not rotate with the rotation of the body which creates these, which is however contradicted by many authors, which is also counter contradicted by some other authors. In the present state of knowledge we may conclude that it is the free space which is the carrier of electromagnetic fields and small field creating bodies, while translating can not carry electromagnetic fields with them.

Therefore, all the real equations originating from Maxwell's equations are only applicable in free space where the earth moves with a vigorous velocity (30 km/sec.) which is confirmed by the phenomena of aberration as observed by Bradley (1729). Therefore, the real laws enumerated above should be expected to be applicable on the surface of the moving earth with corrections as well as modifications for the velocity of the earth in free space.

But surprisingly,

(i) All the real equations e.g. (34), (36-41), (47), (50), (54), (59), (61), (65-67), are not seen to be affected by the motion of the earth when the experiments were performed on earth to verify them,

(ii) A magnetic field is likely to be observed for a charge or for a system of charges when the system is stationary on the moving earth but such a magnetic field was denied by the Trouton-Noble Experiment (1904).

(iii) The magnitude of magnetic field should appreciably change depending on the position and orientation of a current carrying wire on the moving earth which was not reflected in the results of the relevant experiments,

(iv) The velocity of light in water moving on the surface of the moving earth should appreciably vary with the position and orientation of the Fizeau Apparatus on the moving earth. No such variation was observed,

(v) If a man moves through rain falling straight, he is to lean his umbrella to protect his body from rain due to the motion of rain relative to the motion of the man. Similarly, when light beam (photon rains) comes from the stars, the astronomer to see these stars is to tilt the telescope due to the motion of the

light beams relative to the motion of the earth. This is commonly called rain drop effects for star light. Similar raindrop effects should be observed for light rays coming from a source of light a little above the surface of the moving earth. No raindrop effect is observed at the earth's surface for light rays coming from a source a little above the surface of the earth (Zapffe 1992 [12]). Where as such effects are seen for star rays as observed by Bradley (1729).

(vi) The Michelson-Morley type experiments should register a definite fringe-shift. But it is well confirmed that all optical phenomena produced by terrestrial sources are independent of the motion of this planet. All Michelson-Morley type experiments performed in air stationary on the moving earth, with light rays (Kennedy Thorndike Experiment, 1932), with microwaves (Cederholm et al Expt. 1958) with laser beams (Jajeva et al. Expt. 1964), laser frequency locking techniques (Brillet and Hall Expt. 1979[13], Hils and Hall Expt. 1990[14]) register the null results for two ways velocity of light[15]. For one way velocity of light (Will 1992[16], contra Bay and White 1989[17]), Riis et al (two photon absorption Expt. 1989[18]) and Krisher et al (NASA Jet Propulsion laboratory Expt. 1990[19]) have confirmed isotropy of space near the surface of the moving earth.

(vii) The Michelson-Morley Expt. performed on the surface of the earth with starlight (Tomaschek's Expt. 1924) and with sunlight (Miller's Expt. 1925) have also registered null results,

(viii) It has been found that an astronomer after having determined the apparent direction of star-rays and their apparent frequency can predict from these, by ordinary laws of optics and without attending any motion of the earth, the results of all experiments on reflection, refraction, diffraction and interference that can be made with the rays[20].

(ix) Bradley (1728) observed that the stars appear to move in circles (aberate), the angular diameter of these circles being 41 seconds of an arc ($2v/c$ where v is velocity of the earth in free space). But when Airy (1871) observed these stars with a telescope filled with water, with the above analogy he should measure the angle of aberration as $2nv/c = 54$ seconds of an arc where n is the refractive index of water. But surprisingly he measured the same angle 41 seconds of on arc for the aberration of stars when seen though a telescope filled with water.

(x) If a beam of light is split by a splitter and sent in opposite direction by mirrors around the circumference of a turntable, an interference pattern is observed. The turntable is capable of being rotated in the laboratory. If the turntable is rotated, the interference fringe is shifted on the interferometer relative to the stationary turntable position. If the turntable be rotated in the opposite direction, the fringes moves to the opposite side. Fringe-shift masured is dependent on the angular velocity of the turntable, but independent of the velocity of the earth. This is the celebrated Garress (1912) – Sagnac (1913) – Pogany (1928) Expt, confirmed by Dufour and Prunnier (1942) and Macek and Davis (1963) with ring laser.

All those paradoxical results clearly imply that the earth (probably all large heavenly bodies) carries electromagnetic fields at the near vicinity of its surface[21], though it is proved with certainty that small field creating bodies can not carry them[10,11].

6. Analyses

In the section 5], we have discussed that all the results of electromagnetic experiments performed on the surface of the moving earth demand that the surface of the moving earth is equivalent to free space for our description of electromagnetic phenomena on it. This implies that the earth (probably all other large heavenly bodies) carries electromagnetic fields with its surroundings just like it carries all other physical objects at the near vicinity of its surface.

All those equations [(1) - (67)], though derived for the free space are seen through experiments to be equally applicable on the surface of the moving earth. Motion of the earth does not modify the results of the experiments. Therefore, we have concluded that the earth carries electromagnetic fields along with its surroundings, just like it carries all other physical objects at its surroundings.

The null results of the Michelson-Morley Experiment (1881, 1887), the Michelson-Morley type experiment performed in any medium stationary on earth, two photon absorption Expt. (Riis et al 1989), NASA jet propulsion Lab. Expt. (Krisher et al 1990), the Kennedy-Thorndike Experiments (1932), the Trouton-Noble Experiment (1904), the Tomaschek (1924) and Miller's (1925) Experiments and the Sagnac Effect (1913), Airy's observation on fixed aberration (1871) and astronomers' observation on the laws of reflection, refraction, diffraction, and interference of terrestrial and astral light corroborate this proposition.

In the two photon absorption expt. (Riis et al 1989) and in the NASA jet propulsion lab Expt. (Krisher et al, 1990) the space near the surface of the earth is seen to remain isotropic for one way velocity of light. In the Michelson-Morley type Experiments performed in any medium stationary on earth and in the Kennedy -Thorndike Experiment, the two-way velocity of light remain the same 'c/n' in all directions on earth, if measured on earth, for the electromagnetic fields are carried along with the earth. In the Trouton-Noble Experiment, no magnetic field should be observed on the surface of the moving earth due to the charges stationary on earth as the electromagnetic fields are carried with the earth and in the Sagnac experiment, when the turntable rotates, the interference fringes should be displaced owing to the change in the path lengths of light beams on the surface of the earth but not for the motion of the planet.

In the Fizeau experiment and in the Biot-Savart Experiment, the results should not alter depending on the position and orientation of the Fizeau apparatus and current carrying wire on the moving earth, as the electromagnetic fields are carried along with the earth.

In the phenomenon of fixed aberration observed by Airy, electromagnetic fields and vibrations originating in stars are transmitted to the galactic space, thence to the solar space and thence to the surroundings of the earth which carries electromagnetic fields and vibrations with it. Therefore, the relative direction of electromagnetic vibrations outside the surroundings of the earth become the real direction inside the earth's surroundings and therefore, there will be phenomenon of solar space related - aberration as observed by Bradley (1728) and there will be no further aberration as observed by Airy (1871).

This phenomenon of solar space related - fixed aberration as seen from the earth clearly indicates that the earth is carrying electromagnetic fields and vibrations just like it carries all other physical forces and vibration on its surface and the sun's action on the electromagnetic fields is the same as the earth's[22].

Absence of rain drop effect for light ray coming from a little above the near vicinity of the surface of the earth as cited by Zapffe (1987)[23], is in complete harmony with this implication.

Moreover, the above proposition further implies that under such a situation, induction takes place only when the inductor or the inducted body moves either in free space or moves on the surfaces of the large heavenly bodies. No induction is possible when the inductor or the inducted body moves along with the large heavenly bodies whatever high movements the large heavenly bodies may have with respect to the stars. This is confirmed by observations which show that on earth, all optical phenomena like reflection, refraction, diffraction, interference etc., produced from terrestrial or astral sources are independent of the motion of this planet. The same explanation can easily be extended to Tomaschek (1924) and Miller's Experiment (1925) where the Michelson-Morley experiment has been performed with starlight and sunlight and to a host of experiments cited by Jo'zef Wilczyn'ski (1994)[24], A.G.Kelly[25], R. Manaresi[26], C.A. Zaffe[23] in their excellent papers.

Now as the electromagnetic fields are carried with the surface of the moving earth, in the Sagnac experiment, the speed with which the light beam catches the mirrors on the turntable in the direction of rotation is $c-v = c-\omega R$ where ω is the angular velocity of the turntable and R is its radius.

For the beam travelling in the opposite direction the light beam catches the mirror with the speed $c + v = c + \omega R$. For the first beam, time taken to travel around the circumference $\tau_1 = \frac{2\pi R}{c - v}$ and for the

second beam $\tau_2 = \frac{2\pi R}{c+v}$. Therefore, $\Delta\tau = \tau_1 - \tau_2 = 2\pi R\ (\frac{1}{c-v} - \frac{1}{c+v}) = \frac{4\pi R v}{c^2} . \frac{1}{1-\frac{v^2}{c^2}} \approx \frac{4A\varpi}{c^2}$ where A is the area of the turntable and from Newtonian view point, the result is dependent on the angular velocity of the turntable but independent of the location of the observer and the motion of this planet The calculation given above exactly matches with the results of the experiment. Sagnac effect denounces special relativity directly and upholds the view that the earth is really carrying, electromagnetic fields at the near vicinity of its surface.

7. Conclusion

Thus the conclusion that the earth (as well as all large heavenly bodies) carries electromagnetic fields at the near vicinity of its surface just like it carries all other objects at its surroundings - will explain all electrodynamic phenomena on the moving earth and overthrow the special relativity from electrodynamics at a stroke[27].

References

[1] Lorentz, H.A.: The Theory of Electron (Dover Publications. Inc. New York, 1909, 1915), 35-36.

[2] Whittaker, E.T.: A history of the theories of Aether and Electricity (Thomas Nelson and sons Ltd. 1951)1, 307-309.

[3] Miller, A.I.: Albert Einstein's Special Theory of Relativity (Springer, 1981), 98-99.

[4] i] Heaviside, O.: Electrician, Dec. 7, 1888, 148.
ii] Heaviside, O.: The Phil. Mag., 1989; 324-339.

[5] Thomson, J.J.: The Phil. Mag, 1989, 1-14.

[6] Hajra S.: PIRT, 1998, Supple. Papers, Edited by Dr. M.C. Duffy, 149-157.

[7] Searle, G. F.C.: The Phil. Mag., 1897, 329-339.

[8] Lorentz, H.A.: The Theory of Electrons (Dover Publications. Inc., New York, 1909, 1915), 57-61.

[9]Kompaneyets, A.S.: Theoretical physics (Foreign Language Publishing House, Moscow, 1961), 105.

[10] Alvager, T. et al : Physics Lett, 12 (1964), 260.

[11] Brecher, K. : Phy. Rev. Lett, 39 (1977), 1051-54.

[12] Zapfee, C.A. : Ind. J. of theo. Phy., 40 (1992), 145-148.

[13] Brillet, A. and Hall, H.L. : Phy. Rev. Lett., 42 (1979), 549-52.

[14] Hils, D. and Hall, J.L. : Phy. Rev. Lett. 64 (1990), 1697 – 1700.

[15] Excepting a few contradictions from the days of D.C. Miller 1928 (Astro Phy. J. 28, 352-368). Selleri has suggested an anisopropy of the space near the surface of the earth from the analysis of unusual effect in the Miller's Expt. (1925), Kennedy-Thorndike Expt. (1932), Jaseva-Javan – Murray – Townes Expt. (1963) (Selleri's, F. : On the anisotropy observed by Miller and Kennedy & Thorndike) and some authors are drawing attention to the anisotropy of the space near the surface of the earth from the analyses of the results of Couvoisier's Expt. (1953), Silvertooth – Jacobs Expt. (1983), Marinov Expt. (1987), Marinov and Wesley Expt. as cited by Wilczyn'ski (Ind. J. of Theo. Phy. 43, 1995 : 269-77) which do not seem to have any strong foundation at the present moment. Some electric and magnetic experiments in the form of the Trouton-Noble Expt. 1904 are required to confirm them. Anisotropy in the upper atmosphere of the earth has also been observed by Hefele-Keating (1972) and Smoot et al (1977) which should be studied carefully. However, upper atmospheric anisotropy is likely to be observed from the consideration of the study presented in this paper. Some anisotropy of c +ωR , c-ωR type of the space near the surface of the earth (ω = angular velocity of the earth and R is the radians of the earth) has been suggested from the Michelson-Gale Exp. (1925), experiments of Saburi et al (1976), Brillet and Hall Expt. (1979) as analysed by Aspden (1981), the experiment of Bilger et al (1995), GPS

time synchronisation cited by Kelly (1996) and Marmet (PIRT-2000, edited by Dr. M.C. Duffy) which require serious attention though those seem to be highly improbable from electromagnetic viewpoint.
Any electromagnetic fields are carried along with the moving earth at the near vicinity of the surface of the earth which indicates that the velocity of light is subject to the influence of the gravitational fields of the earth and this has been confirmed by many experiments. Therefore, it is likely that the centrifugal force and especially the Coriolis force originating from the rotation of the earth should also act on the propagation of light which have not been taken into consideration for the explanation of the results of the Michelson-Gale type experiments as proposed by Michelson-Gale, Kelly, Marmet and others.
If there were any (c+v), (c-v) effects for the rotation of the earth on the velocity of light as measured on earth, the magnetic field due to a system of charges slowly moving eastward on the equator would be twice the magnetic field of the same system of charges moving similarly at the 60° latitude.
More interestingly, if (c+v), (c-v) effects would be correct for the rotation of the earth, the direction of the magnetic field would remain the same in both the cases when the charges move either eastward or westward in the equator or in many other places on the earth.
Similarly, if a wire is kept stretched in the equator of the earth (or in many other places) east to west and current changes its direction inside the wire, the magnitude of the magnetic field at any point should differ, all of which are improbable.
Therefore, the explanation of the Michelson-Gale type experiments as proposed by Michelson-Gale, Kelly, Marmet and others are improbable.
[16] Will, C.M.: Phys. Rev., D 45 (1992), 403.
[17] Bay, Z. and White, J.A: Phys. Rev. Lett., 62 (1989), 841.
[18] Riis, E. et al Phy. Rev. Lett., 60 (1988), 81.
[19] Krisher, T.P. et al.: Phy. Rev., D 42 (1990), 731.
[20] Lorentz, H.A.: The theory of Electrons (Dover Publication Inc., New York, 1909, 1915), 176.
[21] A very interesting study to confirm / abandon special relativity has been initiated by Spavieri and Contreas, 1986 [Nuovo Cimento, 91B (1986), 143-155] and Spavieri and Bergamaschi, 1989 [Nuovo Cimento, 104B, (1989), 497-511] analysing deviations of starlight when refracting through a prism and then through air. Arago's expts. indicate a non-null result. If these non-null results are not experimental errors, Spavieri-Contreas and Bergamaschi think that special relativity collapses and Stokes-Plank-like ether theory would be confirmed.
[22] When light starts from an astral source, it propagates with respect to the stars and enter in the galactic regions, where it propagates with respect to the galaxy.Then, it comes to the solar system, where it propagates with respect to the sun and finally travels towards the earth. Therefore the angle of the aberration of stars ≈ 2v/c, where v and c are measured with respect to the sun.
[23] Zapffe C.A. : Some excellent papers published in eighties and nineties of the past century in Ind. J. of Theo. Phy. And in Toth-Maation Rev.
[24] Wilczyn'ski, J. : A host of excellent papers in the eighties and nineties of the past century in Ind. J. of Theo. Phy.
[25] Kelly, A.G. : A New Theory on the behaviour of light, Monograph 2 (1996), IEI.
[26] Manaresi, R. : Relativity principle and Lorentz contraction are incompatible (the paper has been kindly send to me by the author).
[27] Apart from Tangherlini (1961), Marinov (1974), Chang (1983), Podlaha, Tian and Li (1990), a host of contributors have raised important objections in the foundation of relativity and have tried to reduce absurdity in relativity as per the writings of W.M.Honig, A.G.Kelly, J.Levy, R.Manaresi, P.Marmet, W.Petry, V.Pope, Z.Reut, F.Selleri and C.R.Whitney as per PIRT conference 2000, London, edited by Dr. M.C. Duffy.

Geometrized scalar gravity as a new physical paradigm

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The viable scalar theory of gravitation is proposed on the profound geometrical ground and idea that all the geometry is generated by universal gravitating scalar field ϕ. By applying at first the integrability conditions to general spacetime deformation tensor (being the Lee derivative of metric $g_{\mu\nu}$ in timelike ξ-directions: $\pounds_{\xi} g_{\mu\nu} \equiv \xi_{\mu;\nu} + \xi_{\nu;\mu} = L_{\mu\nu}$, $\xi^{\mu} = \xi u^{\mu}$, $u_{\alpha}u^{\alpha} = 1$), a fundamental identity {master equation} involving the geometric scalar $\xi = (\xi_{\alpha}\xi^{\alpha})^{1/2}$ and Ricci tensor $R_{\mu\nu}$ is found**:**

$$R_{\alpha\beta}u^{\alpha}u^{\beta} = -\xi^{-1}\Box\xi + u_{\alpha;\beta}u^{\alpha;\beta} + \xi^{-1}f_{\alpha}u^{\alpha}, \quad f^{\mu} \equiv -\frac{1}{2}\left(L^{\alpha}{}_{\alpha}\right)^{;\mu} + L^{\mu\alpha}{}_{;\alpha}, \quad \Box\xi \equiv \xi_{;\alpha}{}^{;\alpha}.$$

So, for if $L_{\mu\nu} = 0$, $L_{\mu\nu} = \Phi g_{\mu\nu}$ or $L_{\mu\nu} = \Psi h_{\mu\nu}$, $h_{\mu\nu} = g_{\mu\nu} - u_{\mu}u_{\nu}$, we input the Killing, conformal or space-conformal symmetries, etc. A scalar ξ and metric $g_{\mu\nu}$ are considered as functions of ϕ due to $R_{\alpha\beta}u^{\alpha}u^{\beta}$ be proportional to energy density of (at least) the scalar field ϕ as *unremovable* content of spacetime. In the Killing case it follows $\xi = e^{\phi}$, so at the feeble static limit $\xi^{-1}\Delta\xi \cong \Delta\phi$ we get *directly* the Poisson equation of Newtonian gravity. In *Lagrangian* version of theory there is *no* need to use the *Ricci scalar* R at all. From typical solutions of scalar gravity equations with cosmological constant Λ it follows that the usual (not the low-energy limit of superstring theories) *scalar field is of tachyonic nature*, and effective (imaginary) mass of tachyonic carriers is no more $10^{-65}g \approx 10^{-33}eV$. The *scalar charges* (sources) proves to be the usual *masses*, so all the particles and gauge fields should live in dynamical equilibrium with scalar condensate (having in general the vortex component as well). *Gravity is always a secondary effect with respect to scalar field.* It is shown *the origin of such a field* can be explained *as a direct neutral superposition of electric fields of both signs.* There exist very compact objects but *no* more *black-holes* in such approach, and *no* need to consider the *gravitational waves* propagating with speed of light and to quantize them. All the 'crucial effects' are to be confirmed, new approach to lensing effects is found, and the usual quantum effects in strong (classical) field remain to be meaningful. Instead of the *black–hole thermodynamics* we successfully develop the *well-defined scalar thermodynamics.* The preferable cosmological scenario is a *triple* electroweak-type *bounce* (instead of a single planckian big bang) at the beginning of a given evolution era of our oscillating Universe, which gives rise to three generations of leptons (with three peaks in neutrino sea) and other particles but without any susy, monopoles and other exotic matter behind and far behind the electroweak limit. Some comments:

Comments to 'Geometrized Scalar Gravity (GSG) as a new paradigm'.

The survey above consists of four main parts: I Mathematical Foundations, II Thermodynamic Conceptions, III General Properties of Scalar Field, and IV New Scalar Paradigm:

I. The principal idea of GSG implies that properties of vacuum are generated by scalar field. Mathematical foundations of such a theory are represented by the generalized Killing-type equations and integrability conditions for those, from the one hand, and the modified Raychaudhuri equation, from the other. Both these approaches give rise to one and the same main

master-equation given above. A corresponding geometric Lagrangian of non-Gilbert type does produce the third way to get such equation. This Lagrangian goes over into the field one, and, simultaneously, master-equation goes over into the principal scalar field equation, through the algebraic replacement of covariant time-time projection of Ricci tensor by the appropriate (Einstein-type or other) combination of scalar field energy-momentum tensor components projected on the basic timelike congruence. The other matter fields can also be included as usual.

II. The thermodynamic consistency of the theory is investigated ab initio. By analogy with anti-deSitter space (i.e. with a negative sign of cosmological constant) the antiscalar field conception is proposed for the case when only a negative sign of scalar field energy-momentum tensor in the Einstein-type equations proves to be thermodynamically admissible. In non-static case the antiscalar approach leads in particular to so-called phantom field concept applied to the recent dark energy problems. A correspondence between the black-hole thermodynamics and alternative classical scalar vacuum thermodynamics is manifested for the different cosmic scales. This is achieved with no use of such notion as 'horizon' because in scalar gravity the role of that is now pertaining to any appropriate equipotential surface (only the intensity of field has a meaning).

III. On the base of electrostatic balanced and corresponding scalar field solutions of admissible Einstein-type equations it is shown that the primary origin of the background scalar field can naturally be explained as a simple neutral but gravitating superposition of quasi-electrostatic fields of both signs. As a consequence all the masses play a role of scalar charges, and no another scalar charge besides mass can exist. The tachyonic character of such a field is also demonstrated on the ground of inevitable appearance of the imaginary masses in typical solutions of scalar gravity equations. There are no vacuum Gilbert-Einstein's equations, no black holes in this approach, and no gravitational waves, and so there is no necessity to quantize them. Nevertheless there is no reason to exclude all the usual quantum effects in strong (classical) gravitational fields.

IV. To appreciate the meaning and origin of scalar gravity method it is worth to point out on such eternal problems in Gravitation as those of energy, singularities, the origin and structure of cosmological term, dark matter and energy, existence and interpretations of black-holes, worm-holes and other exotic curvilinear coordinate effects, existence of new predicted type of matter – gravitational waves, quantization of those and reality of gravitons, existence and meaning of "highest dimensions" in brane/string/M-theories with a long tail of concomitant scalar fields and topological defects, the gauge status of gravity, etc. In Particle Physics it is not much better situation with a permanent unrevealing of Higgs bosons (a base for the famous Standard Model being much more successful than justified), a deep incomprehension of true nature of elusive neutrinos, a huge discrepancy between the Planck and EW vacuum scales on the one hand and gravitational vacuum density scale from the other (hierarchies problems), a feeble validity of GUT and TOE ideas, and so on. Nevertheless, as we believe, much of these obstacles could be or resolved, or at least abounded, by taking a paradigm of gravity as a secondary effect with respect to the unique, ubiquitous and well-defined scalar field filling the Universe. It could also be responsible for the origin of all other gravitating matter including the gauge fields and neutrinos. Being of tachyonic nature, both gravity proper and our free scalar field should in principle be not quantizable in a canonical sense. Thus they surely manifest the fundamental boundary to Relativity and Quanta.

Clock Synchronization and Finsler Structure of a Flat Anisotropic Space-Time

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This paper is concerned with the signal method of clock synchronization. A new relation for the proper time is deduced. New transformations of coordinates in two-dimentional space-time are obtained. Space-time has a structure of a flat anisotropic Finsler space at the general nonstandard clock synchronization. Different cases of clock synchronization between inertial systems are considered.

1. Introduction

The signal method of clock synchronization in relativistic mechanics is just one hundred years in 1898. This method was first suggested by Poincare [1] under consideration of a problem of simultaneity of distant events within an inertial reference systems. Further Einstein [2] used Poincare method in his special theory of Relativity. Poincare [3] was the first who considered formalism of four-dimensional space-time. He found invariants of Lorentz group in four-dimensional space-time. Poincare formulated significant physical principle that description of physical processesy in real world is connected with convention of definition of space-time geometries. Finally, Minkowsky [4] used formalism of four-dimentional Poincare geometry and suggested the pseudoeuclidean geometry for events in reference systems. The signal method of Poincare found a good application in this geometry.

The results of experimental investigation [5] of a spectrum of superhigh energy primary cosmic protons points the conclusion that the Lorentz transformations fail at $\left(1-v_0^2/c_0^2\right)^{-1/2} \approx 5\cdot 10^{10}$. This means that the Lorentz-factor $\left(1-v_0^2/c_0^2\right)^{-1/2}$ in the traditional relativistic mechanics needs further generalization.

The traditional approach uses standard Poincare-Einstein clock synchronization, for which one has the form-invariant expression for proper time in the inertial system (K)

$$t_0^2 = t^2 - x^2/c_0^2 \tag{1.1}$$

Physical processes get isotropic description in two-dimentional Minkowsky space-time. Unidirectional light speed is isotropic and invariant quantity $c_+ = c_- = c'_+ = c'_- = c_0 = 3\cdot 10^8\, m/sec$ in inertial systems (K) and (K').

A promising direction of investigation is the relativistic kinematics in Finsler space-time [6]. Different relativistic techniques in Finsler geometry were considered by Beem [7], Pimenov [8, 9], Bogoslovsky [10, 11], Ishikawa [12], Zaripov [13, 14], Jiam-Miin Liu [15] and others.

We should note studies by Bogoslovsky [10, 11] where two-dimentional Finsler space-time is considered with the following expression for the proper time

$$t_0^2 = \left(\frac{t - x/c_0}{t + x/c_0}\right)^{-r}\left(t^2 - \frac{x^2}{c_0^2}\right), \qquad -1 < r < 1 \tag{1.2}$$

where r is the constant parameter. At $r = 0$ we have (1.1).

The aim of the present work is to consider Finsler structure of space-time under general non-standard closk synchronization [16-19]. Unidirectional speed of light has non-isotropic and non-invariant quantity [16]

$$c_+ = \frac{c}{1+\varepsilon}, \quad c_- = \frac{c}{1-\varepsilon}, \quad \frac{x}{c_+} + \frac{x}{c_-} = \frac{2x}{c}, \quad (-1 \le \varepsilon \le 1) \tag{1.3}$$

$$c'_+ = \frac{c'}{1+\varepsilon'}, \quad c'_- = \frac{c'}{1-\varepsilon'}, \quad \frac{x'}{c'_+} + \frac{x'}{c'_-} = \frac{2x'}{c'}, \quad (-1 \le \varepsilon' \le 1) \tag{1.4}$$

Physical processes have anisotropic description. Parameters ε and ε' shows time anisotropy in systems (K) and (K').

In present paper strict physical substantiation of Finsler structure of two-dimentional space-time is given, including expression (1.2). New transformations of space-time coordinates are obtained. Under certain assumptions the known result follow from the derived relations.

2. Transformation of Space-Time Coordinates in a Flat Finsler Geometry

Let us consider two frames (K) and (K'). Relative velocities of motion of systems v_+ и v'_- are determined in scales units of systems (K) and (K'), respectively [16]

$$v_+ = \frac{v}{1+\varepsilon v/c}, \; v_- = \frac{v}{1-\varepsilon v/c}, \; \frac{x}{v_+} + \frac{x}{v_-} = \frac{2x}{v}, \tag{2.1}$$

$$v'_+ = \frac{v'}{1+\varepsilon' v'/c'}, \; v'_- = \frac{v'}{1-\varepsilon' v'/c'}, \; \frac{x'}{v'_+} + \frac{x'}{v'_-} = \frac{2x'}{v'}, \tag{2.2}$$

$$\left(1-\varepsilon\frac{v_+}{c}\right)\left(1+\varepsilon\frac{v}{c}\right) = \left(1+\varepsilon'\frac{v'_-}{c'}\right)\left(1-\varepsilon'\frac{v'}{c'}\right) = 1. \tag{2.3}$$

The space-time coordinate transformations can be defined by the "k" coefficient method [13, 16]. Let us write the relations

$$\begin{aligned} &(c'/c_0)^{1/2}(t' - x'/c'_+) = k_+(c/c_0)^{1/2}(t - x/c_+), \\ &(c'/c_0)^{1/2} k'_-(t' + x'/c'_-) = (c/c_0)^{1/2}(t + x/c_-), \end{aligned} \tag{2.4}$$

where $t = t_2(x)$ and $t' = t_2(x')$. Coefficients $k_+(c/c')^{1/2}$ and $k'_-(c'/c)^{1/2}$ describe Doppler Effect in the x direction. One can get from (2.4) the following relations

$$k'_-\frac{c'}{c_0}\left[\left(t' - \frac{\varepsilon' x'}{c'}\right)^2 - \frac{x'^2}{c'^2}\right] = k_+\frac{c}{c_0}\left[\left(t - \frac{\varepsilon x}{c}\right)^2 - \frac{x^2}{c^2}\right], \tag{2.5}$$

$$k_+ k'_- \frac{t' + x'/c'_-}{t' - x'/c'_+} = \frac{t + x/c_-}{t - x/c_+} \tag{2.6}$$

At $x' = 0$ we have $x = v_+ t$, and at $x = 0$ we have $x' = -v'_- t'$. Then from (2.6) we obtain

$$k_+ k' = k^2 = \frac{1 + v_+/c_-}{1 - v_+/c_+} = \frac{1 + v'_-/c'_+}{1 - v'_-/c'_-} \tag{2.7}$$

for the connection of relative velocities v_+ and v'_-. From (2.7) follows the relations

$$c\left(\frac{1}{v_+} - \frac{1}{c_+}\right) = c'\left(\frac{1}{v'_-} - \frac{1}{c'_-}\right), \; \left(\frac{v}{c} = \frac{v'}{c'} = \frac{v_0}{c_0}\right), \tag{2.8}$$

$$\frac{v'_-}{c'}=\frac{v_+}{c}\left[1-\frac{(\varepsilon+\varepsilon')v_+}{c}\right]^{-1},\quad \frac{v_+}{c}=\frac{v'_-}{c'}\left[1+\frac{(\varepsilon+\varepsilon')v'_-}{c'}\right]^{-1}, \tag{2.9}$$

$$\left(1-\frac{(\varepsilon+\varepsilon')v_+}{c}\right)\left(1+\frac{(\varepsilon+\varepsilon')v'_-}{c'}\right)=1. \tag{2.10}$$

From (2.7) we have the group properties of the transformations of unidirectional velocities

$$k\left(\frac{x}{t}\right)=k(v_+)k\left(\frac{x'}{t'}\right),\ k\left(\frac{x'}{t'}\right)=k(w'_+)k\left(\frac{x''}{t''}\right),\ k\left(\frac{x}{t}\right)=k(z_+)k\left(\frac{x''}{t''}\right),\ k(z_+)=k(v_+)k(w'_+). \tag{2.11}$$

Here w'_+ and z_+ is the velocity of a third system (K'') relative to systems (K) and (K'), respectively. Consider the equality

$$\frac{1+v_+/c_-}{1-v_+/c_+}=\frac{1+v/c}{1-v/c}=\frac{1+v_0/c_0}{1-v_0/c_0} \tag{2.12}$$

from which it follows that the coefficient k is invariant on anisotropic and isotropic describing of physical processes. Then from (2.11) we get the group properties of the transformations of average velocities over the closed path

$$k(z)=k(v)k(w'),\ k(z_0)=k(v_0)k(w'_0) \tag{2.13}$$

Relations (2.13) can be written in a traditional form

$$\frac{z}{c}=\frac{v/c+w'/c'}{1+vw'/cc'},\quad z_0=\frac{v_0+w'_0}{1+v_0\,w'_0/c_0^2} \tag{2.14}$$

The second relation in (2.14) is Poincare-Einstein law of velocity addition in special theory of relativity ($\varepsilon=\varepsilon'=0$, $c=c'=0$).

Direct and inverse transformations can be obtained from (2.4) of the matrix $\mathbf{A}_+$ and its inverse matrix $\mathbf{A}'_-$

$$\begin{pmatrix}t'\\x'\end{pmatrix}=\mathbf{A}_+\begin{pmatrix}t\\x\end{pmatrix}=\sqrt{cc'}\begin{bmatrix}1/c'_- & 1/c'_+\\-1 & 1\end{bmatrix}\begin{bmatrix}k_+ & 0\\0 & 1/k'_-\end{bmatrix}\begin{bmatrix}1 & -1/c_+\\1 & 1/c_-\end{bmatrix}\begin{pmatrix}t\\x\end{pmatrix}, \tag{2.15}$$

$$\begin{pmatrix}t\\x\end{pmatrix}=\mathbf{A}'_-\begin{pmatrix}t'\\x'\end{pmatrix}=\sqrt{cc'}\begin{bmatrix}1/c_- & 1/c_+\\-1 & 1\end{bmatrix}\begin{bmatrix}1/k_+ & 0\\0 & k'_-\end{bmatrix}\begin{bmatrix}1 & -1/c'_+\\1 & 1/c'_-\end{bmatrix}\begin{pmatrix}t'\\x'\end{pmatrix}, \tag{2.16}$$

where matrix $\mathbf{E}$ is the unity matrix and $A=\det\mathbf{A}_+=\dfrac{k_+}{k'_-}$, $A'=\det\mathbf{A}'_-=\dfrac{k'_-}{k_+}$, $\mathbf{A}_+\mathbf{A}'_-=\mathbf{E}$, $A\cdot A'=1$.

From (2.7) one gets the relations $k_+=\sqrt{A}k$, $k'_-=\sqrt{A'}k$.

Let us consider a class of reference systems, for which the relations $\sqrt{A}\neq 1$ or $\sqrt{A'}\neq 1$ is valid. Then from (2.15) and (2.16) we have the general transfomations of space and time

$$\frac{x'}{\sqrt{c'}}=\sqrt{\frac{A}{c}}\frac{x-v_+t}{\alpha_+},\quad \alpha_+=\left[\left(1-\frac{v_+}{c_+}\right)\left(1+\frac{v_+}{c_-}\right)\right]^{1/2}, \tag{2.17}$$

$$\sqrt{c'}t'=\frac{\sqrt{Ac}}{\alpha_+}\left\{t\left[1-\frac{(\varepsilon+\varepsilon')v_+}{c}\right]-x\left[\frac{v_+}{c_+c_-}+\frac{\varepsilon-\varepsilon'}{c}\right]\right\}. \tag{2.18}$$

The inverse transformations is

$$\frac{x}{\sqrt{c}}=\sqrt{\frac{A'}{c'}}\frac{x'+v'_-t'}{\alpha'_-},\quad \alpha'_-=\left[\left(1+\frac{v'_-}{c'_+}\right)\left(1-\frac{v'_-}{c'_-}\right)\right]^{1/2}, \tag{2.19}$$

$$\sqrt{c}t = \frac{\sqrt{A'c'}}{\alpha'_+}\left\{t'\left[1+\frac{(\varepsilon'+\varepsilon)v'_-}{c'}\right]+x'\left[\frac{v'_-}{c'_+c'_-}-\frac{\varepsilon'-\varepsilon}{c'}\right]\right\}. \tag{2.20}$$

From (2.5) one obtains the equality

$$\frac{c'}{c_0}\left[\left(t'-\frac{\varepsilon' x'}{c'}\right)^2-\frac{x'^2}{c'^2}\right] = A(v_+)\frac{c}{c_0}\left[\left(t-\frac{\varepsilon x}{c}\right)^2-\frac{x^2}{c^2}\right] \tag{2.21}$$

Consider the third system (K'') and get, according to (2.21), the following functional equation $A(z_+) = A(v_+)A(w'_+)$. Quantity $A(v_+)$ is a coefficient of contraction or extension of space-time coordinates of system (K).

Taking the group property (2.11) into account we can obtain

$$A(v_+) = [k(v_+)]^{-2r} = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r},\ k_+ = k^{1-r},\ k'_- = k^{1+r},\quad -1<r<1. \tag{2.22}$$

where r is a constant parameter. Then from (2.17) and (2.18) we can obtain the direct transfomations

$$\frac{x'}{\sqrt{c'}} = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2}\frac{x-v_+t}{\sqrt{c}\alpha_+}, \tag{2.23}$$

$$\sqrt{c'}t' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2}\frac{\sqrt{c}}{\alpha_+}\left\{t\left[1-\frac{(\varepsilon+\varepsilon')v_+}{c}\right]-x\left[\frac{v_+}{c_+c_-}+\frac{\varepsilon-\varepsilon'}{c}\right]\right\}. \tag{2.24}$$

Taking (2.6) into account, one can get from (2.21) form-invariant expression for the proper time

$$t_0^2 = \frac{c}{c_0}\left(\frac{t-x/c_+}{t+x/c_-}\right)^{-r}\left(t-\frac{x}{c_+}\right)\left(t+\frac{x}{c_-}\right) = \frac{c'}{c_0}\left(\frac{t'-x'/c'_+}{t'+x'/c'_-}\right)^{-r}\left(t'-\frac{x'}{c'_+}\right)\left(t'+\frac{x'}{c'_-}\right), \tag{2.25}$$

which can be written as

$$\begin{aligned} t_0^2 &= \frac{c}{c_0}\left\{\frac{[(t-\varepsilon\, x/c)-x/c]^2}{(t-\varepsilon\, x/c)^2-x^2/c^2}\right\}^{-r}\left[(t-\varepsilon\, x/c)^2-x^2/c^2\right] = \\ &= \frac{c'}{c_0}\left\{\frac{[(t'-\varepsilon'\, x'/c')-x'/c']^2}{(t'-\varepsilon'\, x'/c')^2-x'^2/c'^2}\right\}^{-r}\left[(t'-\varepsilon'\, x'/c')^2-x'^2/c'^2\right] \end{aligned} \tag{2.26}$$

For the standard Poincare-Einstein clock synchronization ($c'=c=c_0$, $\varepsilon'=\varepsilon=0$) expression for the proper time (1.2) follows from (2.26), considered by Bogoslovsky [10, 11].

At $r=0$ we obtain the equality $A(v_+) = A(v'_-) = 1$ for coefficients of contraction or extension. Then obtained results coincide with the known ones on the general nonstandard clock synchronization [16]. At $A(v_+) = A'(v'_-) = 1$ one can obtain the orthogonal transformations of space-time coordinates.

Thus, we have Finsler structure of two-dimensional space-time under general nonstandard clock synchronization. The value $c' = cA(v_+)$ or $c = c'A'(v'_-)$ leads to the transformations

$$\frac{x'}{c'} = \frac{x-v_+t}{\alpha_+},\quad t' = \frac{1}{\alpha_+}\left\{t\left[1-\frac{(\varepsilon+\varepsilon')v_+}{c}\right]-x\left[\frac{v_+}{c_+c_-}+\frac{\varepsilon-\varepsilon'}{c}\right]\right\}, \tag{2.27}$$

from which follows the form-invariant expression

$$(t-\varepsilon x/c)^2 - x^2/c^2 = (t'-\varepsilon' x'/c') - x'^2/c'^2. \tag{2.28}$$

As a result, transformations (2.27) are obtained, those estabilish a singlevalued grid (2.28) of two-dimensional space-time in (K) and (K'). Space-time with the grid (2.28) was considered in [16].

3. Synchronization of Clocks Between Systems

Consider two clocks, situated at points $x'=0$ and $x=0$ in systems (K') and (K), respectively. The moves with the relative velocities v_+ and v'_- and show the times $t'(0)$ and $t(0)$. The initial readings and unit scales can differ between systems (K) and (K'). Therefore, necessity arises to compare the course of clock and synchronization in systems (K') and (K).

Let us first consider the class of reference systems with relative simultaneity of distant events. Let us define the conventional physical principle: *moving clocks measures the proper time*. Then from (2.25) we obtain the relations

$$c' = c = c_0\,, \tag{3.1}$$

$$t_0 = t'(0) = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2} \sqrt{\left(1-\varepsilon v_+/c_0\right)^2 - v_+^2/c_0^2}\; t\big|_{x=v_+t}\,, \tag{3.2}$$

$$t_0 = t(0) = \left(\frac{1-v'_-/c'_-}{1+v'_-/c'_+}\right)^{-r/2} \sqrt{\left(1+\varepsilon' v'_-/c_0\right)^2 - v'^2_-/c_0^2}\; t'\big|_{x'=-v'_-t'}\,. \tag{3.3}$$

Clock synchronization between systems determines the allowed class of reference systems with the nonstandard synchronization of Reichenbach-Grünbaum [16]. Average speed of light over the closed path is an invariant quantity. From (2.23) and (2.24) we shall get the transformations

$$x' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2} \frac{x-v_+t}{\alpha_+}\,, \tag{3.4}$$

$$t' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2} \frac{1}{\alpha_+}\left\{t\left[1-\frac{(\varepsilon+\varepsilon')v_+}{c_0}\right] - x\left[\frac{v_+}{c_+c_-}+\frac{\varepsilon-\varepsilon'}{c_0}\right]\right\}. \tag{3.5}$$

If the simultaneity condition in systems (K) and (K') are defined with the same values $\varepsilon'=\varepsilon$, then from (3.4) and (3.5) we have the transformations

$$x' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2} \frac{x-v_+t}{\alpha_+}\,, \tag{3.6}$$

$$t' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{-r/2} \frac{1}{\alpha_+}\left\{t\left[1-\frac{2\varepsilon v_+}{c_0}\right] - \frac{v_+}{c_+c_-}x\right\}, \tag{3.7}$$

obtained by Petryszhyn [20] and Lyakhovitsky [21]. At $r=-\varepsilon$ from (3.6) and (3.7) we get the transformations

$$x' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{\varepsilon/2} \frac{x-v_+t}{\alpha_+}\,, \tag{3.8}$$

$$t' = \left(\frac{1+v_+/c_-}{1-v_+/c_+}\right)^{\varepsilon/2} \frac{1}{\alpha_+}\left\{t\left[1-\frac{2\varepsilon v_+}{c_0}\right] - \frac{v_+}{c_+c_-}x\right\} \tag{3.9}$$

considered by Asanov [22].

For the standard Poincare-Einstein clock synchronization $(\varepsilon' = \varepsilon = 0)$ we have the transformations

$$x' = \left(\frac{1+v_0/c_0}{1-v_0/c_0}\right)^{-r/2} \frac{x - v_0 t}{\sqrt{1-v_0^2/c_0^2}}, \quad t' = \left(\frac{1+v_0/c_0}{1-v_0/c_0}\right)^{-r/2} \frac{t - v_0\, x/c_0^2}{\sqrt{1-v_0^2/c_0^2}} \tag{3.10}$$

discussed by Pimenov [9] and Bogoslovsky [10, 11].

Further, let us consider the class of reference systems with the absolute simultaneity of distant events. At $\Delta t = 0$ the equality $\Delta t' = 0$ is required. From (2.18) we get the equality

$$\varepsilon' = \varepsilon + \frac{v_+ c}{c_+ c_-} = \frac{\varepsilon + v/c}{1 + \varepsilon v/c}. \tag{3.11}$$

Then we obtain the relations between clock readings

$$t'(0) = \sqrt{A\frac{c}{c'}}\, \alpha_+\, t\big|_{x=v_+ t}, \qquad t(0) = \sqrt{A'\frac{c'}{c}}\, \alpha'_-\, t'\big|_{x'=-v'_- t'}. \tag{3.12}$$

Let us define the next conventional principle: *moving clock measures the time*

$$t'(0) = \sqrt{A}\; t\big|_{x=v_+ t}, \; t(0) = \sqrt{A'}\; t'\big|_{x'=-v'_- t'}. \tag{3.13}$$

Reference frames are defined with the general nonstandard synchronization [16]. From (2.17) and (2.18) we get the following transformations

$$x' = \sqrt{A}(x - v_+ t), \;\; t' = \sqrt{A}\, t, \tag{3.14}$$

$$v_+ = v'_-, \; c' = c\alpha_+^2, \;\; c = c'\alpha_-'^2, \;\; A(v_+) = e^{-2av_+/c_0}$$

for which we have the group properties of unidirectional velocities transformations

$$\frac{x'}{t'} = \frac{x}{t} - v_+, \;\; \frac{x''}{t''} = \frac{x'}{t'} - w'_+, \;\; \frac{x''}{t''} = \frac{x}{t} - z_+, \;\; z_+ = v_+ + w'_+. \tag{3.15}$$

Relations (3.15) could be derived from the property (2.11).

Then from (3.14) we have transformations

$$x' = e^{-av_+/c_0}(x - v_+ t), \qquad t' = e^{-av_+/c_0} t, \tag{3.16}$$

where a is a constant parameter.

For the standart Poincare-Einstein clock synchronization in system (K) we have the transformations

$$x' = e^{-av_0/c_0}(x - v_0 t), \qquad t' = e^{-av_0/c_0} t, \tag{3.17}$$

obtained by Petryszyn [20].

Using of (3.15), from (2.21) we get the form-invariant expression for the proper time

$$t_0^2 = \frac{c}{c_0} e^{-2ax/c_0 t}\left[(t - \varepsilon x/c)^2 - x^2/c^2\right] = \frac{c'}{c_0} e^{-2ax'/c_0 t'}\left[(t' - \varepsilon' x'/c')^2 - x'^2/c'^2\right] \tag{3.18}$$

In conclusion let us consider the special case. One takes (2.12) into account and has the following value for the unidirectional velocity

$$v_+ = \frac{\gamma c_0}{2} \ln \frac{1+v_0/c_0}{1-v_0/c_0}, \tag{3.19}$$

where $\gamma = r/a$ is a constant parameter. We use value $\gamma = 1$ and get from (3.15) the law of velocity addition

$$z_+ = v_+ + w'_+ = \frac{c_0}{2}\ln\frac{1+v_0/c_0}{1-v_0/c_0} + \frac{c_0}{2}\ln\frac{1+w'_0/c_0}{1-w'_0/c_0} = \frac{c_0}{2}\ln\frac{1+z_0/c_0}{1-z_0/c_0}. \tag{3.20}$$

From (3.20) we have the traditional Poincare-Einstein law of velocity addition (2.14).

Let us consider the case when the general standard clock synchronization ($\varepsilon = 0$, $c \neq c_0$) is set in reference system (K) [16]. Then in system (K) we have isotropic description and from (3.17) we get the following transformations

$$x' = e^{-av/c_0}(x - vt), \quad t' = e^{-av/c_0} t. \tag{3.21}$$

Average velocities over the closed path have values

$$v = \frac{c_0}{2} \ln \frac{1 + v_0/c_0}{1 - v_0/c_0}, \qquad c = \frac{vc_0}{v_0} = \frac{c_0^2}{2v_0} \ln \frac{1 + v_0/c_0}{1 - v_0/c_0} \tag{3.22}$$

In the moving system (K') we have anisotropic description with $\varepsilon' = v_0/c_0$ and $c' = c(1 - v_0^2/c_0^2)$.

Analogous results (3.20) and (3.22) were obtained by Jiam-Miin Liu [15] in his Finslerian Structures in Gravity - Free Space.

4. Conclusions

In this paper Poincare's signal method of clock synchronization (1898) is discussed. New transformations of space-time coordinates are obtained in two-dimensional space-time in the case when the determinant of transformation does not equal a unity. Investigation of coefficient of contraction or extension $A(v_+)$ has great significance for definition of the type of space-time geometry.

Let us consider the clock synchronization at point $\mathbf{x} = (x, y, z)$ of three-dimensional space. Then generalizing the relation (2.26) one gets the distance between two nearest points $(\mathbf{x}, c_0 t)$ and $(d\mathbf{x}, c_0 dt)$ in four-dimensional space-time

$$ds^2 = c_0^2 dt_0^2 = \left\{ \frac{\left[\sqrt{n} c_0 dt - \frac{(\mathbf{e} + \mathbf{q}) d\mathbf{x}}{\sqrt{n}} \right]^2}{\left(\sqrt{n} c_0 dt - \frac{\mathbf{e} d\mathbf{x}}{\sqrt{n}} \right)^2 - \frac{d\mathbf{x}^2}{n}} \right\}^{-r} \left[\left(\sqrt{n} c_0 dt - \frac{\mathbf{e} d\mathbf{x}}{\sqrt{n}} \right)^2 - \frac{d\mathbf{x}^2}{n} \right] \tag{4.1}$$

Here $n = c/c_0$ is the refraction coefficient for light in system (K); $\mathbf{e} = (\varepsilon_x, \varepsilon_y, \varepsilon_z)$ is the time anisotropy vector under nonstandard clock synchronization; $\mathbf{q} = (q_x, q_y, q_z)$ is the space-time anisotropy vector with the norm $\mathbf{q}^2 = 1$ [10]. At $n = 1$ and $\mathbf{e} = 0$ we have Poincare-Einstein clock synchronization. Then (4.1) coincides with the value of distance considered by Bogoslovky [10, 11].

For reference system with the absolute simultaneity we can generalize (3.18) and get the following distance in four-dimensional space-time

$$ds^2 = c_0^2 dt_0^2 = e^{-2\mathbf{a} d\mathbf{x}/c_0 dt} \left[\left(\sqrt{n} c_0 dt - \frac{\mathbf{e} d\mathbf{x}}{\sqrt{n}} \right)^2 - \frac{d\mathbf{x}^2}{n} \right]. \tag{4.2}$$

Here $\mathbf{a} = (a_x, a_y, a_z)$ is the space-time anisotropy vector. Thus, two types Finsler flat anisotropic space-time is derived.

In conclusions consider the case where grid of four-dimensional space-time in first type are

$$dt^2 - d\mathbf{x}^2 / c_0^2 = dt'^2 - d\mathbf{x}'^2 / c_0'^2 . \tag{4.3}$$

Synchronization of course of clock determines the allowed class of reference systems with the general standard synchronization ($c_0 \neq c'$ and $\varepsilon = \varepsilon' = 0$ [16]). Then we have the speed of light in (K')

$$c' = c_0 \left[\frac{\left(1 - \mathbf{q}\mathbf{v}_0 / c_0\right)^2}{1 - \mathbf{v}_0^2 / c_0^2} \right]^{-r} . \tag{4.4}$$

Analogous in second type we obtain the relations

$$dt^2 - d\mathbf{x}^2 / c_0^2 = \left(dt' - \mathbf{e}' d\mathbf{x}' / c'\right)^2 - d\mathbf{x}'^2 / c'^2 , \; c' = c_0 e^{-2\mathbf{a}\mathbf{v}_0 / c_0} . \tag{4.5}$$

According (4.4) and (4.5), the value of average speed og light determine of vectors anisotropy $\mathbf{q}$ or $\mathbf{a}$ and the velocity $\mathbf{v}_0$. This speed c' be necessary determine in the experiment.

References

[1]H.Poincare, *Rev. Metaphys. Morale*, **6**, 1-13 (1898).

[2] A.Einstein, *Ann. Phys.*, **17**, 891-921 (1905).

[3] H.Poincare, *Rend. Circolo Mat.* Palermo, **21**, 129-176 (1906).

[4] H.I.Minkowski, *Phys. Ztschr.* Bd 10, 104-107 (1909).

[5] G.B.Khristiansen, *Cosmic Rays of Superhigh Energies* (in Russian, Moscow University Press, Moscow, 1974).

[6] H.Rund, *The Differential Geometric of Finsler Spaces* (Springer-Verlag, Berlin, 1959); H.Busemann, *Metric Methods in Finsler Spaces and in Foundations of Geometric* (Princeton University Press, Princeton, 1942); G.S.Asanov, *Finsler Geometrie, Relativity and Gauge Theories* (D.Reidel Publishing Company, Dordrecht, 1985).

[7] J.K.Beem, *Canadian Journ. of Math.*, **22**, 1035-1039 (1970).

[8] R.I.Pimenov, *Kinematic Spaces (Mathematical Theory of Spacetime)*, (Plenum Press, New York, 1970).

[9] Pimenov R.I., *Anisotropic Finsler extention of General Relativity as an order Structure,* (in Russian, Komi Branch Academi of Sci. USSR Press, Siktivkar, 1987)

[10] G.Yu.Bogoslovsky, *II Nuovo Cimento*, **40B**, 99-115 (1977).

[11] G.Yu.Bogoslovsky, *II Nuovo Cimento*, **40B**, 116-134 (1977).

[12] H.Ishikawa, *Journ. of Math. Physics*, **22**, 995-1004 (1981).

[13] R.G.Zaripov, in *Gravitation and Theory of Relativity***,** (in Russian, Kazan University Press, N 29, Kazan,1992) pp.64-71.

[14] R.G.Zaripov, *Apeiron,* **4**, 114-117. (1997).

[15] Jian-Miin Liu, *Galilean Electrodynamics,* **8**, 43-48 (1997).

[16] R.G.Zaripov, *Galilean Electrodynamics,* **9**, 63-72 (1998).

[17] R.G.Zaripov, *Galilean Electrodynamics,* **10**, 57-60 (1999).

[18] R.G.Zaripov, *Galilean Electrodynamics,* **11**, 29-35 (2000).

[19] R.G.Zaripov, *Galilean Electrodynamics,* **11**, 63-68 (2000).

[20] H.Petryszyn, *Inst. Mat. Fiz. Teor., Politechniki Wroclawskiey Kommunicat*, N12, Wroclaw (1973).

[21] V.N.Lyachovisky, *Commun. I.T.E.P.* Moskow, (in Russian, 1987).

[22] G.S.Asanov, *Herald of Moskow University, series 3, Physics and Astronomy,* N4, Moscow, **36**, 7-13 (in Russian, 1995).

Relativistic analogies in the classical mechanics

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The present paper is the review of results, obtained by various authors, testifying that there are analogues of relativistic effects within the framework of the classical mechanics. Such effects were observed, in particular, at a movement of topological solitons (kinks, dislocations) in solids. These effects are described by the formulas, similar to the formulas of the special theory of relativity, but they contain sound velocity instead of velocity of light. It is shown that there is a number of non-linear systems, enabling the supersonic motion of solitons. Such solitons are mechanical analogues of tachyons.

1. Introduction

At first sight it would seem that the special theory of relativity (STR) cannot have the classical analogue. However, there are the classical particle-like objects, solitons [1], which are the solutions of the Lorentz-covariant equations. Similar to the relativistic particles, these solitons have a continuous and limited spectrum of velocities $0 < v < v_s$ where v_s is a sound velocity. The movement of these solitons is accompanied by effects, similar to the relativistic ones. Among these effects are Lorentz reduction of width of a moving soliton, velocity dependence of soliton energy according to the Lorentz law [1] etc. The formulas, describing these effects, are similar to the formulas of STR but they contain sound velocity instead of velocity of light. The origin of these formulas is explained as follows. The effects of STR are based on finiteness of the velocity of information transfer. In the mechanics of deformable solids the information concerning soliton movement is transferred by sound waves. Therefore in the formulas of the soliton theory the sound velocity occupies the place of velocity of light.

This analogy takes place both for solitons in one-dimensional systems and for dislocations in three-dimensional continua. However the classical mechanics also demonstrates more complicated relativistic effects. There are two velocities of information transfer in isotropic solids: velocities of longitudinal and transverse sound waves. Besides there are the gradient nonlinearity and dispersion in solids. As a result of these features the atomic equations of motion do not Lorentz-covariant ones. However, there are the effects caused by finiteness of sound velocities in these solids. We have the right to use the name 'relativistic' for referring to these effects. Some non-Lorentz-covariant equations have the supersonic soliton solutions. These solitons are mechanical analogues of tachyons. The discussions which follow point out that their existence does not break the causality principle and other decrees of nature.

It is known that any optical effect has an acoustic analogue. It is less known that this analogy has a continuation. There is an analogue of classical electrodynamics (i.e. theory, describing not only distribution of electromagnetic waves but also their interaction with charged particles) within the framework of the classical mechanics. Kosevich [2] has shown that such an analogue is the dynamical theory of dislocations, i.e. topological solitons in a crystal lattice. The dislocations correspond to electrical charges, and the fields of elastic deformations and mechanical stresses are analogues of an electromagnetic field. Later Musienko and Koptsik [3] has shown that the dynamical theory of dislocations can be formulated as the gauge theory. The ambiguity of potential of the dislocation elastic field ('gauge freedom') is connected to the ambiguity of displacements around a dislocation. Unfortunately, we can not discuss this interesting problem in more detail for reasons of space. For an extended discussion see our papers [3, 4].

In order to prevent misunderstanding we would like to explain the meaning of the term 'soliton'. The mathematicians say that soliton is the localized particle-like solution of integrable nonlinear system of equations having finite energy [5]. Topological solitons are a special case because they have the topological charge. The physicists usually use more wider definition and say that topological soliton is the localized object having the topological charge [6]. We use this definition.

The aim of this paper is to summarize relativistic effects in the classical mechanics, to compare them to the analogous effects of STR, to discuss their origin, to show that there are the 'mechanical' relativistic effects which do not have analogues in STR up till now, in particular, supersonic solitons. It is natural to suppose that such solitons are analogues of tachyons.

2. Relativistic effects in dynamics of solitons in one-dimensional systems

Let us consider classical one-dimensional Frenkel–Kontorova model of a dislocation [7]. The chain of particles of mass m, connected by linear springs of rigidity k, interacts with sinusoidal potential. The chain can be stretched (or compressed) in such a way that the number of particles is less (or more) than number of potential wells per unit. Such a configuration refers to as a kink (or, accordingly, antikink). Let a is a period of potential, U_n is a displacement of particle n relative to well n, $u_n = U_n /a$. The equation of motion of the chain is

$$m\,\partial_t^2 u_n - k\left(u_{n+1} - 2u_n + u_{n-1}\right) + A\sin 2\pi u_n = 0$$

where $\partial_t \equiv \partial/\partial t$, A is a constant, $-\infty<n<\infty$. In continuous approximation we obtain the sine-Gordon equation [1]

$$\frac{1}{c^2}\partial_t^2 u - \frac{1}{a^2}\partial_n^2 u + \frac{A}{k\,a^2}\sin 2\pi u = 0 \tag{2.1}$$

where $c = a(k/m)^{1/2}$ is the sound velocity in the chain. Let $a = 1$, $A = 1$, $k = 1$. We shall designate continuous spatial variable through x. Then the kink (topological soliton) solution of the eq. (2.1) is

$$u(x,t) = \frac{2}{\pi}\operatorname{atan}\exp\frac{x - vt}{\gamma} \tag{2.2}$$

where $\gamma = (1 - v^2/c^2)^{1/2}$, v is the kink velocity. From formula (2.2) follows that the significant part of changing u occurs in the narrow region near the kink centre. The width of this region is

$$L = \left(1 - \frac{v^2}{c^2}\right)^{1/2}. \tag{2.3}$$

This is the width of the kink. It depends on the kink velocity according to the Lorentz law. However, the formula (2.3) contains the sound velocity instead of velocity of light.

The kinetic energy of the chain is

$$T = \frac{m}{2}\sum_n \left(\partial_t u_n\right)^2. \tag{2.4}$$

The potential energy of the chain is

$$U = \frac{k}{2}a^2\sum_n \left(u_{n+1} - u_n\right)^2 + \frac{Aa^2}{2\pi}\sum_n \left(1 - \cos 2\pi u_n\right) \tag{2.5}$$

The total energy of the kink is $E = T + U$. In continuous approximation the sums in (2.4-2.5) are replaced by integrals. Inserting the solution (2.2) in these integrals we obtain [1]

$$E = \frac{E_0}{\left(1-\frac{v^2}{c^2}\right)^{1/2}} \tag{2.6}$$

where E_0 is the kink rest energy. From formula (2.6) follows that if the kink velocity v is significantly less than the sound velocity c then its kinetic energy $T=Mv^2/2$ where M is the kink mass. It is connected to the kink rest energy E_0: $E_0 = Mc^2$.

The interactions of atoms in real solids are more complicated than the sinusoidal one. Their account results in breaking Lorentz-covariance of atomic equations of motion. Solutions of such equations are supersonic solitons. Toda [8] pioneered in analytical investigation of supersonic dynamical solitons in an one-dimensional lattice. He considered the chain of particles with exponential interaction (the Toda lattice). Let $\rho_n \equiv u_{n-1} - u_n$ is a reduction of distances between the neighboring particles of mass m, caused by their displacements u_n. The equations of motion of particles is

$$Mb\, \partial_t^2 \rho_n = k\, [\exp (b\, \rho_{n+1}) + \exp (b\, \rho_{n-1}) - 2 \exp (b\, \rho_n)].$$

The soliton solution of these equations is

$$\rho_n = \frac{1}{b} \ln\left\{1 + \frac{\mathrm{sh}^2(qa)}{\mathrm{ch}^2[q(na - vt)]}\right\} \tag{2.7}$$

where the soliton velocity

$$v = \frac{c}{qa}\,\mathrm{sh}\,(qa), \tag{2.8}$$

$c = a(k/M)^{1/2}$ is velocity of longitudinal sound waves in a harmonic chain, i.e. at $b = 0$. Parameter q characterises the reciprocal soliton width $q \approx 2\pi/L$ where L is the soliton width. From formula (2.8) follows that the acoustic solitons (2.7) are always supersonic ones. If $v \to c$ then the soliton amplitude tends to zero and its width goes to infinity. Thus, such a soliton can not overcome the sonic barrier in this chain. These solitons are born with supersonic velocities. It is interesting to note that the similar behaviour was predicted for tachyons [9].

If the soliton width L is much above the lattice parameter then we can use the continuous approximation. Then the solution (2.7) looks like $\rho(x,t) = \frac{1}{b}\left[\frac{\mathrm{sh}(qa)}{\mathrm{ch}(q\zeta)}\right]^2$, where $\zeta = x - vt$. The soliton energy is $E = T + U$ where the kinetic energy is

$$T = \frac{M}{2a}\int [\partial_t u(\zeta)]^2 d\zeta = \frac{2Mv^2\,\mathrm{sh}^4(qa)}{3qa^3 b^2},$$

the potential energy is

$$U = \frac{2k\,\mathrm{sh}^4(qa)}{3qab^2}\left[1 + \frac{4}{15}\mathrm{sh}^2(qa)\right].$$

If the velocity of the supersonic soliton goes to the sound velocity then its energy tends to zero. Thus, the properties of the Toda solitons in the supersonic region are opposite to the properties of the soliton solutions of Lorentz-covariant equations in the subsonic region where the energy of the soliton tends to infinity if its velocity goes to sound velocity and the soliton width tends to zero. In addition to the Toda soliton, other non-topological solitons, e.g., soliton solutions of the Boussinesq and Korteweg–de Vries equations, have the similar properties. The question arises as to whether there are the solitons, unifying these two extremes? It turns out that appropriate one-dimensional models exist. In order to construct such a model it is necessary to insert a non-linear chain, enabling dynamical

solitons, in an external periodic field (the substrate field). Then these solitons acquire the topological charge. In such a case particles at the chain ends are in minima of the substrate potential. If the interaction with the substrate is rather weak then the solitons can keep the 'generic' property of dynamical solitons that is supersonic motion.

3. Supersonic topological solitons

Kosevich and Kovalev [10] pioneered in investigation of supersonic topological solitons. They considered an anharmonic chain of particles in sinusoidal potential. In continuous approximation the equation of motion of the chain is

$$\partial_t^2 u - c^2\left[\partial_x^2 u + \frac{a^2}{12}\partial_x^4 u + \frac{\pi^2}{2}(\partial_x u)^2\partial_x^2 u\right] + \frac{2\pi}{am}U\sin\frac{2\pi u}{a} = 0 \tag{3.1}$$

where *m, c, a, U* are constants. The solution of this equation is a kink, having velocity *v*,

$$u = \frac{2a}{\pi}\operatorname{atan}\exp\left(-\frac{x-vt}{L_K}\right) \tag{3.2}$$

where the kink width L_K (curve 3 in fig. 1) is the solution of the biquadratic equation

$$48\pi^2 U L_K^4 + 12ma^2(v^2-c^2)\,L_K^2 - mc^2a^4 = 0 \tag{3.3}$$

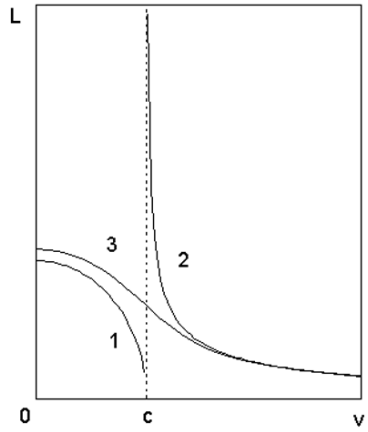

Fig. 1. The widths of various solitons *L* vs. their velocity *v*: 1 – for the sine-Gordon kink (the Lorentz law); 2 – for the supersonic dynamical solitons; 3 – for the Kosevich–Kovalev soliton.

If the interaction of the chain with the substrate is stronger than the interaction between the particles, i.e. U » mc^2, then the last item in the left side of eq. (3.3) can be neglected. In such a case the soliton width–velocity relationship has the Lorentz form (curve 1 on fig. 1)

$$L_L = \frac{ac}{2\pi}\sqrt{\frac{m}{U}\left(1-\frac{v^2}{c^2}\right)}.$$

The second extreme case is a free chain without a substrate (*U*=0). Then the soliton width–velocity relationship has the form which is typical of supersonic dynamical solitons (curve 2 on fig. 1)

$$L_D = \frac{a}{2\sqrt{3\left(\frac{v^2}{c^2}-1\right)}}.$$

Thus, the function $L_K(v)$ for the Kosevich–Kovalev topological soliton is 'matching' two solutions: the Lorentz function $L_L(v)$ for subsonic topological solitons and the function $L_D(v)$ for supersonic

dynamical solitons. Soliton (3.2) has a continuous spectrum of velocities in the range from zero to infinity. The sound velocity is not the singular point for this soliton.

We would like to note that there is not tachyonic theory which would admit the similar 'desingularization' near the velocity of light. The question arises as to whether there are the analogues of soliton (3.2) in the non-linear field theories? If the velocity of such a soliton is small then it should behave like a usual particle following Lorentz laws. However if its velocity will verge towards the velocity of light then the deviation from the Lorentz behaviour should increase. At last the soliton should get the superluminal velocity. During the passage soliton velocity through the velocity of light any soliton characteristics should not approach zero or infinity.

Kosevich and Kovalev [10] also have considered the similar model using another potential of particles' interaction. Potential of interaction with the substrate was chosen in the polynomial form. Then the equation of motion of the chain is

$$\partial_t^2 u - c^2\left[\partial_x^2 u + \frac{a^2}{12}\partial_x^4 u - 3\beta\,\partial_x u\,\partial_x^2 u\right] + \frac{2U}{m\,a^4}u(a-u)(a-2u) = 0. \qquad (3.4)$$

This equation describes, in particular, dynamics of a bistable molecular chain. Eq. (3.4) has the soliton solution

$$u = \frac{a}{1+\exp\dfrac{3\beta(x-vt)}{a}} \qquad (3.5)$$

This soliton can move at the single velocity

$$v = c\sqrt{1+\frac{3\beta^2}{4}-\frac{2U}{9mc^2\beta^2}}$$

If $\beta^4>8U/(27mc^2)$ then this velocity is supersonic. The discrete spectrum of velocities is the general property of many supersonic topological solitons.

Later this model was investigated by Savin [11] who modified the substrate potential. Then eq. (3.4) has changed into the following

$$\left(1-v^2\right)\partial_\zeta^2 u + \frac{1}{12}\partial_\zeta^4 u - 3\beta\,\partial_\zeta u\,\partial_\zeta^2 u - 4Gu\left(u^2-1\right) = 0 \qquad (3.6)$$

where $\zeta = x - vt$, $a = 1$, $c = 1$, $m=1$. If the substrate is absent ($G = 0$) then eq. (3.6) evolves into the Boussinesq equation after an integration. Its solution is a supersonic dynamical soliton

$$\varphi = \frac{1-v^2}{\beta}\operatorname{sech}^2(q\zeta)$$

where v is the soliton velocity, $\varphi\equiv\partial_\zeta u$. In the presence of the substrate this soliton has the topological charge

$$Q(v) = u(+\infty)-u(-\infty) = \int_{-\infty}^{+\infty}\varphi(\zeta)d\zeta = -\left[\frac{4\left(v^2-1\right)}{3\beta^2}\right]^{1/2} = -1. \qquad (3.7)$$

If the chain contains N identical dynamical solitons then eq. (3.7) looks like $NQ(v_N)= -1$. Then the velocity of each soliton

$$v_N = \left[1+\frac{3}{4}\left(\frac{\beta}{N}\right)^2\right]^{1/2}.$$

Thus, at the limit $G \to 0$ the topological solitons of this model have limited discrete supersonic spectrum of velocities. The sound velocity is the accumulation point of this spectrum. Certainly, the supersonic kink can move at other velocities which are absent in the discrete spectrum. The numerical

modeling has shown [11] that in such a case the kink radiates sound waves until it reaches the nearest velocity from the discrete spectrum. Then the radiation ceases.

It should be realized that the supersonic solitons differ essentially from the superluminal electromagnetic solitons [12] which propagate in non-equilibrium media only and do not transfer information. Unlike them, the supersonic topological solitons propagate in equilibrium systems and transfer information. The study of these solitons may be useful for the solution of the tachyonic problem.

It is widely believed that it is impossible to observe the tachyons in the real world because their existence should result in paradoxes: breaking the causality principle, imaginary energy of particles etc. The considered analogy allows to make some hypotheses for the tachyonic properties. Really, the theory of solitons in solids is similar to STR in the subsonic region. Any relativistic effect has an analogue in the theory of solitons. This statement is true both for one-dimensional systems and for three-dimensional ones. However in the theory of solitons in solids we can repeat the usual arguments against an existence of tachyons. Really, the absolute value of parameter c in the Lorentz root is not important for these arguments: c can be equal to the velocity of light or the sound velocity. These arguments result in the conclusion that the supersonic solitons can not exist. However the considered examples of such solitons contradict this conclusion. Analytical studies and numerical modeling of supersonic solitons show that the supersonic movement do not lead to appearance of imaginary soliton energy and breaking the causality principle. The explanation of this contradiction is simple. Near the sound velocity and in the supersonic region the soliton equations are not Lorentz-covariant. Hence, they can have the supersonic solutions. The standard anti-tachyonic arguments are unfounded in such a case as they are based on using the Lorentz transformations in the supersonic region where these transformations are inapplicable.

4. Relativistic effects in the dislocation dynamics

Let us consider a straight screw dislocation in a three-dimensional crystal. Dislocation is parallel to an axis z and move at velocity v along an axis x (fig. 2).

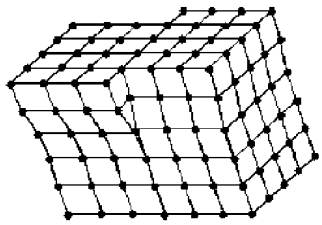

Fig. 2. The screw dislocation in a three-dimensional crystal.

Let us use continuous approximation and consider isotropic and linear elastic medium. Then the displacements of medium points u_3 around of the dislocation are the solutions of the equation

$$\mu\left(\partial_1^2 + \partial_2^2\right)u_3 = \rho\,\partial_t^2\,u_3 \tag{4.1}$$

on condition that

$$\oint_L du_3 = b.$$

Here μ is the shear modulus, ρ is the density of the medium, b is magnitude of the Burgers vector. The dislocation solution of eq. (4.1) is [7]

$$u_3 = \frac{b}{2\pi}\,\mathrm{atan}\,\frac{\gamma y}{x - vt} \tag{4.2}$$

where $\gamma=(1 - v^2/c_t^2)^{1/2}$, $c_t = (\mu/\rho)^{1/2}$. The point of origin is situated on the dislocation. The displacements u_1 and u_2 are equal to zero.

Let us use the cylindrical co-ordinates. The single non-zero component of mechanical stress tensor, created by the dislocation, is

$$\sigma_{\varphi z}(r,\varphi)=\frac{\mu b\gamma}{2\pi r\left(\cos^2\varphi+\gamma^2\sin^2\varphi\right)}$$

where $r^2 = (x - vt)^2 + y^2$, an angle φ is counted off an axis x. The electric intensity of an infinite straight charged rod, parallel to an axis z and moving at velocity v along an axis x, has the same dependence on co-ordinates and velocity.

The kinetic energy of the screw dislocation

$$T=\frac{\rho}{2}\int\left(\partial_t u_3\right)^2 dV=\frac{E_0 v^2}{2\gamma\, c_t^2}$$

where the rest energy of the dislocation

$$E_0=\frac{\mu b^2}{4\pi}\ln\frac{R}{r_0},$$

R and r_0 are limits of integration over r. As a rule R is equal to the distance apart the dislocation and crystal surface, r_0 is the lattice parameter. Potential energy of the screw dislocation

$$U=\frac{1}{2}c_{idfh}\int\partial_d u_i\,\partial_h u_f\,dV=\frac{E_0}{\gamma}\left(1-\frac{v^2}{2c_t^2}\right)$$

where c_{idfh} is the tensor of elastic modules. The total energy of the screw dislocation

$$E=T+U=\frac{E_0}{\left(1-\frac{v^2}{c_t^2}\right)^{1/2}}.$$

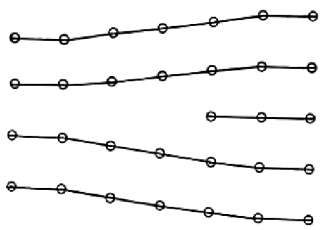

Fig. 3. An edge dislocation in a two-dimensional crystal.

Let us consider an edge dislocation in a two-dimensional crystal (fig. 3). Let us use continuous approximation and consider isotropic and linear elastic medium. Then the displacements of medium points around of the edge dislocation, having the Burgers vector $\boldsymbol{b} = (b, 0)$ and moving at the velocity $\boldsymbol{v} = (v, 0)$, are the solutions of the system of equations [7]

$$\mu\,\partial_j\partial_j u_i+(\mu+\lambda)\partial_i\partial_j u_j=\rho\,\partial_t^2 u_i \tag{4.3}$$

on condition that

$$\oint_L du_i = b.$$

Here λ is a Lame constant. There are longitudinal and transverse sound waves in the 2D crystal. Their speeds never coincide. Therefore the formulas of dislocation dynamics are not Lorentz-covariant. The dislocation solution of eq. (4.3) is [13]

$$u_1(x,y,t)=\frac{bc_t^2}{\pi v^2}\left[\operatorname{atan}\frac{y\left(1-\frac{v^2}{c_l^2}\right)^{1/2}}{x-vt}+\left(\frac{v^2}{2c_t^2}-1\right)\operatorname{atan}\frac{y\left(1-\frac{v^2}{c_t^2}\right)^{1/2}}{x-vt}\right],$$

$$u_2(x,y,t)=\frac{bc_t^2}{2\pi v^2}\left\{\frac{\frac{v^2}{2c_t^2}-1}{\left(1-\frac{v^2}{c_t^2}\right)^{1/2}}\ln\left[(x-vt)^2+\left(1-\frac{v^2}{c_t^2}\right)y^2\right]+\left(1-\frac{v^2}{c_l^2}\right)^{1/2}\ln\left[(x-vt)^2+\left(1-\frac{v^2}{c_l^2}\right)y^2\right]\right\} \quad (4.4)$$

where $c_l = [(\lambda+2\mu)/\rho]^{1/2}$ is the speed of longitudinal sound waves. The point of origin is situated on the dislocation. The mass M and energy E_0 of an edge dislocation are related by the formula [13]

$$E_0=\frac{Mc_t^2}{1+\frac{c_t^4}{c_l^4}}.$$

Thus, in the 2D soliton theory Lorentz transformations are not the universal tool for calculation of the soliton characteristics. The most physicists suppose that it is impossible to construct the relativistic non-Lorentz-invariant theory. But such a theory already exists and has experimental confirmations. It is the dynamical theory of dislocations. There are analogues of any effect of STR within the framework of this dislocation theory. Thus, we can use the term 'relativistic' for this theory. However the dynamical theory of dislocations transforms into the Lorentz-invariant one at the limit $c_l \to \infty$.

All these results are correct for a straight edge dislocation in the 3D crystal. Its Burgers vector is normal to a dislocation line. The Burgers vector of a screw dislocation is parallel to this line. In the general case the angle this vector makes with the dislocation is not equal neither 0 nor 90^0. The dislocation can be edge and screw at different points of its line. The theory of screw dislocations is Lorentz-invariant and the theory of edge dislocations is not Lorentz-invariant. Hence, there is the continuous transition from Lorentz-covariant formulas to non-Lorentz-covariant ones. This transition follows from the general dislocation theory. So, formulas (4.2, 4.4) can be obtained from the Mura formulas [14] for the dislocation distortions. These relations look much simpler if we use 4D co-ordinates [3]

$$\beta_{jn}(x_f)\equiv\partial_n u_j(x_f)=\frac{C_{iabd}}{c_t^2}e_{nhag}\int_\Omega J^h{}_i{}^g(x'_f)\partial_d G_{bj}(x_f-x'_f)d\Omega' \quad (4.5)$$

where $b, i, j = 1, 2, 3$; $a, d, f, g, h, n = 0, 1, 2, 3$; $c_t = (c^{1212}/\rho)^{1/2}$, $\partial_0 \equiv c_t^{-1}\partial_t$,

$$C_{iabd}=\begin{cases} c_{iabd} & \text{if } a=1,2,3;\\ -\delta^{bi}\delta^{ad}\rho c_t^2 & \text{if } a=0,\end{cases}$$

c^{iabd} is the 3D tensor of elastic modules, δ^{bi} is a Kroneker symbol, e_{nhag} is an antisymmetric Levi-Civita tensor, $e^{0123} = 1$,

$$J^h{}_i{}^g(x'_f)=\tau^h b_i V^g\delta(x'_f-x_f^0)$$

is the tensor of dislocation flux density, τ^h is a vector tangent to the dislocation line, $V^g = (c_t, -\boldsymbol{V})$ is a 4D dislocation velocity, $\boldsymbol{V}$ is a 3D vector of dislocation velocity, $\delta(x'_f - x^0_f)$ is the delta function, x^0_f are

co-ordinates of the dislocation line, $d\Omega' = dV'd(c_t t')$ is a 4D differential of spatio-temporal volume, G_{bj} is the tensorial Green function of the equations of the classical linear theory of elasticity.

The listed relativistic effects are consequences of signal delay in soliton motion. It appears from this that the physical quantities, connected to the signal delay in soliton motion: dislocation fields of elastic deformations and mechanical stresses, soliton energy etc., depend only on the soliton velocity through Lorentz (or another relativistic) law. All other quantities, which are not connected to the signal delay (for example, lattice parameters), do not depend on the soliton velocity.

The examined analogy between STR and soliton dynamics allows us to revise the conventional opinions about an origin of relativistic effects. Contrary to prevailing view the analogues of all effects of STR can be obtained within the framework of the classical Newtonian mechanics. Relativistic effects in soliton theory are much more complicated than their electrodynamical analogues. The question arises as to whether it is possible to continue analogy between STR and soliton theory in the superluminal region. This issue remains open up till now. However this analogy allows us to advance a hypothesis that there are superluminal solitons within the framework of the nonlinear field theories.

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References

[1] G.L. Lamb, Jr., *Elements of Soliton Theory* (J. Wiley & Sons, New York, 1980).
[2] A.M. Kosevich, *Usp. Phys. Nauk*, **84**, 579-609 (1964) [*Soviet Physics–Uspekhi*, **7**, 837-854 (1964)].
[3] A.I. Musienko and V.A. Koptsik, *Kristallografija*, **40**, 438-445 (1995) [*Crystallography Reports*, **40**, 398-405 (1995)].
[4] A.I. Musienko, *J. Nonlin. Math. Phys.*, **8**, Suppl., 219-224 (2001) (also as electronic preprint on the web at http://xxx.lanl.gov/abs/hep-th/0006212).
[5] L.D. Faddeev and L A. Takhtajan, *Hamiltonian Methods in the Theory of Solitons* (Springer, Berlin, 1987).
[6] V.I. Sanyuk, in *Physical encyclopedia*, vol. 5, edited by A.M. Prokhorov (Bolshaya Rossijskaya Encyclopedia, Moscow, 1998), pp. 134-142 (in Russian).
[7] J.P. Hirth, J. Lothe, *Theory of Dislocations* (McGraw-Hill, New York, 1968).
[8] M. Toda, *Theory of Nonlinear Lattices* (Springer, Berlin, 1981).
[9] E. Recami, *Rivista Nuovo Cimento*, **9**, 1-178 (1986).
[10] A.M. Kosevich and A.S. Kovalev, *Solid State Commun.*, **12**, 763-765 (1973).
[11] A.V. Savin, *Zh. Eksp. Teor. Fiz.*, **108**, 1105-1116 (1995) [*JETP*, **81**, 608-614 (1995)].
[12] S.V. Sazonov, *Usp. Phys. Nauk*, **171**, 663-677 (2001) [*Physics–Uspekhi*, **44**, 631-644 (2001)].
[13] J. Weertman and J.R. Weertman, in *Dislocations in Solids*, vol. 3, edited by F.R.N. Nabarro (North-Holland, Amsterdam, 1980), pp. 1-59.
[14] T. Mura, *Micromechanics of Defects in Solids* (Martinus Nijhoff, Dordrecht, 1987).

Some problems of description of space-time structure in microworld[1]

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The different approaches to the description of quantum space-time structure are discussed. In general this problem consists of two parts: description of quantum space-time itself and its projection on classical space-time. The known interactions have the same properties, which define topology on the arbitrary set. It is pointed out that discrete topology corresponds to the absence of any interaction. It is not contradicts to the possibility of the fundamental length existence, which may said about non archimedean nature of quantum physics. Some problems of description of possible quantum fluctuation of space-time topology are discussed also.

Обсуждаются различные подходы к описанию структуры пространства-времени в микромире. В общем случае эта проблема состоит из двух частей: (а) описание структуры собственно пространства-времени микромира и (б) описание его проекции на классическое пространство-время, в терминах которого описываются результаты экспериментов и строятся современные модели квантовой теории поля. Одними из наиболее популярных гипотез, обсуждаемых в связи с рассматриваемой проблемой, являются гипотеза о существовании элементарной длины, гипотеза о дискретной структуре пространства-времени в микромире и гипотеза о флуктуациях топологической структуры пространства или пространства-времени в микромире (гипотеза пространственной или пространственно-временной пены). На основании (1) аналогии между определениями топологической структуры на произвольном множестве и свойствами известных взаимодействий и (2) ставшим уже стандартным предположении о том, что структура пространства-времени должна определяться наполняющими его материей и полями, сделан вывод о том, что дискретная структура соответствует отсутствию каких-либо взаимодействий. Отмечено, что это заключение не противоречит предположениям о возможном существовании фундаментальной длины, которое, в случае его справедливости, может, в частности, свидетельствовать либо о неархимедовости метрической структуры пространства-времени микромира, а возможно, и всей квантовой физики, либо о том, что наблюдаемая локально-евклидова топологическая структура классического пространства-времени является асимптотической. В заключении рассмотрены некоторые проблемы описания возможных квантовых топологических флуктуаций (пространственно-временной пены).

1. Introduction

The description of space-time structure on quantum level is unresolved problem which attracts attention of many authors. The interest to this problem is connected with the problems of quantum physics and unified theories. The especially considerable interest to the problem of quantum space-time is observed during last years: in electronic lanl-archive, for instance, it may be found several hundreds of preprints on this subject. The concepts of discrete structure of space

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or space-time [1] and the space-time foam [2, 3] (quantum fluctuations of space-time topology) are among the most popular hypothesis which are discussed in connection with this subject. The concepts of discrete structure of space-time are based usually on the hypothesis of the fundamental length l_{fund} existence. On the other hand, the last supposition is connected with (a) the possibility to obtain, using the fundamental physical constants, the constant with dimension of length (the so-called Plank length $l_{Plank} = 10^{-33}$ cm), (b) Heisenberg uncertainty correlation and (c) some other physical reasons [1]. The space-time is considered in the framework of this hypothesis as a mesh like is used in numerical methods of partial differential equations solving or in Regge calculus [4]. Some other structures are also discussed. The characteristic feature of all existing approaches to discrete models of space time is supposition what the set of events is finite or countable.

In the following section we try to demonstrate that suppositions about discreteness of space-time, existence of fundamental length and finiteness or countability of the event space are independent. We try to argue also, that discrete structure corresponds to the absence of interactions.

In the third section we discussed some problems of description of topological fluctuation (space or space-time foam). The space-time foam hypothesis or hypothesis about possible topological fluctuations (topological changes) of space (or space-time) on very small scales was firstly introduced by Wheeler in its geometrodynamics [3]. This hypothesis includes also suppositions about influence of small-scale topological fluctuations on the macroscopic picture of particle and fields configurations, including the models of mass without mass, charge without charge and so on [3]. For simplicity we shall discuss space-time foam hypothesis in its initial form, which supposes that during local topological transformations space-time remains locally smooth. We consider the main problems of topological changes description, the connection between the topology change and the fields configurations and briefly discuss some difficulties in realization of Wheeler's hypothesis about mass without mass, charge without charge and so on [3].

Some concluding remarks are presented in the final section.

2. Discrete space-time and fundamental length

We begin our discussion from the consideration of the of the term "discrete space-time".

The concepts of discrete space-time are usually associated with the supposition that the set of its points is finite or countable. However the "number"of points is characteristic of its cardinality, while the discreteness is characteristic of topology, which is introduced on this set.

There are three equivalent definitions of topology and topological space in the modern mathematics.

Let X is arbitrary set. The following definition of topology on X is the most known and may be found in any textbook on topology, differential geometry and mathematical foundation of general relativity (see for instance [5–7]):

Definition 1: Let $\tau = U : \ U \subset X$ is some set of subsets of X. The set τ is called *topology* on the set X and the pair (X, τ) is called topological space is the following properties are satisfied:

1 Both $\emptyset \subset \tau$ and $X \subset \tau$, where $\emptyset$ denotes the empty set;

2 the union of arbitrary countable subset of elements of τ is belong to τ;

3 the intersection of arbitrary finite subset of elements of τ is belong to τ.

The subsets $U \subset X : U \in \tau$ are called *open* subsets by definition. For any open set $U \in \tau$ the adjoint subset $F = X \setminus U$ is called *closed*. The empty set $\emptyset$ and the set X are open and closed simultaneously. The topology τ which is defined by above definition is called an *open topology* on the set X.

The next definition, which introduces the so-called closed topology on the set X, is reciprocal to previous one:

Definition 2: Let $\sigma = F : \ F \subset X$ is some set of subsets of X. The set σ is called *topology* on the set X and the pair (X, σ) is called topological space is the following properties are satisfied:

1 Both $\emptyset \subset \sigma$ and $X \subset \sigma$, where $\emptyset$ is denoted the empty set;

2 the union of arbitrary finite subset of elements of σ is belong to σ;

3 the intersection of arbitrary countable subset of elements of σ is belong to τ.

The subsets $F \subset X : \ F \in \sigma$ are called *closed* subsets of X.

The third definition is less known [8]:

Definition 3: The set X is called a topological space if there is a binary relation δ on X, such that for arbitrary two point $x \in X$ and $y \in X$ and for arbitrary two subsets $A \subset X$ and $B \subset X$ the following conditions are satisfied:

1 $x \neg \delta \emptyset$ for $\forall x \in X$;

2 $x \delta x$ for $\forall x \in X$;

3 if $x \delta A$ and $A \subset B \subset X$ then $x \delta B$;

4 if $x \delta A \bigcup B$ then $x \delta A$ or $x \delta B$;

5 if $\forall y \in B \ y \delta A$ and $x \delta B$ then $x \delta A$.

The subset $A \subset X$ is called closed if $\forall x \delta A \Longrightarrow x \in A$ and subset $U \subset X$ is called open if its adjoint is closed.

These definitions are equal in the sense that the structure is topology in the sense of one definition is a topology in the sense of other two definitions also.

We shall need also in the following two definitions [5].

Definition 4: Let τ is some topology on X. The topological space (X, τ) is called Hausdorff space if for arbitrary two points $x \in X$ and $y \in X$ there are two open non intersecting subsets $U_x \subset X$ and $V_y \subset X$ $(U_x \bigcap V_y = \emptyset)$ such that $x \in U$ and $y \in V$.

Definition 5: Let τ is some topology on X. The subset $\tau_b \subset \tau$ is called the base of topology τ if $\forall U \in \tau$ there are exists such subsets $V_i \subset X$, $V_i \in \tau_b$ that $U = \bigcup\limits_i V_i$.

It is easy to verify that following bases introduce (equivalent) topologies on the real line $\mathbf{R}$ in the sense of above definitions:

1 The set of open intervals (a, b), where $a < b$ and $a \in \mathbf{R}, : \mathbf{b} \in \mathbf{R}$;

2 The set of closed intervals $[a, b]$, where $a < b$ and $a \in \mathbf{R}, : \mathbf{b} \in \mathbf{R}$;

3 The set of subsets of the form $x \leq a$ $(x \geq a)$ where $x \in \mathbf{R}$ and $a \in \mathbf{R}$.

The above definitions admit different topologies on the same set X. Among them are *trivial topology* which contains only two open sets: the empty set $\emptyset$ and the set X itself: $\tau = (\emptyset, : X)$ and *discrete topology* for which the set τ besides of the empty set $\emptyset$ and the set X itself consists of all subsets of the set X. The base of the discrete topology on X is the set X itself.

It is clear that discrete topology may be introduced on the set of arbitrary cordiality including both finite, countable and non countable sets. In particular the discrete topology may be introduced on continual set as well as on the finite or countable sets. It must be noted also that the discrete topology is the only Hausdorff topology which may be introduced on the finite or countable set.

Now, let us discuss the connection between topology and metric and between topology and the properties of interactions.

First, as in may be seen from above definitions, the topological structure on a set has no direct connection with some metric on the same set: introduction of metric is a useful but not the only tool for introduction topology. The metric on some set may be both consistent and inconsistent with topology on it. Hence there is no direct connection between topology of space-time and possibility of fundamental length existence. The possible consequences of fundamental length existence will be discussed later.

Second, any interaction defines for any point its nontrivial (non empty) neighborhood or nontrivial region of influence. By another words, any interaction between points of some set X defines on this set (or on some set M which may be called as a set of positions of elements of X) some topology, whose base does not reduce to the set X (or to the set M). Hence, the topology which consists with some interaction cannot be reduced to the discrete topology. In other words, the discrete topology corresponds to the absence of any interactions. It is clear that space-time without interactions has no physical meaning.

Although there is no direct connection between topological and metric structures on the same set, the illusory contradiction between above conclusion and the numerous suppositions about fundamental length existence requires some explanations or comments. It is possible to indicate several possibilities. First, it must be pointed out that the field of rational numbers has non unique closure: it has the only Archimedean closure, which is the field of real numbers, and infinite set of non Archimedean numbers (see, for example, [9]). Therefore the possible existence of fundamental length may indicate non Archimedean nature of quantum space-time. Second, because the results of experiments are usually described in the terms of classical space-time, the possible existence of fundamental length may corresponds not to the quantum space-time, but to the projection of quantum space-time on the classical one. Of cause, the last possibility means also that classical and quantum space-times have nonequivalent structures. At last, in the framework of usual consideration of classical theories as a limiting case at $\hbar \to 0$ of corresponding quantum theory, the classical space-time may be considered as some averaging and its topology may be considered in asymptotic sense also (about asymptotic topology see, for example [10]). In this case fundamental length (if it exists) may define the scale of averaging and on the smaller scales the topology of space of events may differ from classical one, in particular, quantum space-time may have the same or higher cardinality than classical one.

Because of the absence of some experimental or theoretical foundations it seems impossible to discussed these or other possibilities in more details or formulate some final theory or hypothesis on this subject.

3. Space-time foam or topological fluctuations hypothesis

As it is follows from previous section, the changes of space or space-time topology may include, in general case, *(i)* the change of initial set of points, i.e. its cardinality, and/or *(ii)* specification (or changing if topology changes on the same set) of the subsets, which define the base of topology.

For simplicity we shall consider this problem in the simplest case of smooth manifolds. It is known, what to describe the manifold structure the following data must be given: *(i)* the finite or countable set of coordinate maps and the order of their junction; (*ii*) the set of functions which connect the coordinate systems of different maps in their intersections [6, 11]. Conditions *(i)* define the topology of manifold and the conditions *(ii)* define its smooth structure. These data must be given before the solution of any equations. They may be considered as constraints or as a part of boundary conditions which are given "by hand" because they are not follow from some fundamental principles of current physics. To see this, consider the action integral of some field theory (both classical and quantum in its path integral form) in four-dimensional space-time in the following general form

$$S = \int\limits_{\mathbf{M}} L(\Phi_A, \Phi_{A,\alpha})d^4\sigma, \tag{1}$$

where $L(\Phi_A, \Phi_{A,\alpha})$ is the Lagrangian which depends on the fields potentials Φ_A and their derivatives, A is the cumulative index and $d^4\sigma$ is the invariant volume element which in the local coordinates $\{x^\alpha, \alpha = 0, ..., 3\}$ has the standard form

$$d^4\sigma = \sqrt{-g}d^4x = \sqrt{-g}dx^0 \wedge ... \wedge dx^3 \tag{2}$$

where "$\wedge$" is the exterior product of differential forms, $g = \det \|g_{\alpha\beta}\|$ and $g_{\alpha\beta}$ is the metric tensor of Lorentzian signature $diag(+, -, -, -)$ on $\mathbf{M}$.

In the equality (1) the integration is carried out over the full manifold $\mathbf{M}$, so both action S and corresponding Feynman amplitude $\exp\{iS/h\}$ are the functionals of both field variables Φ_A and the manifold itself.

To formalize the action dependence upon the manifold structure, let us consider some atlas $\mathbf{U} = \{\mathbf{V}_k\}$ of $\mathbf{M}$, i.e. a finite or countable covering of $\mathbf{M}$ by coordinate maps $\mathbf{V}_k$, such that every $\mathbf{V}_k$ and every intersection $\mathbf{V}_{k_1} \cap \mathbf{V}_{k_2}$ for all k, k_1 and k_2 are diffeomorphic to the unit cube $\mathbf{D}^4$ of the Euclidean space $\mathbf{R}^4 : \mathbf{M} = \bigcup\limits_{k\in\mathbf{P}} \mathbf{V}_k$, where $\mathbf{V}_k \sim \mathbf{D}^4$ and $\mathbf{P} \subset \mathbf{N}$ is a subset of the set $\mathbf{N}$ of natural numbers, which numerate the elements of the covering $\mathbf{U}$. Let $\{x^\alpha_k\}$ are the local coordinates in the region $\mathbf{V}_k$ and $\left\{x^\alpha_{i_0...i_l}\right\}$ are some local coordinates in the intersection $\mathbf{V}_{i_0} \cap ... \cap \mathbf{V}_{i_l}$ (which is also diffeomorphic to $\mathbf{D}^4$). In any intersection $\mathbf{V}_{i_0} \cap ... \cap \mathbf{V}_{i_l}$ the field potentials Φ_A must satisfy to the natural consistency conditions which may be considered as an additional constraints.

In the atlas $\mathbf{U}$ the integral (1) may be rewritten in the following form [12]

$$S = \sum_{k\in P} \int\limits_{V_k} L(\Phi_A(x_k), \Phi_{A,\alpha}(x_k))\sqrt{-g}d^4x_k -$$
$$\sum_{k<l} \int\limits_{V_k\cap V_l} L(\Phi_A(x_{kl}), \Phi_{A,\alpha}(x_{kl}))\sqrt{-g}d^4x_{kl} + ...$$
$$+(-1)^K \sum_{i_0<...<i_K} \int\limits_{V_{i_0}\cap...\cap V_{i_K}} L(\Phi_A(x_{i_0...i_K}), \Phi_{A,\alpha}(x_{i_0...i_K}))\sqrt{-g}d^4x_{i_0...i_K} \tag{3}$$

where $K < \infty$ because of standard supposition about paracompactness of space-time manifold $\mathbf{M}$ [7].

For the following formalization of the action integral (1) consider the set $\mathbf{\Lambda_U}$ of subsets of the set $\mathbf{P}$ such that $(i_0, ..., i_k) \in \mathbf{\Lambda_U}$ if and only if $\mathbf{V}_{i_0} \cap ... \cap \mathbf{V}_{i_k} \neq \emptyset$, where $\emptyset$ denotes the empty set. The set $\mathbf{\Lambda_U}$ which satisfies to such condition is called a nerve of the covering $\mathbf{U}$ [13] and its elements of type $I_k = (i_0, ..., i_k)$ are known as k-dimensional simplexes [13]. The zero-dimensional simplexes, i.e. elements of the type $I_0 = i_0 \in \mathbf{P}$, are vertexes. It is follows from definition that if $I_k \in \mathbf{\Lambda_U}$ and $J_l \in I_k$, where $l < k$, then $J_l \in \mathbf{\Lambda_U}$. This property shows that the nerve $\mathbf{\Lambda_U}$ of the covering $\mathbf{U}$ is a particular case of the abstract simplicial complex [13]. Such constructions are widely used in the algebraic topology, in particular, in Cech cohomology theory whose connections with topological quantization was discussed, for instance, in [14] and recently in [15].

With the nerve $\mathbf{\Lambda_U}$ of the covering $\mathbf{U}$ we may associate the system of its characteristic functions which will be denoted as $\mathbf{F_\Lambda} = \left\{ f^0_{\mathbf{\Lambda}}, ..., f^K_{\mathbf{\Lambda}} \right\}$ [12,16] where the functions $f^l_{\mathbf{\Lambda}} = f^l_{\mathbf{\Lambda}}(I_l) \in \mathbf{F_\Lambda}$, $0 \leq l \leq K$, are defined on the set of natural numbers as follows

$$f^l_{\mathbf{\Lambda}} = \sum_{i_0 < ... < i_l} a^{I^l}_{\mathbf{\Lambda}} f^0_{i_0} \wedge ... \wedge f^0_{i_l} \tag{4}$$

where $i_m \in \mathbf{N}$, $\mathbf{N}$ is the set of natural numbers,

$$f^0_i = f^0_i(j) = \delta_{ij} \tag{5}$$

and

$$a^{I_l}_{\mathbf{\Lambda}} = 1, \quad \text{if} \quad I_l \in \mathbf{\Lambda}_U, \quad \text{and} \quad a^{I_l}_{\mathbf{\Lambda}} = 0, \quad \text{if} \quad I_l \notin \mathbf{\Lambda_U} \tag{6}$$

It is follows from definitions that the correspondence of the system $\mathbf{F_\Lambda}$ of characteristic functions of the nerve $\mathbf{\Lambda_U}$ with atlas $\mathbf{U}$ is one-to one and hence $\mathbf{F_\Lambda}$ defines the topology (but not the smooth structure) of the manifold $\mathbf{M}$ as well as its atlas $\mathbf{U}$.

Using the definitions (4)-(6) we may rewrite equality (3) as follows [12,16]

$$S = \sum_{k=0}^{K} (-1)^k \sum_{i_0 < ... < i_k} S^{I_k}_k f^k_{\mathbf{\Lambda}}(I_k) \tag{7}$$

where

$$S^{I_k}_k = \int\limits_{\mathbf{V}_{i_0} \cap ... \cap \mathbf{V}_{i_k}} L(\Phi_A(x_{i_0...i_k}), \Phi_{A,\alpha}(x_{i_0...i_k}))\sqrt{-g} d^4 x_{i_0...i_k} \tag{8}$$

Equalities (7), (8) together with definitions (5)-(6) formalize the dependence of the action integral upon the topology of space-time. They make possible to do several observations.

First, equalities (7) and (8) formalize the above statement that the topology of manifold play the role of the additional constraint because the action (7) contains the system $\mathbf{F_\Lambda}$ by linear manner without any signs of $\mathbf{F_\Lambda}$ changes. It is necessary to note, that the representation (7)-(8) of the action functional (1) does not contain any information about smooth structure of the manifold (in particular, the connection between coordinates $\{x^i\}$ and $\{x^j\}$ in the intersection $\mathbf{V}_i \cap \mathbf{V}_j$). For briefness, we do not consider here the modification of the representation (7)-(8) which make possible to serve the information about manifold smoothness.

Second, the topology of arbitrary manifold may be coded by the system $\mathbf{F_\Lambda}$. This system may be done finite for the compact manifolds. Some another methods of the manifold structure coding are described in [17], but they are less suitable for our purpose. It is known, that independently

of the method of the manifold structure coding in three or more dimensions, the set of codes which describe all manifolds of the given dimensionality is infinite with infinite subset of codes which define the given manifold. Moreover, if dimension of manifold is three or more then there is no so simple classification of manifold structures as in two dimensions [17].

Third, any changes in the topology of manifold $\mathbf{M}$ may be represented as corresponding changes of the system $\mathbf{F}_{\mathbf{\Lambda}}$. Really, to change the topology of $\mathbf{M}$ it is necessary to change its atlas $\mathbf{U}$, i.e. the order in which the coordinate maps $\mathbf{V}_i$ are joined with each other and their number. The change of atlas $\mathbf{U}$ induce the change of its nerve $\mathbf{\Lambda}_{\mathbf{U}}$ and hence the system $\mathbf{F}_{\mathbf{\Lambda}}$, because the correspondences $\mathbf{U} \leftrightarrow \mathbf{\Lambda}_{\mathbf{U}} \leftrightarrow \mathbf{F}_{\mathbf{\Lambda}}$ are one-to-one. However the representation of the action functional S in the form (7) does not contain any sign of the $F_{\mathbf{\Lambda}}$ changes. Moreover, such representation does not contains any information about joining conditions in the intersections of coordinate maps. So, any changes of space or space-time topology may be done in this scheme only "by hands" and does not follow from the general formalism. Therefore the standard methods of the current field theory (both classical and quantum in its path-integral form), which are based on the usage of the action functional S in the form (1) or (7), does not permit to describe the dynamical change of the topology of space-time. To make possible such description it is necessary to use functionals which contain not only $\mathbf{F}_{\mathbf{\Lambda}}$ but also some objects that may be called as "discrete derivatives" of $\mathbf{F}_{\mathbf{\Lambda}}$ (as an example of such objects may be considered the operators $\rho^{\pm}_{I_k}$, which were introduced in [12, 16] and may be interpreted as the operators of creation and annihilation of the simplex I_k). The introduction of such objects is equivalent to introduction of some non local (topological) interaction which has no analogies in the current field theory. Therefore it is almost hopeless to solve this problem directly. Nevertheless, it is possible to investigate some possibilities in the construction of the consecutive topology change theory and its main features in the scope of the standard theories.

In the above the general 4-dimensional form of the action integral was considered while in the context of the topology change description the usage of some (3+1)-decomposition would be more suitable. It is easy to see that such decomposition does not change result: (i) both in general four-dimensional form and in the parameterized (3+1) form the topological variables are the discrete valued functions, and (ii) the action functional S contains the topological variables only as parameters (or constraints).

In the context of (3+1)-decomposition of space-time some restricted class of topological changes may be described in the framework of so-called Lorentz cobordism [18] or usual Morse cobordism theory [11]. In the first case resulting space-time has no singularities but contains closed time-like curves [18], while in the second case the point-like singularities occur [19]. It is important, that in both cases space-time is defined as solution of non hyperbolic problem, whose physical meaning is not obvious both in 4-dimensional and multidimensional theory [20].

In recent years some attempts were made to describe simplified version of topology change in which the final configuration is not a smooth manifold but a stratified manifold, which may be obtained >from initial space-time by means of identification of some pairs of points, which leads to so-called minimalist wormhole [21]. To this end the two point function on space-time $W(x, y)$ is introduced and it is supposed that the points of space-time is identified iff this function is equal to zero, i.e. iff $W(x, y) = 0$ (for details see [21], where equations for simplified form of $W(x, y)$ is also introduced). Unfortunately, both in paper [21] and in following papers on the same subject there is no influence of function $W(x, y)$ on the fields on manifolds is considered. As a result the statements of these papers that equality $W(x, y) = 0$ mean identification of points x and y are wrong. Really, to identify two arbitrary points $x \in M$ and $y \in M$ of some

manifold (or topological space) M it is necessary to exclude from the space $F(M)$ of function (of arbitrary type) on M all functions which are nonequal in the points x and y. For instance, to transform the real line $\mathbf{R}$ into circle S^1 all non-periodic functions must be excluded from the ring of all functions on the real axis.

Unfortunately, there are no tools in modern theoretical and mathematical physics which may realize this procedure.

4. Conclusion

To conclude this discussion some final remarks are necessary.

In this report we have discuss some problems of space-time descriptions at small (quantum) scales. We have argue, that hypothesis about discrete structure of space-time at small scales is unphysical because it mean the absence of any interactions. So, the procedure of discretization, which is useful for numerical methods is not suitable for construction of quantum space-time theory. We consider also some problems of topology change description. We have saw, that any interaction on the set of events introduces some topology on this set. The known physical interactions introduce the same topology on the space of events, which are equivalent to each over and to the topology, which is defined by space-time metric. For topological change (on the same set of events) it is necessary, but not enough, to have some interaction, whose topology does not equal to the topology of over interactions. Moreover such interaction must acts on the manifold structure, i.e. on the space of smooth functions on the set of events.

All the considered problems are mostly physical problems and their solution is impossible without new experimental data and the deep reconsideration or revision of the measuring procedures.

Список литературы

[1] Д.И. Блохинцев, Пространство-время в микромире. М., Наука, (1970); А.Н. Вяльцев, Дискретное пространство-время, М., Наука, (1965); D. Finkelnstein, Phys. Rev., (1969), **184**, 1261; Phys. Rev. D., (1972), **D5**, 2922; D.D. Reid, Canadian J. of Phys., (2001), **79**, 1-16.

[2] S.W. Hawking, *Nuclear Physics*, 1978, **B138**, 349.

[3] J.A. Wheeler, *Ann. Phys.*, NY, 1957, **2**, 604; Geometrodynamics (New York: Academic Press), 1962; Einstein's Vision (Berlin: Springer) 1968.

[4] J. Frohlich, Regge calculus and discretized gravitational functional integral, Preprint, Zurich, 1990; J. Ambjorn, J. urkiewich and R. Loll, (2001), Nucl. Phys. B., **B610**, 347-382.

[5] Yu.G. Borisovich, N.M. Blisnyakov, Ya.A. Israilevich, T.N. Fomenko, Introduction to topology, M., Nauka, 1995 (in Russian);

[6] S. Kobayashi, K. Nomizu, Foundation of differential geometry, v. 1-2, New-York - London, 1963.

[7] S.W. Hawking, G.F.R. Ellis, The Large Scale Structure of Space-Time, Cambridge U.P., 1973.

[8] P.S. Aleksansandrov, Introduction to the set theory and general topology, M., 1977 (in Russian).

[9] V.S. Vladimirov, I.V. Volovich and E.I. Zelenov, p-Adic Analysis and Mathematical physics, World Sci., Singapoure, 1994.

[10] A.N. Dranishnikov, Uspekhi Mat. Nauk, (2000), **55**, N 6, 71.

[11] J. Milnor, The Morse theory, *Annals of Mathematical Studies,* v. 51 (Prinston: Prinston University Press)

[12] M.Yu. Konstantinov, *Izv. vuzov. Fizika*, 1983, 4, p. 55 (in Russian)

[13] P.J. Hilton, S. Wayle, Homology Theory. An introduction to algebraic topology, (Cambridge University Press), 1960

[14] O. Alvarez, *Common. Math. Phys.*, (1985), **100,** ,279-309.

[15] V.N. Efremov and N.V. Mitskevich, Diskrete model of spacetime in terms of inverse spectra of T_0 Aleksandroff topological spaces, gr-qc/0301063.

[16] M.Yu. Konstantinov, Abstracts of GR-10, Padova, 1983, vol. 2, p. 1107.

[17] B. A. Dubrovin, S. P. Novikov, A. T. Fomenko, The Modern Geometry. Methods of the Gomology Theory, Moscow, Nauka, 1984. (in Russian)

[18] P. Yodziz, Gen. Relat. and Gravit., 1973, **4**, 299.

[19] M.Yu. Konstantinov, Topological transitions in classical gravitation theory: scalar-tensor formalism, in: "Problems of gravitation theory and elementary particles, v. 16", Moscow, Energoatomizdat, 1985.

[20] M.Yu. Konstantinov, Int. J. Mod. Phys. D., (1998), v. 7, N 1, pp. 1-17.

[21] Lee Smolin, Fermions and Topology, 17 pages, preprint CGPG-93/9-4, gr-qc/9404010

Computer simulation of the gravitational interaction in binary systems

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The problem about a motion of a planet in a field of a binary star with the different mass components is considered in the paper. The classic problem about two fixed centers of gravity is utilized as model. The addition of the gravity potential term is utilized for the relativistic correction for more massive component. The term gives rise to rotational displacement of a perihelion of a planet. The program for computer simulation is generated on the basis of the given model. It is shown, that the relativistic correction gives in magnification of area of inconvertible orbits, as contrasted to nonrelativistic viewing. The relativistic correction increases of a planets presence probability for binary stars. The approximate analytical viewing of the given problem is yielded. The problem about two fixed centers of gravity in a classic mechanics is solved analytically for Newtonian potentials. The precise analytical solution does not exist because of the additional term in the relativistic case. It is shown, that the relativistic correction give in time shift, strains of trajectories and to oscillating corrections in the starting conditions in the classic mechanics solution.

1. Introduction

In a general relativity (GR) the problem of a motion of a trial particle in a field of other body is solved analytically for any velocities of a trial body. It is showed, that the relativistic effects give rise to rotational displacement of a perihelion of a particle. The given effect is appeared by occurrence of the additive term in gravity potential [1].

In case of two bodies of comparable mass of the equation analytically also are solved [2,3]. In case of a binary star the motion of a trial body in a field of its components is reduced to a problem of three bodies, which one generally analytically is not solved neither in a classic mechanics, nor in GR. Therefore generally given problem should be solved or analytically approximately, or with the help of computer simulation. The given problem is solved, for example, for three bodies of the Sun, Earth and moon approximately [4]. The relativistic correction in gravity potential of the Sun is the additive term. The gravitational field of the Earth is considered in Newtonian approach. In the given model are discarded the relativistic term of a field of the Earth and cross relativistic term of a field of the Earth and Sun. The given term arises because of nonlinearity of the equations GR. However, when mass of the second body is much less than mass first and both they much more than mass third, the given model is suitable [4].

The problem about two fixed centers of gravity in a classic mechanics is solved analytically for Newtonian potentials [5,6,7]. The precise analytical solution does not exist because of the additional term in the relativistic case. But, it is possible to analyze as relativistic effects will change character of a motion of a body, which is obtained from the classic mechanics solution.

2. The basic equations

In the paper two fixed centers of gravity of miscellaneous mass are considered. More light is featured by Newtonian gravity potential. More massive is featured by two terms: by Newtonian gravity potential and relativistic term. The trial particle with small mass moves in these fields. The equations of motion of a particle look like:

$$\frac{d^2x_\alpha}{dt^2} = -G\frac{m_1{}^*(x_\alpha - x_{\alpha 1})}{(\sum_\alpha (x_\alpha - x_{\alpha 1})^2)^{3/2}} - 6G^2\frac{m_1{}^{2*}(x_\alpha - x_{\alpha 1})}{c^2(\sum_\alpha (x_\alpha - x_{\alpha 1})^2)^2} - \\ -G\frac{m_2{}^*(x_\alpha - x_{\alpha 2})}{(\sum_\alpha (x_\alpha - x_{\alpha 2})^2)^{3/2}} \tag{1}$$

Here x_α - the coordinates of a particle of mass m, $x_{\alpha 1}$ - the coordinates of more massive first center of gravity of mass m_1, $x_{\alpha 2}$ - the coordinates of the second center of gravity of mass m_2, G - gravitational constant, c - speed of light.

We view the cases, when

$$m_1 \geq 10 * m_2 \tag{2}$$

Then it is possible to neglect the relativistic correction from the second center of gravity, since it is proportional to a quadrate of mass, and relativistic correction from the cross term of both centers of gravity.

The first and third terms in a right member of the equation (1) are correspond to attractive forces called in customary Newtonian potentials. The second term in a right member grows out of the registration of the relativistic correction. It responds for a rotational displacement of a perihelion of an orbit of a planet for a single star [1].

If for a particle, whose motion is featured the motion equations to write a Hamiltonian in elliptic variable, will appear that variable for the given Hamiltonian are not parted [7]. If to consider only for Newtonian potential, the problem is solved analytically.

3. Simulation results

The simulation has shown following. Disregarding of relativistic term a particle (planet) or experiences librations around of more massive center. Or, depending on initial parameters (initial position and velocity) it can fall on one of stars, usually more massive , or take the flight in outside space. In case of simulation with correction for of relativistic term a general tendency: three possible such as behaviour of a particle did not change. But the range of values of initial parameters was a little increased, at which one there are librations. It is shown, that the relativistic term dilates area of inconvertible orbits.

4. Approximate analytical solving

Let's consider the given problem analytically. For this purpose we shall take into account, that the relativistic term is small. Without it the problem precisely analytically is solved in elliptic coordinates [5,6,7].

Let r and r' - there will be distances from a particle (planet) of mass m up to masses m_1 and m_2. Let's enter elliptical coordinates λ and μ:

$$\lambda = \frac{1}{2}(r + r'), \qquad \mu = \frac{1}{2}(r - r') \tag{3}$$

Distance between masses m_1 and m_2 is equaled 2F. We select an axis x directional on direct, pairing these two mass. The axis y is guided perpendicularly axes x and intercrosses it between masses apart F from each.

In coordinates λ and μ: we obtain a Hamiltonian by the way

$$H = H_0 + H_1 \tag{4}$$

where H_0 - it is a reference Hamiltonian [7] disregarding of relativistic term:

$$H_0 = \frac{1}{2}\left(\frac{m}{\lambda^2 - \mu^2}\right)\left\{(\lambda^2 - F^2)\dot{\lambda}^2 + (F^2 - \mu^2)\dot{\mu}^2\right\} - \\ - \frac{Gm((m_1 + m_2)\lambda - (m_1 - m_2)\mu)}{\lambda^2 - \mu^2} \tag{5}$$

H_1- it is the correction to a Hamiltonian called by the relativistic term

$$H_1 = -\frac{6G^2 m m_1^2}{c^2(\lambda + \mu)^2} \tag{6}$$

Let's note, that in the equation of Hamilton - Jacobi for a Hamiltonian H_0 variable are parted. But in equation of Hamilton - Jacobi for Hamiltonian H variable are not parted because of availability of the cross terms in H_1.

Let $\lambda_1 > \lambda_2$ and $\mu_1 > \mu_2$ roots of the corresponded equations:

$$h\lambda^2 + G(m_1 + m_2)\lambda + \alpha = 0,$$
$$h\mu^2 + G(m_1 - m_2) + \alpha = 0 \tag{7}$$

where h and α - usual integration constants of the equation of Hamilton - Jacobi.

Then the trajectories are set is parametric for variable λ and μ:

$$\lambda = \frac{\lambda_1(\lambda_2 - F)sn^2u + \lambda_2(\lambda_1 - F)cn^2u}{(\lambda_2 - F)sn^2u + (\lambda_1 - F)cn^2u},$$
$$\mu = c\frac{(\mu_1 + F)sn^2v - (\mu_1 - F)cn^2v}{(\mu_1 + F)sn^2v + (\mu_1 - F)cn^2v} \tag{8}$$

Where u and v are set by following expressions:

$$u = \sqrt{(\lambda_1 - F)(\lambda_2 + F)}\theta,$$
$$v = \sqrt{(\mu_1 - F)(\mu_2 + F)}(\theta - \theta_0) \tag{9}$$

here θ - parameter.

For variable x and y is obtained:

$$x = snu, \qquad y = snv \tag{10}$$

In the formulas (8) and (10) modules of elliptic functions of Jacobi in expressions for λ and x are set as:

$$k^2 = \frac{2F(\lambda_1 - \lambda_2)}{(\lambda_1 - F)(\lambda_2 + F)} \tag{11}$$

And the module of elliptic functions of Jacobi in expressions for μ and y is set as

$$\chi^2 = \frac{2F(\mu_1 - \mu_2)}{(\mu_1 - F)(\mu_2 + F)} \tag{12}$$

The time is determined as

$$t - t_0 = \int \frac{\lambda^2}{\sqrt{2(\lambda^2 - F^2)(h\lambda^2 + G(m_1 + m_2)\lambda + \alpha}} d\lambda -$$
$$\int \frac{\mu^2}{\sqrt{2(F^2 - \mu^2)(h\mu^2 + G(m_1 - m_2)\mu + \alpha}} d\mu \tag{13}$$

Or noting connection between λ and μ:

$$t - t_0 = \int \frac{\lambda^2 - \mu^2}{\sqrt{2(\lambda^2 - F^2)(h\lambda^2 + G(m_1 + m_2)\lambda + \alpha}} d\lambda \tag{14}$$

Thus, we know dependence λ and μ from the time.

In connection with that variable are not parted in the equation for H, the usual perturbation techniques give in not solved analytically equations system. Therefore we shall take advantage of an original method. We substitute the obtained values λ and μ in the equations of Hamilton - Jacobi for H. Thus we gain the correction for h, which one is meant h_1:

$$h_1 = -\frac{6G^2 m m_1^2}{c^2(\lambda + \mu)^2} \tag{15}$$

The given value h_1 should be toted with h and to substitute in (7), (13) - (15).

Let's note, that h is initial energy of system. Thus has appeared, that the relativistic term deforms trajectories and shifts a motion on variable in time. As we have considered periodic trajectories, the

registration of relativistic effects gives in the in batches oscillating corrections to the starting conditions. Such oscillations give in effects such as gyration of a perihelion of a planet.

5. Conclusion

The problem about a motion of a planet in a field of a binary star with components of different mass is considered. As model the classic problem about two fixed centers of gravity will be utilized. The addition of the term in gravity potential are used. as the relativistic correction for more massive center. On the basis of the given model the program for computer simulation is generated. It is shown, that the relativistic correction gives in magnification of area of inconvertible orbits, as contrasted to by nonrelativistic viewing. Thus, the relativistic correction increments probability of a presence of planets for binary stars. The approximate analytical viewing of the given problem is yielded. It is shown, that the relativistic correction give in time shift, strains of trajectories and to oscillating corrections in the starting conditions in the classic mechanics solution.

References

1. T. Levi-Civita, Fondamenti di meccanica relativistica, Bologna, Nicola Zanichelli, 1928.
2. L. Infeld and J. Plebanski, Motion and relativity, New York, Pergamon Press, 1960.
3. V. Fock, The theory of space, time and gravitation, London, Pergamon Press, 1959.
4. A.S. Eddington, The mathematical theory of relativity, Cambridge, University press,1924.
5. C. Jacobi, Vorlesungen uber Dynamik, Berlin, 1884.
6. P. Appel, Traite de Mecanique Rationelle, t.1, Paris 1896.
7. L.A. Pars, A treatise on analytical dynamics, London, Heinemann, 1964.

The Bohr's solution of quantum relativistic weak gravitation problem for Moon- Earth system.

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A new method of Moon-Earth system analysis for weak gravitation field is presented together with some estimates of quantum parameters of these system. Such model, that use in this article, can give important information about linkage of distances between levels in solid states and in this system and information about nature of large periods in this system. Analysis of this model shown that relativistic mechanic is very important for correct considering of two bodies quantum gravitation problem.

1. Introduction

The Coulomb's law for dot charged bodies and the Newton's law of gravity for dot masses are similar [1]. Therefore consider opportunity of gravitational analogue of the Bohr's atom of hydrogen. To analyze this possibility, we estimate such attitudes as the attitude of electron and nucleus charge, attitudes of masses of the Moon (m_1) and the Earth (m_2), the attitude of electronic and nuclear radiuses (r_e,r_n) to distances between them (r_{en}) and the attitude of radiuses of the Moon (r_1) and the Earth (r_2) to distance between them. We have received the following numerical estimations 1:1, 1:120, 1:10000, and 1:100. Therefore we also have numerical analogy. We investigate a s-state as a trajectory of the Moon in system of readout the Earth is a circle with a sufficient degree of accuracy [2].

What distinctions we should take into account ? The Bohr's rule of quantization is

$$rp = \hbar N, \qquad N= 1,2,3,..., \tag{1}$$

where r is radius of Bohr's orbit, p is impulse, $\hbar$ is z-projection of photon moment. Recently [3] the Bohr's rule of quantization was used for estimations of black hole structure [3]. In the quantum theory of gravitation an interaction carrier is graviton. His z- -projection of impulse moment is equal $2\hbar$ [4] and consequently instead of (1) we have

$$rp = 2\hbar N, \qquad N= 1,2,3,\ldots \tag{2}$$

2. Analysis of problem with endless velocity of light

In the first place we consider the model with endless velocity of light c and the Newton's gravitation potential. The mechanical energy E` (E-mc^2 in relativistic case) is

$$(E`/m_1c^2)=(1/2)(\xi^2-1/\eta), \tag{3}$$

where ξ is (p/ m_1c), η is (r/r_g), r_g is a gravitation radius, $r_g = 2G\, m_2/c^2$, G is the Newton's gravitation constant. Then equality (2) is

$$\Lambda\, \xi\, \eta = N, \qquad N= 1,2,3,..., \tag{4}$$

where $\Lambda= Gm_1m_2/ \hbar c$ is gravitation analogue of fine structure constant. Using uantization equality (4) and principle of extremuma for E` we have gotten the formula of ξ

$$\xi_N = (1/2)(\Lambda/N), \qquad N= 1,2,3,..., \tag{5}$$

where ξ_N is dimensionless impulse, and formula for η

$$\eta_N = 2(N /\Lambda)^2, \qquad N= 1,2,3,... \tag{6}$$

where η_N is dimensionless radius. Consider case N= 1. Then we have

$$\eta_1 = 2 / \Lambda^2 , \quad (7)$$

where $\Lambda = m_1 m_2/(m_{PL})^2$, $m_{PL} = (\hbar c/G)^{1/2}$ is Plank`s mass and $m_{PL} = 2.08 \ 10^{-8}$ kg. So the first Bohr's radius is very little ($m_1 = 7.35 \ 10^{22}$kg, $m_2 = 5.98 \ 10^{24}$ kg) , r_1 about 10^{-127} m. Gravitation radius of the Earth about 9 10^{-3} m. It means that radius trajectory of the Moon mass center is more smaller of gravitation radius but it is impossible. Therefore assumption about endless velocity of light is wrong.

3. Analysis of problem with end velocity of light

Energy of a particle in gravitation field is $E/mc^2 = (1- \beta^2)^{-1/2}$, where m is weight of particle, g_{00} is a component of the metric, $\beta = v/c$, v is velocity of particle. For gravitation field with a central symmetry g_{00} is known and $g_{00} = 1- (r_g/ r)$ [5]. Therefore we have

$$(E`/m_1c^2)=(1-1/\eta)^{1/2}(1+ \xi^2)^{1/2} - 1. \quad (8)$$

Using equality (4) and principle of extremuma for E` we have gotten the formula of ξ quantization

$$\xi_N = (1/3)(N/\Lambda)(1-(1-3(\Lambda/N)^2)^{1/2}). \quad (9)$$

As ξ mustn` t have imaginary part, we have critical value of quantum number N

$$N_{c0} = E (3^{1/2} \Lambda), \quad (10)$$

Where E (x) is a function determining an integer part of number x . Dimensionless value of Bohr's radius is

$$\eta_N = 3 / (1-(1-(N_c/ N)^2)^{1/2}), \quad (11)$$

where $N_c = 3^{1/2} \Lambda$. As $N >>> N_c$ in weak gravitation field, we have formula with sufficient degree of accuracy for η parameter

$$\eta_N = 6(N_c/ N)^2 (1- (1/4) (N_c/ N)^2 + 0). \quad (12)$$

and the energy spectrum is equality

$$E_N =- (1/24) (1 + (1/12) (N_c/ N)^2)) (N_c/ N)^2 m_1 c^2, \quad (13)$$

If velocity of light is endless the energy spectrum is

$$E_N = - (1/24) (N_c/ N)^2 m_1 c^2 . \quad (14)$$

4. Estimations and conclusions

Using formulas (10) and (11) and the following values of parameters system the Moon-Earth $m_1 = 7.33*10^{22}$kg, $m_2 = 5.98*10^{24}$kg fnd $r = 3.84*10^8$ m and physical constants $\hbar = 1.05457*10^{-34}$ kg*m^2 *s^{-2} , $c = 2.99792*10^8$ m* s^{-1} , $G = 6.673*10^{-11}$ kg^{-1}m^{3}s^{-2}, we have the following estimations $N_{c0} = 1. 6$ **10^{63} , $N_1 = 1.36*10^{68}$** .,$E_1 = m_1 c^2$ and $E_2 = m_2 c^2 >> E_{N1} = - 1.76 * 10^{28}$ kg* m^2 s^{-2} , $E(N_1 +1) - E(N_1) = 2.59*10^{-40}$ kg* m^2 s^{-2} or $1.6* 10^{-21}$ eV.

The Bohr's hamiltonian (8) at transition from classical variables to operators forms a basis for construction of the theory of indignations on the basis of the ordinary quantum mechanics.

So, we shall receive, that the usual quantum mechanics may be used for the analysis of an oscillatory spectrum in system the Moon-Earth as Rydberg`s atom.

References

[1]. L. Brillouin Relativity reexamined, Academic Press, N.Y. and London (1970).

[2]. V.N. Zharkov, Astronom. Vestnik, 34, 3 (2000).

[3]. S. Hod, Phys. Rev. Lett. 81, 4293 (1998).

[4]. M. Fiers, W. Pauli, Proc. Soc., 173, 211 (1939) or J. Weber , General Relativity and Gravitation Waves, W.A. Benjamin., Inc., N.Y. (1961).

[5]. L. Landau , Lifshitz, The field theory, Moscow, Nauka, (1988).

О движении фермиона в гравитационном поле неоднородно распределенной массы

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On the fermion motion in the gravitational field of masses distributed ununiformly

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The regions of the finite and infinite motions of relativistic classical particles and one-half spin fermions are determined with the help of effective potential curves. The particles are traveling in the centrally symmetrical gravitational field of masses distributed according to the Schuster law for star globular clusters. The metric tensor components are obtained as the solution of the Einstein equations. Expressions and curves of g_{11}-, g_{00}-functions are presented and discussed. The effective potentials u_1 for a classical particle and u_2 for a fermion are obtained with a help of the Hamilton-Jacobi equation and of the Dirac equation accordingly. The comparison of u_1- and u_2-expressions shows that they coincide for the radial motion and in the part depending on the orbital angular momentum squared. Besides u_2 depends on spin properties: quantum number k, taking on positive and negative values has an effect on the u_2-magnitude to a large extent; the fourth member in the u_2-expressoin describes the spin-orbit interaction. Therefore, the gravitational field is not the geometry only. The effective potential curves for radial motions (fig.3) and for motions with nonzero angular momentum (fig.4) are discussed. In particular the gravitational capture does not take place for any angular momentum in contradistinction to the Schwarzschild field of the point mass.

С помощью эффективного потенциала определены области финитного и инфинитного движения релятивистских классической частицы и фермиона половинного спина, находящихся в стационарном центрально-симметричном гравитационном поле. Поле создается пылевидным веществом, неоднородно распределенным в конечной области пространства. Компоненты метрического тензора для этой области получены в результате решения соответствующих уравнений Эйнштейна, поле вне – описывается метрикой Шварцшильда.

Вещество, являющееся источником гравитационного поля, распределено в пределах сферы радиуса R по закону Шустера для звездных шаровых скоплений, [1],

$$\rho(r) = \rho(0)\left(1 + (r/r_0)^2\right)^{-\frac{5}{2}},$$

где r_0 - параметр, характеризующий распределение плотности вещества $\rho(r)$, r -координатный радиус. Тензор энергии-импульса вещества, рассматриваемого как сплошная среда, равен $T_i^k = (p+\varepsilon)\, u_i\, u^k - p\, \delta_i^k$, [2]. Пренебрегая взаимодействием частиц вещества, полагаем давление в такой среде равным нулю: p=0, [2]. Центрально-симметричное распределение вещества и возможные радиальные движения типа пульсаций, характерные для некоторых звездных скоплений, позволяют для внутренней области $r \le R$ и внешней области $r \ge R$ искать метрику в виде:

$$ds^2 = e^{\nu} c^2 dt^2 - r^2 (d\theta^2 + \sin^2\theta d\varphi) - e^{\lambda} dr^2,$$

где $x^0=ct$; $x^1=r$; $x^2=\theta$; $x^3=\varphi$ и $g_{00}=e^{\nu}$; $g_{11}=-e^{\lambda}$; $g_{22}=-r^2$; $g_{33}=-r^2 \sin^2\theta$. В сопутствующей системе отсчета, где компоненты 4-вектора скорости $u^i=dx^i/ds$ равны $u^r=u^\theta=u^\varphi=0$, $u^0 \neq 0$, тензор энергии-

импульса имеет во внутренней области ($r \le R$) одну компоненту, отличную от нуля: $T_0^0=\varepsilon=\rho(r)\,c^2$, а во внешней области ($r \ge R$) – все $T_i^k=0$.

Решение известной системы уравнений Эйнштейна для центрально-симметричного гравитационного поля [2] с указанными выше условиями имеет вид:

$$g_{00}(0 \le r \le R) = e^{\nu(o \le x \le x_0)} = \left(1-\frac{x_g}{x_0}\right)\cdot \exp\left(\int_x^{x_0} \frac{\alpha x \cdot dx}{\alpha x^2 - (1+x^2)^{\frac{3}{2}}}\right),$$

$$g_{00}(r \ge R) = e^{\nu(x \ge x_0)} = \left(1-\frac{x_g}{x}\right);$$

$$-g_{11}(0 \le r \le R) = e^{\lambda(0 \le x \le x_0)} = \left(1-\frac{\alpha x^2}{(1+x^2)^{\frac{3}{2}}}\right)^{-1},$$

$$-g_{11}(r \ge R) = e^{\lambda(x \ge x_0)} = \left(1-\frac{x_g}{x}\right)^{-1},$$

где $x=r/r_0$, $x_0=R/r_0$, $\alpha=(8\pi G/3c^2)\,\rho(0)\,r_0^2$, $x_g = r_g(M)/r_0=\alpha\, x_0^3(1+x_0^2)^{-3/2}$, $r_g(M)$-гравитационный радиус полной массы M вещества $M=(4\pi/3)R^3\,\rho(0)(1+(R/r_0)^2)^{-3/2}$.

Из приведенных выражений видно, что, во-первых, внутреннее решение непрерывно переходит во внешнее решение Шварцшильда, и, во-вторых, g_{00} и g_{11} могут иметь особенности при некоторых соотношениях между задаваемыми параметрами, такими как $\rho(o)$, r_0, R. Например, если постоянная α, которую можно интерпретировать как отношение гравитационного радиуса массы $m(r_0,\rho(0))=(4\pi/3)r_0^3\rho(0)$ к размеру r_0 ядра вещества, равна $(3\sqrt{3})/2$: $\alpha^*=r_g(r_0,\rho(0))/r_0=(3\sqrt{3})/2$, то внутренняя метрика имеет особенность при $x^*=\sqrt{2}$. При каждом $\alpha>\alpha^*$ в метрике возникает по две сингулярности с особой областью между ними, для которой изменяется сигнатура метрики.

Соблюдение условия на определитель метрического тензора $|g_{ik}|<0$ накладывает ограничения на значения параметров системы, которую можно описать с помощью полученного решения. Далее будем рассматривать движение пробных частиц в гравитационном поле, для которого выполняются условия: $\nu \le 0$, $\lambda \ge 0$, $\lambda(r=0)=0$, при $r \to \infty$ $\nu \to 0$, $\lambda \to 0$, т.е. $e^{\nu} \le 1$, $e^{\lambda} \ge 1$ [1], в частности, это возможно при $\alpha<\alpha^*$.

Графики функций $\rho(x)$, $g_{00}(x)$, $g_{11}(x)$ для $x_0=5$, $\alpha= 1,2$ представлены на рис.1,2.

Эффективный потенциал для классической частицы найдем, воспользовавшись уравнением Гамильтона-Якоби, которое в рассматриваемом случае имеет вид:

$$e^{-\nu}\left(\frac{\partial S}{c\partial t}\right)^2 - e^{-\lambda}\left(\frac{\partial S}{\partial r}\right)^2 - \frac{1}{r^2}\left(\frac{\partial S}{\partial \theta}\right)^2 - \frac{1}{r^2\sin^2\theta}\left(\frac{\partial S}{\partial \varphi}\right)^2 - m^2c^2 = 0$$

где S – действие, m – масса частицы.

В качестве плоскости, в которой происходит движение и которая проходит через центр поля, выберем плоскость $\theta=\pi/2$. Для постоянного гравитационного поля t и φ являются циклическими координатами, поэтому полная энергия частицы E_0 и ее момент импульса L будут по-

стоянными величинами, что позволяет известным способом (см.[2]) найти S, а затем закон изменения радиальной скорости, измеряемой локальным наблюдателем в собственном времени:

$$v^r = \frac{dr}{d\tau} = e^{-\frac{\nu}{2}}\frac{dr}{dt} = ce^{\frac{-\lambda}{2}}\left(1-\left(\frac{U_{eff1}(r)}{E_0}\right)^2\right)^{1/2}$$

$U_{eff1}(r)$ – так называемая эффективная потенциальная энергия, а

$$u_1^{\,2} = \left(\frac{U_{eff1}(r)}{mc^2}\right)^2 = e^{\nu}\left(1+\frac{L^2}{m^2c^2r^2}\right) = e^{\nu}\left(1+\frac{a^2}{x^2}\right) -$$

эффективный потенциал для классической частицы,

$$a = L/mcr_0$$

-безразмерный момент импульса классической частицы.

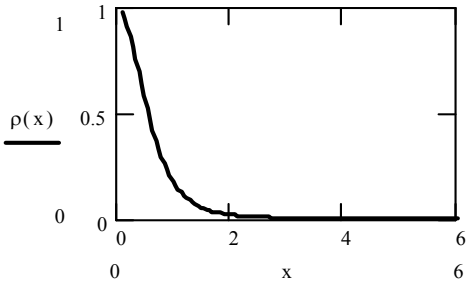

Рис.1

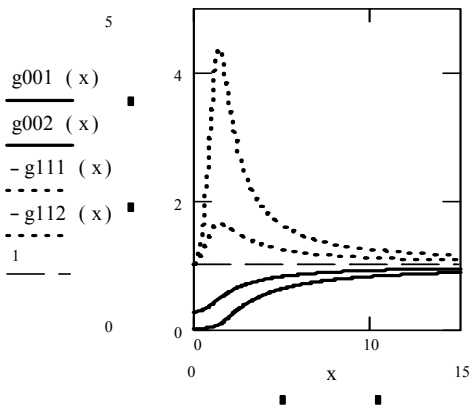

Рис.2

Потенциальные кривые движения фермиона найдем, воспользовавшись уравнением Дирака

$$i\hbar\gamma^{\alpha}\nabla_{\alpha}\Psi + mc\Psi = 0$$

[3]:где Ψ - волновая функция с трансформационными свойствами спинора, ∇_{α}- знак ковариантной производной, γ^{α} - матрицы Дирака. Для описания взаимодействия фермиона с гравитационным полем используем тетрадный формализм, т.е. запишем уравнение Дирака в системе локально-ортогонального репера. В центрально–симметричном поле угловая зависимость волновой функции такая же, как и для поля кулоновского типа. А уравнения для радиальных функций имеют вид:

$$\left[e^{-\frac{\lambda}{2}}\frac{d}{dr}+\frac{(1-k)}{r}\right]R_1(r) = \frac{1}{\hbar c}\left[E_0e^{-\frac{\nu}{2}}+mc^2\right]R_2(r)$$

$$\left[e^{-\frac{\lambda}{2}}\frac{d}{dr}+\frac{(1+k)}{r}\right]R_2(r) = \frac{1}{\hbar c}\left[-E_0e^{-\frac{\nu}{2}}+mc^2\right]R_1(r),$$

где $k=\pm1,\pm2,\pm3,$. при.$k>0$ $k=l+1,$ при $k<0$ $|k|=l,$ l – орбитальное квантовое число ($j=|k|-1/2$).

Квадрирование уравнений для $R_1(r)$ и $R_2(r)$ дает выражение для эффективного потенциала

$$u_2^{\,2}=e^{\nu}\left[1+\frac{b^2(k^2-1)}{x^2}\right]+e^{\nu-\frac{\lambda}{2}}\frac{b^2(1\mp k)}{x^2}-e^{-\frac{\lambda}{2}}\frac{b^2(1\mp k)}{2x}\frac{de^{\nu}}{dx}$$

где $b=\dfrac{\hbar}{mcr_0}$ – безразмерная единица измерения момента импульса фермиона, ± означает: знак (-) для $R_1(r)$, знак (+) для $R_2(r)$.

Сравнение выражений для u_1 и u_2 показывает, что они совпадают при радиальном движении и в части, зависящей от квадрата орбитального момента импульса. Однако, u_2 определяется также и спиновыми свойствами микрочастиц: квантовое число k, принимая положительные и отрицательные значения, существенно влияет на его величину, а четвертое слагаемое в u_2 описывает спин-орбитальное взаимодействие. Таким образом, гравитационное поле не сводится только к геометрии.

В качестве иллюстрации решений рассмотрим некоторые особенности потенциальных кривых.

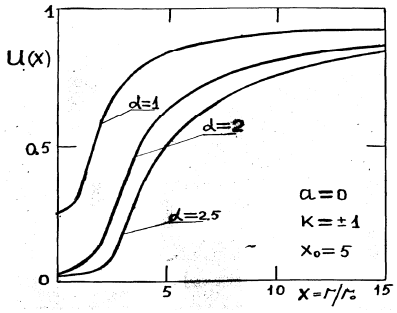

Рис.3

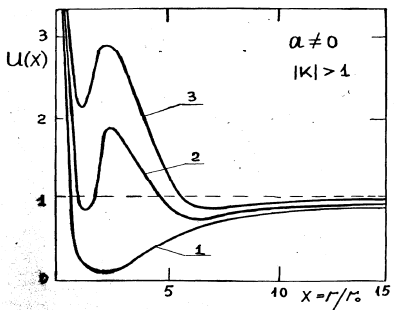

Рис.4

В случае радиального движения, когда $a=0,$ $k=\pm1,$ $j=1/2,$ потенциальные кривые (рис.3) имеют вид потенциальной ямы. Классическая частица с энергией $(E_0/mc^2)<1$ может совершать колебательное движение относительно центра $r=0,$ амплитуда которого определяется шириной ямы. Для квантовой частицы это также будет область связанных состояний, без “утечки”. При $(E_0/mc^2)>1$ классическая частица свободно проходит через центр, тогда как для фермиона коэффициент прозрачности ямы отличен от единицы.

При малых моментах импульса потенциальные кривые похожи на ньютоновские с соответствующими особенностями (рис.4, кривая 1). Для больших моментов (рис.4, кривые 2,3) потенциальные кривые имеют по два минимума и один максимум, что означает наличие двух областей финитного движения. Для фермиона, кроме того, существует вероятность туннелирования сквозь потенциальный барьер конечной ширины как при движении к центру, так и от центра. Однако, гравитационный захват ни с малым моментом, ни с большим не происходит, как в поле точечной массы (решение Шварцшильда).

Литература

1.Холопов П.Н. Звездные скопления. – М.:”Наука”.1981.

2.Ландау Л.Д., Лившиц Е.М. Теория поля.-М.:”Наука”. 1988.

2.Зоммерфельд А. Строение атома и спектры. Том 2. - М.: Гос. Изд-во Технико-Теоретической лит-ры.1956.

Специальная теория относительности и современное высшее профессиональное образование

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Новое тысячелетие, как веха в развитии человеческой цивилизации, знаменует новые задачи в области развития производительных сил общества и образования подрастающих поколений, нацеливает на объективную позицию к научно-техническому прогрессу и развитию культуры. В изменчивом мире появляющихся и исчезающих задач и проблем каждый участник производства должен уметь быстро перестраиваться на профессиональном уровне, быть готовым к мобильному принятию технических и организационных решений и способным решать глобальные проблемы человечества и планеты.

Озабоченные сохранением и дальнейшим развитием современной цивилизации, общественная мысль, наука и образование в поиске пути решения настоящих и перспективных проблем человеческого общества приходят к идее получения подрастающими поколениями фундаментального общенаучного образования. Человечество вступило в эпоху научно-теоретического стиля мышления, благодаря которому оно способно раскрывать глубинные функциональные связи и отношения, существующие в природе и обществе.

Ректор МГТУ им. Н.Э. Баумана академик И.Б. Федоров, отмечая усиление в высшем профессиональном техническом университетском образовании фундаментальной подготовки, которая ранее была характерной только для классических университетов, подчеркивает, что цель фундаментального образования состоит в синтезе знаний, формирующем цельное представление о процессах, происходящих в окружающей природе и обществе, и способствовании дальнейшему развитию личности /4, 192/.

В целях повышения уровня фундаментального образования принята Программа государственной поддержки интеграции высшей школы России и Российской академии наук. Программа направлена на развитие совместных фундаментальных исследований РАН и высшей школы, повышение качества фундаментального образования, подготовку и издание современных учебников, развитие новых форм подготовки кадров в области фундаментальных наук и др.

Образование передает новым поколениям накопленный «багаж» в виде социальной памяти, фиксирующей достигнутый уровень социально-исторического развития человеческого общества. Социальная память, включающая социальные знания и социальную информацию, представленный на категориальном уровне способ мышления, социальное сознание, социальные ценности, охватывающие важнейшие стороны общественной жизни, является осуществляемым обществом с помощью специальных институтов, посредством соответствующих носителей и средств процессом фиксации, систематизации и хранения теоретически обобщенного коллективного опыта человечества, добытого им в процессе развития науки, философии, искусства, знаний и образных представлений о мире. Хранящиеся в книгах и других средствах социальной памяти сведения тем или иным путем выдаются образованием для «переписывания» в память индивидов и, следовательно, для использования людьми в их разнообразной деятельности. Вне опоры и использования социальной памяти, вне взаимодействия с научным знанием и материальным производством, другими сторонами жизни общества, невозможны современная коммуникация и познание, решение актуальных проблем человечества, развитие самой цивилизации /1; 75/.

Целью всякого познания является преобразование природы и совершенствование человеческой практики на основе научных достижений. Образование, ориентированное в целом на эту главную цель, решает прежде всего задачи воспроизведения действительности в мышлении, формирования соответствующей своему времени мировоззрения, которые являются не-

обходимыми компонентами познания и исследовательской деятельности, инструментом преобразования мира и человеческого общества. Социальная память нацеливает образование на ознакомление студентов с основополагающими идеями современного естествознания и культуры, на выработку диалектического, теоретического и креативного стиля мышления, на овладение методологией научного познания и понимания модельных знаний и опыта на различных ступенях научного познания и образования.

Трудно переоценить роль учения Эйнштейна об относительности, составляющего «золотой фонд» наследия современной человеческой цивилизации, в вопросах содержания и передачи новым поколениям социальной памяти. Возникновение и становление, в частности, специальной теории относительности, как попытки устранить противоречия в принципе относительности Галилея, сопровождалось развитием и разрешением ряда принципиальных научных проблем – концепции мирового эфира, современного понимания пространства и времени, их преобразований при переходе от одной системы отсчета к другой и т. д. Поистине неоценима роль теории относительности в формировании современного мышления, для которого специальная теория относительности явилась мощным инструментарием, вскрывшим неожиданные факты, такие как многомерное пространство, относительность одновременности, преобразования времени при переходе к другим системам отсчета и др. Преодолевая с помощью специальной теории относительности ограниченность обыденной наглядности, фундаментальное естествознание строит научную картину мира, синтезирующую в себе естественнонаучную, философскую, техническую и другие картины и схемы мироздания, в единую модель природы, которая дает наиболее общее и систематизированное знание об окружающем мире. Чтобы познавать и преображать мир, решать новые проблемы и оценивать свои дела и поступки, специалисту нужна подобная научная и практическая интегративная система, из которой он мог бы черпать нужные сведения для выполнения и оценки своей производственной деятельности, находить все необходимое для формирования и самовыражения своей позиции, для удовлетворения культурно-духовных потребностей.

Необходимо подчеркнуть роль специальной теории относительности как базового компонента фундаментального естествознания. Она имеет предметом своего приложения и Вселенную в целом, и закономерности микромира. Важна ее роль в формировании современного мировоззрения. Раскрывая концептуальный базис естественнонаучного мировоззрения, специальная теория относительности становится фундаментом базовых моделей естествознания, которые составляют основу любой специальной дисциплины.

Современное образование, характеризуемое гуманистической направленностью, в центр образовательно-воспитательного процесса ставит личность учащегося, в противовес прочно укоренившемуся обезличенному и бессубъектному учебному процессу, оторванного от реального мира и реальной жизни. Феномен взаимодействия воспитанника с воспитателем, сверстниками, другими людьми, а также с предметами, в ходе которого происходит трансформация «элементов» культуры из общественно-социальной формы в индивидуально-психическую, есть форма диалектического процесса отражения окружающего мира человеком. Существенным моментом отражающей способности человека, как самой развитой и совершенной отражательной системы, является его активность. Активность субъекта деятельности диалектического отражения есть способность его функционально выделять из продуктов взаимодействия характеристики, относящиеся к объекту-оригиналу, соответствующие ему, и функционально исключать характеристики, зависимые от носителя отражения. Высший способ проявления принципа отражения — это способ вскрытия и передачи качественных характеристик, которые, будучи обусловленными внутренним строением, внутренним взаимодействием частей объекта отражения, могут не передаваться путем непосредственного взаимодействия одного объекта с другим.

Педагогическое взаимодействие не достигнет своего педагогического воздействия (воспитания, обучения, развития), если объект воздействия — обучающийся и воспитуемый — не проявит своей «отражательной» активности, не выступит в качестве субъекта этого воздействия, не перейдет в стадию субъектности. Субъектность — это сложная интегративная ха-

рактеристика личности, отражающая ее активно-избирательное, инциативно-ответственное, преобразовательное отношение к самой себе, к своим поступкам, действиям, учебной познавательной деятельности, к людям, к миру и жизни в целом.

Активная субъектная позиция реализуется не сама по себе, а в деятельности человека, направленной на удовлетворение своих потребностей. Люди не находятся в каком-то отношении к природе, событиям, культуре, а активно действуют, овладевая ими с помощью действия и формируя свое отношение к действительности. Деятельность есть специфическая форма отношения человека к окружающему миру, содержание которой составляет его целесообразное изменение и преобразование с целью удовлетворения своих потребностей. Благодаря творческой активности субъекта в деятельности, во взаимодействии с объектами отражения происходят приращение информации о вещном мире, изменения в личностном плане. Деятельность индивида, личности как раз и является тем механизмом, который позволяет преобразовать совокупность внешних влияний в собственно развивающие изменения, в новообразования личности, в профессиональную квалификацию. На предшествующем этапе исторического развития образования имел место «знаниевый» подход, который исследовал «функционирование» знаний, закономерности их наиболее эффективной передачи и присвоения учащимися. Сами знания обособлялись и отделялись от их носителя, обучающихся, приводящих их в движение. В настоящее время уровень научного прогресса, проникновение в сущность явлений и процессов природы и общественной жизни достиг такой глубинной стадии обобщения, что извлечение и использование научной информации вне активности человека и его деятельности, стали невозможными.

По сути дела между фундаментальным теоретическим познанием, являющимся условием дальнейшего развития современной цивилизации на Планете в настоящий период, и прагматизмом производственной практики существует пропасть, непреодолимая для формальной логики и компьютерных технологий. В фундаментальных обобщениях не содержится указаний на получение конкретных рецептов профессиональной деятельности. Добыть их применительно к решению конкретных научно-производственных проблем и задач можно только в рамках человеческого поиска, человеческой деятельности, благодаря присутствию фактора человеческой личности. Это подтверждается, например тем, что несмотря на широкие возможности современных компьютеров, не удается решить ставшие «вечными» такие задачи, как задача полного исследования динамики трех тел или задачу хотя бы среднесрочного прогноза погоды. Деятельность человека относится к феномену, в котором соединяются теоретические достижения современной науки и потребности общественной практики.

Отсюда вытекает требование к субъектно-деятельностной педагогике высшего профессионального образования о насущной необходимости обучения и воспитания в выпускниках вуза «искусства» субъектной деятельности, как объективного условия выживания и развития современной цивилизации, успешного поиска решения современных глобальных и каждодневных проблем человечества.

В настоящее время период жизни знаний сократился до 3 – 5 лет. Прикладные аспекты наук и их теорий быстро устаревают, обесцениваются и не отвечают динамичности современного производства. Высокой устойчивостью к различным переменам обладают фундаментальные знания. Отличающаяся необходимым прагматизмом профессиональная подготовка специалиста побуждает его обратиться в своей в профессиональной деятельности к фундаментальной основе практических знаний. Чем больше видов деятельности с меньшими затратами может осуществить работник, тем идеальнее система его подготовки. Приоритетным фактором здесь становится сам субъект, в деятельности которого только и могут соединиться фундаментальные знания с инженерной и гуманитарной практикой, обрести единство эти две полярности: фундаментальные знания – предметная деятельность субъекта – профессиональная подготовка.

Субъектно-деятельностный подход в решении научных и учебных проблем имеет два плана своей реализации: стратегический и тактический /2/. Первый соотносится с выбором между возможными альтернативами решения проблемы, второй намечает путь осуществле-

ние выбранного решения. Стратегический замысел выражает теоретическую основу метода решения научной или учебной проблемы. Вне «философии» теории относительности нельзя осуществить глубокий поиск решений новых задач современной цивилизации и будущих поколений. В силу важности этого тезиса М.Э. Омельяновский, известный исследователь философии современной физики, настаивает на изучении теории относительности, как основополагающей теории современной физики, составляющей ее фундамент.

Уходя своими корнями в механику, электродинамику и оптику, учение А. Эйнштейна об относительности представляет собой не просто развитие соответствующих разделов курса физики, а решает в учебном курсе свои специфические образовательные задачи:

- выдвигает и формирует фундаментальные физические принципы: относительности, эквивалентности, соответствия, конечности скорости передачи сигнала (близкодействие), постоянство скорости света в вакууме;
- устанавливает новые факты: зависимость массы тела от скорости, связь массы и энергии, изменении размеров и формы движущихся тел и др.;
- формирует релятивистские представления о пространстве и времени;
- раскрывает понятие четырехмерного (многомерного) пространства в физике;
- составляет основу инженерных расчетов (циклотрон, регистрация и определение скоростей быстрых частиц и др.)
- служит инструментом преодоления обыденной (ограниченной) наглядности и развивает научно-теоретический стиль мышления.

Подтверждением справедливости высказанной позиции служит создание основателем психологической теории деятельности А.Н. Леонтьевым многомерного образа мира, как интегрального, синтетического образования познавательной сферы человека. Образ мира, как система, выступает в качестве целого и исходного по отношению к любому частному акту познания и в то же время является результирующим образованием, сохраняющим и накапливающим любые новые знания. Развивая четырехмерный мир теории относительности, А.Н. Леонтьев вводит пятое квазиизмерение образа мира, системы, в которой представлен не только физический мир материальных объектов, но и раскрытые совокупной общественной практикой значения, или означенности, вещного мира, идеальные формы существования предметного мира, в которых запечатлена сама деятельность человека, объективирующаяся в своем продукте и наделяющая его «тем предметным содержанием, которое она объективно несет в себе» /3/.

Дидактические принципы изучении специальной теории относительности ориентируют нас на фундаментализацию естественнонаучного знания, на формирование теоретического мышления и современного научного мировоззрения, на привлечение специальной теории относительности к общепрофессиональной инженерной подготовке и формированию профессиональной компетентности современного специалиста.

Литература

1. Колеватов В.А. Социальная память и познание. М., 1984.
2. Купавцев А.В. Деятельностная технология обучения: человечная и человеческая.- В сб. Технические университеты как центры формирования инженерной элиты XX1 века. /Материалы 4-х Энгельмейровсих чтений. -М.: Изд-во МГТУ им. Н.Э. Баумана. 2001. С.204-209.
3. Леонтьев А.Н. Психология образа. – Вестник МГУ, сер.14. Психология, 1979 №2 с.3-13.
4. Федоров И.Б., Еркович С.П., Коршунов С.В. Высшее профессиональное образование.- М., МГТУ, 1998. -368с.

Phase problem algoritms solution application to the determination of the gravitation fields phase structure

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In this report we review the application of the phase problem solution original algorithm to the determination of wave field phase by the spatial power spectrum also called as angular power distribution (APD). For the gravity waves laser interferometry the phase problem solution method application of may be useful due to the gravity waves investigations are based on the interfering light waves intensity distribution measurement. These measured data processing leads to resume about the presence or absence of the interfering light waves phase change and at the end to calculate both gravity waves phase structure and their sources.

The phase problem solution algorithm developed by the author allows not only to restore the interfering wave field phases by the APD, but to determine their sources distribution. The calculating aspects investigation of the developed algorithm done by the test problems solution achieves it high precision and measurements errors stability not only in the interfering intensity measuring, but on later numerical processing. Such suggested algorithm performances are the consequence of the used phase problem solution method based on assumption of the interfering waves sources spatial distribution finiteness.

The mathematical basis of the developed algorithm is constituted the numerical decision of a Riemann - Gilbert- boundary-value problem, to which a problem of a phase wave field by APD definition is managed to bring. The offered algorithm program realization of the phase problem decision is reduced to consecutive application of the measured intensity distribution values array on the interference pattern several Fourier transformations. Use of effective fast Fourier transform algorithms the allows to achieve high speed of a wave fields phase distribution accounts by the intensity values array and as consequence to apply the developed algorithm in real time mode directly in laser interferometry installations intended for experimental detection of a gravitational waves existence.

In this report the application of researched algorithm for adjustments and diagnostics of mirrors, lenses of experimental laser gravitational waves interferometry installations, and also mirror and lenses aerials, phased antenna arrays of the radio telescopes, and other various elements of the precision and adaptive optics are discussed.

Gravitational ether as the origin of the Universe and IFE

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Einstein's theory of gravitational ether is a basis for a simple understanding of the interaction between matter and universal space as well as the evolution of life.

1. Introduction

One of the main questions of cosmology is: where does matter come from in the *inflation phase* of the big bang and where does it go when it disappears at the centre of black holes. According to Stephen Hawking in the *inflation phase the* energy of matter that is positive and the gravitational energy which is negative are multiplied. Its sum always remains zero: E matter (Em) + E gravitational (Eg) = 0. In a similar way as (-1) + (1) = 0, (-2) + (2) = 0, (-3) + (3) = 0 (1).

Describing the energy of matter as positive, the energy of gravitation as negative and explaining the *inflation phase* with the equations above does not answer the question of the appearance of matter. Equations that function in mathematics do not necessarily function in physics. Mathematics only describes reality and can not explain it. Einstein says: "As far as the laws of mathematics refer to reality, they are not certain; and as far as they are certain, they do not refer to reality" (2).

2. Discussion

Einstein's idea of the ether in the Morgan Manuscript was that cosmic space can not be imagined without the existence of gravitational ether: "The ether of the general theory of relativity differs from the one of earlier optics by the fact that it is not matter in the sense of mechanics. Not even the concept of motion can be applied to it. It is furthermore not at all homogeneous, and its state has no autonomous existence but depends on the field-generating matter. Since in the new theory, metric facts can no longer be separated from true physical facts, the concepts of space and ether merge together. Since the properties of space appear as determined by matter, according to the new theory, space is no longer a precognition for matter; the theory of space (geometry) and of time can no longer be presupposed prior to actual physics and expounded independently of mechanics and gravitation" (3).

Interaction between matter and non homogeneous gravitational ether can be described with the roundness of Riemann's four-dimensional geometry. The bigger the star is, the rounder the space around it is, and the higher the density of gravitational ether. In the centre of the black hole density, pressure and temperature are infinite. All atomic and subatomic particles disintegrate into gravitational energy. In singularity of the black hole only gravitational ether persists. Gravitational ether is never created and never destroyed, unchangeable. It can not be described with terms as "movement", "time" and "entropy". Its only characteristic that can be described mathematically is the density.

For Einstein, four-dimensional physical space, fulfilling the functions of the ether, was something

primary to matter consisting of particles. During the lecture at the University of Nottingham, Einstein would say: "The strange conclusion to which we have come is this-that now it appears it appears that space will have to be regarded as a primary thing and that matter is derived from it, so to speak, as a secondary result. We have always regarded matter as a primary thing and space as a secondary result. Space is now having its revenge, so to speak, and is eating up matter. But this is still a pious wish. " (4).

From the statement above arises the idea that in black holes matter disappears into gravitational ether and in the big bang matter appears back from it. In the universe energy is constantly circulating, it is never created or destroyed. The amount of gravitational energy and the energy of matter is constant:

E gravitational (Eg) + E matter (Em) = E constant (Ek).

In the first moment after the big bang Em = 0, Eg = Ek. In the subsequent moments of the inflation phase Eg is structured into Em, and the transformation is over when Eg and Em are balanced: Eg = Em (Eg = Ek/2, Em = Ek/2). With the formation of black holes, when the transformation of Em into Eg starts, Em is falling towards zero (Em --> 0), Eg is rising towards Ek (Eg --> Ek) (5).

The density of gravitational ether is increasing with the disintegration of matter into it. This process increases the gravitational forces between the galaxies, the speed of the expansion of the universe slows down. At a certain point the expansion stops and the universe starts to collapse in an enormous black hole that explodes into a new big bang where gravitational ether transforms back into matter.

The energy of matter and gravitational energy are in a *permanent dynamic equilibrium*. Big bangs are cyclic, the universe is a self-renewing system. It has no beginning and no end. The increase of the entropy of matter after the big bang is only temporary. In black holes the energy of matter disintegrates back into pure unstructured gravitational energy that has no entropy.

Pressure, temperature and density in singularity of the big bang and black holes, have infinite values. The physical circumstances of transformation "gravitational ether --> matter" and back are infinite. This presents one of the matters that create difficulties in the unification of General Relativity and Quantum Mechanics.

Experiments in weightlessness show that functioning of organisms is related with the changes of the strength of gravitational field, with the other words, with the changes of the density of gravitational ether.

Spaceflight induces a cephalad redistribution of fluid volume and blood flow within the human body and space motion sickness, which has a problem during first few days of spaceflight, could be related to these changes in fluid status and in blood flow of the cerebrum and vestibular system (6).

In weightlessness there is a decreased activity of spinal ganglia neurons of the hypothalamic nuclei producing arginine, vasopressin and growth hormone releasing factor. Structural changes of the somatosensory cortex and spinal ganglia suggest a decreased afferant flow to the somatosensory cortex in microgravity. The results characterise the mechanisms of structural adaptation to a decreased afferant flow in microgravity by the neurons in the hemisphere cortex and brain stem nuclei. So, under microgravity there is a neuron hypoactivity (7).

Microgravity has a direct influence on bone fracture healing because of poor production of bone callus in microgravity : there is an increase volume of osteoid and a decrease in the number and activity of osteoblasts (8).

Experiments with Lumbricus Teresticus have shown that the weight of a worm is greater when alive than when it is dead. Gravitational ether is denser around living organisms than around dead ones.

For a living organism to function additional density of gravitational ether is needed. Preliminary experiments have been carried out at the Bio-technical Faculty, Ljubljana, Slovenia in June 1987. Measurements have been performed on a Mettler Zurich M5 scale. Six test-tubes were filled with three millilitres of a water solution made out of meat and sugar. Four test-tubes were used and a fungus was put into two of the test-tubes. All of test tubes were welded airtight. The weight difference between test-tubes was measured for ten days. After three days of growth, the weight of test-tubes with the fungus increased (on average) 34 micrograms and in last seven days remains unchanged. The experiment was carried out in sterile circumstances. Here the biomass is increasing by incorporating nonliving substances and could be represented by the following equation:

$$Fg_{dead} + dFg = Fg_{living}$$

where Fg_{dead} is the weight of the nonliving organisms, dm is the change of the weight of the system, and Fg_{living} is the weight of the living organisms.

In another experiment, two test-tubes were filled with 5 grams of Californian worms with distilled water. All of the test-tubes were then welded airtight. The weight difference between test-tubes was measured for 5 hours. At the end of the first hour there was no appreciable difference but at the end of the second and third hour there was an decreased weight of 4.5 micrograms on average. This weight then remained stable for the next 2 hours most likely due to there no longer being any living organisms. This change in weight due to the change of organisms from a living condition to a nonliving one could be shown with the following equation:

$$Fg_{living} = Fg_{dead} + dFg \ .$$

These experiments were repeated from August to September of 1988 at the Facilty for Natural Science and Technology, Ljubljana. Two Mettler Zurich scales, type H20T were used in the measurements. Identical results were obtained. A test-tube was filled with 70 grams of live Californian worms and a small test-tube was filled with 0.25 ml of 36% water solution of formaldehyde. The control test tube contained 70 ml of distilled water with a small test tube of formaldehyde inside. Both test tubes were welded, wiped clean with 70% ethanol, and put into the weighing chamber of the balance. Approximately one hour was allowed for acclimatization. Later both test-tubes were measured three times at intervals of five minutes. Then the test tubes were turned upside down to spill the solution of formaldehyde and again they were measured seven times at intervals of fifteen minutes. The weight of the test-tube with the worms was found to have increased in the first 3 minutes after the poisoning on average for an average weight of 60 micrograms and it then went down. Fifteen minutes after poisoning, the weight diminished on average by 93.6 micrograms.

This last experiment was repeated twelve times. The standard deviation amounted to 16 micrograms. The pressure in both test tubes was one atmosphere for the entire duration of the experiment as well as the temperature remaining unchanged. Neither the pressure nor the temperature could have therefore been the cause for the change in the weight.

In 1997the results of the experiments were published in the "Newsletter" nr. 18-19 of Monterey Institute for Study of Alternative Healing Arts, California. On March 3rd 1998, Dr. Shiuji Inomata from Japan informed the editor (S. Savva) that Dr. Kaoru Kavada got similar results using rats as the experimental organism, again in a closed system (9).

Duncan MacDougall experiments have shown decreasing of the man's weight at the time of death; density of gravitational ether is stronger on the living man than on the same dead one (10).

Studies by Penrose and Hameroff (11,12) suggest that the force of quantum gravity acting on the mass of neurones within the brain may be responsible for the emergence of consciousness. The process is fundamentally related to the influence of quantum gravity on microtubule networks within the neurones. Loss of the weight at the time of man's death supports studies by Penrose and Hameroff; it shows that gravitational force is acting on the mass of living neurones stronger as on the mass of the same death ones.

All events in the universe including evolution are happening in gravitational ether. Gravitational ether can be described as a four-dimensional reality in which three-dimensional material objects are floating. Gravitational ether do not finishes at the surface of an object or living organism, its also extends inside of it.

In gravitational ether time does not runs. Time can only be experienced, and not clearly perceived as can matter and cosmic space. There is no clear evidence of existence of time as a physical entity. The question arises: How can time be experienced without being perceived? The answer is given by analysing the rational way of experiencing. The eyes perceive a stream of irreversible change of an event: change A is transforming into B, B is transforming into C and so on. Once elaborated by the mind, the irreversible stream of change is experienced as a linear running of time. Change A as past, B as present, C as future. Time is created by the rational part of the mind. Through time the perception of change of an object or an event can be elaborated scientifically. From the point of the gravitational ether makes no sense that the morning is before the evening, that the father is born before the son.

Immanuel Kant was right in saying that time exists only as a structure of the mind: space and time are not realities existing in themselves. In a word, they are subjective forms (13). Linear time and three-dimensional space exist only as structures of the rational scientific mind. Through these mental structures science can elaborate an event

In scientific experiments we observe constant stream of irreversible physical, chemical and biological change. Change X1 is transforming into X2, change X2 into X3 and so on. Time exists only as a stream of this change. We can measure with clocks its duration, speed, and numerical order.

Change run slower in those parts of the universe where the density of gravitational ether is bigger and the gravitational force is stronger. Experiments confirm that the clocks near the sea in Venice are running slower than on the mountain Monte Rosa. Gravity is stronger near sea level than on the top of Monte Rosa. To be empirically consequent, one has to admit that by means of elementary perception (sight), it is only possible to conclude that the speed of clocks on Monte Rosa is faster than in the city of Venice. It is only by indirect reasoning that we interpret different clock speeds as if they mean different speeds of time stream. By observing the clocks it is only possible to state that the speed of irreversible change is slower where the density of gravitational field is higher, and is faster where the density of gravitational field is smaller. Time exists only as stream of change (14).

Time considered as stream of change, gets totally integrated with space. The image of space-time develops into the image of timeless gravitational ether in which change is running. As gravitational ether has no entropy matter has a permanent tendency to evolve in living organisms with higher organisation. Gravitational ether is an universal force that creates life. The evolution of life can be described as a cosmic negantropic process in which the entropy of living matter decreases towards a state of non entropy of gravitational ether.

This idea is supported by the fact that the whole universe is in a state of chemical evolution. In all observable cosmic space basic organic molecules that are necessary for the development of life have

been discovered. On the planets similar to our planet chemical evolution continues into biological evolution. In the universe there are many planets with solar systems similar to ours. Life could also develop there. Universe is physically homogeneous, circumstances for life to develop are same in the whole universe. Evolution of life on the planet Earth can be understood as a part of an universal process.

There are some hints that life could have been brought to earth from elsewhere in our solar system or even galaxy. Life tends towards more complexity but Homo Sapiens is not the pride of creation but rather an accident in evolution. Humans may disappear as they have come but evolution will continue. Life as such is not an accident but a function of our universe (15).

3. Conclusion

The Universe is a timeless phenomena composed of one type of energy. When this energy is structured it appears as matter, when it is unstructured it appears as the gravitational ether that cosmic space is made of. Gravitational ether is essential for living organisms to function and for life to evolve.

References

1. Stephen Hawking, 1988, The Brief History of Time, Routlege & Kegal Paul
2. Einstein, 1982, Sidelights on Relativity, Dover Publications
3. Ludwik Kostro, 2000, Einstein And The Ether, Aperion, p.78
4. Ibid., p.124
5. Sorli,2002, Conscious Experience of The Universe, The Journal of Psychospiritual Transformation, Numer 3, PSRI PRESS, New York
6. US Army ,1993, Madigan Army Medical Centre, Tacoma, Washington " Cerebral blood velocity and other cardiovascular responses to 2 days head down tilt. Journal of Applied Physiology, 74 (1): 319-25
7. Krasnov IB,1994, Institute of Biomedical Problems, Moscow, Russia. "Gravitational neuromorphology", Advances in Space Biology and Medicine 4:85-110,
8. Durnova G.N., Burkovskaia TE. Voraotnikova E.V., Kaplanskil A.S., Arustamov O.V., 1991,"The effect of weightlessness on fracture healing of rats flown on Biosatellite Cosmos 2044", Kosmicheskaia Biologiia I Aviakosmosmicheskaia Meditsina 25 (5):29-33,
9. Sorli, 2001, Additional Roundness of Space-Time and Unknown Vacuum Energies in Living Organisms, Frontier Perspectives, Vol.10, Nr. 2
10. Duncan MacDougall M.D. 1907, 'Hypothesis Concerning Soul Substance Together with Experimental Evidence of The Existence of Such Substance', American Medicine, April (www.artbell.com/duncan.html)
11. Penrose, Roger,1994, 'Shadows of the Mind' (Oxford) pp377-391.
12. Hameroff, Stuart,1994, 'Quantum Coherence in Microtubules: A Neural Basis for Emergent Consciousness?' , Journal of Consciousness Studies Vol.1, No.1, pp91-118
13. Immanuel Kant, 1999, Critique of Pure Reason, Amazon
14. Sorli, 2002, Time as Stream Of Change, Journal of Theoretics, Vol. 4, Nr. 6
15. Stephan Krall, 2002, Life and Light, Network, Journal of Scientific And Medical Network, Num.78

On radiating star subsurface

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1. Introduction

The Einstein equations are insufficient for star models with the interior energy-momentum tensor of the perfect isotropic fluid and more general energy-momentum tensors. The number of unknown functions occurs to be greater then the number of equations. The additional assumptions are required. These assumptions can often significantly simplify the model equations and permit to obtain their exact solutions as in the works [1] and [2]. But only few of such simplifying speculations have clear physical interpretation. Another approach for the problem is then possible. One can use more physically certain additional equations by refusing the exact solutions for the sake of the approximate ones.

The total star model must describe both its interior and exterior parts. Then the junction of these two space-times becomes very important. We use the well-known Darmois junction formalism [3] as it has been described in [4]. It gives us more information then the Singe-O'Brien matching conditions [5] which are also satisfied.

For the exterior space-time we choose the Vaidya solution described in the radiating Bondi coordinates by the metric

$$ds^2 = (1 - \frac{2M(u)}{r})du^2 + 2dudr - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{1}$$

and the energy-momentum tensor of radiation

$$T^{(rad)}_{\rho\sigma} = -\frac{\dot{M}}{4\pi r^2} l_\rho l_\sigma, \tag{2}$$

where u, r, θ, φ are consequently time, radial and angle coordinates, $M(u)$ is the mass of the star and l_ρ is an isotropic geodesic vector,

$$l_\rho l^\sigma = 0, \;\; l_{\rho;\sigma} l^\sigma = 0. \tag{3}$$

Here and after dot means a derivative by the time coordinate u and semicolon means the covariant derivative. For the mass M independent from time the metric (1) turns to the interior Schwarzschild solution. The Singe-O'Brien junction of Vaidya solution with different exact interior solutions has been researched for example in [1], [6]-[7] and Darmois junction - in [2], [9]. There are also papers for Darmois matching on thin shells, see for example [10].

We choose the interior metric in general spherically symmetric form

$$ds^2 = e^{2\beta(u,r)}(1 - \frac{2m(u,r)}{r})du^2 + 2e^{\beta(u,r)}dudr - r^2(d\theta^2 + \sin^2\theta d\varphi^2) \tag{4}$$

with two unknown metric functions $\beta(u,r)$ and $m(u,r)$. We search these functions as the series in $\delta(u) = (R(u)-r)/R(u)$, where $R(u)$ is the radius of a star. The solution acquired in such a form describes the subsurface of star well enough but fails in its inner layers.

The system of units with the light velocity and the Newtonian gravitational constant equal to unity is used. The Einstein equations in this system take form

$$G_{\rho\sigma} = -8\pi T_{\rho\sigma}, \tag{5}$$

where $G_{\rho\sigma}$ is the Einstein tensor and $T_{\rho\sigma}$ is the energy-momentum tensor.

2. Method description

The stress-energy tensor for the interior part of the model we choose in the form

$$T_{\rho\sigma l} = T_{\rho\sigma}^{(fl)} + \frac{L_{in}(u,r)}{4\pi r^2} l_\rho l_\sigma + T_{\rho\sigma}^{(stress)}, \tag{6}$$

where $T_{\rho\sigma}^{(fl)} = (\mu+p)u_\rho u_\sigma - pg_{\rho\sigma}$ is the tensor of the perfect isotropic fluid, μ is its energy density, p is a pressure, $u^\sigma = dx^\sigma/ds$ is a 4-velocity, $u^\sigma u_\sigma = 1$, $L_{in}(u, r)$ is an arbitrary function determining the energy production,

$$T_{\rho\sigma}^{(stress)} = \xi(u,r)\gamma_{\rho\sigma} \tag{7}$$

is a tensor respondent to the anisotropy [1] caused by the radiation passing through the fluid and the origin of tangent stresses, $\gamma_{\rho\sigma}$ is in its core the metric of 2-sphere so as $\gamma_{\rho\sigma}dx^\rho dx^\sigma = d\theta^2+\sin^2\theta d\varphi^2$, $\xi(u, r)$ is an arbitrary function.

We don't use the frame of reference comoving with the stellar fluid and the 4-velocity has the form

$$u^\alpha = \frac{[1, v(u,r), 0, 0]}{\sqrt{(1-2m/r)e^\beta + 2v(u,r)e^\beta}}, \tag{8}$$

where the 3-velocity of the fluid $v(u, r)$ isn't equal to zero and on the junction surface differs with the velocity of this surface as opposed to [2] and [9]. So the radiation of the star partially emerges due to the surface sublimating - transformation of the surface particles into the radiation. The surface sublimating for homogeneous model is considered in [7].

The Einstein equations for the energy-momentum tensor (6) and metric (4) take the form

$$\frac{\dot m}{r^2} - \frac{m'}{r^2}e^\beta(1-\frac{2m}{r}) + \frac{4\pi\mu(v+e^\beta(1-2m/r))^2}{2v+e^\beta(1-2m/r)} + \frac{4\pi p v^2}{2v+e^\beta(1-2m/r)} + \frac{8\pi L_{in}e^{-\beta}}{r\text{^}2} = 0; \tag{9}$$

$$\frac{m'}{r^2} - \frac{4\pi\mu(v+e^\beta(1-2m/r))}{2v+e^\beta(1-2m/r)} - \frac{4\pi p v}{2v+e^\beta(1-2m/r)} = 0; \tag{11}$$

$$\frac{\beta'}{r}e^{-\beta} - \frac{4\pi(\mu+p)}{2v+e^\beta(1-2m/r)} = 0; \tag{12}$$

$$rm'' + r^2e^{-\beta}\dot\beta' - r^2(1-\frac{2m}{r})(\beta''+\beta'^2) + 3rm'\beta' - (r+m)\beta' + pr^2 + \xi = 0. \tag{13}$$

We postulate the coordinate u to be equivalent to the time of the Galilean observer in the Vaidya space-time. The junction surface Σ is given with $r = R(u)$.

The Darmois conditions for the joining of two parts of the stellar model consist in continuity of the quadrics

$$g_{ij}d\xi^i d\xi^j = \frac{\partial x^\rho}{\partial\xi^i}\frac{\partial x^\sigma}{\partial\xi^j} g_{\rho\sigma}d\xi^i d\xi^j \tag{14}$$

and

$$K_{ij}d\xi^i d\xi^j = \frac{\partial x^\rho}{\partial\xi^i}\frac{\partial x^\sigma}{\partial\xi^j} n_{\rho;\sigma}d\xi^i d\xi^j \tag{15}$$

on Σ where ξ^i are coordinates on Σ in our case (u, θ, φ) and n_ρ is the normal to Σ. They lead to the continuity of metric functions on the surface and the equation

$$\dot m|_R + (3R\dot R + 1 - \frac{2M}{R})m'|_R + R\dot R\dot\beta|_R - R((1-\frac{2M}{R})^2 + 3(1-\frac{2M}{R})\dot R + \dot R^2)\beta'|_R = 0. \tag{16}$$

This equation along with Einstein equations allow to find values of the pressure and the first derivatives of the metric coefficients on the surface:

$$p_R = \frac{\zeta \Delta L}{8\pi R^2}; \tag{17}$$

$$\dot{m}\big|_R = 2\pi\zeta\mu_R R^2 \dot{R}^2 + \frac{1}{2}(\zeta\Delta L - 8\pi\mu_R R^2)\dot{R} - L_{out}; \tag{18}$$

$$m'\big|_R = -2\pi\zeta\mu_R R^2 \dot{R} - \frac{1}{2}(\zeta\Delta L - 8\pi\mu_R R^2); \tag{19}$$

$$\dot{\beta}\big|_R = -2\pi\zeta\mu_R R\dot{R}; \tag{20}$$

$$\beta'\big|_R = 2\pi\zeta\mu_R R, \tag{21}$$

where $\Delta L = L_{out}\text{-}L_{in}|_R$, $L_{out} = -dM/du$ is the luminosity of the star and

$$\frac{2}{\zeta} = 1 - \frac{2M}{R} + 2\dot{R} + \frac{\Delta L}{4\pi\mu_R R^2}. \tag{22}$$

Anisotropy $\xi(u, r)$ doesn't appear in these equations.

The Einstein equations include the second derivatives of the metric functions in the linear way. All elder derivatives on the surface are constrained by the linear equations and can be found easily. The approximate solution is written in the form of the Taylor series:

$$m(u,r) \approx M(u) - R(u)m'(u,r)\big|_R \delta(u) + \frac{1}{2}R(u)^2 m''(u,r)\big|_R \delta(u)^2; \tag{23}$$

$$\beta(u,r) \approx -R(u)\beta'(u,r)\big|_R \delta(u) + \frac{1}{2}R(u)^2 \beta''(u,r)\big|_R \delta(u)^2; \tag{24}$$

$$\mu(u,r) \approx \mu(u,r)\big|_R - R(u)\mu'(u,r)\big|_R \delta(u). \tag{25}$$

This solution depends on four arbitrary functions $L_{in}(u, r)$, $\xi(u, r)$, $R(u)$ and $M(u)$ and also on the choice of the equation of state.

The velocity of the fluid on the surface

$$v_R = \dot{R} + \frac{\Delta L}{8\pi\mu_R R^2} \tag{26}$$

is equal to the velocity of the surface itself only with $\Delta L = 0$. This means that due to the sublimating the surface moves separately from the particles generating it. In particular the absolute value of dR/du can be greater then the speed of light.

3. The model with constant radius and luminosity

Consider the results we obtain with the discussed method. As the energy density of the stellar sub-surface is small unlike the density of the inner layers of the star we choose the politropic equation of state for our example:

$$p = p_0 (\frac{\mu}{\mu_0})^\gamma. \tag{27}$$

Arbitrary functions we choose in the simplest form, R, ΔL and L_{out} are constant and positive then the total mass of the star decreases linearly:

$$M = M_0 - L_{out} u. \tag{28}$$

The assumption of the constant radius and luminosity are applicable for any radiating model but only in a small enough time periods.

In this case the equations for the derivatives (13)-(16) take the simpler form of

$$\dot{m}\big|_R = -L_{out} < 0; \tag{29}$$

$$m'|_R = -4\pi(p_R - \mu_R)R^2 > 0; \tag{30}$$

$$\dot{\beta}|_R = 0; \tag{31}$$

$$\beta'|_R = \frac{16\pi^2 \mu_R p_R R^3}{\Delta L} > 0. \tag{32}$$

One can get an implicit dependence of all unknown functions from the time u by taking the energy density μ_R as the parameter and evaluating u through it:

$$u = \frac{1}{L_{out}}\left(-\frac{R\chi}{2} - \frac{\Delta L}{8\pi\mu_R R} + \frac{\Delta L}{8\pi p_R R}\right), \tag{33}$$

where $\chi = 1\text{-}2M_0/R$. The time $u = 0$ corresponds to the initial value of the surface energy density μ_R equal to

$$\mu_0 = \frac{p_0 \Delta L}{\Delta L - 4\pi\chi p_0 R^2}. \tag{34}$$

The total time derivative of the density is equal to

$$\frac{d\mu_R}{du} = \frac{8\pi R L_{out}}{\Delta L} \cdot \frac{p_R^2 \mu_R^2}{p_R^2 - v_s^4 \mu_R^2}, \tag{35}$$

where $v_s^2 = (dp/d\mu)_R$ is the square of the sound velocity on the surface. This derivative is negative while the total time derivatives of functions $m'|_R$ and $\beta'|_R$ are positive.

Since for constant radius and luminosity we consider only the small time intervals the time dependence of the stellar characteristics is not so interesting as their dependence from seven parameters of the star M_0, L_{out}, R, ΔL, p_0, γ and $\xi(0, R)$, and possible restrictions for these parameters.

The first constraint follows from the requirement of the limited and nonnegative initial the energy density on the surface:

$$\Delta L - 4\pi\chi p_0 R^2 > 0. \tag{36}$$

The second constraint arises from the physical condition of the energy density decreasing from center to surface, namely the derivative μ' must not be greater then zero everywhere inside the star including its surface. In the obtained approximation. μ' depends on the time only.

Let's now $p_0 = (\Delta L/4\pi\chi R^2)(1\text{-}\delta p)$, where δp is an arbitrary small positive quantity so that the condition (31) is satisfied. Then the sign $\mu'|_R$ concurs to the first approximation by δp with the sign of the expression

$$(\chi(1-\chi) + 2L_{out})\gamma + \chi(1-\chi) - 6L_{out})\delta p. \tag{37}$$

So for $\mu'|_R < 0$

$$\gamma > \frac{\chi(1-\chi) - 6L_{out}}{\chi(1-\chi) + 2L_{out}}\delta p. \tag{38}$$

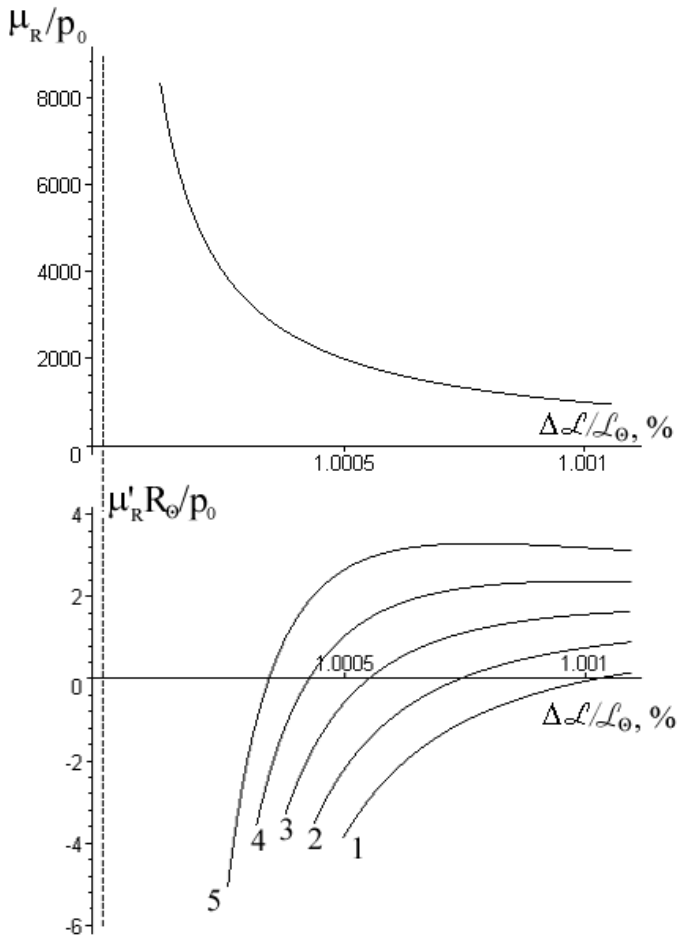

Fig. 1 Density and its first derivative on the surface vs. the parameters $\xi(0, R)$ and ΔL.
1: $\xi(0, R) = 0$; 2: $\xi(0, R) = L_\Theta \cdot 10^{-33}$; 3: $\xi(0, R) = 2L_\Theta \cdot 10^{-33}$; 4: $\xi(0, R) = 3L_\Theta \cdot 10^{-33}$; 5: $\xi(0, R) = 4L_\Theta \cdot 10^{-33}$.

This inequality holds true for any $\delta p < 1$ as for the physically appropriate polytropic equations of state $\gamma > 1$. Then one can always select the relative departure of initial pressure from critical value $\Delta L/4\pi\chi R^2$ small enough for the density to decrease near the surface. But for the greater deviations of expression (31) from zero the numerical approach shows that the derivative $\mu'|_R$ becomes positive (fig. 1). That is the conditions of the nonnegative energy density and nonpositive first derivative of the energy density are opposed to each other and the parameters of the model take on values only in a certain range.

We considered the new method of radiating star subsurface construction based on the Darmois junction of an interior general spherically symmetric space-time with the exterior space-time of Vaidya. The method permits to investigate the interplay of such integral stellar characteristics as mass, radius and luminosity with the subsurface values of the energy density, pressure and metric coefficients. The expressions for the components of the metric tensor obtained in the form of Taylor series by powers of $R(u)$-r one can regard as the approximate solution of Einstein equations. For the case of constant luminosity and radius we have succeed with the help of computer system Maple to find the values of the second derivatives of metric functions and the linear approximation for density and pressure. We have found the constrains for the stellar parameters which follows from the nonnegative and non-increasing the energy density. These restrictions must be also maintained in every time moment for the models with variable radius and luminosity.

References

1. Baranov A.M., Paklin N.N. On interior source of the Vaidya solution // Izv.Vuz.(Fizika), 1988. - No.3. - P. 36.
2. Herrera L., Jimenez J., Ruggeri G.I. Evolution of radiating fluid spheres in General Relativity//Phys. Rev. D, **22**, 10 (1980) 2305.
3. Darmois G. Les equations de la gravitation Einsteinienne//Memorial des science Mathematiques, Fascicule XXV. Gauthier-Villairs. Paris, 1927
4. Musgrave P., Lake K. Junctions and thin shells in general relativity using computer algebra I: The Darmois-Israel Formalism // Class. Quantum Grav., **13** (1996) 1885--1900.
5. Singe J.L. Relativity: General Theory. - Amsterdam,1960.
6. Baranov A.M., Koksharov A.I., Paklin N.N. The evolution radiating relativistic sources. 1. Nonpascal energy-momentum tensor. The generalization of homogeneous model //Dep.VINITI AS USSR 06.12.89, No.7226 - B 89 (in Russian).
7. Baranov A.M., Koksharov A.I., Paklin N.N. The evolution radiating relativistic sources. 2. The schwarzschildlike model //Dep.VINITI AS USSR 06.12.89, No.7197 - B 89 (in Russian).
8. Baranov A.M., Paklin N.N. The evolution radiating relativistic sources. 3. The generalization of Tolmen's IV solution //Dep.VINITI AS USSR 02.07.90, No.3704 - B 90 (in Russian).
9. Fayos F., Senovilla J.M.M., Torres R. Spherically symmetric models for charged radiating stars and voids: Theoretical approach//arXiv: gr-qc/0206076
10. Villas da Rocha J.F., Anzhong Wang, Santos N.O. Gravitational Collapse of Perfect Fluid//Phys.Lett., **A255** (1999) 213-220
11. Herrera L., Nunes L.A. Evolution of radiating spheres in general relativity: a seminumerical approach.- Merida: Universidad de Los Andes, 1990.

Extra gravitational tug

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A simple application of Newton's Gravitational force, improved by the Autodynamics Universal Gravitation, explains the extra gravitational force that slows down Pioneer 10/11, Galileo and Ulysses spacecraft, and the Ulysses anomaly. This constant acceleration acting on the spacecraft towards the Sun direction has a magnitude of ~ 8.5 x 10^-8 cm/s^2. So far, the alternative proposed here appears to be the only one that explains the phenomenon, because a number of possible causes have been rule out, specially the most notorious of them, the Neutrino and General Relativity among many others. Will be shown that, taking the solar mass as 1.9891 10^33 g, we need only increase the solar mass by 1.690735 10^26 g to explain the current acceleration found. This is only a partial solution because the full one involves the solar increasing mass per unit of time.

Autodynamics (AD) Universal Gravitation postulates that the absorption of Pico-Gravitons[1] by gravitational force, or gravitational acceleration. Was proved by AD that this extra gravitational force also results in the planets' acceleration in their orbital motion, known as the planetary perihelion advance and the binary star precession. All the stellar bodies in the Universe causes the Setting X as the fraction that dividing M⊙ gives the increasing mass per time unit, the expression for X is:

$$X^{\frac{4}{t+2}}\left(1+\frac{1}{X}\right) = \left[\frac{(t+2)\,\phi}{2\,G\sqrt{\frac{M_\Theta}{r^3}}}\right]^{\frac{2}{t+2}} \qquad \textbf{(1)}$$

Where t = time, ϕ = angular displacement, G = gravitational constant, M ⊙ = solar mass, r = radio or distance.

Taking 43" per century as the most secure observational value for Mercury's perihelion advance, X is calculated and the solar increasing mass is known.

1.- AD's actual solution.

1-1.- Classical Mechanics:

$$F = m\,a \qquad \textbf{(2)}$$

$$F = m\,\frac{M_\Theta\,G}{r^2} \qquad \textbf{(3)}$$

F = force, m = mass, a = acceleration, M_Θ = solar mass, G = gravitational constant, r = distance.

1-2.- Autodynamics Mechanics.

$$F' = m'\,a' \qquad \textbf{(4)}$$

$$F' = m'\,\frac{M'_\Theta\,G}{r^2} \qquad \textbf{(5)}$$

The [′] means "increment."

From **(2)** and **(3)**

$$a = \frac{M_\Theta\,G}{r^2} \qquad \textbf{(6)}$$

From **(4)** and **(5)**

$$a' = \frac{M'_\Theta\,G}{r^2} \qquad \textbf{(7)}$$

Dividing **(7)** by **(6)** and using

$$M'_\Theta = M_\Theta + M_\Theta in \qquad \textbf{(8)}$$

$M_\Theta in =$ Solar increasing mass.

We have:

$$\frac{a'}{a} = \left(1+\frac{M_\Theta in}{M_\Theta}\right) \qquad \textbf{(9)}$$

$$\text{or} \qquad \frac{a'}{a} - 1 = \frac{M_\Theta in}{M_\Theta} \qquad \textbf{(10)}$$

In equation **(9)**, the second term inside the parentheses represents the AD increasing mass that it is "constant" in a short time interval, and equation

(10) gives the AD acceleration regarding the Classical Mechanics value. If **a = 1** (reduction to unit) equation **(10)** is

$$a'' - a = \frac{M_{\Theta} in}{M_{\Theta}} \qquad \textbf{(11)}$$

If a = 1 a'' = 1.001 000 108 1 **(12)**

If a = a a'' = a + 8.1 10^{-8} **(13)**

(See example in Endnote)

The acceleration difference is constant and equal to the ratio between the increasing mass and the initial mass.

As was mentioned in the Abstract, taking the solar mass as **1.9891 10^{33} g** one increment of **1.690735 10^{26}g** eliminate the spacecraft slow-down. A different condition will be explained further.

2.- AD's permanent solution.

The real solution is obtained by applying the solar increasing mass per unit time starting when the spacecraft is lunched. In equation **(9)** or **(10)** it is possible to see that the relation between Classical Mechanics and AD Mechanics is constant and proportional to the increasing difference between actual solar mass and the solar mass value as consequence of Pico-Graviton[2] absorption given by equation **(1)**. Of course, a more complicated calculation is needed when the spacecraft is traveling inside the solar system, because it is necessary take into account the extra "attraction" provoked by other planets in the neighborhood, via their own increasing mass. This could increase the "extra tug" to larger values as is mentioned for Ulysses[3].

3.- Comments.

In their paper[3] the authors say: "… to speculate on the possibility that the origin of the anomalous signal is a new physics[10]. This is true even though the probability is that some standard physics or some as-jet-unknown systematic will be found to explain this acceleration. The probability is of interest in itself giving that we have found no plausible explanation so far."

It is a new Physics in the sense that it is AD's Physics. Will be"Standard Physics." AD know the "as-jet-unknown."

The following statement[3] is relevant regarding AD: "Is it dark matter or a modification of gravity? Unfortunately, neither easily works."

It is a modification of gravity. Technically speaking is a dynamical modification of the Newton equation.

They conclude: "taking account the accuracy of the ephemeris" only a variation of "a few times 10^{-6} M_{Θ} is allowed."

The AD increasing mass is only **1.690735 10^{-7} g**, that is one order of magnitude smaller. This, of course, is in this particular situation. In a real case this value will decrease significantly. If a fraction of this acceleration is explained by conventional physics, for example radiation, but a remaining is left, AD's explanation is still valid. Theoretically, this value could be reduced to around a 10 % or even less. The real solution, as was mentioned above, is to apply the increasing mass when the spacecraft is launched.

The quotes made by an author of the original paper[4]: "Milgrom and Sanders, respectively, have shown that a force such as this would produce an anomalous perihelion shift in Mercury and Icarus, respectively, that is not seen (if the force were gravity)," is irrelevant for AD. The force doesn't produce any "anomaly." The force produces the perihelion advance. "The perihelion advance doesn't exist as "spontaneous generation." The perihelion advance exists because a force (energy) produces it.

While a New Paradigm is unknown to the general community, it is difficult to see the truth. To show this we will quote John Ries,[5] a planetary scientist at the University of Texas at Austin:

"I cannot believe a new gravitational force is involved, because that should affect the motion of the planets."

This is precisely what AD said at the beginning "…this extra gravitational force also results in the

planet acceleration in their orbital motion known as the planet perihelion advance."

This also is related to the quotation in the original paper (equation 3 in page 3 right column):

"The anomalous acceleration is too large to have gone undetected in planetary orbit, particularly for Earth and Mars. NASA's Viking mission provided radio-ranging measurements to an accuracy of about 12 m [15,16]. If a planet experiences a small, anomalous, radial acceleration, a_A, its orbital radius r is perturbed by"

Equation 3 holds in the circular orbit limit and the following explanation hold also in the same condition.

1.- The solar increasing mass will increase the planet centripetal force and the planet will start to move in the Sun's direction. The vectorial composition between the planet's tangential velocity and the centripetal velocity will be larger that the tangential velocity and an extra centrifugal force will push the planet back close to its orbit.

2.- Simultaneously with the solar increasing mass, the planet has its own increasing mass, and an extra centrifugal force is created that equal the centripetal force created by the Sun. The Planet is in its original orbit. Its larger velocity explains its perihelion advance.

In an elliptical orbit the two phenomena co-exist in a complicated manner: The planet follow its orbit suffering an extra acceleration that we call perihelion advance. This will be further analyzed in detail.

All this is related to the Ulysses anomalies. It is traveling from 1.3 AU at perihelion to near Jupiter at 5.4 AU.

The Ulysses anomaly has a simple explanation. The increasing solar and planetary mass develops a new celestial mechanics.

The spacecraft flying in the Sun's direction suffers an extra increasing "attraction" that provokes an extra increasing velocity. When the spacecraft is receding from the Sun, the solar mass continues to increases and the spacecraft will suffer a slow-down in velocity. This is true until it starts to suffer the Jupiter "attraction" due to its increasing mass, given to the spacecraft an extra velocity. When the spacecraft is receding from Jupiter it suffers an extra slow-down because Jupiter, like the Sun, is constantly increasing its mass. Of course this is related to Newton's static gravitational force.

The spacecraft cycle will start again when it flies to the Sun, but with a very important difference regarding the previous cycle: The solar mass is larger than in the previous cycle and the increasing mass is also larger.

This explains the "anomalies." These "anomalies" exist, of course, because the Newton equation doesn't describe the phenomenon correctly. We are confronting a new dynamical law of gravitation.

Of course, we are investigating where is the limit, or eccentricity, between the perihelion advance and the spacecraft or planets slow-down.

If the force discovery by JPL's team is confirmed, even if the values are smaller, it is possible, if the new celestial dynamics is correct, that the spacecraft never will escape from the Solar System. They will travel for thousand of years as planets or comets of the Sun.(Hyperbolic orbit to elliptical orbit).

References

1.- R. L. Carezani, Autodynamics. Fundamental Basis for a New Relativistic Mechanics. SAA, 801 Pine Ave. # 211, Long Beach, CA. 90813. USA

2.- U. J. Balis. Private communication. Pico-Graviton mass =~ 8.08 10^-82 kg, density =~ 6.7 10^53 particles/m^3 and velocity =~ 27 c.

3.- J. D. Anderson et al. "Indication, from Pioneer 10/11, Galileo, and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration." At http://xxx.lanl.gov code gr-qc/9808081 28 Aug. 1998. Phys. Rev. Lett. 81, 2858(1998).

4.- M. M. Nieto. Private communication October 20, 1998.

5.- C. Seife, "If the force is with them....". New Scientist, 12 September 1998.

Graviton Velocity and Gravity Speed

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Even though there exists a profuse literature on Gravitation velocity, the question is: Does this Gravitation velocity exist? Newton's gravitation velocity is instantaneous, equivalent to an infinite velocity, and his equation for Gravitation is working perfectly. In Einstein's theory, which equates space-time curvature to Gravitation, the propagation velocity has no meaning. Contradicting himself, Einstein accepted that Gravitation propagates at Light velocity, which is postulated as the maximum velocity in the Universe. Autodynamics, a Quantum Relativistic Mechanics accepts the Graviton – Le Sage's mundane particle - as Gravitation's carrier and accepts the Graviton's velocity, but proves that Gravity is a-temporal and exists in the whole space of the Universe without propagation velocity, and this coincides with Newton's apparently infinite velocity.

1. Introduction

Some months ago SAA received a simple question: **If the Moon does appear spontaneously in the sky, when will it be see on Earth?**

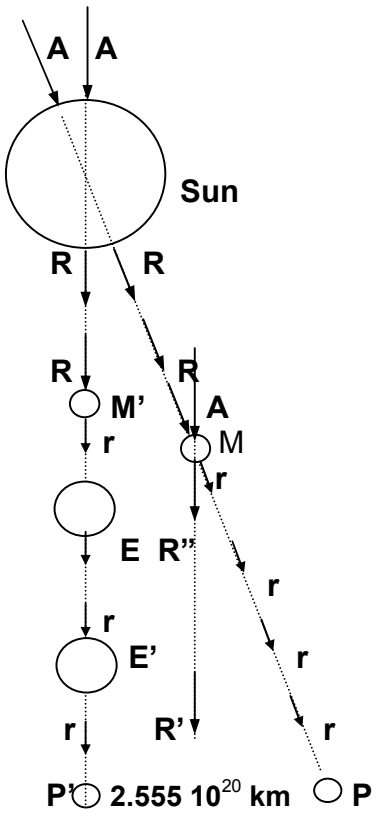

Fig 1

From the AD Universal Gravitation[1] point of view based on the Pico-Graviton, the answer is simple: The Moon will be detected by Gravity perturbation earlier than visually because the Graviton travels 27 times [2] faster than Light. Taking the distance to the Moon as 384 000 km, the light from there will take 1.28 seconds to arrive at the Earth but the Graviton will arrive in 0.0474 seconds. Is this always true? Is it true that with the Graviton traveling at 27 c, gravity itself is traveling at this same velocity?

2. AD's gravitation mechanism

The answer is **NO**. Gravity is a-temporal, it is always present and consequently its velocity appears to be infinite.

This problem, from the theoretical point of view, is identical to the "Special Relativity Failure of Simultaneity." In AD, there is no failure, and the problem doesn't ever arise.

Gravity propagation at-a-distance doesn't exist in AD Universal Gravitation because there is no Gravity propagation. What happens if the Moon doesn't "appear" spontaneously and existed before? To show the interaction clearly, a figure will be used.

If the Moon began its existence 1 million years ago, its gravitational influence reached a distance of 2.555 10^{20} km. This number, of course, is obtained by multiplying Moon's age x

Days in a Year x Hours in a day x Minutes in an Hour x Seconds in a Minute x Light speed x Graviton velocity.

Of course, if the Moon "appears spontaneously" the "Gravity" (Graviton) velocity is equal to 27 c, but for a system that exists for a long time, the Gravity velocity has no meaning. We can see this in Figure 1. Letting **A** be the gravitons received by the Sun and **R** be the gravitons leaving the Sun after absorption, this quantity arrives to the Moon at position M, from the time that the system exists. After the absorption by the Moon, the Graviton quantity **r** will continue traveling at 27 c to reach the distance of 2. 555 10^{20} km, at position P. When the Moon reaches position M' and the Earth is at E, nothing changes. If the Earth is at position E' the situation is exactly the same because the gravitational influence doesn't change except at a distance equal to 2.555 10^{20} km. That is, if the Earth doesn't exist at position E' the same situation is repeated as with the Moon at position M.

Of course, if the Moon at position M directly receives (as constantly happens) the Pico-Graviton quantity A, after absorption the Pico-Graviton R' > R will propagate the Moon's Gravitation to the same distance of 2.555 10^{20} km.

It is necessary always to have in mind that Gravitation exists between two bodies because those two bodies interchange Pico-Graviton after respective absorption. For example, the Sun and the body at point P' or the Moon at point M and the body at point P. The mutual process of Graviton absorption is what is called Gravitation. If the P body doesn't exist, M has no Gravity with respect to point P, as an abstract point. Without respective absorption by two bodies there is no gravitational force between those two bodies.

At position X-X, R from the Sun is traveling at 27 c but R exists all the way to the Earth and this happens from the time that the Sun and Earth first existed, from the time that Sun and Earth were formed.

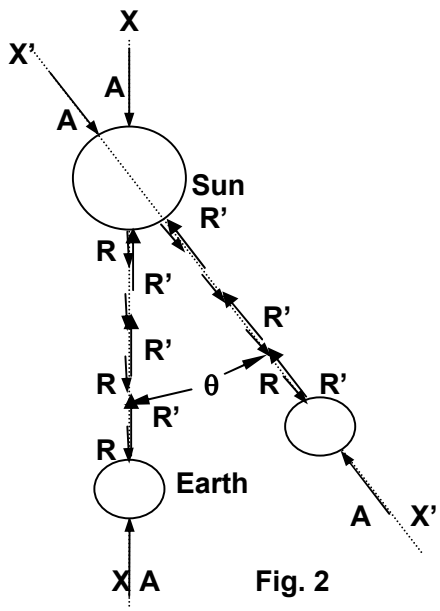

Fig. 2 represents the eternal action of Gravitation, that is, showing that Gravitation from the Sun to Earth or vice-versa is not actually transmitted: it exists, it is always present because Graviton action is constant and has existed forever

At position X'-X' everything is the same and really the angle θ is only the differential dθ or Δθ as infinitesimal change. Actually, any gravitational force from the Sun to Earth or vice-versa is transmitted. Gravitation between the Sun and Earth exists without anything transmitting it.

Gravitation between two bodies is not transmitted, it exists, but Gravitation perturbation travels at 27 c, if we accept this velocity as the Pico-Graviton velocity

To observers on Earth and all observers up to a distance of 2.555 10^{20} km in the first example, Gravity is always a-temporal, that is, apparently the Gravity's velocity is infinite. The same is shown in Fig. 2.

There is no attraction between the Sun and the Moon. The force produced by Pico-Gravitons on the Sun acts only on the Sun and the force produced by Pico-Graviton on Earth acts only on the Earth. Each force depends on the relative re-

lation between both bodies' Pico-Graviton absorption. Gravity is a local phenomenon without transmission. The Earth is falling in the Sun's direction because the local force pushes it and it orbits because it has tangential velocity. (Motion creation through increasing mass in AD's Universal Gravitation)

Reversing the argument, the conclusion supports the Graviton existence as a cause of Gravity. Gravitation based on the Graviton also explains the Spacecraft Slowdown [3], The Allais' Anomaly [4] and the Moon receding from the Earth [5].

3. Conclusion

Because in AD the Universe is eternal, the Gravity of each body is present throughout the Universe with respect to any other body. Consequently, there is no velocity of Gravitation propagation, there is no finite or infinite velocity: Simply, that velocity doesn't exist.

Of course any Gravity perturbation (Graviton quantity) will travel at the Graviton velocity. For example, Fig. 1, if Earth exists at position E', r, after Earth's absorption of gravitons, will be r'. This r' traveling at 27 c needs 1 million year to arrive at position P' but between Earth and the body at this position Gravitation existed eternally.

Close to the truth, with its infinite velocity, is the Newtonian conception, event though it includes no Gravity propagation. Nevertheless, using the Newtonian equation, all calculations supposing that Gravity is an instantaneous action give correct results.

AD goes a step further, showing that gravity is a-temporal. The only future problem is to measure the Pico-Graviton velocity.

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[1] .- R. L. Carezani. *Autodynamics. Fundamental Basis for a New Relativistic Mechanics.* SAA. 801 Pine Ave. # 211, Long Beach, CA, USA. LCCCN 9886262 and ISBN 0-p665533-0-4 or at http://www.autodynamics.org.

[2].- This velocity was first obtained theoretically by U. J. Balis, a researcher now at Harvard. This value equals the empirical value found by Carezani in calculating the average density of matter in the Universe.

[3].- Researchers at NASA apparently discovered and confirmed that the Spacecraft Pioneer 10/11, Galileo and Ulysses suffer an acceleration in the Sun's direction larger than that calculated by Newton's Gravitation. It is constant and it is not distance-dependent.
J. D. Anderson et al, *Indications from Pioneer 10/11, Galileo and Ulysses Data, of an Apparent Anomalous, Weak, Long-Range Acceleration,* Phys. Rev. Lett. 81, 2858(1998). URL http://xxx.lanl.gov, code gr-qc/9808081, 28 Aug 1998.

[4].- M. Allais, *Aero Space Engineering,* Set. 1959, p. 46-52; Oct. 1959, p. 51-55; Nov. 1959 p. 55; C.R.A.S (Fr.), 247, 1958, p. 1428. The Allais observation involves a pendulum's period variation and angular deviation.

[5].- J. G. Williams, X. X. Newhall and J. O. Dickey, *Relativity parameters determined from Lunar Laser Ranging*, Phys. Rev. D, 53, 6730(1996). They measured the Moon receding from the Earth.

The Principle of Equivalence

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A fundamental tenet of General Relativity, the Equivalence Principle, states that the ratio of Gravitational Mass to Inertial Mass is equal to all different bodies.

Even though this principle had been tested in the Laboratory, it has only recently been tested for bodies large enough to have a significant part of their mass coming from Gravitational self-energy.

A requirement of the so-called Strong Equivalence Principle is that all bodies fall with equal acceleration in a Gravitational Field, contributing the Gravitational Energy equally to the Gravitational and Inertial mass.

Inertial Mass is the mass opposed to the change of motion and Gravitational mass is the mass accelerated by the Gravitational Energy. But this Gravitational Energy according to the Equivalence Principle contributes equally to the Inertial and Gravitational Mass.

To Autodynamics (AD) Universal Gravitation, the Equivalence Principle is absolutely irrelevant because the Inertial Motion doesn't exist in the Cosmos. AD postulates that only Gravitational Systems exist.

1. Introduction

Modern Physical Science supports many different concepts and postulates introduced artificially to explain what is irrelevant to explain, because what is postulated doesn't really exist in Nature.

The SR Failure of Simultaneity doesn't exist, as AD proved. This failure exists only in SR because its equations – really Lorentz's equations – fail to describe the physical phenomenon, aggravated by the Light velocity as a limit speed in the Cosmos.

AD show through its equations that that failure doesn't exist, showing conceptually that Simultaneity depends on the information propagation velocity and it is avoided by the Light constant velocity.

Event though GR equates Gravitation to Space-Time-Curvature where the Gravitation velocity should be irrelevant, Einstein contradicts himself, accepting that Gravitation velocity is equal to the Light velocity, which he takes as the maximum velocity in the Universe. Also GR has no Mechanism or Machinery to explain the Gravitational Energy.

AD shows that the Gravitation velocity doesn't exist as it is explained in the other paper presented to this International Conference. (Graviton velocity and Gravity Speed).

The most recent astronomical observation is the Lunar Laser Ranging,[1] which permits relating Gravitational Physics with the Equivalent principle, and the conclusion is that it is confirmed.
As we will show further, the Equivalence Principle between Inertial mass and Gravitational mass doesn't exist, because the Inertial Motion doesn't exist in the Cosmos.

2. AD's Universal Gravitation

According to AD's Universal Gravitation [2] the Sun's gravitational mass increases every second in 2.047 570 7 10^{16} gram/second, to yield an extra gravitational acceleration equal to 1.029 395 555 10^{-17} cm/s^2. (See "E4.Universal Gravitation" on page 197)

If a body with **inertial mass** of 1 gram is being accelerated by gravitation, its **gravitational mass** will increase by the quantity equal to 1.030 483 519 74 10^{-17} gram.

This difference cannot be detected by our current technology, but this is not a historical problem because in Nature, in the Cosmos, no place exists without gravitation.

Consequently, there doesn't exist any difference between **inertial mass** and **gravitational mass,** because in the **Cosmos,** no **inertial systems** [3] exist. All systems in the **Cosmos** are **gravitational systems**, that is, they are all **accelerated systems.** [4]

This is not strange, because it is well known that in **AD Kinematics doesn't exist;** motion without expending energy doesn't exist. All phenomena are **dynamic.**

AD postulates that **Gravitation has no velocity**: The **Graviton has velocity** but **Gravitation has none**. Gravitation exists, but is not transmitted; it is a-temporal.

2. Conclusion.

Actually the famous **GR Principle of Equivalence** between **inertial mass** and **gravitational mass** doesn't exist in AD, because in Nature all systems are **gravitational systems**, that is, **accelerated** systems and consequently **inertial systems don't exist** in the Cosmos.

References

[1].- Lunar Laser Ranging: A Continuing Legacy of the Apollo Program, by J. O. Dickey et al., Science, vol. 265 22 July 1994, 482-490.

[2].- "Autodynamics. Fundamental Basis for a New Relativistic Mechanics" SAA, 801 Pine Ave. # 211, Long Beach, CA 90813. ISBN 0-9665533-1-4 htpp://autodynamics.org

[3].- Experiments in flying Muons on Laboratory show that the particle is deviated by the Gravitational Force. This means that the Muon Inertial Linear Motion is subject to lateral acceleration provoked by the Gravitational Force. If the particle fallow the line of the Gravitational Field the System has increasing velocity, that is, it has constant acceleration, which means that it is not an Inertial System.

[4].- "E7.- Extra Gravitational Tug." On page 215 in the AD's book.

Интеферометр контроля формы плоских оптических поверхностей для прецизионных оптических экспериментов

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Существующие технологические процессы изготовления высокоточных оптических деталей позволяют выявлять дефекты формы поверхностей только после операции полирования. Только тогда, используя интерферометрические методы контроля, выявляют дефекты поверхностей, устранять которые приходится дополнительным полированием («доводкой»), зачастую, с потерей технологического базирования и вероятностью появления новых дефектов форм.

Анализ методов контроля формы плоских поверхностей оптических деталей как с помощью механических средств (шаблонов, сферометров, координатометров и т.п.), так и оптических (метод оптической струны и др.) позволили выбрать метод и принципиальную схему контроля.

Рекомендован интерференционный метод контроля с реализацией схемы в виде лазерного интерферометра с дифракционными решетками. Использование такого интерферометра позволяет контролировать форму шлифованных поверхностей, исправляя дефекты еще до проведения самой длительной и трудоемкой операции полирования. Сформулированы научные и технические задачи, которые необходимо решить для устранения аналога прибора.

Опытный образец интерферометра, в отличие от аналога, выполнен в виде накладного прибора, устанавливаемого непосредственно на контролируемую поверхность оптической детали. Доказана рациональность такого исполнения, исходя из сохранения технологического базирования при смене зон контроля, а также удобства и простоты юстировки схемы прибора при смене контролируемых зон поверхности оптической детали.

Разработана и реализована функциональная схема опытного образца прибора. Схема позволяет осуществлять как визуальный, так и интерферометрический контроль интерференционной картины. Данные фотометрического контроля вводятся непосредственно в ЭВМ с последующей обработкой и получением характеристик в виде P-V и RMS.

Реализация разработанной конструкции показала простоту и удобство юстировки, высокую помехозащищенносъ и устойчивость работы прибора непосредственно на участке обработки оптических деталей.

Ожидается в 3-4 раза увеличение производительности и сокращение времени изготовления поверхностей высокоточных оптических деталей по сравнению с существующими.

Пути повышения КПД импульсных лазеров для систем высокоточной навигации

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Низкие значения световых коэффициентов полезного действия твердотельных лазеров во многом определяются неэффективностью накачки, несовпадением спектров излучения ламп накачки спектрам поглощения материалов активных элементов.

В работе проведен анализ твердотельных лазеров на кристаллах рубина, алюмоиттриевого граната, на стекле с ниодимом.

Рассмотрены спектры излучения ртутных, ксеноновых, криптоновых ламп накачки. Рассмотрены основные конструкции квантронов. Сделан вывод, что наиболее часто встречающим-

ся осветителем в квантронах является эллиптический цилиндр, когда в одном из фокусов эллипса размещен активный элемент, а в другом – лампа накачки.

Оценивая целесообразность такой конструкции, отмечается либо полное отсутствие, либо неэффективное применение фильтрации излучения (в первую очередь, теплового) ламп накачки.

Исследуется, предлагается и оценивается экспериментальная конструкция, когда эллиптический цилиндр осветителя выполняется составным, а на его плоские поверхности наносятся многослойные интерференционные покрытия в виде полосовых фильтрующих систем, пропускающих только длины волн областей поглощения материала активного элемента. На внешние поверхности эллиптических полуцилиндров наносятся многослойные интерференционные покрытия переменной оптической толщины, отражающие длины волн областей поглощения материала активного элемента. Отмечается, что некоторые технологические сложности нанесения покрытий компенсируются повышением стабильности работы лазеров и их оптических характеристик.

Автоматизированный контроль асферической оптики, используемой в системах космической локации

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Создание качественных оптических систем космической локации неразрывно связано с повышением требований к точности изготовления оптических деталей.

Особенно остро эта проблема стоит для оптических деталей с асферическими поверхностями, которые позволяют существенно улучшить качество оптического изображения, повысить оптические характеристики, упростить конструкцию приборов, что влечёт за собой уменьшение их габаритов и массы.

Однако сложность изготовления и контроля асферики препятствует широкому использованию такой оптики.

При разработке схем устройств контроля параметров деталей желательно использовать объективные бесконтактные методы измерений; автоматизировать процесс измерений, предусмотреть возможность проведения измерений в условиях участков цехов и заводских лабораторий.

Все эти требования можно обеспечить полностью или частично при использовании метода лазерного зондирования. Пучок света лазера падает на поворотное зеркало, закрепленное на валу шагового двигателя, и задаёт зондируемые координаты контролируемой поверхности. Зеркало смещено с оси зондируемой детали, но находится в параксиальной области поверхности на расстоянии радиуса кривизны центральной зоны.

Отразившись от поверхности, пучки анализируются также в центральной области вблизи центра кривизны центральной зоны с помощью координатора, разработанного на базе фазового растрового оптико-электронного преобразователя. Размер контролируемой зоны определяется размером пучка лазера.

Величина шага дискретного наклона зеркала может быть выбрана любой и соответствует величине ожидаемых дефектов формы поверхности, пропорциональных, в общем случае, размеру инструмента, используемого при асферизации. Контроль всей поверхности осуществляется последовательным зондированием радиальных зон разворотом самой детали на 360 град.

Экспериментальные исследования показали возможность контроля формы поверхностей оптических деталей с погрешностью до 5 угловых секунд угла отклонения нормали.

Гилометродинамика

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Гилодинамика (движущаяся гиле (материя) и ее атрибуты) это метанаучный уровень представлений, а применительно к теоретической физике наиболее приемлемым вариантом ее проявления является гилометродинамика, т.е. это гилодинамика с выделением категории “мера”, которая является атрибутом материи и собственно наиболее характерна для научной теории. Мера как атрибут материи (материального объекта) отражает качественно – количественную систему взаимосвязей внутри явлений, включающая в себя функциональные зависимости внутри объекта, а также функциональные зависимости целого и частей. Мера рассматривается и как средство измерения, и как имманентная граница объекта, т.е. граница количественных изменений данного качества. По сути дела, гилометродинамика основывается на классической геометродинамике (теория относительности (А. Эйнштейн), геометродинамике и квантовой геометродинамике (Дж. А. Уилер), является их обобщением и метауровнем в представлениях (уход от геометрии к гилометрии). Ибо в представлениях как о мега- так и микромире понятие “геометрия” (землемерие), т. е. геоцентризм, уже неприемлемо (в развитии принципа негеоцентризма). В объективной реальности, по существу, имеет место (в силу принципа неисчерпаемости материи “в глубь” и “вширь”) мерометризация материальных объектов с различной метрикой и топологией, т. е. гилометродинамика.

Анализируя свое основное уравнение классической геометродинамики $R_{ik} - 1/2 g_{ik} R_{ik} = -T_{ik}$, где R_{ik} – обозначает скаляр римановой кривизны, T_{ik} – тензор энергии материи в феноменологическом представлении, Эйнштейн охарактеризовал его следующим образом: левая часть – это само совершенство (изящный мрамор), а правая часть – плохое дерево (феноменологическое представление), которое далеко не полно отражает все известные свойства материи[1]. Может сложиться мнение, что в отсутствие материи все g_{ik} равны нулю и нет понятия интервала ($ds^2 = g_{ik} dx^i dx^k$), в отсутствии материи отсутствует пространственно-временной континуум и как вывод: тесной связи пространства и материи не содержится в формализме классической геометродинамики. На самом деле это чисто геометродинамическая трактовка, гилометродинамическая же трактовка говорит о том, что речь идет не о формализме, а лишь об его определенной интерпретации, когда отсутствие материи ассоциируется с равенством нулю тензора материи T_{ik}. Однако тензор материи далеко не полно представляет в ней материю. Кроме неизвестных форм и состояний материи в нем не учтены энергия флуктуаций вакуумов различных полей и энергия самого гравитационного поля. Поэтому выводы о том, что равенство нулю тензора материи T_{ik} свидетельствует об “отсутствии материи” несостоятельны. Равенство нулю тензора материи T_{ik} не эквивалентно “отсутствию материи” и не сопровождается равенством нулю всех g_{ik}, исчезновением понятия интервала и пространственно-временного континуума. Согласно геометродинамики и гилометродинамики, если бы материя и ее движение исчезли, то больше не было бы ни пространства, ни времени.

Главной проблемой гилометродинамики является определение тензора материи T_{ik} внутри масс и, как частный случай, для точечных объектов (ядро, “элементарные” частицы), при этом определение тензора Эйнштейна необходимо связать с гилометрией. По сути дела, это целое направление исследования, которая дает возможность рассматривать любую физическую теорию не как формальную, а как физическую, наполненную глубоким качественным

[1] См.: Физика и реальность. – М.: Наука, 1965. С. 50.

содержанием. В классической геометродинамике Эйнштейна, определение компонент тензора материи T_{ik} как функции от координат возможно только в той области, где эти компоненты равны нулю. В области же, занятой материей, задание тензора материи T_{ik} требует знания метрики, а метрика же сама зависит от материи. Поэтому определение компонент тензора материи в области, где они отличны от нуля, может быть произведено только совместно с определением фундаментального тензора g_{ik} . Определение фундаментального тензора g_{ik} во втором приближении для области вне масс позволяет построить тензор материи T_{ik} в функции от координат и времени, что в свою очередь, позволяет определить фундаментальный тензор g_{ik} также и внутри масс. Это дает возможность для формулировки условий, обеспечивающих единственность решения.

Гилометродинамика предполагает, что при определении тензора Эйнштейна и фундаментального тензора g_{ik} необходимо ввести гилометрическую координатную систему, которая по своим свойствам ближе всего подходит к фоковской гармонической координатной системе. Различие заключается в трактовке малости отклонения метрики внутри и вне материи от евклидовой, которая является функцией ньютонова потенциала тяготения. Соответственно и определение тензора материи T_{ik} будет гилометрически связано с определением фундаментального тензора g_{ik} как внутри материи, так и вне нее в динамике. Гилометродинамическая интерпретация пространственно-временной картины в физике является, по сути дела, логическим продолжением и обобщением известного в физике знаменитого метода геометризации. В конечном счете, речь идет об онтологических аспектах проблемы геометризации и гилометродинамической интерпретации физики, выражающих взаимоотношения между пространством-временем и материей в объективной реальности – онтологический принцип гилометродинамики, и об их гносеологических аспектах, выражающих то взаимоотношение, которое существует между понятиями пространство-время и материи в научной теории – гносеологический принцип гилометродинамики.

Представление о том, что метрика вносится в пространство-время извне масштабными объектами – это есть, с одной стороны, следствие ньютоновской интерпретации пространства-времени, а с другой – отождествления онтологического и гносеологического аспектов геометризации, гилометризации. Другими словами – нет четкого различия между реальным пространством-временем и абстрактным. Пространство-время само по себе, а так же без и вне материи, существует лишь как абстракция, а материя существует объективно реально лишь в единстве с ним. Отождествление онтологических и гносеологических аспектов проблемы приводит многих к выводу о том, что А. Эйнштейн в классической геометродинамике и Дж. Уилер в квантовой геометродинамике, говорили о полном сведении физики к геометрии. Однако анализ показывает, что оба в своих представлениях имели в виду гносеологический (теоретико-познавательный) аспект геометризации физики и материи отводили определяющую роль во взаимоотношении с пространством-временем (онтологический (сущностный) аспект).

Гилометродинамика основывается на не отождествлении материи и геометрии, гилометрии, а также онтологических и гносеологических аспектов такой интерпретации. Гилометрия предполагает многочисленные проявления метрических и топологических свойств пространства-времени, в том числе и различные виды геометрии как евклидову, так и неевклидовы. Метрика может служить в качестве основной единицы измерения всех характеристик материальных объектов. Так в уилеровской геометродинамике такой единицей служит один сантиметр, а в гилометродинамике субатомных и субъядерных взаимодействий предлагается ввести один ферми с тем, чтобы можно было количественно описывать соответствующие процессы. При гилометродинамическом описании ядерной материи в хромодинамических представлениях осуществляется в терминах теории расслоенных пространств. Однако это

только промежуточный этап дальнейшей гилометродинамической интерпретации теории сильных взаимодействий.

И еще, в развитии основных идей гилометродинамики необходимо отметить, что в хроногилометрическом подходе при рассмотрении физических взаимодействий в различных представлениях имеет место гилометродинамическая детерминация, т. е. обусловленность пространственно-временных форм многомерных и сложных топологических образований материального Мира и микромира процессами формирования материальных объектов.

С открытием неевклидовых геометрий проблема соотношения материи, пространства-времени и других ее атрибутов стала приобретать характер гилометродинамических построений. Построение "воображаемых" геометрий, отказ от гносеологии рационализма означал постановку вопроса о новом обосновании гилометрии (геометрии) путем введения онтологического и гносеологического принципов гилометродинамики. Построение неевклидовых геометрий привело к возникновению в математике такой же ситуации, как и в геометрии до ее аксиоматизации. Образование отдельных геометрических систем никого не устраивало, подобно тому, как не удовлетворяли Евклида эмпирические законы землемерия. Это объективно привело к теоретико-групповому подходу (Ф. Клейн). Группа преобразований в многообразии, которая требует развить теорию инвариантов, – формулировка задачи теоретико-группового подхода к геометрии. Так называемая "Эрлангенская программа" дала точную математическую интерпретацию понятию "движения" посредством математических преобразований, построенных на основе теории групп, освободила геометрию от чувственно-наглядного образа. Возникло представление о многообразии, развитое из геометрии без пространственной картины, несущественной для чистой математики. Геометрия стала пониматься как система аксиом, связывающая свойства отношений универсальных элементов. "Эрлангенская программа" Ф. Клейна – это теория однородных пространств, и она очень близка к концепции Ньютона: структура этих пространств определяется исключительно аксиомами[2].

Другой интересный подход к геометрии был предложен Б. Риманом – общая теория неоднородных пространств. В основе этой теории лежит понятие "расстояние", а в программе Ф. Клейна – понятие "равенство". Риман рассматривает расстояние между двумя бесконечно близкими точками как квадратный корень из квадратичной формы дифференциалов координат: $dS^2 = g\mu\nu dx^\mu dx^\nu$. Он вводит понятие о положительной "кривизне", характеризующую геометрию Римана, как частный случай гауссовской "кривизны", которая в свою очередь полностью характеризует внутреннюю геометрию искривленной поверхности. Определение кривизны пространства предшествует определению расстояния между двумя бесконечно близкими точками, поскольку кривизна непосредственно связана с коэффициентами $g^{\mu\nu}$. Лишь значение кривизны в каждой точке пространства дает возможность определить его геометрию, поэтому исключительно аксиоматическое построение геометрии оказывается невозможным. Риман ищет основу метрических отношений в физике, выходя, таким образом, за пределы чистой геометрии. По этому поводу он писал: "Нужно пытаться объяснить возникновение метрических отношений чем-то внешним – силами связи, действующими на это реальное", т. е. "то реальное, что создает идею пространства"[3]. Обращая внимание на те факты, которые не могут быть объяснены классическими представлениями о пространстве и времени, Риман писал: "Решение этих вопросов можно надеяться найти лишь в том случае, если, исходя из ныне существующей и проверенной опытом концепции, основа которой положена Ньютоном, станем постепенно ее совершенствовать, руководствуясь фактами, которые ею объяснены быть не могут… здесь мы стоим на пороге области, принадлежащей дру-

[2] См.: Об основаниях геометрии. – М., 1956. С.402
[3] См.: Там же. С. 324.

гой науке - физике, и переступать его не дает нам повода сегодняшний день”[4].

По этому пути пошло дальнейшее развитие теоретической физики. Была создана теория относительности, которая определила физический смысл коэффициентов $g^{\mu\nu}$ в квадратичной форме $dS^2 = g^{\mu\nu}dx^{\mu}dx^{\nu}$ и тем самым в особенной, специфической форме демонстрирует общую зависимость метрических свойств пространства-времени от соответствующих свойств материальных взаимодействий, их обусловленность “силами связи”, которая была предсказана Риманом. Развитие геометрии с необходимостью выводило ее за собственные границы, отделило геометрический смысл от физического смысла и в то же время связало их. Возникла проблема соотношения геометрии и физики. Об онтологическом статусе геометрии как евклидовой, так и неевклидовой был очевиден и Ньютону, Лобачевскому, Гауссу, Риману в той или иной степени. Геометрия столь же апостериорна, как и физика. Гаусс по этому поводу писал: “Геометрию приходится ставить не в один ранг с арифметикой, существующей чисто a priori, а скорее с механикой”[5]. Исходную посылку научного знания об изучении материальных процессов в пространстве-времени по своему интерпретирует А. Пуанкаре. Он считает, что единственным объектом научного познания являются материальные процессы, тогда как пространство существует только как абстрактное многообразие и является предметом лишь математического исследования. Чистая геометрия не изучает свойства внешнего мира, а физика в свою очередь не изучает свойства пространства. Без отношения к геометрии невозможно понять физические процессы, хотя геометрия и не есть физика. Очевидно, что здесь отношение пространства и материи имеет не онтологический, а гносеологический характер, другими словами, является не объективно реальным отношением, а отношением языка и объективной реальности. Для Пуанкаре геометрия – это предпосылка научной теории, независимая от свойств описываемого объекта. В опыте проверяются лишь совместно геометрия и физика и, следовательно, возможно деление на геометрию и физические законы в одних и тех же опытных данных. В этом проявляется конвенционализм Пуанкаре: отрицание онтологического характера геометрии и физических закономерностей и интерпретация их как условных соотношений[6].

В противоположность конвенционализму Пуанкаре для Эйнштейна геометрия не является языком, независимым от свойств описываемого объекта. Она имеет онтологический характер, ее “выбор” должен учитывать опытные данные. “Мы можем рассматривать ее фактически как самую древнюю ветвь физики, - писал Эйнштейн, – ее утверждения покоятся существенным образом на выводах из опыта, а не только на логических заключениях… Такому пониманию я придаю особое значение, поскольку без него я не смог бы установить теорию относительности”[7]. “Дуализм” физических теорий, который является естественнонаучным основанием конвенционализма, в общей теории относительности удалось преодолеть. В ней проблема решается на основе гилодинамики, положения об атрибутивности пространства-времени по отношению к материи. “Согласно общей теории относительности, – писал Эйнштейн, – метрические свойства пространства-времени причинно не зависят от того, чем это пространство-время наполнено этим последним”[8]. И далее следует уточнение: “… геометрические свойства пространства не самостоятельны: они обусловлены материей”. Общая теория относительности доказала объективность пространства-времени, установив однозначную связь между геометрическими характеристиками пространства-времени и физическими характеристиками гравитации. Переход при построении ОТО от евклидовой геометрии к римановой геометрии предопределило развитие гилометродинамики. В основаниях этой теории

[4] См.: Там же.
[5] См.: Там же. С. 103.
[6] См.: Философские проблемы современного естествознания. – Киев, 1986. Вып. 61. С. 25.
[7] См.: Эйнштейн А. Собр. науч. тр. В 4-х тт., – М., 1966. Т. 2. С. 87
[8] См.: Там же. С. 408.

лежит положение о том, что материя определяет как пространство-время, так и гравитацию, которые являются свойством движущейся материи. Материальные взаимодействия проявляются в соответствующих свойствах (метрических, топологических, симметрии) пространства-времени.

Учет онтологического принципа гилометродинамики при решении проблемы соотношения материи и ее атрибутов является необходимым критерием, но не достаточным для решения проблемы соотношения гилометрии (геометрии) и физики. Кажущееся противоречие между целью познания и его результатом снимается при учете онтологического принципа гилометродинамики в диалектическом единстве с гносеологическим принципом гилометродинамики. Так, если материя определяет пространство-время, свойства материальных взаимодействий проявляются в соответствующих свойствах пространства-времени (онтологический аспект), то познание структуры материи, свойств материальных взаимодействий есть цель (гносеологический аспект), а понятие пространства-времени, будучи более абстрактным, является исходным на соответствующей стадии физического познания. На стадии восхождения от абстрактного к конкретному, с помощью понятия пространства-времени, в научной теории воссоздается исследуемая структура материи. По метрическим, топологическим и другим свойствам пространства-времени можно судить о соответствующих характеристиках материальных взаимодействий, другими словами по гилометрии (геометрии) судят о физике.

Осуществление программы гилометризации физической науки приводит к тому, что в теории физические свойства известных взаимодействий описываются на внутреннем по отношению к теории языке. Информативность гилометрического языка оказывается достаточно полной, чтобы описывать соответствующие материальные эффекты (например, гравитацию) в пределах гилометрии. Гилометрическая интерпретация гравитации обеспечивает относительную логическую независимость теории от опыта, наделяет ее такой степенью логической связности и общности, что делает корректным сопоставление ее как целого с опытом. В основе этого лежит программа гилометризации физики: онтологический аспект – материя – определяет пространство-время и гравитацию, другие свои атрибуты, а свойства материальных взаимодействий проявляются в свойствах пространства-времени и гравитации; гносеологический аспект – по свойствам пространства-времени и других атрибутов можно судить о соответствующих свойствах материальных объектов и взаимодействий (по гилометрии можно судить о соответствующей физике).

Обратимся к идее объединения всех фундаментальных взаимодействий, которое остается наиболее актуальной и, в частности, применительно к атомному ядру. Кроме определяющего сильного взаимодействия в нем имеет место электромагнитное между заряженными микрочастицами (протоны, “цветные” кварки и глюоны), при распаде ядер и частиц проявляется слабое взаимодействие. Основой для этого объединения служат общие характеристики электрослабого и КХД полей: они являются квантовополевыми, обладающими локальной калибровочной симметрией динамических переменных. С другой стороны эти взаимодействия имеют аналогию с классической геометродинамикой (общей теорией относительности). Смысл этой аналогии заключается в том, что геометродинамику можно рассматривать с позиции локальной симметрии, но не калибровочных полевых переменных, а лоренцевых преобразований пространства-времени. Именно в этом заключена эвристическая роль гносеологического принципа гилометродинамики, основу которого составляет синтезирование идей геометродинамики с достаточно емкими обобщениями теории калибровочных полей[9].

Несомненно, что одним из важных аспектов такого синтеза является идея гилометризации, геометризации физики начиная с работ Г. Вейля. Смысл в том, что формализм теории

[9] См.: Философские проблемы современного естествознания. – Киев, 1986. Вып. 61. С. 25.

калибровочных полей по своей структуре эквивалентен формализму теорий расслоенных пространств. Последняя рассматривает сложное многообразие, в основе которого носитель – база, обладающая неевклидовой и даже неримановой структурой. В каждой точке базы задаются касательные пространства и сечения, и имеющие уже более простые структуры, которые определяются гилометрией (геометрией) базы, а точнее структурой ковариантной производной любых геометрических объектов на касательных пространствах и сечениях по параметрам базы. Связь между двумя теориями заключается в том, что поля, характеризующие частицы-источники, можно рассматривать как сечения расслоенного пространства, базой которого является четырехмерное пространство-время. Калибровочные поля в свою очередь описываются гилометрическими (геометрическими) характеристиками – связностями расслоенного пространства. Динамическая симметрия, локализация которой требует введения "компенсирующих" калибровочных полей, является группой симметрии слоя, а "компенсирующая" производная, с помощью которой в лагранжиане поля и в полевых уравнениях учитывается переход от "глобальной" динамической симметрии к "локальной" и взаимодействие исходных полей с калибровочными отождествляется с ковариантными производными в расслоенных пространствах[10].

Общий метод расслоенных пространств, т. е. формализм теории калибровочных полей не выясняет пространственно-временной природы слоев, дает всего лишь абстрактное геометрическое описание этой инвариантности, как и для других симметрий. И в этом смысле калибровочные теории являются феноменологическими теориями, родившимися на основе экспериментов на ускорителях микрочастиц. Возникает проблема выявления внутренней симметрии, локализация которой приводит к соответствующему калибровочному полю, как геометрической симметрии какой-либо структуры реального пространства-времени. Риманова геометрия, метрический тензор которой T_{ik} наделяет нетривиальной метрической структурой только базу M, тогда как касательные пространства $T(x)$ он снабжает лишь тривиальной метрической структурой псевдоевклидова пространства. Из этого следует, что такая геометрия не дает возможности для конструктивного анализа проблемы общих метрических свойств касательных пространства, а, следовательно, и всего касательного расслоения. Необходимо использование методов теории метрических пространств более общих, чем римановы, что соответствует содержанию гносеологического принципа гилометродинамики.

Пути решения поставленной проблемы заключаются в применение более общих представлений, в которых известные подходы калибровочных полей и гилометрии (геометрии) входят органически, как составные части таких представлений. К примеру, можно применить известное финслерово пространство, являющееся непосредственным обобщением риманова.

Теория финслеровых пространств как одно из проявлений гилометрии обладает богатыми внутренними структурами, необходимыми для введения различных полей. Однако попытки использования финслеровой геометрии для описания фундаментальной структуры мира микрочастиц редки и далеко не все возможности этой теории до конца изучены. Она диктует необходимость пересмотра структуры пространства-времени уже в классической науке[11]. Общая теория пространств Финслера восходит к знаменитой работе Римана "О гипотезах, лежащих в основаниях геометрии"[12]. В ней обсуждаются различные возможности метризации n-мерного многообразия и уделяется особое внимание метрике, задаваемой положительным квадратным корнем из положительно определенной квадратичной дифференциальной формы. Эта метрика лежит в основе римановой геометрии. Далее предполагается, что метрической функцией может служить также и положительный корень четвертой степени из дифференциальной формы четвертого порядка. Указанные метрические функции обла-

[10] См. например: Философские проблемы классической и неклассической физики. – М., 1998. С. 159.
[11] См.: Рунд Х. Дифференциальная геометрия финслеровых пространств. – М., 1981.
[12] См.: Об основаниях геометрии. – Казань, 1984. С. 67.

дают с математической точки зрения следующими общими свойствами: они положительны, однородны первой степени по дифференциалам, а также являются выпуклыми функциями дифференциалов. С точки зрения физики финслерова геометрия описывает неоднородное пространство – пространство с выделенными направлениями. Так в квантовой хромодинамике каждый кварк имеет только одну направленную от него к соседнему кварку линию, вдоль которой может распространяться одномерное взаимодействие. Другими словами, кварки имеют направленную валентность, т. е. они одновалентны. Пространственноподобные конфигурации, обладающие полной упорядоченностью и, следовательно, определенными взаимодействиями между кварками, состоят только из пар кварков или из триад. Такой вывод следует из предположения о характере поведения глюонного поля в пространстве *Г*-геометрии. В силу дискретности $\Gamma_4(x)$-пространства пространственноподобные интервалы не могут быть меньше элемента длины. Поэтому кварки не могут находиться в одной точке. Тем самым отпадает известная трудность относительно статистики кварков: различные положения кварков можно рассматривать как их различный “цвет”. Другими словами, различие в “цвете” кварков может быть отождествлено с различием в положении этих кварков. Формализм финслеровой геометрии не получил широкого применения в теоретической физике, так как требование однородности пространства-времени является одним из фундаментальных в современной физике[13].

Подход, основанный на финслеровой геометрии, дает простой ответ на вопрос о том, что представляет собой внутреннее, изотопическое пространство в плане касательного расслоения действительного пространства-времени? Изотопическое пространство является не чем иным, как индикатриссой финслеровой структуры пространства-времени. Такой подход дает возможность единого гилометрического описания различных калибровочных полей и ядерной хромодинамики. Гравитационное и сильное взаимодействия объединяются согласно формуле – риманова метрическая структура + изотопическая инвариантность = финслерова метрическая структура с индикатриссой нулевой кривизны. Теория финслеровой геометрии можно рассматривать и в соответствии с принципом взаимности, который определяет, что любой физический закон в *x*-пространстве имеет “инверсный образ” в импульсном *p*-пространстве. Этот принцип приводит к требованию введения нетривиальной метрики в *p*-пространстве, что опять непротиворечивым и единым образом может быть описано только финслеровой геометрией. Этот принцип автоматически приводит к конечности микрочастиц, к принципу квантования, указывает на физический смысл конечности планковского действия h[14].

Количественный анализ применимости обобщенных геометрий (гилометрии) применительно к конкретным видам взаимодействий требует специальной физико-математической проработки и представляет собой сложный исследовательский механизм, выходящий за рамки данной работы. Однако качественный анализ дает основание для оптимистической оценки применимости принципов, лежащих в основаниях теорий обобщенных геометрий.

Представляет интерес то, что достаточно абстрактная модель мира живого может быть построена, если попытаться описать биологическую изменчивость через гилометродинамику (геометродинамику) вероятностного пространства. Наиболее общей теорией в физике оказался геометродинамический подход Эйнштейна в общей теории относительности. Дальше последовали идеи Уилера о квантовой геометродинамике. Оказалось возможным ввести представления о флюктуации топологии на малых расстояниях. Разъяснительная сила геометродинамического подхода увеличилась - он охватил и электромагнитные явления[15]. Так

[13] См.: Философские проблемы классической и неклассической физики. – М., 1998. С. 158; Блохинцев Д. И. Труды по методологическим проблемам физики. – М.: МГУ, 1993. С. 229.
[14] См.: Блохинцев Д. И. С. 176.
[15] Уилер Дж. Предвидение Эйнштейна. – М., 1970; Брайс С. де Витт Квантовая гравитация // В мире науки, 1984, № 2. С. 50.

возникла в физике новая научная программа гилометродинамического моделирования биологической изменчивости, основанная на представлении о том, что все множество морфофизиологических признаков мира живого как-то соотнесено с числовой осью[16]. Динамика определяется не перемещением конкретной точки, фиксируемой на оси, а приданием различных весов участкам этой шкалы. Вводится в рассмотрение плотность вероятности. Геометродинамика раскрывается через теорему Бейеса, обретающую статус вероятностного силлогизма. Так удается объяснить все многообразие биологической изменчивости и дать представление о собственном - биологическом времени. Мир живого раскрывается через геометрический образ.

Для обоснования научной теории представляет интерес гилометрическая интерпретация пространственно-временных моделей, в генезисе представлений известная еще и как метод геометризации в физике[17]. Разные аспекты проблемы гилометрической (геометризации) интерпретации физики, так или иначе, сводятся к двум. Речь идет по существу о том, что означает гилометризация физики, а именно, о том, что она играет определяющую роль гилометрии (геометрии) по отношению к физике. Первичность метрических и топологических свойств пространства-времени по отношению к соответствующим не пространственно-временным свойствам материальных явлений или же возможность по гилометрии (геометрии) судить о физике, по метрическим и топологическим свойствам пространства-времени судить о соответствующих не пространственно-временных свойствах материальных явлений. Отождествление атрибутивного взаимоотношения между свойствами пространства-времени и соответствующими свойствами материальных явлений, которое существует лишь на одном из этапов построения теории, с действительным взаимоотношением между ними в процессе теоретического познания и тем более в объективной реальности. Это, по существу, является гносеологическими корнями предположения об определяющей роли метрических и топологических свойств пространства-времени по отношению к непространственно-временным свойствам материальных явлений, об определяющей роли гилометрии в теоретической физике. Тот факт, что по метрическим и топологическим свойствам пространства-времени можно судить о непространственно-временных свойствах материальных явлений, в соответствующей интерпретации, порождает иллюзию об определяющей роли первых по отношению ко вторым в самой объективной действительности, однако к подлинному процессу гилометризации (геометризации) физики такое отождествление онтологического и гносеологического аспектов проблемы никакого отношения не имеет. А. Эйнштейн – автор теории относительности, и его последователи, применяя геометрическую интерпретацию в физике, всегда имели в виду лишь гносеологический аспект проблемы, возможность по геометрии судить о физике, по метрическим и топологическим свойствам пространства-времени судить о не пространственно-временных свойствах материальных явлений[18].

А. Эйнштейн был глубоко убежден в том, что в объективной действительности во взаимоотношении между пространством-временем и материей определяющую роль играет материя. “Согласно общей теории относительности, – писал он,– геометрические свойства пространства не самостоятельны: они обусловлены материей”[19]. ”Нельзя, – подчеркивал он, – ... сводить физику к геометрии”[20]. Даже в геометродинамике Дж. Уилера, которая в научной и

[16] Ближе к абстрактно-геометрическому описанию Мира живого подошел известный биофизик Н. Рашевский (топологический подход) и его ученик Р. Розен (подход с позиций теории категорий и функторов): Rashevsky N. Topology and Life: Search of General Mathematical Principles in Biology and Sociology // Bulletin of Mathematical Biophysics, 1954, 16, 4, 317.

[17] См.: Эйнштейн и философские проблемы физики XX века. – М., 1979. С. 172; Методология научного познания и физика. – М., 1985. С. 341; Барашенков В. С. Проблемы субатомного пространства и времени. – М., 1979. С. 174.

[18] См.: Эйнштейн А. О специальной и общей теории относительности. Собр. научн. тр. – М., 1965. Т. 1. С. 587.

[19] См.: Методология научного познания и физика. – М., 1985. С. 343.

[20] См.: Эйнштейн А. Автобиографические заметки. Собр. научн. тр. – М., 1967. Т. 4. С. 280.

философской литературе интерпретируется как попытка полного сведения физики к геометрии, при более внимательном рассмотрении фактически реализуется не онтологический аспект проблемы геометризации физики, а лишь ее гносеологический аспект. Однако, в последующем геометродинамика, вернее геометродинамическая интерпретация все больше вела к противоположному подходу – эйнштейновскому. "Не геометрия, а затем квантовый принцип, а наоборот, сначала квантовый принцип, а уже затем геометрия"[21], – провозглашает Дж. Уилер. Речь идет о противоположных подходах в построении теории: на пути от гилометрии (геометрии) к физике или же, наоборот, от физики к гилометрии (геометрии). Оба эти подхода находятся в единстве, однако второй принял характер гилометрической (геометрической) интерпретации в физике, так как процесс абстрагирования с необходимостью приводит на определенном этапе к пространственно-временным понятиям, с помощью которых в теории затем воссоздаются соответствующие непространственно-временные свойства материи и ее движения.

Следует четко различать представление о реальном пространстве-времени, существующем вне и независимо от какой бы то ни было теории, от абстрактного пространства-времени. Так при переходе к теории "элементарных" частиц, в основе которых лежат калибровочные поля структуры материальных взаимодействий, ставится в соответствие не структура реального пространства-времени, а структура некоего абстрактного пространства-времени. Геометрическое описание материальных взаимодействий осуществляется в терминах теории расслоенных пространств, пространство-время Минковского выступает как база расслоенного пространства, свойства которой не зависят от свойств слоя. Другими словами номиналистический характер гилометризации (геометризации) здесь состоит в том, что все взаимодействия рассматриваются не как следствие неевклидова характера реального пространства-времени, а как следствие неевклидовой природы концептуального расслоенного "пространства", являющегося своеобразной надстройкой над реальным пространством-временем. Последнее играет роль базиса расслоенного "пространства" и обычно имеет "плоский" характер[22]. Таким образом, речь идет о геометризации физики в буквальном смысле слова не как о методе, позволяющем по структуре реального пространства-времени определенной области объективной действительности воспроизвести структуру материальных взаимодействий, а о методе, позволяющем описать структуру материальных взаимодействий с помощью (подобной пространственно-временной) структуры абстрактного пространства-времени, т. е. о гилометризации. Геометрическая интерпретация в терминах расслоенных пространств рассматривается лишь как промежуточный этап на пути адекватного описания свойств реального пространства-времени – гилометризации.

Следует заметить, что процесс геометризации, применение геометродинамической интерпретации физики носит ограниченный характер, ибо в топологических моделях уже видна глубокая связь с непространственно-временными свойствами материальных явлений, что дает повод говорить о не геометродинамической интерпретации, а о гилометродинамической интерпретации как более общем пространственно-временном представлении. Как частный (предельный) случай можно рассматривать геометродинамическую интерпретацию неевклидового характера. Исходным философским основанием для этого служит качественная бесконечность материального Мира в своих пространственно-временных и непространственно-временных проявлениях.

[21] См.: Уилер Дж. Предвидение Эйнштейна. – М., 1970. С. 95.
[22] См.: Бранский В. П. Теория элементарных частиц как объект методологического исследования. – Л., 1989. С. 219.

STR and boundary of the Mendeleev's periodic system stability

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Consider the non-relativistic model of atom with one electron, the nucleus charge is $Z|e|$, where $|e|$ is module of electron charge and Z is the whole number of all elements of the Mendeleev`s periodic system. Then the most possible distance between electron and nucleus is $r_1 Z^{-1}$, where r_1 is the first Bohr radius. The nuclear radius is approximately $r_p A^{1/3}$, where A=Z+N and N is number of neutron in nucleus, r_p is proton radius ($r_p \approx 1.4*10^{-15}$ m)[1]. So ratio $(A/Z)^{1/3}$ approximately equal 1.3 (from Mendeleev`s periodic system (computer modeling)), we have equation for evaluation of $Z_{\max}$

$$1.3\, r_p Z^{1/3} = r_1 z^{-1} \tag{1}$$

The ratio $(r_1 / r_p)/1.3$ is approximately $3.8*10^4$ and we have

$$Z_{\max} \approx 2.2*10^3 \tag{2}$$

The evaluation (2) is very strongly differs from number of elements in the Mendeleev's periodic system. But it this moment we have only a little more than 100 elements in this system and synthesis of elements with number more than 100 is very difficult [1]. Therefore we take part the end velocity of electron in atom. Then we have the following equation for electron energy E in s-state of atom in our terms

$$(E / m_e c^2) = (1+\xi^2)^{1/2} - 1 - (Z/\eta), \tag{3}$$

where $\xi = p / m_e c$, p – impulse of electron and m_e is the mass of electron, c is light velocity, $\eta = r / r_e$, r is distance between electron and nucleus, $r_e = e^2 / m_e c^2$, r_e is electron classic radius. Then the Bohr's rule of quantization is

$$\Lambda \xi \eta = N, \qquad N=1,2,3,.. \tag{4}$$

where $\Lambda = e^2 / \hbar c$, Λ is constant of thin structure and Λ is determined the geometry of atom sphere [2]. Using principle of energy extremuma and conditional (4) we derive from equation (3)

$$(E_1 / m_1 c^2) = (1 - (Z\Lambda / N)^2)^{1/2} - 1 \tag{5}$$

The equality is exact energy spectrum of electron in Coulomb's field for s–state [3]. We suppose that stability of atom is stability of 1s-state of atom. Then N=1 and we have

$$Z_{\max} = E(1/\Lambda) \tag{6}$$

Where E(x) is the integer part of x. So Λ is equal $7.29735308*10^{-3}$ [4], then

$$Z_{\max} = 137 \tag{7}$$

If Z equal $Z_{\max} + 1$ we have from (4) for such atom life time

$$\tau = \hbar / \operatorname{Im} E_1 \tag{8}$$

The numerical estimation with use of equality (8) gives about $1.085 * 10^{-20}$ s. This size of time of life is characteristic for absolutely astable nuclear conditions. For this conclusion the essential assumption about spherical symmetry in system a electron-nucleus was used. This time is less than time of formation of a nucleus which is accepted at a level about 10^{-12} s.

We consider case when Coulomb potential replaced by potential, which does not address in infinity at zero value of radius. Such potential turns out, for example, if the nucleus is not the dot charged object. We have made computer modelling. Applying instead of a Bohr`s rule the rule of quantization of the Bohr - Zommerfeld is possible to consider cases of the extended electronic cloud and non-spherical symmetric nucleus In both cases we have gotten that the number of elements of Mendeleev's periodic system of the order 100.

References

[1] Valentin L.Physique of quantum noyaux et particules, v.1.,Hermann, 75015 , Paris,1982.

[2] Thirring V.E. Principles of quantum electrodynamics, Academic Press inc., Publishers, N.Y. London, 1958

[3] Landau L.D , Lifshitz E.M. Quantum Electrodynamics, 4th ed., Fissmatlit, Moscow, 2001

[4] Cohen E.R., Taylor B.N. The 1986 CODATA Recommended Values of the Fundamental Physical Constant//Journal of Research of the National Bureau of Standarts. 1987. vol. 92. № 2. P. 85-95.

The uncertainty principle is untenable

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By re-analyzing Heisenberg's Gamma-Ray Microscope experiment and the ideal experiment from which the uncertainty principle is derived, it is actually found that the uncertainty principle can not be obtained from them. It is therefore found to be untenable.

Key words: uncertainty principle; Heisenberg's Gamma-Ray Microscope Experiment; ideal experiment

1. Ideal Experiment 1. Heisenberg's Gamma-Ray Microscope Experiment

A free electron sits directly beneath the center of the microscope's lens (please see AIP page http://www.aip.org/history/heisenberg/p08b.htm or diagram below). The circular lens forms a cone of angle **2A** from the electron. The electron is then illuminated from the left by gamma rays--high energy light which has the shortest wavelength. These yield the highest resolution, for according to a principle of wave optics, the microscope can resolve (that is, "see" or distinguish) objects to a size of **dx**, which is related to and to the wavelength **L** of the gamma ray, by the expression:

$$dx = L/(2\sin A) \quad \textbf{(1)}$$

However, in quantum mechanics, where a light wave can act like a particle, a gamma ray striking an electron gives it a kick. At the moment the light is diffracted by the electron into the microscope lens, the electron is thrust to the right. To be observed by the microscope, the gamma ray must be scattered into any angle within the cone of angle **2A**. In quantum mechanics, the gamma ray carries momentum as if it were a particle. The total momentum p is related to the wavelength by the formula,

$$p = h / L, \text{ where h is Planck's constant.} \quad \textbf{(2)}$$

In the extreme case of diffraction of the gamma ray to the right edge of the lens, the total momentum would be the sum of the electron's momentum **P'x** in the **x** direction and the gamma ray's momentum in the **x** direction: $P' x + (h \sin A) / L'$, where L' is the wavelength of the deflected gamma ray.

In the other extreme, the observed gamma ray recoils backward, just hitting the left edge of the lens. In this case, the total momentum in the **x** direction is: $P''x - (h \sin A) / L''$.

The final **x** momentum in each case must equal the initial **x** momentum, since momentum is conserved. Therefore, the final **x** momenta are equal to each other:

$$P'x + (h \sin A) / L' = P''x - (h \sin A) / L'' \quad \textbf{(3)}$$

If **A** is small, then the wavelengths are approximately the same, $L' \sim L'' \sim L$. So we have

$$P''x - P'x = dPx \sim 2h \sin A / L \quad \textbf{(4)}$$

Since **dx = L/(2 sinA)**, we obtain a reciprocal relationship between the minimum uncertainty in the measured position, **dx**, of the electron along the **x** axis and the uncertainty in its momentum, **dPx**, in the **x** direction:

$$dPx \sim h / dx \text{ or } dPx\, dx \sim h. \quad \textbf{(5)}$$

For more than minimum uncertainty, the "greater than" sign may added.

Except for the factor of **4pi** and an equal sign, this is Heisenberg's uncertainty relation for the simultaneous measurement of the position and momentum of an object.

2. Re-analysis

To be seen by the microscope, the gamma ray must be scattered into any angle within the cone of angle **2A**. The microscope can resolve (that is, "see" or distinguish) objects to a size of **dx**, which is related to and to the wavelength **L** of the gamma ray, by the expression **(1)**. This is the resolving limit of the microscope and it is the uncertain quantity of the object's position. The microscope can not see the object whose size is smaller than its resolving limit, **dx**. Therefore, to be seen by the microscope, the size of the electron must be larger than or equal to the resolving limit.

But if the size of the electron is larger than or equal to the resolving limit **dx**, the electron will not be in the range **dx**. Therefore, **dx** can not be deemed to be the uncertain quantity of the electron's position which can be seen by the microscope, but deemed to be the uncertain quantity of the electron's position which can **not** be seen by the microscope. To repeat, **dx** is uncertainty in the electron's position which can **not** be seen by the microscope.

To be seen by the microscope, the gamma ray must be scattered into any angle within the cone of angle **2A**, so we can measure the momentum of the electron. **dPx** is the uncertainty in the electron's momentum which can be seen by microscope. What relates to **dx** is the electron where the size is smaller than the resolving limit. When the electron is in the range **dx**, it can not be seen by the microscope, so its position is uncertain. What relates to **dPx** is the electron where the size is larger than or equal to the resolving limit. The electron is not in the range **dx**, so it can be seen by the microscope and its position is certain. Therefore, the electron which relates to **dx** and **dPx** respectively is not the same. What we can see is the electron where the size is larger than or equal to the resolving limit **dx** and has a certain position, **dx = 0**.

Quantum mechanics does not rely on the size of the object, but on Heisenberg's Gamma-Ray Microscope experiment. The use of the microscope must relate to the size of the object. The size of the object which can be seen by the microscope must be larger than or equal to the resolving limit **dx** of the microscope, thus the uncertain quantity of the electron's position does not exist. The gamma ray which is diffracted by the electron can be scattered into any angle within the cone of angle **2A**, where we can measure the momentum of the electron.

What we can see is the electron which has a certain position, **dx = 0**, so that in no other position can we measure the momentum of the electron. In Quantum mechanics, the momentum of the electron can be measured accurately when we measure the momentum of the electron only, therefore, we have gained dPx = 0.

And, $dPx\ dx = 0$. **(6)**

3. Ideal experiment 2. Single Slit Diffraction Experiment

Suppose a particle moves in the **Y** direction originally and then passes a slit with width **dx**(Please see diagram below) . The uncertain quantity of the particle's position in the **X** direction is **dx**, and interference occurs at the back slit . According to Wave Optics , the angle where No.1 min of interference pattern is can be calculated by following formulae **(1)** and **(2)**. So the uncertainty principle can be obtained $dPx\ dx \sim h$ **(5)**

4. Re-analysis

According to Newton first law , if an external force in the **X** direction does not affect the particle, it will move in a uniform straight line, (Motion State or Static State) , and the motion in the **Y** direction is unchanged .Therefore , we can learn its position in the slit from its starting point.

The particle can have a certain position in the slit and the uncertain quantity of the position is **dx =0**. According to Newton first law , if the external force at the X direction does not affect particle, and

the original motion in the Y direction is not changed , the momentum of the particle int the X direction will be **Px=0** and the uncertain quantity of the momentum will be **dPx =0**. This gives **(6)**

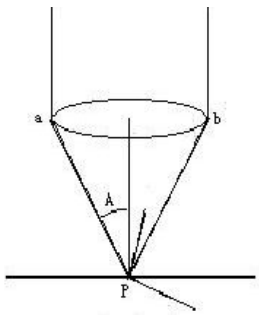
No experiment negates NEWTON FIRST LAW. Whether in quantum mechanics or classical mechanics, it applies to the microcosmic world and is of the form of the Energy-Momentum conservation laws. If an external force does not affect the particle and it does not remain static or in uniform motion, it has disobeyed the Energy-Momentum conservation laws. Under the above ideal experiment , it is considered that the width of the slit is the uncertain quantity of the particle's position. But there is certainly no reason for us to consider that the particle in the above experiment has an uncertain position, and no reason for us to consider that the slit's width is the uncertain quantity of the particle. Therefore, the uncertainty principle, **(5)** which is derived from the above experiment is unreasonable.

5. Conclusion

From the above re-analysis, it is realized that the ideal experiment demonstration for the uncertainty principle is untenable. Therefore, the uncertainty principle is untenable.

Reference

1. Max Jammer. (1974) The philosophy of quantum mechanics (John wiley & sons , Inc New York) Page 652. Ibid, Page 673. http://www.aip.org/history/heisenberg/p08b.htm

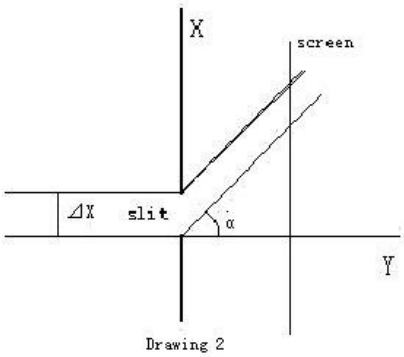


Drawing 1

Experiment of Heisenberg Gamma-Ray Microscope

Pa and Pb are the extreme forward scattering and extreme backward scattering

2A is the field angle formed between the diameter of the lens and the object



Drawing 2

Experiment of single slit diffraction

⊿X is the width of the slit

α is the angle where NO.1 min of interference pattern is

Fizeau experiment, aberration of starlight and Einstein: Relativity (issues)

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Einstein had repeatedly said (only after 1907) that the **Fizeau experiment** had "influenced him the most" and that it was "crucial", and likewise said Fizeau and Aberration "were enough", to guide him to "create the theory of relativity".

Now, the $(1- 1/n^2)v$ term of Fresnel-Fizeau which was known in 1905 was only of first order, which was not relevant then, and could not be harbinger of relativity which released the new formula for composition of velocities, which in 1907, von Laue showed, explained the Fizeau experiment, in fact went beyond the first order. Other related matters and analysis are presented in the paper. (We are looking forward to Russia, which is experimenting in this higher area).

In the case of **aberration** of starlight also, both in theory and astronomical observations, the order of accuracy is low and is of the order v/c, where v is the average velocity of the earth in the orbit from perihelion to aphelion. It is subject to a small correction due to the rotation of the earth around its axis. The maximum aberration observed at zenith is 20.4955 arc-sec. The theoretical calculation gives the figure 20.503 arc-sec.

The relativistic theory of composition of the velocity of light(s) and orbital velocity (v) as pointed out by Einstein himself is approximate and the calculation is based on flat Euclidean plane for the composition of velocities. The velocity of light (c) compounded with the orbital velocity (v) is to be c again, which is possible only on a curved surface, perhaps, only the surface of a sphere. Calculation of maximum aberration is made assuming that the "velocity-space" is like that of a sphere, which yields a (curved) length of 20.503 arc-sec.

There is need for getting more accurate observations of orbital velocity and the aberration itself.

1. Fizeau experiment and Einstein

The experiment of Fizeau to determine the velocity of light in a moving medium (water), as measured from the rest-system of the laboratory had the greatest effect on Einstein in the development of the special theory of relativity. In a talk entitled, "How I created the theory of relativity" given at Kyoto University in December 1922 he said, " It was more than seventeen years ago that I had an idea of developing the theory of relativity for the first time. While I cannot say exactly where that thought came from, I am certain it was contained in the problem of the optical properties of moving bodies."[1]

In conversation with Shankland[2] in Feb. 1950, Einstein told him that the experimental results, which had influenced him most, were the observations on the velocity of light in moving water: in fact he emphasized, "They were enough." In a statement sent by Einstein[3] in 1952 at the time of the centenary of Michelson's birth, Einstein again said that he was guided by the result of the Fizeau-experiment and the phenomenon of aberration. In his book The Meaning of Relativity, Einstein[4] asserted, "No other theory has satisfactorily explained the facts of aberration, the propagation of light in moving bodies (Fizeau), and phenomena observed in double stars (de Sitter)". In fact, Einstein[5] regarded the result of the Fizeau experiment as a "crucial test" in favour of the theory of relativity. Reminiscing about his conversations with Einstein in 1950-54, Shankland[6] wrote in 1973 that Einstein considered the phenomenon of the effect of moving water on the speed of light as a major observational fact for this work on special relativity, and that the close relation of the Fizeau experiment to the relativistic velocity addition theorem was the prime reason for Einstein's interest in this subject,

However, Einstein[7] did not mention the Fizeau experiment or any other, specifically, in his paper of 1905. He mentioned only "the unsuccessful attempts to discover any motion of the earth relative to the light-medium…", and the Fizeau experiment was not one such.

It might be mentioned that in the same paragraph, from which we have quoted above, Einstein said that as has already been shown to "the first order of small quantities, the laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good." There is no reference at all to sources in which the validity obtains for all orders in v/c. The supporting experiments, aberration and Fizeau Experiment, quoted by Einstein were also established to O(v/c), although the possibility of verifying higher order terms existed. Indeed such $O(v^2/c^2)$ experiments had been performed before Einstein wrote his paper in 1905, such as Michelson's.

According to the usual presentation, the result of Fizeau's experiment is to be regarded as a purely kinematic consequence of two relative motions. No detailed account of the physical processes involved is considered necessary. The result to be expected theoretically follows at once from the 'relativistic' formula for compounding two velocities.

The velocities compounded are:-

i) The velocity of light if observed in the rest frame of the moving medium which is taken to be the same as in the medium, when it is at rest relative to the source of light, that is, c/n, where c is the velocity of light in vacuo, and n is the refractive index of the medium; and (ii) the (second) velocity, v, of the medium relative to the source of light. The addition-theorem of velocities applies generally only when the two velocities are in the same line, as first pointed out by Pauli[8], and this is the case we consider here. When the direction of the light-ray and the wave-normal differ, the law of parallelogram of velocities does not apply, as also in the case of aberration. Incidentally, in the present case, one of the velocities, pertains to an entity which has a wave-nature, characterized by frequency(ν), whether it is regarded as a photon or as a field, while the second velocity pertains to particulate matter, the de Broglie wave-length of which is smaller compared to that of light.

The velocity of light in moving water, c_{mov} , as observed in the rest-frame of the laboratory, is then given by the compounding formula:

$$\mathbf{c_{mov} = (c/n+v)/(1+v/nc)} \qquad \textbf{(1)}$$

and more symmetrically,

$$\mathbf{= c(c+nv)/(v +nc)} \qquad \textbf{(2)}$$

where v is the velocity of water relative to the laboratory. We have taken the case when the velocity of water is opposite to the direction of c and c/n. When the water is moving towards the source of light, -v is to be substituted for v.

The second term in equation (1), $(1+v/nc)^{-1}$, may be expanded in a binomial series to give:

$$\mathbf{c_{mov} = c/n + (1-1/n^2)v - (1-1/n^2)v^2/nc} \qquad \textbf{(3)}$$

The Fresnel's theoretical formula, which the Fizeau experiment verified, consists of the first two terms on the right hand side of equation (3), viz.,

$$\mathbf{c_{mov} = c/n + (1-1/n^2)v} \qquad \textbf{(4)}$$

While the medium moves with velocity v relative to the source of light, the change in the velocity of light in the moving medium as measured in the rest system of the source is less, being v, assuming that n is greater than unity.

We shall comment on the derivation of this formula by Fresnel later. What we should remark here is tht the Fresnel formula permits velocities greater than the velocity of light, c. For example taking n for sodium light ($\lambda = 0.589 \times 10^{-6m}$) as 4/3 in the case of water, we find that for velocity of the medium greater than 4c/7, c_{mov} is greater than c. Fresnel's formula cannot, therefore, be said to be in conformity with the theorem of the constancy of the velocity of light or the theorem that the velocity of light is an impassable limit from which Lorentz Transformation and the relativistic formula for compounding collinear velocities follow.

It needs must be stated that c_{mov} is the signal velocity and not the phase velocity of light in the medium (water). It is the velocity of light measured over the distance from the point of entry into the moving medium to the point of exit, in the Fizeau's experiment.

If one was not in possession of the velocity addition formula of the special theory, it would be difficult for one to see how the factor $(1-1/n^2)$ of the Fresnel formula could lead to the principle of relative motion and, conversely, how the principle of relative motion could lead to a knowledge of this factor.

It would be reasonable to say that if an experiment is relevant to a new and unknown theory, it would contain some recognizable elements of connection and correspondence. In fact, for more than forty years, Fizeau's experiment satisfied one of the then prevailing hypothesis that the putative aether was to have some effect on the velocity of light in a moving transparent medium. The velocity measured by Fizeau in moving water conformed to the expected relation:

Velocity of light in a moving medium = its velocity in air + k x velocity of medium relative to light-source, **or,**

$$c_{mov} = c - kv, (k<1), \tag{5}$$

based on the hypothesis mentioned above.

Fizeau measured the value of k in water as about 0.44, not zero, nor unity, but something in between, approximately verifying Fresnel's formula for it. Indeed Poincare[9] was so impressed by the result of Fizeau's experiment that he said in the year 1900, that one believes one can touch the aether with one's fingers – with this result in hand!

Moreover, the Fresnel-Fizeau result was of first order, i.e. v/c effect, but what was needed in the year 1904-5 was a theory which would explain results of experiments dependent on the v^2/c^2 term.

So the Fizeau result could not have inspired Einstein in a logical and constructive way, but as a stimulus which went far beyond, in fact outside of, its related possibilities.

No one – not even Einstein - talked of the connection between the Fizeau's experiment and the velocity addition formula of the special theory until 1907, when Laue[10] showed that the Fresnel formula follows as an approximation to order (v/c), from the relativistic velocity-addition formula in a straightforward way. From what we said above, it is apparent that the Fresnel formula is a good approximation at small values of the velocities of moving media ($v/c << 1$). For large velocities, the relativistic terms $O(v^2/c)$ and higher, arise. It may be possible to verify the relativistic formula, using solid transparent bodies and current technologies for the similar term of the velocity-addition formula.

2. The Fresnel Formula

The formula derived by Fresnel[11] some forty years before its experimental verification by Fizeau in 1859, regarding the velocity of light in moving media, was really a lucky dip in the unknown. Fresnel made some, rather gratuitous assumptions to arrive at his formula. These were:

i) The density of aether is proportional to the square of the refractive index, n. This made sure the appearance of n^2 in the formula.

So, if ρ is the density of the aether in vacuum, the density of aether within matter, ρ, is given by the equation.

$$\rho' = n^2\rho. \tag{6}$$

ii) When the medium is in motion, a part of the aether is carried along or dragged by the medium.

The density of aether carried along is $\rho' - \rho$ or $(n^2 - 1)$, while the rest of the aether of density, ρ, remains stationary.

iii) The velocity of the centre of gravity of the aether within the medium which is carried along is then

$$[(n^2 - 1)/n^2)]v = (1 - 1/n^2)v \tag{7}$$

iv) The velocity of light in the medium was then assumed to increase by this amount.

Since the Fresnel formula was at that time regarded as the backbone of Fizeau's experimental result, one can hardly visualise, even in retrospect, how it moved Einstein in arriving at the special theory of relativity from this result.

3. Derivation using L.T.E.

The velocity addition formula of the special theory is itself usually derived with the help of the Lorentz Transformation Equations (L.T.E.).

We derive the formula for the velocity of light in a transparent moving medium directly from the L.T.E.

Let S(x,t) be the inertial reference system in which the light source and the observer are at rest, and S´(x´, t´) the system of the medium moving with velocity v relative to S in the direction which coincides with X and X´ axes. We have suppressed the y, y´, z and z´ coordinates because the events of interest are located on the coincident X and X´ axes. The clocks of each of the two systems are synchronized with the clock at the origin of each system.

A pulse of light is sent in the X – X´ direction through the moving medium.

The two events of interest are the entry of a pulse of light (a photon) into the moving medium and its exit.

Event 1: The pulse leaves the two origins at the one end of the medium at time t = t´ = 0, and x=x´=0.

Event 2: The pulse travels a distance x in S in time t and x´ in S´ in time t´ and reaches the other end of the medium, in motion with velocity v.

We know that if n is the refractive index of the medium in its rest-frame S´,

$$\mathbf{x'/t' = c/n\ ,\ or\ \ x' = ct'/n.} \qquad \textbf{(8)}$$

Using (inverse) L.T.E., going from system S´ to S, we get

$$\mathbf{x = \beta(x' + vt'),\ t = \beta\,(t' + vx'/c),\ \beta = (1 - v^2/c^2).}$$

Substituting from equation (8), for x´, we get straightway

$$\mathbf{x = \beta\,(c/n + v)t',\ t = \beta\,(t' + vt'/nc) = \beta\,(1 + v/nc)t'} \qquad \textbf{(9)}$$

Therefore,

$$\mathbf{c_{mov} = x/t = c\,(c + nv)/(nc + v),}$$

which is obviously the velocity of light in the medium, c_{mov}, as measured from the rest system of the observer, S, as in equation (2). β has disappeared.

We show in Appendix 1 that the same formula for the velocity of light in a medium when the medium is in motion relative to the emitter of light can be derived without using the relativistic compounding formula for two collinear velocities. We consider the fact that the photons which interact with some of the molecules, are carried along by these molecules (or atoms) of the moving transparent medium. The physical point to note is that the interactions of matter and radiation are not instantaneous. We shall also comment in this connection, on the "extinction theorem" for light travelling through transparent media, which is used to derive n, the refractive index, and is a gross property of matter in the aggregate, and a critical and historical note in Appendix 2.

References

1. Y.A.Ono, trans. of A.Einstein's lecture given in Kyoto University on 14 Dec. 1922, "How I created the theory of relativity", Phys.Today 35, 45-47 (1982).
2. R.S.Shankland, "Conversations with Albert Einstein", Amer. J.Phys. 31, 47-57 (1963).
3. R.S.Shankland, "Michelson-Morley Experiment", Amer.J.Phys.31, 47-57 (1963).
4. A.Einstein, Relativity, The Meaning of Relativity, Methuen & Co., London, 4th ed., 1950, p.25.
5. A.Einstein, Relativity, The Special and The General Theory, Smith, 1931, Chapter 13 end.

6. R.S.Shankland, "Conversations with Albert Einstein: II", Amer. J.Phys. 41, 895-901 (1973). Incidentally, Shankland approvingly refers to the related work of the present author (p.901, reference 3).
7. A.Einstein (& others), The Principle of Relativity, Dover Pub., p.37.
8. W.Pauli, Theory of Relativity, trans. by G.Field, Pergamon Press, 1958. P.114.
9. Quoted by: A.Pais, Subtle is the Lord, Oxford Univ. Press, New York, 1982, p.127. (see p.136, giving reference to Poincare's work.)
10. M.von Laue, "Propagation of light in moving bodies on the Principle of Relativity", Ann.d.Phys. 23, 989-990 (1907).
11. A.J.Fresnel, Ann. Chim. Phys. 3, 57 (1818).

APPENDIX 1 Derivation of formula c_m=c(c+nv)/(nc+v), without using relativistic velocity addition formula

I think "refractive index" could be understood fully only if the photon or particulate concept and the electromagnetic-wave aspect are both used. Let me quote J.A.Wheeler from his article (J.A.Wheeler, The young Feynman, Physics Today 42(2), 24-28(1989)).

"…how could we understand in terms of scattering, and nothing but scattering the propagating of a photon through a medium of variable refractive index….. refractive index as a cumulative consequence of many scattering processes…as a tool to add up scattered waves.." Let us note that visible light has a wave-length 3800-7000x10^{-8} cm and the water molecule is as small as 2.6x10^{-8} cm.

Equation 2 may be derived on the consideration that a photon in its rectilinear motion through transparent media, has a weak interaction with some molecules involving delay in transmission.

Obviously the internal electronic (not electrical) structure of transparent media is such that no excitations of the structure exists which match the frequencies of light (photons).

Even the outermost valence electrons of transparent media are so tightly bound that only photons of higher energy in the ultraviolet region could excite these electrons. The energy of photons in the visible range goes from about 1.5 to 3.0 eV, while the energy required for a free valence electron in ice, for example, is 4 eV. (Ice is therfore opaque to ultraviolet radiation.) In the case of insulators such as glass, quartz and diamond, excitation resonances do not obtain in the visible ranges of frequencies.

So in the case of transparent media, there is no absorption, followed by emission of photons by interacting with electrons.

However, in the case of transparent media the electric field-vector (E) or light radiation can act on the dipoles formed by the charges on the electrons and the nuclei of the atoms or molecules, as a whole, without absorption-emission process which is not possible. It is a kind of elastic interaction, but it is not instantaneous but takes finite time in each of these random interactions, say, τ, on an average per interaction. There is some time-delay due to this kind of scattering.

Since the energy of a photon itself is $h\upsilon$ the limit to the time of interaction is $\tau > 1/\upsilon$,because according to the uncertainly principle; $\boldsymbol{\tau \times h\upsilon > h}$ **, or** $\boldsymbol{\tau > 1/\upsilon}$**.**

For light, the interaction time is therefore more than 10^{-13} sec.

As is well-known, when interacting with matter, a photon has a point interaction. (For example, a radio wave with λ = 30m or $h\upsilon$ = 6.6 x 10^{-20} erg, is received by an electron of dimension ~ 3 x 10^{-13} cm.)

It suffices to make just one assumption, that a photon does have an interaction with the molecules (or atoms) of transparent media over a discrete period of time, without changing its direction on an average, as is experimentally confirmed, after entering the medium in motion. Otherwise there could be no change in the velocity of light within such media. So, the passage of photons through such media is subject to delays at the interaction points, travelling at the vacuum-velocity, c, between interactions.

Let, *l* be the average distance between two successive points within the slab, where interactions and delays in transmission take place. Also, let τ be the average time of one interaction. Consequently, the

average velocity in the medium is reduced from c to some velocity, $c_m = c/n$ ($n > 1$ is then refractive index), inside the quartz slab, while between the interactions the velocity remains c.

We assume that during the interactions, the photon energy ($h\upsilon$) is carried pickaback by the molecules of the interacting medium (the quartz slab), moving in the direction of the ray, with the two faces of the slab, perpendicular to the direction of motion (and the ray). In fact, delay takes place in all scattering and absorption-and-emission processes, more or less. (No one knows exactly how the photon comes to rest and then resumes its velocity c as in the case of a mirror, for example).

Now to the derivation of the formula. $v\tau$ is the distance travelled in the time τ, while a photon is interacting with the molecules of the medium (quartz slab) which in turn are in motion with velocity v, carrying the photon with itself. The average velocity of light in the moving medium as measured in the rest system of the light source is then

$$\mathbf{C/n = \frac{\text{distance between, two interactions + distance travelled during interaction}}{\text{time of travel at velocity c + time of travel with velocity v}}}.$$

$$= \frac{l\,(1 + v/c) + v\tau}{l\,(1 + v/c)\,/\,c + \tau}\,. \qquad \textbf{(1)}$$

It is (+v/c) when the medium (quartz) is travelling away from the source of light. It would be (-v/c) when the medium is moving towards the source, and light has to travel over a distance less than *l*.

When the medium is at rest relative to the source of light, the photons only interact with some molecules, randomly, with each interaction lasting on an average, for time τ. In this time the distance travelled by a photon is equal to *l* only (because the medium is not moving and the molecules are at rest, averaged over time relative to the source of light). So, $\mathbf{c/n} = \dfrac{l}{l/c + \tau}$.

So, $$l = c\tau\,/\,(n - 1) \qquad \textbf{(2)}$$

This means l/τ is a constant equal to c/(n-1). Substituting for *l* in equation (1), we get the desired equation thus:

$$\mathbf{c/n = \frac{l\,(1 + v/c) + v\tau}{l\,(1 + v/c)\,/\,c + \tau} = \frac{c\tau(1 + v/c)\,/(n - 1) + v\tau}{c\tau(1 + v/c)\,/(n - 1)\,c + v\tau} = \frac{c(1 + v/c) + v\,(n - 1)}{(1 + v/c) + (n - 1)} = c(c + nv)\,/\,(nc + v).}$$

l and τ have disappeared! We needed them as physical entities, surely, in order to get the analysis going to find the velocity in the moving medium.

APPENDIX 2 EXTINCTION THEOREM

In order to explain reduction in the velocity of light in transparent media, P.P.Ewald in 1915, for crystalline, and C.W. Oseen in his doctorate thesis in 1916, for isotropic media, further refined by Hoeck in 1939, showed that the change in the free-space velocity of light entering a medium (including interstellar medium) takes place in a "boundary" layer over an "extinction length". The cause of this is the electric-vector field of the incident radiation which interacts with the field of permanent dipoles, plus the induced dipoles (generally, a tensor field depending on the configuration of the negative and positive charges inside the molecules). The induced dipoles arise from the displacement of the electrons relative to the nucleus under the electric field of the radiation. (The magnetic field-vector of the radiation. (The magnetic field-vector of the radiation produces a negligible effect.)

The total effect of the neighbouring excited molecular dipoles is to generate a new oscillating field around them. This induced field consists of two parts, and most curiously, one of the parts exactly "extinguishes" the primary field travelling with velocity c. The other part has a different velocity, c/n, but has the same frequency, and it is propagated in the same direction as the primary field. As we shall see, this causes some problems.

Another wonder of the theory is that it correctly leads to the formula for the polarizability of a molecule in terms of the refractive index, n, given earlier by Lorentz and Lorenz, empirically.

L.Rosenfeld explained the work of Ewald and Oseen in great detail in his book, Theory of Electrons, North-Holland Pub. Co., 1951, and it finds an honourable place in the classical book of Max Born and E.Wolf, Principles of Optics, Pergamon Press, 1959, and briefly in the Handbuch de Physik, vol. iv, p.99, 1962, in an article on Principles of Electrodynamics and Relativity, by Melba Phillips.

I hope that I have explained the concept but before I criticize it, let me fortify myself with the following excerpts from the authoritative book of Born and Wolf just mentioned:

"It will, however, be shown that the dipole field $E^{(d)}$ may be expressed as the sum of two terms, one of which obeys the wave equation in the vacuum and cancels out exactly the incident wave, whereas the other satisfies the wave equation for progression with velocity c/n. the incident wave may therefore be regarded as extinguished at any point within the medium by interference with the dipole field and replaced by another wave with a different velocity (and generally also a different direction) of propagation. This result is known as the extinction theorem and was established first for crystalline media by EWALD and for isotropic media by OSEEN!"”(Page 99.)

"It is to be noted that the extinction of the incident wave is brought about entirely by dipoles on the boundary of the medium and the polarization P is set up entirely by mutual action of neighbouring dipoles." (Page 100)

Obviously, in the theorem mentioned above the time delays in the process of interactions between the medium and the wave with velocity v/n, are not taken into account, which would cause further retardation in the velocity of light. Thus the extinction and re-emission process is assumed to be instantaneous, and the only interaction. Why does the wave with velocity c/n not interact further with the medium? There are other difficulties, as well.

1. Is c/n the velocity of light in empty space after extinction? This violates the principle of invariance of the velocity of light.
2. How is the velocity of light restored to the normal value c, at exit from the medium into free space?
3. What exactly happens to the energy of the photons in the change from velocity c to c/n, and vice-versa? Is the energy $h\nu$ at velocity c/n the same as at c?
4. What is the momentum of the photon at velocity c/n? Is it $h\nu n/c$? Does the momentum suddenly change at exit, to $h\nu/c$?

Similar questions arise when we take light as electromagnetic propagation. We have, of course, assumed polar elastic interactions of photons with the molecules over a small period of time, individually, the aggregate of which is substantial, leading to a lower average value of the velocity of light in a moving medium. And this seems to be unobjectionable.

Remarks about the energy flux and momentum of electromagnetic field

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The paper re-examines the problem about the energy flux and momentum of electromagnetic (EM) field. It presents a number of physical reasons, which indicate that the Poynting vector does not describe the energy flux of a non-radiating EM field. The corrected expression for the energy flux for such a kind of EM field has been derived, which has a clear physical meaning: the non-radiating EM field simply moves together with its source. Besides, it naturally produces the equality $u_{EM} = m_{EM}c^2$ for the energy density u_{EM} and mass density m_{EM} of EM field. Simultaneously a related problem of momentum of the EM field has been considered. It has been concluded that EM radiation and non-radiating EM field are different physical entities: in the first case a momentum is defined via the Poynting vector; for the non-radiating EM field a transformation of mechanical to EM momentum and vice versa for a closed system occurs in accordance with the known requirement $\vec{P}_G = \text{const}$, where $\vec{P}_G = \vec{P}_M + \sum_i q_i \vec{A}_i$ is the generalized momentum of the system ($\vec{P}_M$ is the mechanical momentum, q is the charge, and $\vec{A}$ is the vector potential). Some physical inferences from the obtained results are discussed.

1. The energy flux in a non-radiating electromagnetic field

It is known that the density of energy of EM field is defined by the expression

$$u = \varepsilon_0 \left(\frac{E^2}{2} + \frac{c^2 B^2}{2} \right) \tag{1}$$

($\vec{E}, \vec{B}$ are the electric and magnetic fields, respectively), and a local validity of the energy conservation law requires that the partial time derivative of a density of EM energy

$$\frac{\partial u}{\partial t} = \varepsilon_0 \vec{E} \frac{\partial \vec{E}}{\partial t} + \varepsilon_0 c^2 \vec{B} \frac{\partial \vec{B}}{\partial t}. \tag{2}$$

should be equal to the energy flux across the boundary of that volume and a transmission of energy to matter. Considering Eq. (2), Poynting proposed to use the Maxwell equations to evaluate the field partial time derivatives [1]:

$$\frac{\partial \vec{B}}{\partial t} = -\nabla \times \vec{E}, \tag{3a}$$

$$\frac{\partial \vec{E}}{\partial t} = c^2 \left(\nabla \times \vec{B} \right) - \frac{\vec{j}}{\varepsilon_0}, \tag{3b}$$

Then the substitution of Eqs. (3) into Eq. (2) leads to the familiar equation

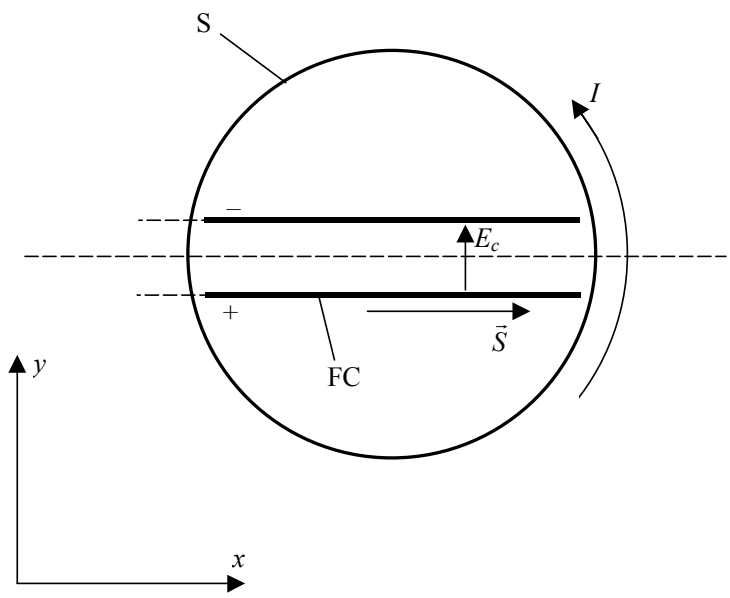
$$\frac{\partial u}{\partial t} + \nabla \vec{S} + \vec{E}\vec{j} = 0 \tag{4}$$

($\vec{j}$ is the current density), where

$$\vec{S} = \varepsilon_0 c^2 \vec{E} \times \vec{B} \tag{5}$$

is the Poynting vector, which defines the energy flux.

Eqs. (4-5) perfectly describe the EM radiation: the direction of $\vec{S}$ coincides with direction of EM wave propagation, and the term $\vec{j}\vec{E}$ corresponds to an absorption of EM radiation by charged particles. At the same time, for quasi-stationary (non-radiating) EM field, the Poynting vector sometimes gives strange pictures of the energy flux. One of familiar problems of this kind is a straight line with current: the vector $\varepsilon_0 c^2 \vec{E} \times \vec{B}$ is orthogonal to the line, and it draws in energy not from a source of current (which would be natural to assume), but from surrounding space. Another curiosity is seen in Fig. 1: the charged parallel plate capacitor is inside the elongated solenoid. One



can see that the Poynting vector $\varepsilon_0 c^2 \vec{E} \times \vec{B}$ is

Fig. 1. Parallel plate capacitor FC inside the solenoid S with the current *I*. The electric field of FC lies along the axis *y*, the magnetic field of S is directed along the axis *z*.

directed along the axis *x*, and the energy flux suddenly appears near the left end of the capacitor, and suddenly disappears near the right end of capacitor. It seems also very strange. And what is more important, such the energy fluxes in non-radiating EM fields were never detected in the experiments. Finally, there is the most serious objection against the applicability of Eq. (4) to a non-radiating EM field: such a field is not absorbed by charged particles, and the change of their kinetic energy $\vec{j}\vec{E}$ should be exactly equal to the change of their potential energy in EM field. However, the potential energy of charged particles has been lost in Eq. (4).

These physical problems, arising under application of Eq. (5) to a non-radiating EM field, are issued, in the author's opinion, by incorrect physical interpretation of the energy balance equation for a non-radiating EM field. Namely, we hasten to interpret Eq. (4), while its transformation to the final expression is still not completed for the non-radiating EM field. In particular, due to a mutual canceling of the changes of kinetic and potential energy of charged particle in the non-radiating EM field, a physically meaningful energy balance equation should contain neither kinetic, nor potential energy of particle. And such an equation is expected to give a true expression for the energy flux in the non-radiating EM field.

In order to transform the energy balance equation to such a final form, we use the known relationship

$$\vec{B} = \frac{\vec{v} \times \vec{E}}{c^2} \tag{6}$$

for the electromagnetic field, produced by a charged particle moving at the constant velocity $\vec{v}$. Then

$$\begin{aligned} \nabla\vec{S} &= \varepsilon_0 c^2 \nabla\left(\vec{E} \times \vec{B}\right) = \varepsilon_0 c^2 \left[\vec{B}\left(\nabla \times \vec{E}\right) - \vec{E}\left(\nabla \times \vec{B}\right)\right] = -\varepsilon_0 c^2 \left[\vec{B}\frac{\partial \vec{B}}{\partial t} + \vec{E}\left(\nabla \times \left(\frac{\vec{v} \times \vec{E}}{c^2}\right)\right)\right] = \\ &= -\varepsilon_0 c^2 \left[\vec{B}\left(\frac{\vec{v} \times \partial\vec{E}/\partial t}{c^2}\right) + \frac{\vec{E}\vec{v}\left(\nabla\vec{E}\right)}{c^2} - \frac{\vec{E}(\vec{v}\nabla)\vec{E}}{c^2}\right] = -\varepsilon_0 \vec{B}\left(\vec{v} \times \frac{\partial\vec{E}}{\partial t}\right) - \vec{E}\vec{j} + \varepsilon_0 \vec{E}(\vec{v}\nabla)\vec{E}. \end{aligned} \tag{7}$$

Here we used the vector identity $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}\left(\vec{a}\vec{c}\right) - \vec{c}\left(\vec{a}\vec{b}\right)$, as well as the equality

$\vec{v}\left(\nabla\vec{E}\right) = \frac{\vec{v}\rho}{\varepsilon_0} = \frac{\vec{j}}{\varepsilon_0}$,

where ρ is the charge density. Further, using Eq. (3b), we can write

$$\vec{B}\left(\vec{v} \times \frac{\partial\vec{E}}{\partial t}\right) = \vec{B}\left[\vec{v} \times \left(c^2\left(\nabla \times \vec{B}\right) - \frac{\vec{j}}{\varepsilon_0}\right)\right] = -c^2 \vec{B}(\vec{v}\nabla)\vec{B} \tag{8}$$

(under transformation of Eq. (8) we again use the identity $\vec{a} \times \left(\vec{b} \times \vec{c}\right) = \vec{b}\left(\vec{a}\vec{c}\right) - \vec{c}\left(\vec{a}\vec{b}\right)$, and take into account that the vectors $\vec{v}$ and $\vec{B}$ are orthogonal to each other, so that $\vec{v}\vec{B} = 0$. From Eqs. (7) and (8) we derive

$$\nabla\vec{S} = \varepsilon_0 \vec{E}(\vec{v}\nabla)\vec{E} + c^2 \vec{B}(\vec{v}\nabla)\vec{B} - \vec{j}\vec{E}. \tag{9}$$

Substituting $\nabla\vec{S}$ from Eq. (9) into Eq. (4), we obtain:

$$\frac{\partial u}{\partial t} + \varepsilon_0 \vec{E}(\vec{v}\nabla)\vec{E} + c^2 \vec{B}(\vec{v}\nabla)\vec{B} = 0. \tag{10}$$

In its turn, one can easy prove that $\varepsilon_0 \vec{E}(\vec{v}\nabla)\vec{E} + c^2 \vec{B}(\vec{v}\nabla)\vec{B} = \nabla\left[\vec{v}\left(\frac{E^2}{2} + \frac{c^2 B^2}{2}\right)\right] = \nabla(\vec{v}u)$, and Eq. (10) transforms to

$$\frac{\partial u}{\partial t} + \nabla\vec{S}_U = 0. \tag{11}$$

Thus, we deduced a desired form of the energy balance equation for the non-radiating EM field, which does not contain a change of energy of charged particles. (As we mentioned above, physically it means that the non-radiating EM field is not absorbed by particles). Here

$$\vec{S}_U = \vec{v}u \tag{12}$$

is known as Umov vector. According to the obtained Eq. (11), just this vector describes the energy flux in a non-radiating EM field. We see that Eqs. (12), (11) have a simple physical meaning: a non-radiating EM field uniformly moves together with charged particle.

One can show that in case of many charged particles, produced a non-radiating EM field,

$$\vec{S} = \varepsilon_0 \sum_i \vec{v}_i \frac{\left(\vec{E}_\Sigma, \vec{E}_i\right)}{2} + \varepsilon_0 c^2 \sum_i \vec{v}_i \frac{\left(\vec{B}_\Sigma, \vec{B}_i\right)}{2}, \tag{13}$$

where

$$\vec{E}_\Sigma = \sum_i \vec{E}_i, \ \vec{B}_\Sigma = \sum_i \vec{B}_i \tag{14}$$

are the resultant electric and magnetic fields created by the charged particles. If all particles move uniformly at the constant velocity $\vec{v}$, Eq. (13) transforms to

$$\vec{S} = \varepsilon_0 \vec{v}\left(\frac{\vec{E}_\Sigma^{\,2}}{2} + c^2 \frac{\vec{B}_\Sigma^{\,2}}{2}\right).$$

Here the resultant fields again move uniformly with the system of charged particles.

We see that in case of non-radiating EM field the energy flux depends not only on vectors $\vec{E}$ and $\vec{B}$ (characteristics of the field), but on $\vec{v}$, too (characteristics of a charged particle). This simply reflects an obvious fact that such a field is always attached to its source. On the contrary, a EM radiation exists independently on its sources, and the conventional expression for Poynting vector (Eq. (5)) contains the characteristics of the field solely.

Now let us consider a single electron. In this case the energy flux of its EM field is defined by Eq. (12), and Eq. (11) becomes mathematically equivalent to the continuity equation for the charge density ρ and current density $\vec{j}$. The momentum density of the electron's EM field is computed as

$$\vec{p}_{EM} = \frac{\vec{S}_U}{c^2} = \vec{v}\frac{u}{c^2}, \tag{15}$$

where $\vec{v}$ is the velocity of the electron. Then, introducing the density of EM mass of electron m_{EM}, we immediately obtain

$$u = m_{EM} c^2, \tag{16}$$

in full accordance with the Einstein expression. Integrating Eq. (16) over all space, we get

$$U_{EM} = M_{EM} c^2. \tag{17}$$

The Eqs. (16), (17), just obtained, naturally resolve the known problem of "4/3" [2] in a relationship between the EM energy and EM mass of electron, when the Poynting vector is applied for the non-radiating EM field of moving electron. A resolution of this problem by Eqs. (16), (17) makes classical electrodynamics a fully self-consistent and self-sufficient theory.

In addition, one can easily see that u and $\vec{S}_U$ compose a four-vector, which can be termed the four-vector of EM energy, $E_{EM\,i} = u\dfrac{dx_i}{dt}$.

Further, it is naturally to propose that the EM momentum $\vec{p}_{EM} = \vec{S}_U / c^2 = \vec{v}\,u / c^2$ represents a proper attribute of a charged particle (like its EM mass), and it can be transformed to a mechanical momentum only under collisions of charged particles. At the same time, it is well-known that the momentum of EM field can be transformed to the mechanical momentum under variation of EM field alone, without any collisions of particles. Hence, a re-definition of the energy flux for the non-radiating EM field requires to reanalyze the problem of momentum EM field. This conclusion can be also confirmed by the particular problem considered in the Appendix A (the modified Shockley-James paradox [3]). This problem demonstrates incorrectness of a widespread opinion that the EM momentum of a non-radiating EM field, when defined via the Poynting vector, is transformed into mechanical momentum of charged particles. Instead, a mutual transformation of mechanical and EM momenta occurs in accordance with the known requirement of constancy of generalized momentum for a closed system of charged particles.

2. About a mutual transformation of the electromagnetic and mechanical momentum

It is known that the three-dimensional Lagrangian of a charged particle in an external EM field has the form $L = -Mc^2\sqrt{1 - v^2/c^2} + q\left(\vec{v}\vec{A} - \varphi\right)$, and the motional equation is [4]

$$\frac{d}{dt}\frac{\partial L}{\partial \vec{v}} = \frac{\partial L}{\partial \vec{r}}, \tag{18}$$

where

$$\frac{\partial L}{\partial \vec{v}} = \vec{P}_M + q\vec{A} = \vec{P}_G \tag{19}$$

is the so-called generalized momentum $\vec{P}_G$ of a charged particle in an EM field. One can easily show that, for a closed system of charged particles, $\vec{P}_G$ has a constant value [1]. Indeed, for the simplest system of two charged particles, the common Lagrangian represents a sum of Lagrangians for each particle:

$$L = L_1 + L_2 = -M_1c^2\sqrt{1-v_1^2/c^2} - M_2c^2\sqrt{1-v_2^2/c^2} + q_1\left(\vec{v}_1\vec{A}_{12} - \varphi_{12}\right) + q_2\left(\vec{v}_2\vec{A}_{21} - \varphi_{21}\right), \tag{20}$$

where q, M, $\vec{v}$ are the charge, mass and velocity, $\vec{A}_{12}$, φ_{12} are the vector and scalar potentials of the particle 2 at the location point of the particle 1, and $\vec{A}_{21}$, φ_{21} are the vector and scalar potentials of the particle 1 at the location point of the particle 2. Substituting Eq. (20) into Eq. (18), we obtain

$$\frac{d}{dt}\vec{P}_\Sigma = q_1\nabla\left(\vec{v}_1\vec{A}_{12}\right) - q_1\nabla\varphi_{12} + q_2\nabla\left(\vec{v}_2\vec{A}_{21}\right) - q_2\nabla\varphi_{21}, \tag{21}$$

where $\vec{P}_{G\Sigma} = \vec{P}_{G1} + \vec{P}_{G2} = \left(\vec{P}_{M1} + q_1\vec{A}_{12}\right) + \left(\vec{P}_{M2} + q_2\vec{A}_{21}\right)$

is the total generalized momentum of the system of two particles. Further, one can easily show that

$$q_1\nabla\left(\vec{v}_1\vec{A}_{12}\right) = -q_2\nabla\left(\vec{v}_2\vec{A}_{21}\right),\ q_1\nabla\varphi_{12} = -q_2\nabla\varphi_{21}. \tag{22}$$

Substitution of Eqs. (22) into Eq. (21) gives

$$\frac{d}{dt}\vec{P}_{G\Sigma} = 0,\ \vec{P}_{G\Sigma} = \text{const.} \tag{23}$$

for the closed system of two particles. The theorem can be easily extended to the case of many particles:

$$\frac{d}{dt}\left(\sum_i \vec{P}_{Mi} + \sum_i q_i\vec{A}_i\right) = 0, \quad \text{or} \qquad \frac{d}{dt}\sum_i \vec{P}_{Mi} = -\frac{d}{dt}\sum_i q_i\vec{A}_i\ . \tag{24}$$

Eq. (24) shows that the total time derivative of resultant mechanical momentum (total mechanical force, acting on the closed system of charged particles due to violation of third Newton law for EM interaction) is equal with the opposite sign to the total time derivative of "momentum" $\vec{P}_A = \sum_i q_i\vec{A}_i$. Hence, under change of non-radiating EM fields, just the momentum $\vec{P}_A$ transforms to the mechanical momentum of the system, but not the momentum $\vec{P}_{EM} = \int \vec{S}/c^2 dV$, obtained via the Poynting vector.

The results just obtained allow us to deduce a number of physical consequences:

1. We notice that the momentum $\vec{P}_A$ of a considered system is not associated with an energy flux across the boundary of that system. It is explained by the fact that the energy flux is a property of the EM field solely, whereas the momentum $\vec{P}_A$ belongs to the whole system "EM field + its sources". At the same time this means a qualitative difference between the conventional momentum of a particle and the momentum $\vec{P}_A$. It is known that the mechanical momentum represents the components of the energy four-vector $\{E,\ \vec{P}\}$. One can easily see that $\vec{P}_A$ forms a four-vector with the potential electric energy of charged particles $\{U_e,\ \vec{P}_A\}$, where $U_e = \sum_i q_i\varphi_i$, φ being the electric potential. For this reason we propose to name $\vec{P}_A$ as "potential" momentum. Another reason

to call $\vec{P}_A$ "potential" momentum is our freedom to add a constant vector to the $\vec{P}_A$, representing the gradient of an arbitrary scalar function *f*. This reflects the gauge invariance of the potentials in classical EM theory. Also, the implied physical meaning of $\vec{P}_A$ makes the vector potential $\vec{A}$ a real physical field, not only in quantum mechanics (e.g., in relation to experiments validating the effect of Aharonov-Bohm [5]), but in classical EM theory as well.

2. Eq. (24) loses its physical meaning in case of EM radiation, when the sources of the EM field, in general, may be absent in an arbitrary space volume. Hence, for that (source-free) kind of EM fields, the momentum density is defined by the conventional expression through the Poynting vector $\vec{S} = \varepsilon_0 c^2 \vec{E} \times \vec{B}$. This shows that non-radiating EM field and EM radiation represent two different physical entities.

3. For a non-radiating EM field the momentum $\vec{P}_{EM} = \int \vec{p}_{EM} dV$ (where $\vec{p}_{EM} = \vec{S}_U / c^2 = \vec{v} u / c^2$) represents a proper attribute of charged particles; hence it can be transformed into mechanical momentum only through collisions of particles.

3. Conclusions

In this paper we have demonstrated that a non-radiating EM field and EM radiation are, in general, qualitatively different physical entities, defined by different expressions for EM energy flux, Eq. (12) and Eq. (5), respectively. Hence, the two kinds of EM fields are characterized by different formal expressions for the momentum density. For the non-radiating EM field of a single charged particle (e.g., an electron), the energy balance equation is mathematically equivalent to the continuity equation for the charge density and current density. Then the relationship between the density of EM mass m_{EM} and EM energy density u acquires the form $u=m_{EM}c^2$, in full accordance with relativity theory. This makes the classical electromagnetic theory self-consistent and eliminates the need for shape-dependent charge structure hypotheses, Poincaré stresses, etc.

For a non-radiating EM field the energy density u and energy flux $\vec{S}_U$ constitute a four-vector, termed the four-vector of EM energy $E_{EM\,i} = u \dfrac{dx_i}{dt}$.

The problem of mutual transformation of mechanical and EM momentum for a non-radiating EM field has been reanalyzed. It has been concluded that such an EM field is characterized by two kinds of momentum: $\vec{P}_{EM} = \int \vec{S}_U / c^2 \, dV$, and "potential" momentum $\vec{P}_A = \sum_i q_i \vec{A}_i$. The momentum $\vec{P}_{EM}$ represents a proper attribute of a charged particle, and it transforms to the momentum of a mechanical system only for collisions of charged particles. Under variation of the EM field alone, a mutual transformation of mechanical momentum $\vec{P}_M$ and EM momentum occurs in accordance with the equality $\vec{P}_M + \vec{P}_A = \text{const}$, for a closed system of charged particles.

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Appendix A: Elongated solenoid + two oppositely charged particles
(modification of the Shockley-James paradox)

Let us consider the experiment depicted in Fig. 2. There is a tall solenoid S and two charged particles with opposite charges +*Q* and -*Q*, fixed upon a circumference co-axial with S, as shown.

The radius of cross-section of the solenoid is r, the distance of the particles from the axis of S is R. The length of S is sufficient to allow neglect of the magnetic field outside it. The solenoid and both particles are mechanically fixed upon a platform P, which is free to move in the xy plane without friction. Let initially the current in S be equal to zero. Then S is connected to a battery, which produces a current $i(t)$ in S. We assume that the current increases from 0 to its maximum value I, and it changes slowly enough to allow neglect of any radiative processes. During this process the magnetic field inside the solenoid also increases from zero to some maximum stationary value B_z. Increase of the current in S induces an azimuthal electric field $2\pi RE(t) = -\frac{dB_z(t)}{dt}\pi r^2$, $E(t) = -\frac{dB_z(t)}{dt}\frac{r^2}{2R}$, and the total force, acting on the charged particles is

$$F_x(t) = 2QE(t) = -Q\frac{dB_z(t)}{dt}\frac{r^2}{R}. \tag{A1}$$

Designating the total time of increase of current in S as T, we obtain a total mechanical momentum, acquired by the platform P along the axis x during this time:

$$P_{Mx} = \int_0^T F_x(t)dt = -\frac{QB_z r^2}{R} = -2QA(R), \tag{A2}$$

where $A(R)=B_z r^2/2R$ is the value of vector potential of S acting at the circumference with the radius R.

If we now decrease the current I to zero in the reverse order, the force in Eq. (A1) changes its sign, and the mechanical momentum of the platform decreases its value from P_{Mx} to 0. So, when we return an electric system (solenoid + charged particles) to its initial state, the platform P also returns to the state $\vec{P}_M$=0. This result reflects the law of conservation of total momentum, where the sum of mechanical momentum of P and the momentum of the electromagnetic field maintains a constant value.

However, the momentum of EM field, in its conventional definition, is always equal to zero for the problem considered. Indeed, a momentum density is customarily defined as $\vec{p}_{EM} = \vec{S}/c^2 = \varepsilon_0\left(\vec{E}\times\vec{B}\right)$. For an infinitely long solenoid, the magnetic field in its rest frame exists only in the inner volume of the solenoid. However, the electric field of the two charged particles does not penetrate inside the conducting solenoid. The electric field exists outside the solenoid, where $\vec{B} = 0$. Hence, the EM momentum $\vec{P}_{EM} = \int \varepsilon_0\left(\vec{E}\times\vec{B}\right)dV$ is equal to zero for the system of solenoid + charged particles.

Thus, the only possibility of satisfying the momentum conservation law is to adopt the "potential" momentum of a charged particle in an EM field $\vec{P}_A = Q\vec{A}$, as the real (physical or observable) momentum of the system. Indeed, the sum of momenta P_{Ax} for both particles, $P_{Ax} = P_{A+} + P_{A-} = QA + (-Q)(-A) = 2QA$, exactly equals the momentum of the platform P_{Mx} with the opposite sign.

References

1. J.D. Jackson, Classical Electromagnetism (Wiley, New York, 1962).
2. See for example, R.P. Feynman, R.B. Leighton and M. Sands, The Feynman Lectures in Physics. Vol. 2 (Addison-Wesley, Reading, Mass., 1964).
3. W. Shockley and R. James, Phys. Rev. Lett. 18, 876 (1967).

4. L.D. Landay and E.M. Lifshits, Teorya Polya (Nauka, Moscow, 1988) (in Russian).
5. Y. Aharonov and D. Bohm, Phys. Rev. 115, 485 (1959).

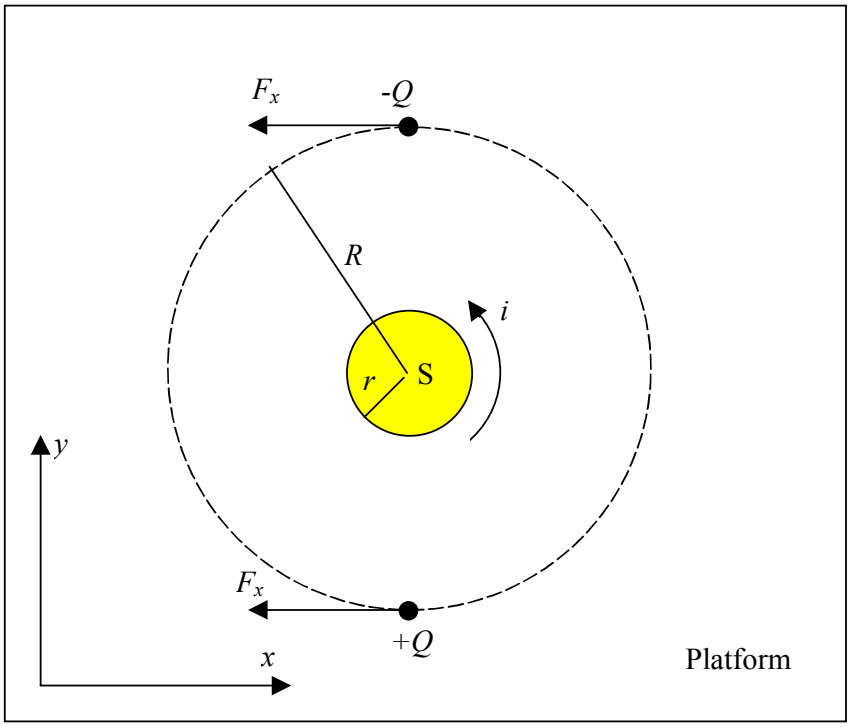

Fig. 2. Two charged particles +*Q* and -*Q* near the elongated solenoid S.

Космологические модели с вращением

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Построена стационарная космологическая модель с вращением для метрики Ожвата-Шюкинга, когда источником гравитационного поля является несопутствующая идеальная жидкость. Построена нестационарная космологическая модель для метрики типа IX по Бьянки, которая характеризуется расширением, вращением и ускорением. Источником гравитационного поля этой модели является сопутствующая анизотропная жидкость.

В настоящее время не теряет своей актуальности вопрос о возможном вращении Вселенной и его связи с вращением галактик. Поэтому сохраняется интерес к построению и исследованию космологических моделей с вращением.

Нами построена стационарная космологическая модель с вращением для метрики Ожвата-Шюкинга [1]:

$$ds^2 = (dt)^2 + R\sqrt{1-2k^2}\,\omega^3 dt - \left(\frac{R}{2}\right)^2 \left\{(1-k)(\omega^1)^2 + (1+k)(\omega^2)^2 + (1+2k^2)(\omega^3)^2\right\}, \quad (1)$$

где $\omega^1 = \cos x^3 dx^1 + \sin x^1 \sin x^3 dx^2$, $\omega^2 = -\sin x^3 dx^1 + \sin x^1 \cos x^3 dx^2$, $\omega^3 = \cos x^1 dx^2 + dx^3$.

$0 \le x^1 \le \pi,\ \ 0 \le x^2, x^3 \le 2\pi,\ \ R, k = const,\ \ |k| < \frac{1}{2}.$

Отметим, что в работе [1] получена космологическая модель с вращением с метрикой (1), заполненная пылью, с космологическим членом. Источником гравитационного поля нашей космологической модели является несопутствующая идеальная жидкость с тензором энергии - импульса $T_{\mu\nu} = (\varepsilon + p)u_\mu u_\nu - pg_{\mu\nu}$, где $\varepsilon > 0$, $u_\mu u^\mu = 1$. Компоненты вектора скорости рассмотрим в следующем виде: $u_0 \ne 0$, $u_1 = 0$, $u_2 = \tilde{u}_2\sqrt{1-2k^2}\cos x^1$, $u_3 = \tilde{u}_3\sqrt{1-2k^2}$. У нас эйнштейновская гравитационная постоянная равна 1. Тогда, ненулевые уравнения Эйнштейна для метрики (1) запишутся в следующем виде:

$$\frac{8k^2-5}{(k^2-1)R^2} = (\varepsilon + p)u_0^2 - p, \quad (2)$$

$$\frac{8k^2-1}{2R(k^2-1)} = (\varepsilon + p)u_0\tilde{u}_2 - p\frac{R}{2}, \quad (3)$$

$$\frac{8k^2-1}{2R(k^2-1)} = (\varepsilon + p)u_0\tilde{u}_3 - p\frac{R}{2}, \quad (4)$$

$$\frac{1}{(k^2-1)} = pR^2, \quad (5)$$

$$-\frac{1}{(k^2-1)} = -pR^2, \quad (6)$$

$$\frac{-(16k^4-2k^2-1)}{4(k^2-1)} = (\varepsilon + p)\tilde{u}_2\tilde{u}_3(1-2k^2) + p\frac{R^2(1+2k^2)}{4}, \quad (7)$$

$$\frac{-(16k^4-2k^2-1)}{4(k^2-1)}=(\varepsilon+p)\tilde{u}_3^2(1-2k^2)+p\frac{R^2(1+2k^2)}{4}, \tag{8}$$

$$\begin{aligned}&\frac{\cos 2(x^3)\sin^2(x^1)k+\sin^2(x^1)(16k^4-2k^2)-16k^4+2k^2+1}{4(k^2-1)}=\\&=(\varepsilon+p)\tilde{u}_2^2(1-2k^2)\cos^2(x^1)+p\frac{R^2(1+k\sin^2(x^1)\cos 2(x^3)+2k^2\cos^2(x^1))}{4}.\end{aligned} \tag{9}$$

Из уравнений (3), (4) вытекает, что $\tilde{u}_2=\tilde{u}_3$, тогда уравнения (7) и (8) будут одинаковы. Из уравнений (5), (6) получаем $p=\frac{1}{\mathrm{R}^2(k^2-1)}$. Подставим p в уравнения (2), (3), (7) и (9), и получим следующую систему уравнений:

$$(\varepsilon+p)u_0^2=\frac{8k^2-4}{R^2(k^2-1)}, \tag{10}$$

$$(\varepsilon+p)u_0\tilde{u}_2=\frac{4k^2}{R(k^2-1)}, \tag{11}$$

$$(\varepsilon+p)\tilde{u}_2^2=\frac{-4k^4}{(k^2-1)(1-2k^2)}. \tag{12}$$

Из условия $u_\mu u^\mu=1$ получим уравнение

$$u_0^2\frac{(2k^2+1)}{2}+2u_0\tilde{u}_2\frac{(1-2k^2)}{R}-2\tilde{u}_2^2\frac{(1-2k^2)}{R^2}=1. \tag{13}$$

Решая систему уравнений (10), (11), (12) при условии (13) находим, что

$$\varepsilon=\frac{8k^2-3}{R^2(k^2-1)},\quad u_0^2=\frac{2(2k^2-1)}{4k^2-1},\quad \tilde{u}_2^2=\frac{2\mathrm{k}^4R^2}{(2k^2-1)(4k^2-1)}.$$

Условия $\varepsilon>0$ □□□, $u_0^2>0$, $\tilde{u}_2^2>0$ □выполняются при $|k|<\frac{1}{2}$.

Для данной модели расширение, сдвиг и ускорение отсутствуют, вращение модели отлично от нуля при k≠0: $\omega=\frac{2\sqrt{2}k^2}{R\sqrt{1-4k^2}\sqrt{1-k^2}}$.

Нами также построена другая космологическая модель. Рассмотрим метрику типа IX по Бьянки вида [2]:

$$ds^2=(dt+A\omega^1)^2-(B\omega^1)^2-C^2((\omega^2)^2+(\omega^3)^2), \tag{14}$$

где A, B, C - функции зависящие от времени,□□ $\omega^1,\omega^2,\omega^3$ есть 1-формы, удовлетворяющие структурным отношениям типа IX по Бьянки.

Представим нашу метрику в тетрадной форме. Используется лоренцевая тетрада с ненулевыми компонентами:

$$\begin{aligned}&e_0^{(0)}=1,\ e_1^{(0)}=-A\sin x^3,\ e_2^{(0)}=A\sin x^1\cos x^3,\\&e_1^{(1)}=-B\sin x^3,\ e_2^{(1)}=B\sin x^1\cos x^3,\\&e_1^{(2)}=C\cos x^3,\ e_2^{(2)}=C\sin x^1\sin x^3,\\&e_2^{(3)}=C\cos x^1,\ e_3^{(3)}=C.\end{aligned} \tag{15}$$

Рассмотрен случай: A=kC, B=αC (k, α =const).

Источником гравитационного поля данной модели является сопутствующая анизотропная жидкость с записанным в тетрадной форме тензором энергии импульса□□

$$T_{ab} = (\varepsilon + \pi)u_a u_b + (\sigma - \pi)\chi_a \chi_b - \pi\eta_{ab},$$

где
$$u_a u^a = 1,\ \chi_a \chi^a = -1,\ \chi^a u_a = 0,\ \varepsilon > 0,\ \sigma > \pi. \tag{16}$$

Предполагаем, что $u^a = \delta_0^a$, $\chi^a = \delta_1^a$.

Тогда тетрадные уравнения Эйнштейна запишем в следующем виде

$$\frac{-(8CC''k^2 + 4C'^2k^2 - 12C'^2\alpha^2 - 3k^2\alpha^2 + \alpha^4 - 4\alpha^2)}{4C^2\alpha^2} = \varepsilon, \tag{17}$$

$$\frac{k(4CC'' - 4C'^2 - \alpha^2)}{2C^2\alpha} = 0, \tag{18}$$

$$\frac{-(8CC''\alpha^2 - 12C'^2k^2 + 4C'^2\alpha^2 + k^2\alpha^2 - 3\alpha^4 + 4\alpha^2)}{4C^2\alpha^2} = \sigma, \tag{19}$$

$$\frac{8CC''k^2 - 8CC''\alpha^2 + 4C'^2k^2 - 4C'^2\alpha^2 + k^2\alpha^2 - \alpha^4}{4C^2\alpha^2} = \pi. \tag{20}$$

Из уравнения (18) находим : $C = \frac{\alpha}{2H} ch(Ht)$ □□□H=const).

Для ε, π и σ получим следующие выражения

$$\varepsilon = \frac{-12C'^2(k^2 - \alpha^2) + k^2\alpha^2 + 4\alpha^2 - \alpha^4}{4C^2\alpha^2},$$

$$\sigma = \frac{12C'^2(k^2 - \alpha^2) - k^2\alpha^2 - 4\alpha^2 + \alpha^4}{4C^2\alpha^2},$$

$$\pi = \frac{12C'^2(k^2 - \alpha^2) + 3k^2\alpha^2 - 3\alpha^4}{4C^2\alpha^2}.$$

Для удовлетворения (16) накладываем условие: $k^2 + 1 < \alpha^2 < k^2 + 4$.

Для данной метрики вычислены кинематические параметры модели. Расширение модели $\theta = \frac{3C'}{C}$, вращение модели $\omega = \frac{k}{2C}$, ускорение $a = \frac{C'k}{C\alpha}$, сдвиг отсутствует.

Литература

1. Ozsvath I., Schucking E.L.// Annal. of Phys.-1969.-v55.-n.1-p.166.
2. Sviestins E. //Gen.Relat.Grav.-1985.-v.17.-n.6.-p.521.

Anomalous Acceleration of Pioneer 10 and 11: Dust Density in the Kuiper Belt

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A previous analysis of Radio Doppler and ranging data from distant Pioneer 10 and 11 spacecrafts indicated an apparent anomalous acceleration. Several hypotheses involving new physical phenomena have been proposed to explain that apparent anomaly. This paper shows that the anomalous acceleration of the spacecrafts Pioneer 10 and 11 in the direction of the Sun is due to the presence of dust in the Kuiper belt, which has been ignored in the calculation. These data provides the first direct measurement of dust density in the Kuiper belt, which is 1.38 x 10^{-19} gr/cc.

1. Introduction

A few years ago, it was observed that Pioneer 10 and 11 were subjected to anomalous constant accelerations equal to α=8. x 10^{-8} cm/s^2, directed toward the Sun. Consequently at a very large distance form the Sun, where the Sun's gravity becomes negligible, the velocities of these spacecrafts are constantly slowing down in their motion through outer space.

A huge effort has been expended looking for possible systematic effects but none has been found, as explained by Anderson et al.(1) The enigma is so profound that new physics has been suggested. Crawford(2) suggests a new gravitational redshift of the radio signal proportional to the distance to the spacecraft. Davis(3) considers the rest mass of the photon. Dark matter and modified gravity is also suggested. Rosales and Sánchez-Gomez(4) propose that this is due to the local curvature in light geodesics in the expanding space-time universe. Østvang(5) claims that the gravitational field of the solar system is not static with respect to the cosmic expansion. Belayev(6) uses a compacted 5th dimension of space to solve the problem. Capozziello and Lambiase(7) argue that this is due to the flavor oscillation of neutrinos. Many other paranormal solutions have been claimed.

We demonstrate here that this anomalous acceleration can be explained using classical physics. Calculation(1) implying that there is an anomalous constant acceleration directed toward the Sun, takes into account a large number of phenomena. There has been a thorough analysis of all the possible sources of internal errors in the spacecraft, too long to be reproduced here. As clearly explained previously by Anderson et al. (1), it seems that all the relevant internal problems have been solved adequately.

Furthermore, there has also been a systematic study of the potential sources of errors, external to the spacecraft. In order to reinvestigate these potential errors, it is necessary to take into account many phenomena explained in detail in the original paper(1). Let us examine here, only these phenomena due to the environment of the spacecraft that led to the anomalous acceleration. The complete list of those external phenomena, which have all been taken into account in the previous calculation(1), is:

1- The pressure of the solar radiation on the spacecrafts. This pressure is due to the exchange of momentum between solar photons and the spacecrafts. This phenomenon produces an acceleration directed away from the sun.

2- The solar wind, which is the pressure of the atoms and ions emitted by the sun pushing the spacecraft away from the sun.

3- The solar corona produces a perturbation in the transmission of the radio signal between the Earth and the spacecrafts. The data obtained by radio communication needs to be analyzed in more detail.

4- In view of the fact that the spacecrafts could hold an electric charge, there is a possibility of deviation of the trajectory by electromagnetic-Lorentz forces especially near Jupiter and Saturn.

5- The deflection of the spacecraft due to a gravitational perturbation due to the "mass of the Kuiper belt".

6- The stability of the reference atomic clocks.

7- The stability of the antenna together with the influence of transmission through the troposphere and ionosphere of the Earth.

This is the complete list of all the external forces, which have been thoroughly analyzed and in which the reader must refer in the original paper by Anderson(1) . However, we show here that one important phenomenon related to the Kuiper belt [5] has been ignored. Although the gravitational perturbation due to the Kuiper belt has been well considered, no account is taken of the momentum transfer of matter of the Kuiper belt on the spacecraft, when the spacecraft is moving through that belt.

2. The Kuiper Belt

In 1951, astronomer Gerard Kuiper suggested that some comet-like debris from the formation of the solar system must exist beyond Neptune. The Kuiper belt is a disk-shaped region past the orbit of Neptune roughly 30 to 100 AU from the Sun(8) containing dust and many small icy bodies. It is now considered to be the source of the short-period comets. The long-period comets are believed to be formed further away in the Oort cloud. The understanding of that region of space is important since the study of the trans-Neptunian asteroids is a rapidly evolving field of research(9), with major observational and theoretical advances in the last few years. Similarly, the phenomenon of disk-shaped region of dust around stars is observed in several solar systems, as recorded on photographs. For example, we see the starlight diffused on its own Kuiper belt around the star Beta Pectoris as seen in figure 2.

In fact, it is some of that dust from the Kuiper belt, which eventually is reaching the Earth neighborhood, since many tons of dust grains(10), including samples of asteroids and comets, fall from space onto the Earth's atmosphere each day. Once in the stratosphere this "cosmic dust" and spacecraft debris joins terrestrial particles. Highflying aircraft with special sticky collectors can capture this dust, as it falls through the stratosphere, before it becomes mixed with Earth dust.

For the first time in the Pioneers 10 and 11 flights, spacecrafts travel through the Kuiper belt. Therefore at last, we have the extraordinary very first opportunity to measure "directly" the density of matter (dust and gases) in the Kuiper belt. We examine here, whether the dust in the Kuiper belt produces a measurable effect on the spacecraft and how sensitive Pioneer 10 and 11 can detect that remnant matter. We show here that these spacecrafts are extremely sensitive to detect a minuscule amount of dust and gases. We show that the direct interaction of the spacecrafts with the dust in the Kuiper belt, leads to a natural explanation of what appeared an anomalous acceleration by the sun.

3. Calculation

Let us examine how the reported constant anomalous acceleration can results from the drag on the spacecraft moving through matter in space. We show that there is a real non-gravitational acceleration of the spacecraft, resulting from the principle of momentum conservation when a moving body interacts with the stationary dust or gases in the media. This produces naturally a slowing down of the spacecraft. At a very large distance from the sun (~75. AU), when the Pioneer 10 and 11 spacecrafts are in the Kuiper belt and move through it, we observe, as should be expected, that they move at an almost constant velocity in a direction away from the sun. We calculate below that the reported[1] "anomalous acceleration", which is an extremely slight change of velocity of the spacecraft, is due to the drag produced by matter belonging to the Kuiper belt.

We know that the mass of Pioneer 10 is M=241. Kg. and its change of velocity[1] per second α (the anomalous acceleration) is 8. $x10^{-8}$ cm/s^2. Therefore, the change of momentum ($\Delta\mu$) of Pioneer 10 per second (which is a force) is then $\Delta\mu/s=M\alpha=241. \times 8 \times 10^{-10} = 1.928 \times 10^{-7}$ Kg. M/s^2.

That change of momentum of Pioneer 10 is due to the collision with dust in the Kuiper belt. Due to its velocity, Pioneer 10 parabolic antenna, which has a radius "R" equal to 1.73 meter, sweeps a cylinder of interstellar dust with a cross section area "A" equal to $A=\pi R^2$. Pioneer 10 velocity "V" is about ~12.2 Km/s. Let us designate "δ" the density of matter (dust + gas) swept by Pioneer 10 in that cylindrical volume. We see that the mass "M" of that cylinder (of dust and gas) swept in one second is $M/s=AV\delta$. The momentum μ of a mass is defined as $\mu=MV$. Therefore Pioneer 10 must absorb a change of momentum per second equal to $\Delta\mu/s=A\delta V^2$, due to the collision with the dust and gas in space. This change of momentum is transferred to the spacecraft, which produces negative acceleration to the spacecraft (the anomalous acceleration). Equating the change of momentum of Pioneer 10 with the change of momentum transferred by the dust gives: $\Delta\mu/s=M\alpha= A\delta V^2$. Therefore, this shows that the dust (plus gas) density "δ" of matter crossed by Pioneer 10 equals 1.38×10^{-16} kg/m^3 or 1.38×10^{-19} gr/cc.

If we consider a gas, this corresponds to an extremely low gas density equal to 1.03×10^{-16} atmosphere. Such a density of gas is already observed in astrophysics inside some nebulae. On the other hand, if we assume fine dust sand particles (arbitrary radius equal to ~50 microns) this density of matter in the Kuiper belt corresponds to one such a tiny grain of dust per 25000. cubic meters of space. This amount of dust in the outer region of the solar system appears quite reasonable remembering that the daily amount of dust falling on Earth is reported as many tons of dust grains per day. We must also notice that the real amount of matter in the Kuiper belt cannot be larger than calculated here, since it would produce a larger, non-observed drag on the Pioneer spacecrafts. We can conclude that according to the Pioneer data, we have the first direct measurement of the density of matter in the Kuiper belt in the regions crossed by Pioneer 10 and 11.

In a more recent paper[11], another slightly larger anomalous acceleration (12. $\times 10^{-8}$ cm/s^2) has also been reported using the Ulysses spacecraft data. However, the Ulysses spacecraft is not traveling in the Kuiper belt (between 30 and 100 AU). The Ulysses spacecraft remains much closer to the Sun between 1 and 5 AU in a region in which the gravitational field of the Sun has already been carefully tested due to the accurately known orbit of planet Mars and some asteroids. At one AU from the sun, the interplanetary dust can even be seen "directly" from Earth when we observe the zodiacal light and the gegenschein[12]. It is calculated that any anomalous gravitational acceleration larger than about 0.1×10^{-8}

cm/s^2 would be measurable on Mars's orbit. Therefore there exists no anomalous acceleration due to gravity at that distance from the Sun.

Furthermore, since the "anomalous acceleration" studied here[1] is due to the interplanetary dust, it is normal that it has a negligible effect on Mars orbit as a consequence of the very large mass of planet Mars with respect to its cross section, when sweeping through the interplanetary dust. In fact, we must realize that if it were due to gravity, these anomalous accelerations would also produce measurable perturbations on the orbit of planets Uranus and Pluto. This has not been reported. The fact that this anomalous acceleration is observed only on bodies having low masses, as on Pioneer 10 and 11, but is missing in massive bodies as Neptune and Pluto shows that its origin is not gravitational. It is the drag due to collision with dust in the Kuiper belt, as should be expected in classical physics due to momentum conservation.

This gives a solution to the problem of the anomalous acceleration of Pioneer 10 and 11 toward the Sun, without the exceedingly improbable hypothesis of new physics[2-7]. It is interesting to see that physics can be explained again without farfetched hypotheses. The understanding of the origin of the anomalous acceleration, as explained here, can help NASA to plan more accurate trajectories. Finally, considering that the Pioneer spacecrafts are submitted to such an acceleration of about $-8. \times 10^{-8}$ cm/s^2, while moving through an enormous Kuiper and Oort cloud, the spacecrafts will absorb that dust due to the mechanism of accretion of dust. After millions of years of accretion, these spacecrafts will become larger and larger in time, while slowing down (unless reaccelerated later by other bodies). Pioneer spacecrafts will become the nucleus of asteroids flying away from the solar system with the interstellar dust.

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References

-1 - "*Study of the anomalous Acceleration of Pioneer 10 and 11*", J. A. Anderson, P. A. Lang, E. L. Lau, A. S. Liu, M. M. Nieto, and S. G. Turyshev, Final Document Cornell, http://arXiv.org/abs/gr-qc/0104064 April 2002

-2 - D. F. Crawford, Cornell, e-print http://arXiv.org/pdf/astro-ph/9904150

-3 - "*Pioneer 10 data yielded another fundamental physics result, a limit on the rest mass of the photon*". L. Davis, Jr. A. S. Goldhaber and M. M. Nieto, Phys. Rev. Lett. 35, 1402 (1975)

-4 - J. L. Rosales and J. L. Sánchez-Gomez, Cornell, e-print http://arXiv.org/abs/gr-qc/9810085

-5 - D. Østvang, Cornell, e-print http://arXiv.org/abs/gr-qc/9910054

-6 - W. B. Belayev, Cornell, e-print http://arXiv.org/abs/gr-qc/9903016

-7 - S. Capozziello and G. Lambiase, Modern Physics Lett. A. 14, 2193, (1999). Cornell, e-print http://arXiv.org/abs/gr-qc/9910026

-8 - Bill Arnett; Oct 7, 2002, http://seds.lpl.arizona.edu/nineplanets/nineplanets/kboc.html

-9 - David Jewitt, Institute for Astronomy, http://www.ifa.hawaii.edu/faculty/jewitt/kb.html, 2680 Woodlawn Drive, Honolulu, HI 96822

-10- http://www-curator.jsc.nasa.gov/curator/dust/dust.htm

-11- "*Indication, from Pioneer 10/11 Galileo, and Ulysses Data, of an apparent Anomalous Weak Long range Acceleration*", Physical Review Letters, John D. Anderson, Philipp A. Laing, Eunice L. Lau, Anthony S. Liu, Michael Nievo and Slava G. Tuyshev), Oct. 5, 1998

-12- *Zodiacal Light and the Gegenschein* http://xansrc.ee.duth.gr/html/nineplanets/medium.html

Философские аспекты геометризации физики*

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Квантовые калибровочные поля можно интерпретировать как связности в расслоениях пространства-времени. Изучение природы расслоений является ключевой задачей при построении единой теории поля. Если оставаться в рамках представлений о 4-мерном пространстве-времени, то путь к унификации всех известных типов взаимодействий лежит через геометрическую интерпретацию их как проявлений искривленно - расслоенного пространства. Эквивалентной этому подходу с математической точки зрения является теория, в основе которой лежит идея о многомерности пространства (теории Калуца-Клейна). В этом случае в фокусе рассмотрения оказываются не расслоения, а искривления в 10- или 11-мерном пространстве-времени. В вопросе выбора из этих эквивалентных математических описаний важное значение приобретает эвристический критерий, указывающий более прочную основу для дальнейшего движения познания. Но в любом случае речь идет о геометризации физики, о придании всем физическим объектам и явлениям геометрической интерпретации.

Распространенной является точка зрения, согласно которой в общей теории относительности устранено различие между материей и метрикой, сама метрика рассматривается как динамическое поле. Может показаться, что именно эта идеология и является основой геометризации. Однако сам факт возникновения слоев или конкретных способов компактификции N- пространственных измерений определяется динамикой эволюции физического вакуума, цепочкой спонтанных нарушений его симметрии. С методологической точки зрения выделение такого объекта как физический вакуум, с присущими ему нетривиальными чертами (нарушение принципа энергодоминантности, постоянство плотности энергии и давления), в качестве основного в физической теории находится в соответствии с методологической установкой о необходимости выделения в развитой научной теории исходной абстракции, того, « с чего следует начинать науку». В ряде работ автора статьи показано, что физический вакуум удовлетворяет всем выявленным в философском дискурсе основным признакам, предъявляемым к исходной абстракции теории: во-первых, является элементом, клеточкой любого физического процесса; во-вторых, эта клеточка несет на себе элементы всеобщего, пронизывает все стороны исследуемого процесса, ибо в любой физический процесс вакуум входит как часть, причем как конкретно-всеобщая часть целостности. В-третьих, именно физический вакуум принимает непосредственное участие в формировании качественных и количественных свойств физических объектов. Такие свойства как спин, масса, заряд проявляются именно во взаимодействии с определенным типом вакуумных конденсатов вследствие перестройки вакуума в момент спонтанного нарушения симметрии. Само качественное различие четырех типов взаимодействий является следствием этих процессов. Так что именно физический вакуум выражает сущность явлений в неразвитом виде. Далее, из вышесказанного понятно, что физический вакуум удовлетворяет и четвертому признаку, то есть содержит в себе противоречия предмета в неразвитом виде. В-пятых, выражает специфику явлений. Достаточно вспомнить хотя бы специфику явления конфайнмента. Специфика макросвойств Вселенной также определяется свойствами конкретного вакуума нашей Вселенной. Само возникновение жизни связано с вполне конкретной цепочкой

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определенным образом нарушенных симметрий исходного вакуума. В-шестых, исходная абстракция должна быть предельной и непосредственной, то есть не опосредоваться другим. Исходная абстракция сама есть отношение. Имеет место оборачивание вакуума: порождая иное, момент самого себя, вакуум оборачивается частью этого момента. И, наконец, вакуум был исторически первым в реальном процессе развития нашей Вселенной.

Тот факт, что вопрос об исходной абстракции в физике может быть уже поставлен, свидетельствует о эволюционно-синергетическом этапе в развитии физической науки. Это позволит, в конечном счете, выписать физику нашего мира на основе принципа восхождения от абстрактного к конкретному.

В плане категориального наполнения рассматриваемых процессов речь идет о диалектике содержания и формы, что означает их одновременное тождество и различие. Можно утверждать, что геометризация физики означает поиски внутренней формы организации конкретного содержания. Если ньютоновские представления о пространстве и времени есть только кажимость (это еще не форма), на что обращали внимание такие мыслители прошлого как Беркли, Кант, Гегель, Мах; четырехмерный мир Эйнштейна - Минковского – внешняя форма существования мира, то придание всем структурным единицам материи геометрической интерпретации есть восхождение к внутренней форме, когда «то, что являет себя как деятельность формы, есть далее, в той же мере собственное движение самой материи… и то, и другое, действие формы и движение материи есть одно и то же… Материя, как таковая, определена или необходимо имеет некоторую форму, а форма – это просто материальная, удерживающая форма» (Гегель). Иными словами, геометризация физики есть выявление внутренней формы как способа организации содержания - вакуума нашей Вселенной. Однако тождество содержит в себе различие. Роль формы в развитии содержания выражается в том, что она организует и выражает содержание, удерживает непрерывные изменения содержания в определенных границах, придает ему определенную устойчивость. При этом именно содержание играет ведущую роль, изменения содержания всегда предшествуют изменениям формы. Форма может быть рассмотрена как структура, обеспечивающая функционирование данного содержания и определяющая границы его возможных изменений. Различие между содержанием и формой указывает на противоречивость их внутреннего единства, что составляет основу развития явления.

Если обратиться к теоретическим прогнозам о будущей эволюции Вселенной, то среди них имеется точка зрения о том, что Вселенная уже сейчас вошла в новую фазу инфляции, что означает в итоге переход ее из одного вакуума в другой. Таким образом методология исследований, основанная на диалектике взаимопревращений категорий содержания и формы, может оказаться востребованной. Это предопределяет необходимость поисков путей синтеза синергетического и диалектического подходов для понимания процессов самоорганизации Вселенной в рамках концепции геометризации.

4D-Time Idea VS Common Minkowski Space-Time

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We offer a novel the model of the real time and space, totally treating them to be a pure four-dimensional time. The treatment is based on Berwald-Moore's Finsler metric function. It is demonstrated that the physical space and time with their characteristics similar to that of the pseudo-Euclidean case can be comprehended to originate from exchanging isotropic signals among different frames of reference, and from significant peculiarities of the adopted Finsler metrics. Such notions as the relative simultaneity, the three-dimensional space, and the speed magnitude have been investigated getting an agreement with either classical or relativistic conception of the relativistic manifold.

1. Introduction

As is well known, the two-dimensional pseudo-Euclidean plane admits several distinctions in kind of formal ways of extending the four-dimensional space. According to one of them, the Minkowski space does emerge, as a result of another - the pseudo-Euclidean space with the signature (+,+,-,-) appears, and as a result of one another - a specific linear space does arise in which the length of vectors satisfies the relation

$$|\boldsymbol{X}|^4=x_1'x_2'x_3'x_4'. \tag{1}$$

with respect to one of the respective bases. Henceforth, we shall call the corresponding space quadranumerical, or the Quadraspace.

At First glance the Quadraspace does not agree with the ideas of the real world because all its unit vectors are seemingly equal in rights, the isotropic space does not reveal any rotational symmetry, and the group of continued linear transformation, similar to the Lorentz transformation, is defined by only 3 operation factors and not by 6. But one should not be in a hurry to carry out any pessimistic conclusions, as we face a representative of a very specific class of spaces usually called Finslerian [1,2]. The properties of the latter space do not resemble the properties of the usual pseudo-Euclidean space, and that is why 4D-time demands a fundamentally new approach. It is worth mentioning that the Finsler Geometry, appeared as an extension of the Riemannian geometry, has been known for a long time and has been investigated in many works. The example (1) is the well-known case called the Berwald-Moore Finslerian metric function [2]. As strange as it will sound, the geometry of the most common Finslerian manifolds, connected with the notion of the linear space, has been studied rather skin-deep. Apparently, this fact is conditioned by lack up to lately of effective methods of their investigating, like the formalism of the scalar product, applied in Euclidean and pseudo-Euclidean geometry. In the previous work [3] we have made an attempt to fill in the respective gap, introducing the notion of the m-linear symmetric form constructed of n vectors, summarizing the notion of the bilinear symmetric form. It was also demonstrated that the space with the metric function (1) corresponds to four-linear symmetric form constructed of 4 vectors A; B; C and D such that with respect to the most convenient basis, the form looks as (A; B; C; D) = 1

$$(\boldsymbol{A},\boldsymbol{B},\boldsymbol{C},\boldsymbol{D})=1/24(a_1b_2c_3d_4+a_1b_2c_4d_3+a_2b_1c_3d_4+\ldots+a_4b_3c_1d_2+a_4b_3c_2d_1). \tag{2}$$

If we place 4 times the same vector A in this form, on analogy by the bilinear space, we get the fourth degree of its length:

$$(\boldsymbol{A},\boldsymbol{A},\boldsymbol{A},\boldsymbol{A})=\boldsymbol{A}^4=a_1a_2a_3a_4,$$

which is tantamount to the case (1).

2. Transversal Directions

There exist 16 particular vectors $e_1,\ldots,e_{16}$ in the Quadraspace which, with respect to the basis used to write the form (2), have the components

$$\begin{array}{ll} \boldsymbol{e}_1\leftrightarrow(1,\ 1,\ 1,\ 1); & \boldsymbol{e}_5\leftrightarrow(-1,-1,-1,-1); \\ \boldsymbol{e}_2\leftrightarrow(1,-1,\ 1,-1); & \boldsymbol{e}_6\leftrightarrow(-1,\ 1,-1,\ 1); \\ \boldsymbol{e}_3\leftrightarrow(1,\ 1,-1,-1); & \boldsymbol{e}_7\leftrightarrow(-1,-1,\ 1,\ 1); \\ \boldsymbol{e}_4\leftrightarrow(1,-1,-1,\ 1); & \boldsymbol{e}_8\leftrightarrow(-1,\ 1,\ 1,-1); \\ & \\ \boldsymbol{e}_9\leftrightarrow(\ 1,-1,-1,-1); & \boldsymbol{e}_{13}\leftrightarrow(-1,\ 1,\ 1,\ 1); \\ \boldsymbol{e}_{10}\leftrightarrow(1,\ 1,-1,\ 1); & \boldsymbol{e}_{14}\leftrightarrow(-1,-1,\ 1,-1); \\ \boldsymbol{e}_{11}\leftrightarrow(1,-1,\ 1,\ 1); & \boldsymbol{e}_{15}\leftrightarrow(-1,\ 1,-1,-1); \\ \boldsymbol{e}_{12}\leftrightarrow(1,\ 1,\ 1,-1); & \boldsymbol{e}_{16}\leftrightarrow(-1,-1,-1,\ 1). \end{array} \qquad (3)$$

These vectors represent an interest because they demonstrate a new quality of relationships among directions, which does not appear in spaces with bilinear form, and acts an extensions of characteristics of the orthogonal property.

If in bilinear space there was only one opportunity to nullify the form (A; B), there are 3 variants in the Quadraspace:

$$(\boldsymbol{A},\boldsymbol{B},\boldsymbol{B},\boldsymbol{B})=0 \qquad (*)$$
$$(\boldsymbol{A},\boldsymbol{A},\boldsymbol{B},\boldsymbol{B})=0 \qquad (**)$$
$$(\boldsymbol{A},\boldsymbol{A},\boldsymbol{A},\boldsymbol{B})=0. \qquad (***)$$

By direct substitution of components of the vectors e_i into (2) we can make sure that every vector of the set is opposed to exactly one vector, it makes pairs that will satisfy *(*)* and *(***)* with 6 others, and with 8 will satisfy *(**)*. Accordingly, the notion that extends the orthogonality characteristics (in the Finsler space theory it is sometimes called the transversality) it stops having a single meaning. On top of all it looses commutativity, as the equality $(A;\ B;\ B;\ B) = (A;\ A;\ A;\ B)$ is not held true with respect to every pair of vectors. Basis, that consists of vectors of one of the grouped fours in the set (3), is in a close similarity to the orthonormal one, and the metric function takes the form:

$$|\boldsymbol{X}|^4=x_1^4+x_2^4+x_3^4+x_4^4-2(x_1^2x_2^2+x_1^2x_3^2+x_1^2x_4^2+x_2^2x_3^2+x_2^2x_4^2+x_3^2x_4^2)+8x_1x_2x_3x_4. \qquad (4)$$

This can be very useful, for example in comparison with the Minkowski space where the interval is conventionally written in terms of mutuality in orthogonal unit vectors.

3. Basic Notions

As the main aim of the work is to build the Finslerian space-time model that would satisfy either classical or relativist conception it would be logical to define common physical notions for the

Quadraspace as close to those in the Minkowski space as possible. That is why, as in the Special Theory of Relativity (STR), we will regard a point in the Quadraspace as some event. The directed non-isotropic lines will be identified with the world lines of the inertial reference frame (IRF). We will give a real number to each pair of points of the world lines, and as in STR we will call it an interval. At the same time we will connect the fourth degree of the interval between two events with the coordinates of the corresponding points by the help of an expression coming from (1):

$$|S|^4=(x_1'-y_1')(x_2'-y_2')(x_3'-y_3')(x_4'-y_4'),$$

or in terms of the differential:

$$|dS|^4=dx_1'dx_2'dx_3'dx_4'.$$

With respect to the basis analogical to the orthonormalized one the differential of the interval is given by the following form:

$$|dS|^4=dx_1^4+dx_2^4+dx_3^4+dx_4^4-2(dx_1^2dx_2^2+dx_1^2dx_3^2+dx_1^2dx_4^2+dx_2^2dx_3^2+dx_2^2dx_4^2+dx_3^2dx_4^2)+ +8dx_1dx_2dx_3dx_4. \quad (5)$$

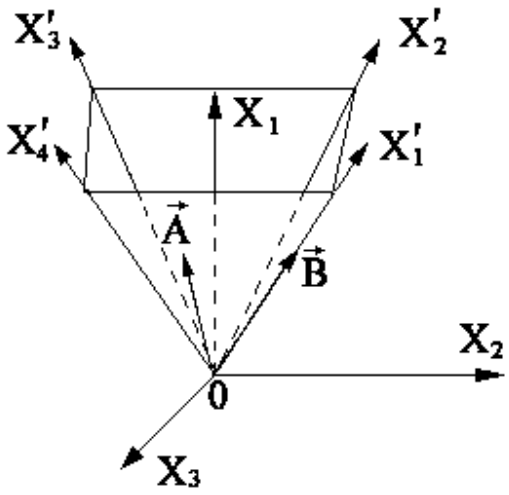
We shall attribute the differential of the interval a physical sense of proper time, that has gone in a frame between two infinitesimally near events. As we have adopted the axiom of the compliance of interval of the Quadraspace to the proper time in the IRF and all transversal coordinates are included into the expression for the interval given by (5) in quite a similar way, there is no reason for us to oppose one direction to three others as it was typical for the Minkowski space. In fact this means that the concerned space is practically equitable in all its directions, as well as the Euclidean space. However, there are differences related with the presence of isotropic vectors in the Quadraspace that are absent in the Euclidean Geometry. Concerning the equitability of linear coordinates in the Quadraspace and their common connection with the notion of proper time, we can say that we face an example of not only space, or space and time, but of a pure multi-dimensional time. It follows from the formula (1) that the Quadraspace has 4 three-dimensional hyperplanes, whose vectors correspond to the zero value of the interval. Such hyperplanes obeys the equations:

$$x'_1=0;\ x'_2=0;\ x'_3=0;\ x'_4=0. \quad (6)$$

In analogy to the Minkowski space we will call the vectors that belong to them isotropic. Isotropic hyperplanes intersect at 6 two-dimensional planes. Those, in their turn, intersect at 4 straight lines. It is natural to try to connect a special basis with the last ones. The basis is the one in which we introduced the metric function (1). In view of uniqueness and its objective preference, we will call it the absolute basis.

Isotropic hyperplanes (6) divide all Quadraspace into 16 simply connected domains, tetrahedral pyramids, from one side opened to the infinity, from the other - with the same vertex. We will call such pyramids the light pyramids. Every light pyramid has 4 edges, that are rays of the coordinates of the absolute basis, that is why the corresponding areas can be defined by the mathematical signs of the bounding half-line. Each light pyramid has at least one common isotropic vector, except for the opposite pyramid, with which they have only one common point. The symmetry axis of the light pyramids agrees in direction with 16 unit vectors $e_1,...,e_{16}$ given by (3). As mentioned above, every light pyramid has one opposite and 14 transversal pyramids. The described characteristics of the simply connected domains of the Quadraspace define the hexadecimalness of its discontinued symmetry.

Let us turn to imagine more distinctly the structure of isotropic surfaces of the Quadraspace, turn to Fig 1 that depicts three-dimensional section of one of the 16 light pyramids. For comparison, we can see analogous section of the light cone of the future Minkowski space on Fig 2.



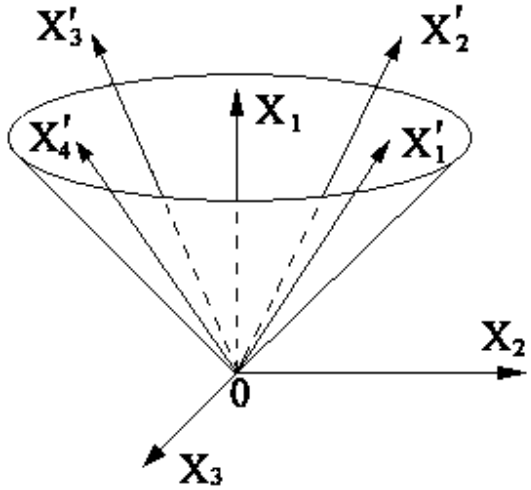


Fig 1: Light pyramid of the future of the Quadraspace. **Fig 2:** Light cone of the future of the Minkowski space.

In this respect it is interesting to note that in the Minkowski space there exist the isotropic vectors that form the bases in a way analogous to that of the absolute basis of the Quadraspace. Namely, the classical form of the Minkowski space in terms of these bases takes the following form:

$$|X|^2 = x_1'x_2' + x_1'x_3' + x_1'x_4' + x_2'x_3' + x_2'x_4' + x_3'x_4',$$

which looks rather strange but in a way similar to the case of the Quadraspace metric form (1).

The 4 sides of the light pyramid are three-dimensional hyperplanes, that remain only as planes on Fig.1 because of suppressing the coordinate x_4. Any line, coming through the centre of the reference system and belonging to the plane can be regardes as a trajectory of a light ray by analogy with the light Minkowski cone. Isotropic subspace surrounding every pyramid does not have rotational symmetry in the Euclidean meaning of the word. That is why at first sight it seems that velocity of light in the Quadraspace must depend on the direction. It is paradoxical, but working in the Finsler space we cannot directly use information obtained in the bilinear geometry. In any case, directions of all isotropic vectors are absolutely equitable in the Quadraspace. The fact that some of them lie on edges - like vector A, and others in the plane - like vector B - is only a problem of the applied method of imagery. This case can be explained by the fact that we have to use Euclidean conception to model the Finsler space, that is why some defects are possible. It is well known that while applying an analogous method in pseudo-Euclidean space using the Euclidean sheet of paper, only affine behavior, common for both spaces, is taken into account. Apparently, though the isotropic vector disparity of the Quadraspace was evident, we came right to the opposite conclusion, that is why its velocities of light are equal in all directions.

If, additionally, we interpret continued linear invariant transformation to its interval as a switch from one IRF to another IRF we can say that in this space as well as in the Minkowski space, velocity of light is independent of the place, direction and speed of an observer. Apparently, metrics of the Quadraspace does not contradict the qualitative experience results of the Michelson-Morly experiments, which in their turn became a push to designing STR.

4. Connection with Hypercomplex Numbers

A peculiarity of the Quadraspace that remarkably distinguishes it from the Minkowski space is an opportunity to set the one-to-one correspondence with commutative-associative algebra and connected with their-connected hypercomlex objects (see [4-9]), these we shall call conventionally the Quadranumbers. We can get Quadranumbers examining the double number algebra against the ring of themselves, or, if it is possible to say so, quadrupling the field of real numbers. In their algebraic structure the Quadranumbers are quite trivial, that evidently became the reason for their ignorance from the community of mathematicians. But the ignorance does not seem to be defensible, especially if we look at them taking into consideration objective Finslerian peculiarities of the procedure of changing geometrical coordinates by physical ones.

5. Symmetry of the Quadraspace

In the Quadraspace, there exist a group of continuous linear transformations that leave their interval invariant. This case let us regard such transformations as congruent and call them the motions. The group of motions of the Quadraspace is an analogue to the ten-parameter Pouncare's group of the Minkowski space, but in contrast thereto is defined by 7 independent parameters. Particularly, 4 parameters are responsible for parallel shift, and the other 3 for hyperbolic turn. It is worth adding a new parameter - the dilatation rate, if we take into consideration that scaling transformation, or in other words - stretching, is also of hyperbolic character. We shall call the obtained eight-parameter system the enlarged group. It splits into 2 four-parameter subgroups - translations and hyperbolic transformations with a fixed point. Quantitative equality of independent parameters of the two types of transformations and their total compliance with arithmetic operations over Quadranumbers (addition and multiplication) describes the fundamentality of the enlarged group and not the common group of motions.

One of the apparent disadvantages of the Quadraspaces is the lack of spatial turns in its motions; in contrast to Pouncare's group of the Minkowski space. But nevertheless the three-parameter group of nonlinear transformations, which is realized in the Quadraspace, is accomplished by an invariant value of a special two vector symmetric form [3]. Just such or similar transformation can be juxtaposed with common physical turns. These trans-formasions are not of invariant type for the four-dimensional metrics of the Quadraspace and that is why they are not motions. But as a matter of fact they can play a specific role, the same as turns do in classical mechanics, as while doing so the three-dimensional spaces that the observer see must be left permanent. Maybe a paradox can be traced in Mach's principle, it consists in non-equitable relativity between the class of uniformly moving physical frames of reference and the class of uniformly rotating physical frames of reference.

6. Physical Space of Observer

It seems that one of the most amusing peculiarities of the Quadraspace is an opportunity to define simple and natural conditions when the observer, living in the world line, can register a four-dimensional space that would be totally congruent in its characteristics with the pseudo-Euclidean space. We can make mathematically sure of this by inserting (5) in the expression of the differential of the interval:

$$|dx_1|>>|dx_2|\approx|dx_3|\approx|dx_4|.$$

Omitting the infinitesimals of third and fourth orders of magnitude, we get:

$$|dS|^4=dx_1^4 -2dx_1^2 (dx_2^2+dx_3^2+dx_4^2).$$

This expression in turn differs from the given below from by the infinitesimals fourth order:

$$|dS|^4=dx_1^4 -2dx_1^2 (dx_2^2+dx_3^2+dx_4^2)+(dx_2^2+dx_3^2+dx_4^2)^2,$$

After extracting the square root it exactly represents the expression for the interval of the Minkowski space.

In the Quadraspace, as well as in the Minkowski space, it is possible to juxtapose the idea of the three-dimensional distance (half of the time needed for the sequential exchange of light signals between 2 rested IRFs) to every pair of events. Geometrically it comes to a triangle, 2 sides of which are isotropic vectors, and the third lies on the world line of the chosen IRF. It is just the physical distance, measured in light-seconds, that is associated with half the interval of the third side. At the same time unlike the analogous construction in the Minkowski space, components of the isotropic vectors of the regular triangle differ not only in sign but also in value.

We can introduce the notion of the respective modulus of the three-dimensional speed for an arbitrary vector of the Quadrapace. To define the modulus case we will take the value of the ratio between three-dimensional distance and 2 static reference frames to half the interval of the time, needed for a signal exchange of the IRFs, that correspond to the vector concerned.

At first glance such construction may seem cumbersome and unusual, but in reality in many aspects it is analogous to the respective constructions in STR. A considerable difference consists in the fact that in the Minkowski space the set of points, equidistant from 2 static events (the space of relatively simultaneous events) forms a hyperplane, and in the Quadraspace this forms a non-linear surface, whose equation reads

$$|(a'_1-x'_1)(a'_2-x'_2)(a'_3-x'_3)(a'_4-x'_4)|=|(x'_1-b'_1)(x'_2-b'_2)(x'_3-b'_3)(x'_4-b'_4)|$$

where a'_i and b'_i stand for the coordinates of fixed events with respect to the absolute basis.

From the accepted definitions it follows that the difference of geometrical (or even better to say chronometrical) and physical coordinates becomes more apparent in the Quadraspace than in the Minkowski space. So, if straight lines can be chosen as the primary ones, then among the four physical coordinates only one coordinate can be linear, namely, that of the proper time. The spatial coordinates are always non-linear as the hyperplane of the conditionally synchronous events is non-linear. But from the point of view of the observer the nonlinear nature of the physical space is practically insignificant, as it is more convenient for the observer to put the measuring on the linear-seeming scale of distance.

$$|dx_1|>>|dx_2|\approx|dx_3|\approx|dx_4|.$$

7. Conclusion

Thus the customary physicist conviction that only the Minkowski space with its all well-accepted characteristics can be the basis to the real world theoretical models is not exactly true. As it has been said above there is one more linear space whose characteristics are not worse, but in a way even better fit our conceptions of the real time and space (in absence of fields). It is also worth trying to make an experiment of changing the Minkowski space by the Quadraspace, at least because of its deep and constructive connection with the classical notion of the number. If we approach the aspect from only philosophical point of view, it is worth admitting that the category of the number is fundamental, and it

is even more general than such global categories as space, time, matter, or interaction. The number may be thought of without time and space, and also independently from matter, or its interaction, but not vice-versa. Maybe it is the deepest conception among all. If we trace the physics evolution history that constantly moved to more and more general sources, it is possible to conjecture that the place of geometrical notions that dominated during the XX'th century can be occupied by the object that is even more general - the number, - being not merely an instrument, that it has been for centuries, but also the most general basis for all the physics.

References

[1] H. Rund: The Differential Geometry of Finsler Spaces (Springer, Berlin, 1959).
[2] G.S. Asanov: Finsler Geometry, Relativity and Gauge Theories (D. Reidel Publ. Comp., Dordrecht, 1985).
[3] D.G. Pavlov: Hypercomplex Numbers, Associated Metric Spaces, and Extension of Relativistic Hyperboloid, gr-qc/0206004.
[4] H. Helmholtz: Über die Thatsachen die der Geometrie zum Grunde liegen, Gottingen, Nachr. 1868.
[5] D.G. Pavlov: Hypercomplex Quadrahyperbolic Numbers and Associated Nonlinear Invariance, gr-qc/021290.
[6] I.L. Kantor, A.S. Solodovnikov: Hypercomplex numbers: An elementary introduction to algebras, Springer, Berlin 1989.
[7] G. Birkho and S. MacLane: Modern Algebra, Macmillan, New York, Third Edition 1965.
[8] B.L. van der Waerden: Modern Algebra, F.Ungar, New York, Third Edition 1950.
[9] A.A. Kassaandrov: Number, Time, Light (in: [http:www.chronos.msu.ru]).

Faster than light signals: myth or reality?

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1. Introduction

Though a number of startling results have been obtained in the recent past on the superluminal phenomena, the phrase, 'faster-than-light' evokes exotic connotations amongst most scientists, partly due to the relativistic world-view a la Einstein, and mainly due to the over-emphasis on the implied counter-intuitiveness in the literature propounding faster-than-light signals. At the popular level, the proponents of tachyons or instant quantum communication have made the subject a sort of science fiction. It was in this background that a comprehensive, scientifically sound and technically accurate account of the whole spectrum of superluminal phenomena was planned culminating into the publication of the monograph [1]. An attempt to offer in depth study and critique on some key issues has been made in my book. The readers of the book would find that there are many interesting problems that could be explored within the conventional paradigm of physics; however more significantly I have tried to argue that a radically new approach at a fundamental level is imperative. This new approach is most likely to emanate from the understanding of the nature of time. Since the detailed discussions and reviews are available in the monograph, the present paper gives a brief overview of the subject in the next section. A critique on the relativity, causality and time, and recent updates since the publication of the book constitute last two sections.

2. Monograph: Over-view and outlook

Superluminal phenomenon is characterized by faster-than-light (i.e. speeds exceeding the vacuum velocity of light, c), apparent or real motion. The subject is nearly a century old one, but a tremendous activity has been witnessed during past decade or so. The scope of the subject is so vast that deep understanding of relativity, quantum mechanics, electromagnetic theory, cosmology, astrophysics and particle physics is required to comprehend its implications. In the monograph we have classified the superluminal phenomena into four scenarios: superluminal electromagnetic wave propagation, tachyons, superluminal signals and quantum nonlocality, and quasars. Since the variable speed of light cosmological models envisage speeds greater than c in the past, this topic in a broader sense also belongs to the superluminal phenomena.

In the first decade of the last century, it was found that both phase and group velocities of light could exceed c in the anomalously dispersive media, and that seemed to contradict the special relativity of Einstein. Sommerfeld-Brillouin theory was put forward to resolve this question, however in recent times investigations on the electromagnetic wave propagation in material media, vacuum (special localized beams), and typical boundary conditions leading to evanescent waves have given rise to the claims of the superluminal behavior. The study and review in the monograph show that in spite of the widely publicized reports, none of them proves faster-than-light signal or energy transmission. We suggest that (1) instead of highlighting "superluminality', it would be scientifically more accurate to term the observed phenomena as 'anomalous time delay effects', (2) a reworking of the Sommerfeld-Brillouin theory for localized vector beams and evanescent modes is a challenging, yet highly desirable

task, (3) a complete quantum field theoretic treatment of the pulse propagation in atomic media remains unexplored, and (4) precision, rigorous error analysis, and repeatability are must for the laboratory experiments.

Since the Bilaniuk et al proposed tachyons thousands of papers have been written on the tachyons, however there is no direct evidence of the existence of such particles. Besides the usual review we have drawn attention to the remarkable analysis of 'such impossible structures' by Eddington, and appearance of faster-than-light possibility in Dirac's model of electron. Recent relevance of tachyons in superstrings is also discussed. The important conclusion of our review is that the hypothetical faster-than-light particles with imaginary mass (i.e. tachyons) belong to a special class of objects, and invoking takchyons in superluminal electromagnetic wave propagation is conceptually flawed.

The discovery of first radio source 3C-273 in 1962 heralded the journey into the mysterious world of quasi-steller radio sources (quasars). Extensive astronomical observations show that apparent superluminal motion within our galaxy, quasars and extra-galactic sources is a real phenomenon. The physical interpretation remains uncertain. In [1] we note that similar to the Roemer's first definite measurement of c in 1675 from astronomical observations, quasars hold promise for new physics in the superluminal scenario.

The landmark 1935 paper of Einstein-Pololsky-Rosen together with the belief in the Copenhagen interpretation of quantum mechanics lead to the quantum correlations with superluminal communication. Unfortunately quantum nonlocality for space-like separations remains controversial subject, and even the original proponents of quantum teleportation admit that faster-than-light communication does not exist. A thorough review is presented in our book.

In the monograph a critical survey of the VSL models is also presented. Though time varying c models are of current interest, in 1980 a simple model was proposed based on the space-time interaction hypothesis [2]. What exactly does it mean to say 'time varying c' in the relativistic cosmology? The question remains unsettled, however a general treatment of the constancy of the fundamental constants exists in the literature, and due to its significance for the superluminal phenomena a thorough review is given in the introductory chapter of the book.

Apart from a balanced review and delineation of the basic issues, the monograph also articulates an alternative unorthodox approach based on the revision of space-time picture in the light of the space-time interaction hypothesis proposed by the author. The reader may find a reasonably detailed account of this hypothesis in the PIRT-1988 Conference Proceedings [3], besides, of course the book [1]. In the next section, some salient points are highlighted.

3. Time and Causality

Exposition of the critiques on the Einstein's relativity can be found in my published articles, and in [1]. Here I make few remarks that appear to me either new or of deep significance. Historically the velocity of light in the Michelson-Morley experiment was interpreted as group velocity, while Einstein in his theory referred to the ray velocity of light. The nature of wave velocity after the investigations of Sommerfeld and Brillouin has not been properly taken into consideration amongst relativists. In Einstein's theory, inertial frame and Newtonian time ordering are presupposed concepts, while the relative or common time of Newton is mis-interpreted as absolute time. For Newton, the absolute time is a metaphysical concept, and Weyl seems to recognize the importance of 'undefined time flow' in relativity. The question of causality is a subtle issue in special relativity, and severe complications arise in general relativity. There are arguments to show that the notion of limiting velocity is not a basic postulate in relativity. We have argued that the causality based on the light-cone structure does not

have any obvious relationship with the past-future causal time ordering. Faced with the paradoxes in Gödel universe, Einstein himself admitted the lack of clarity in the concept of time in general relativity.

In 1988 [3] we pointed out that the most celebrated direct test of relativity, namely the longer life-time of relativistically moving unstable particles, in fact, poses a paradox. The analysis of this problem leads to a new insight : the concept of inertial frame and the Newton's laws of motion have to be revised at a fundamental level. Inertial frames are not equivalent; they differ due to 'unobservable' constant potentials, and the laws of motion begin with the 'pre-momentum' equation. The constant c is just a convenient measurement standard, and the 4-dimensional metric represents the statistical spread of length correlations i.e. the standard deviation. The Newtonian metaphysical absolute time is revised to the causative time of the space-time interaction hypothesis. The physical reality of time proposed in our work has profound implications on the future direction of physics elaborated in [1].

4. Conclusion

The subject of superluminality is an active and exciting field of research: conservative outlook forces one to introduce arbitrary parameters and weird physical interpretations while alternatives (e.g. rejecting Einstein's relativistic time propounded by the author) deserve serious attention. I hope the monograph would stimulate and assist the scientists in this evolutionary phase of the subject.

Another review on the fundamental constants and their variation by Uzan [4], and some papers on Lorentz invariance and photon mass [5] would be of interest to the readers. The analysis of cosmological data shows that the universe is dominated by dark energy, see Perlmutter [6] for a nice review. One of the solutions to explain cosmological observations is based on the proposition of a cosmological constant term. If this lambda-term is not constant, new avenues to address the questions like dark energy and its vanishingly small value at present are opened up [7]. An interesting model is that of unimodular gravity, however the problem of energy-momentum conservation is not adequately emphasized in the literature, see [8]. A recent criticism on VSL models by Ellis and Uzan [9] though begins with a fundamental question on the meaning of c, it fails to appreciate thorough work of Sommerfeld-Brillouin, and Eddington's sharp discussions. Regarding the covariant divergence law for the stress energy tensor, mere specification of a variational principle in the VSL models will not settle the issue as Ellis and Uzan seem to imply. It is possible that a natural geometrical framework for the VSL models may be the Weyl space, see e.g. [10].

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References

[1] S.C. Tiwari, Superluminal Phenomena in Modern Perspective Faster-than-light signals: Myth or reality) (Rinton press, USA 2003)
[2] S.C.Tiwari, e-print gr-qc/0208068
[3] S.C. Tiwari, Proc. PIRT Conference, London (1988)
[4] J-P Uzan, Rev. Mod. Phys. 75, 403 (2003)
[5] P. Wolf et al, Phys. Rev. Lett. 90, 060402 (2003); J.A. Lipa et al, Phys. Rev. Lett. 90, 060403 (2003); J. Luo et al, Phys. Rev. Lett. 90, 08101 (2003); J. H. Mueller et al., Phys. Rev. Lett. 91, 020401 (2003)
[6] S. J. Perlmutter, Phys. Today 56(4), 53 (2003)
[7] M. Ahmad et al, e-print: astro-ph/0209274
[8] S.C. Tiwari, e-print: gr-qc/0310102
[9] G.F.R. Ellis and J.-P Uzan, e-print : gr-qc/0305099
[10] S.C. Tiwari, Phys. Lett. A 135, 315 (1989); Phys. Lett. A 142, 460 (1989)

Could gauge gravitational degrees of freedom play the role of environment in 'extended phase space' version of quantum geometrodynamics?

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In the context of the recently proposed formulation of quantum geometrodynamics in extended phase space we discuss the problem how the behavior of the Universe, initially managed by quantum laws, has become classical. In this version of quantum geometrodynamics we quantize gauge gravitational degrees of freedom on an equal basis with physical degrees of freedom. As a consequence of this approach, a wave function of the Universe depends not only on physical fields but also on gauge degrees of freedom. From this viewpoint, one should regard the physical Universe as a subsystem whose properties are formed in interaction with the subsystem of gauge degrees of freedom. We argue that the subsystem of gauge degrees of freedom may play the role of environment, which, being taken into account, causes the density matrix to be diagonal. We show that under physically reasonable fixing of gauge condition the density matrix describing the physical subsystem of the Universe may have a Gaussian peak in some variable, but it could take the Gaussian form only within a spacetime region where a certain gauge condition is imposed. If spacetime manifold consists of regions covered by different coordinate charts the Universe cannot behave in a classical manner nearby borders of these regions. Moreover, in this case the Universe could not stay in the same quantum state, but its state would change in some irreversible way.

1. Introduction

Quantum geometrodynamics claims to give a description of quantum stage of the Universe evolution, including an explanation of the fact that the behavior of the Universe, initially managed by quantum laws, has become classical. According to Halliwell [1], two requirements must be satisfied for a system to be regarded as classical. The first requirement is that its evolution should be described by classical laws in a very good approximation, and the second requirement involves the notion of decoherence – a transition from a pure state to some mixed state described by diagonal density matrix. There has been a number of works where the notion of decoherence was discussed in the context of quantum cosmology based on the Wheeler – DeWitt quantum geometrodynamics. As well known, the destruction of the off-diagonal terms in the density matrix cannot be regarded in the limits of unitary evolution. A possible way to solve this problem is to consider this destruction as a result of interaction with some environment [2, 3]. The application of this idea to quantum cosmology implies splitting the Universe into two subsystems, one of which is a system under investigation and the other plays the role of environment. It has been suggested to consider some modes of scalar, gravitational and other fields as an environment (see, for example, [1, 4]). However, there is no natural way to split the Universe into two subsystems.

The aim of the present work is to discuss these questions in the limits of recently proposed formulation of quantum geometrodynamics in extended phase space [5 – 11]. In this version of quantum geometrodynamics we quantize gauge gravitational degrees of freedom on an equal basis with physical degrees of freedom. The motivation for it was that it is impossible to separate gauge, or "non-physical" degrees of freedom from physical ones if the system under consideration does not possess asymptotic states, and it is indeed the case for a closed universe as well as for a universe

with rather nontrivial topology. As a consequence of this approach, a wave function of the Universe depends not only on physical fields but also on gauge degrees of freedom. From this viewpoint one should regard the physical Universe as a subsystem whose properties are formed in interaction with the subsystem of gauge degrees of freedom.

The plan of the work is as follows. We remind basic equations of the 'extended phase space' version of quantum geometrodynamics, consider a general structure of a wave function of the Universe and construct a density matrix describing the physical subsystem of the Universe. We then show that under physically reasonable fixing of gauge condition the density matrix may have a Gaussian peak in some variable, say, in a scale factor, while other degrees of freedom (e. g., gravitational waves and matter fields) should be treated quantum mechanically. The important point in this consideration is that the density matrix takes the Gaussian form only within a spacetime region where a certain gauge condition is imposed. In simple cosmological models one can introduce a single gauge condition in the whole spacetime, and the behavior of the Universe can be regarded as classical almost over the whole history of the Universe. Meanwhile, in the case of nontrivial topology spacetime manifold may consist of regions covered by different coordinate charts, so that one should impose different gauge conditions in these regions. Then, the Universe cannot behave in a classical manner nearby borders of these regions. We shall argue that in this case the Universe could not stay in the same quantum state; as a consequence of interaction with the subsystem of gauge degrees of freedom its state would change in some irreversible way.

2. The 'extended phase space' version of quantum geometrodynamics: basic formulas

To investigate a system without asymptotic states we make use of the path integral approach [9, 10]. It is easy to illustrate the crux of the matter for a simple minisuperspace model with a gauged action

$$S = \int dt \left\{ \frac{1}{2} v(\mu, Q) \gamma_{ab} \dot{Q}^a \dot{Q}^b - \frac{1}{v(\mu, Q)} U(Q) + \pi_0 \left(\dot{\mu} - f_{,a} \dot{Q}^a \right) - i w(\mu, Q) \dot{\bar{\theta}} \dot{\theta} \right\}. \tag{2.1}$$

Here $Q = \{Q^a\}$ stands for physical variables such as a scale factor or gravitational-wave degrees of freedom and material fields, and we use an arbitrary parameterization of a gauge variable μ determined by the function $v(\mu, Q)$. For example, in the case of isotropic universe or the Bianchi IX model μ is bound to the scale factor a and the lapse function N by the relation

$$\frac{a^3}{N} = v(\mu, Q). \tag{2.2}$$

$\theta, \bar{\theta}$ are the Faddeev – Popov ghosts after replacement $\bar{\theta} \to -i\bar{\theta}$. Further,

$$w(\mu, Q) = \frac{v(\mu, Q)}{v_{,\mu}}; \quad v_{,\mu} \overset{def}{=} \frac{\partial v}{\partial \mu}. \tag{2.3}$$

We confine attention to the special class of gauges not depending on time

$$\mu = f(Q) + k; \quad k = \text{const}, \tag{2.4}$$

which can be presented in a differential form,

$$\dot{\mu} = f_{,a} \dot{Q}^a, \quad f_{,a} \overset{def}{=} \frac{\partial f}{\partial Q^a}. \tag{2.5}$$

The Schrödinger equation for this model reads

$$i \frac{\partial \Psi(\mu, Q, \theta, \bar{\theta}; t)}{\partial t} = H \Psi(\mu, Q, \theta, \bar{\theta}; t), \tag{2.6}$$

where

$$H = -\frac{i}{w}\frac{\partial}{\partial\theta}\frac{\partial}{\partial\bar{\theta}} - \frac{1}{2M}\frac{\partial}{\partial Q^{\alpha}} M G^{\alpha\beta}\frac{\partial}{\partial Q^{\beta}} + \frac{1}{v}(U - V); \tag{2.7}$$

M is the measure in the path integral,

$$M(\mu, Q) = v^{\frac{K}{2}}(\mu, Q) w^{-1}(\mu, Q); \tag{2.8}$$

$$G^{\alpha\beta} = \frac{1}{v(\mu, Q)}\begin{pmatrix} f_{,a} f^{,a} & f^{,a} \\ f^{,a} & \gamma^{ab} \end{pmatrix}; \quad \alpha, \beta = (0, a); \quad Q^0 = \mu, \tag{2.9}$$

K is a number of physical degrees of freedom; the wave function is defined on extended configurational space with the coordinates $\mu, Q, \theta, \bar{\theta}$. V is a quantum correction to the potential U, that depends on the chosen parameterization (2.2) and gauge (2.4):

$$\begin{aligned} V = {} & \frac{5}{12w^2}\left(w_{,\mu}^2 f_{,a} f^{,a} + 2w_{,\mu} f_{,a} w^{,a} + w_{,a} w^{,a}\right) + \\ & + \frac{1}{3w}\left(w_{,\mu,\mu} f_{,a} f^{,a} + 2w_{,\mu,a} f^{,a} + w_{,\mu} f_{,a}^{;a} + w_{,a}^{;a}\right) + \\ & + \frac{K-2}{6vw}\left(v_{,\mu} w_{,\mu} f_{,a} f^{,a} + v_{,\mu} f_{,a} w^{,a} + w_{,\mu} f_{,a} v^{,a} + v_{,a} w^{,a}\right) - \\ & - \frac{K^2 - 7K + 6}{24v^2}\left(v_{,\mu}^2 f_{,a} f^{,a} + 2v_{,\mu} f_{,a} v^{,a} + v_{,a} v^{,a}\right) + \\ & + \frac{1-K}{6v}\left(v_{,\mu,\mu} f_{,a} f^{,a} + 2v_{,\mu,a} f^{,a} + v_{,\mu} f_{,a}^{;a} + v_{,a}^{;a}\right). \end{aligned} \tag{2.10}$$

The Schrödinger equation (2.6) – (2.10) is derived from a path integral with the effective action (2.1) without asymptotic boundary conditions by the standard well-definite Feynman procedure, thus it is *a direct mathematical consequence* of the path integral. Once we agreed that imposing asymptotic boundary conditions is not correct in the case of a closed universe, *we are doomed* to come to a gauge-dependent description of the Universe.

3. The general solution to the Schrödinger equation and the density matrix

The general solution to the Schrödinger equation (2.6) has the following structure [10]:

$$\Psi(\mu, Q, \theta, \bar{\theta}; t) = \int \Psi_k(Q, t)\delta(\mu - f(Q) - k)(\bar{\theta} + i\theta)dk. \tag{3.1}$$

The dependence of the wave function (3.1) on ghosts is determined by the demand of norm positivity.

Note that the general solution (3.1) is a superposition of eigenstates of a gauge operator,

$$(\mu - f(Q))|k\rangle = k|k\rangle; \quad |k\rangle = \delta(\mu - f(Q) - k). \tag{3.2}$$

It can be interpreted in the spirit of Everett's "relative state" formulation. In fact, each element of the superposition (3.1) describe a state in which the only gauge degree of freedom μ is definite, so that time scale is determined by processes in the physical subsystem through functions $v(\mu, Q), f(Q)$ (see (2.2), (2.4)), while k being determined by initial clock setting. Indeed, according to (2.4), the parameter k gives an initial condition for the variable μ. The function $\Psi_k(Q, t)$ describes a state of the physical subsystem for a reference frame fixed by the condition (2.4). It is the solution to the equation

$$i\frac{\partial\Psi_k(Q, t)}{\partial t} = H_{(phys)}\Psi_k(Q, t), \tag{3.3}$$

$$H_{(phys)} = \left[-\frac{1}{2M}\frac{\partial}{\partial Q^a}\frac{1}{v}M\gamma^{ab}\frac{\partial}{\partial Q^b} + \frac{1}{v}(U-V) \right]\Bigg|_{\mu=f(Q)+k} \tag{3.4}$$

In general, one can seek a solution to Eq. (3.3) in the form of superposition of stationary state eigenfunctions:

$$\Psi_k(Q,t) = \sum_n c_n \Psi_{kn}(Q)\exp(-iE_n t); \tag{3.5}$$

$$H_{(phys)}\Psi_{kn}(Q) = E_n \Psi_{kn}(Q). \tag{3.6}$$

The eigenvalue E corresponds to a new integral of motion that emerges in the proposed formulation as a result of fixing a gauge condition and characterizes the gauge subsystem (see below Eq. (4.2)).

In this paper we shall be interested under what conditions the behavior of the Universe can be regarded as classical. As well known, one of necessary requirements is that a wave function could be represented in the WKB form: $\Psi_{kn}(Q) = C(Q)\exp[iS(Q)]$. However, in our formulation there is an additional requirement. In the classical limit the Universe is described by gauge-invariant Einstein equations, so that all vestiges of gauge fixing should be eliminated. In particular, E must take the zero eigenvalue. Thus, in the classical limit the Universe appears to be in the unique eigenstate with $E = 0$.

Strictly speaking, we need some mechanism of the "reduction" of the wave function (3.5) to the state with $E = 0$. We suppose that such a mechanism involves a specific interaction between gauge and physical subsystems, but we have not been able to give explanation of the mechanism. In this paper we shall assume that, in any region where the quasiclassical approximation exists, the Universe is described by a quasiclassical wave function of the special state with $E = 0$, $\Psi_k(Q,t) = \Psi_{k0}(Q)$, and concentrate on the density matrix of the physical subsystem.

The normalization condition for the wave function (3.1) reads

$$\begin{aligned}
&\int \Psi^*(\mu,Q,\theta,\bar{\theta};t)\Psi(\mu,Q',\theta,\bar{\theta};t)M(\mu,Q)d\mu\, d\theta\, d\bar{\theta}\prod_a dQ^a = \\
&= \int \Psi_k^*(Q,t)\Psi_{k'}(Q,t)\delta(\mu - f(Q) - k)\delta(\mu - f(Q) - k')M(\mu,Q)\,dk\,dk'\,d\mu\prod_a dQ^a = \\
&= \int \Psi_k^*(Q,t)\Psi_{k'}(Q,t)M(f(Q)+k,Q)\,dk\prod_a dQ^a = 1.
\end{aligned} \tag{3.7}$$

The solution (3.1) is normalizable under the condition that $\Psi_k(Q,t)$ is a sufficiently narrow packet over k. Let us emphasize that the dependence of $\Psi_k(Q,t)$ on k is not fixed by the equation (3.3) in the sense that $\Psi_k(Q,t)$ can be multiplied by an arbitrary function of k. One cannot choose the function $\Psi_k(Q,t)$ to be not depending on k, since in this case one would obtain a non-normalizable, non-physical state. It would imply that the gauge condition is fixed absolutely precisely (δ-shaped packet), and such a situation is unrealistic from the physical point of view. We should rather consider a narrow enough packet over k to fit a certain classical $\bar{k}$ value:

$$\begin{aligned}
&\Psi(\mu,Q,\theta,\bar{\theta};t) = \\
&= \frac{1}{\sqrt{2i\alpha\sqrt{\pi}}}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2\alpha^2}(k-\bar{k})^2\right]\Psi_{\bar{k}}(Q,t)\,\delta(\mu - f(Q) - k)(\bar{\theta} + i\theta)\,dk = . \\
&= \frac{1}{\sqrt{2i\alpha\sqrt{\pi}}}\exp\left[-\frac{1}{2\alpha^2}(\mu - f(Q) - \bar{k})^2\right]\Psi_{\bar{k}}(Q,t)(\bar{\theta} + i\theta).
\end{aligned} \tag{3.8}$$

Since our investigation aims in giving description of a physical Universe, we can introduce a density matrix

$$\rho(Q,Q';t)=\int\Psi^*(\mu,Q,\theta,\bar{\theta};t)\Psi(\mu,Q',\theta,\bar{\theta};t)M(\mu,Q)\,d\mu\,d\theta\,d\bar{\theta} \tag{3.9}$$

For the wave function (3.8) the expression for density matrix reads

$$\rho(Q,Q',t)=\frac{1}{\alpha\sqrt{\pi}}\int_{-\infty}^{\infty}\exp\left[-\frac{1}{2\alpha^2}\left(\mu-f(Q)-\bar{k}\right)^2-\frac{1}{2\alpha^2}\left(\mu-f(Q')-\bar{k}\right)^2\right]\times \tag{3.10}$$

$$\times\Psi^*_{\bar{k}}(Q,t)\Psi_{\bar{k}}(Q',t)M(\mu,Q)\,d\mu=$$

$$=\frac{1}{\alpha\sqrt{\pi}}\exp\left(-\frac{1}{4\alpha^2}[f(Q)-f(Q')]^2\right)\Psi^*_{\bar{k}}(Q,t)\Psi_{\bar{k}}(Q',t)\times$$

$$\times\int_{-\infty}^{\infty}\exp\left[-\frac{1}{\alpha^2}\left(\mu-\frac{1}{2}[f(Q)+f(Q')]-\bar{k}\right)^2\right]M(\mu,Q)\,d\mu=.$$

$$=\exp\left(-\frac{1}{4\alpha^2}[f(Q)-f(Q')]^2\right)\Psi^*_{\bar{k}}(Q,t)\Psi_{\bar{k}}(Q',t)M\left(\frac{1}{2}[f(Q)+f(Q')]+\bar{k},Q\right).$$

We have taken into account that only the vicinity of the "point" $\mu=\frac{1}{2}[f(Q)+f(Q')]+\bar{k}$ gives a significant contribution to the integral over μ and replaced μ by its approximate value in the measure $M(\mu,Q)$. The normalization condition for the density matrix is

$$\int\rho(Q,Q,t)\prod_a dQ^a=\int\Psi^*_{\bar{k}}(Q,t)\Psi_{\bar{k}}(Q,t)M\left(f(Q)+\bar{k},Q\right)\prod_a dQ^a=1, \tag{3.11}$$

it corresponds to the condition (3.7).

Thus, we can see that the density matrix contains the factor

$$\exp\left(-\frac{1}{4\alpha^2}[f(Q)-f(Q')]^2\right), \tag{3.12}$$

so if we choose $f(Q)$ to be equal to one of physical variables which we shall denote as q (it may be, for example, a scale factor a), $f(Q)=q$, the expression (3.12) will take a Gaussian form

$$\rho\sim\exp\left[-\frac{(q-q')^2}{4\alpha^2}\right], \tag{3.13}$$

its width $\sqrt{2}\alpha$ being determined by a precision with which we can fix the gauge condition. We shall not discuss here what values α could take; it is a subject of quantum theory of measurements.

At first glance, it may seem that the density matrix has a Gaussian peak only in case of rather specific choice of a gauge condition $\mu = q$. In fact, however, when the gauge condition (2.4) is fixed, it does not mean that some reference frame has been already chosen. By the choice of a reference frame we imply, following to Landau and Lifshitz, imposing some conditions on the metric components $g_{0\mu}$ or, in our simplified model, on the lapse function N. A reference frame is completely fixed only if the choice of the parameterization function (2.2), as well as the gauge (2.4), is made. Let us emphasize again that the choice of gauge variable parameterization and that of gauge condition have an inseparable interpretation, – they are both determined by the construction of a clock, so without loss of generality any gauge condition can be turned to $\mu = q$ by choosing the function v. It is confirmed mathematically by the fact that the Hamiltonian operator in physical subspace (3.4) after substitution $\mu=f(Q)+k$ depends on the function $v(f(Q)+k,Q)$, but not on the functions $v(\mu,Q)$, $f(Q)$ separately. Indeed, in (3.4) the quantum correction V can be presented in the form

$$V\big|_{\mu=f(Q)+k} = \frac{5}{12w^2}\frac{\delta w}{\delta Q_a}\frac{\delta w}{\delta Q^a} + \frac{1}{3w}\frac{\delta^2 w}{\delta Q_a \delta Q^a} + \frac{K-2}{6vw}\frac{\delta w}{\delta Q_a}\frac{\delta v}{\delta Q^a} - \\ - \frac{K^2-7K+6}{24v^2}\frac{\delta v}{\delta Q_a}\frac{\delta v}{\delta Q^a} + \frac{1-K}{6v}\frac{\delta^2 v}{\delta Q_a \delta Q^a}, \tag{3.14}$$

where $\frac{\delta v}{\delta Q^a} = \left(v_{,\mu} f_{,a} + v_{,a}\right)\Big|_{\mu=f(Q)+k}$ is a total derivative with respect to Q^a of the function $v\left(f(Q)+k, Q\right)$, etc. Splitting the procedure of fixing a reference frame into choosing the parameterization and imposing a gauge condition is quite conventional, we shall give some examples in the next section. However, the physical part of the wave function $\Psi_k(Q,t)$, satisfied the equation (3.3), does not depend on this splitting, but on a chosen reference frame as a whole.

The condition $\mu = q$ implies that the gauge subsystem interacts with the physical subsystem through the variable q. As a result of this interaction, the density matrix becomes about diagonal in q. So, the Universe can be regarded as a classical system in any region where the physical part of the wave function $\Psi_k(Q,t)$ can be represented in the WKB form with respect to the variable q and where the gauge condition $\mu = q$ can be imposed. At the same time, the density matrix does not diagonalize in other physical variables (such as gravitational waves and mater fields, with q representing the scale factor), which do not interact with the gauge subsystem and should be treated quantum mechanically. One can assume that these degrees of freedom describe small perturbations on the background of an isotropic universe. A similar approach is adopted, for example, in [1] where the long-wavelength modes of a scalar field play the role of environment while the short-wavelength modes remain quantum mechanical. The authors of the works [1, 4] and others seek for a model in which a density matrix would contain a Gaussian factor. However, it is always questionable what part of the physical Universe should be considered as an environment. On the other side, taking into account the gauge subsystem is thought to be inevitable when constructing mathematically consistent quantum geometrodynamics. Gauge degrees of freedom are not observable directly, but only by its influence on the physical subsystem. The result (3.13), namely, that the density matrix may have a Gaussian peak in some variable under physically reasonable fixing of gauge condition, is quite general. It seems, therefore, to be natural to regard gauge degrees of freedom as the environment for the physical Universe. One could say, in a certain sense, that the physical Universe is a subsystem of itself.

4. The gauge subsystem as a factor of cosmological evolution

When deriving the Schrödinger equation from the path integral with the effective action (2.1), we approximate the path integral on extended gauged set of equations obtained by varying this effective action. The extended set of equations includes ghosts equations and a gauge condition, and equations for physical degrees of freedom also contain gauge-noninvariant terms. So, the gauged Einstein equations look like

$$R_\mu^\nu - \frac{1}{2}\delta_\mu^\nu R = \kappa\left(T_{\mu(mat)}^\nu + T_{\mu(obs)}^\nu + T_{\mu(ghost)}^\nu\right) \tag{4.1}$$

where $T_{\mu(mat)}^\nu$ is the energy-momentum tensor of matter fields, $T_{\mu(obs)}^\nu$ and $T_{\mu(ghost)}^\nu$ are obtained by varying the gauge-fixing and ghost action, respectively. $T_{\mu(obs)}^\nu$ describes the observer (the gauge subsystem) in the extended set of equations.

In particular, the $\binom{0}{0}$-Einstein equation (Hamiltonian constraint) is transformed to the form $H = E$, where H is a Hamiltonian in extended phase space and

$$E = -\int \sqrt{-g}\, T^0_{0(obs)} d^3x\,. \tag{4.2}$$

In quantum theory the modified Hamiltonian constraint leads to a stationary Schrödinger equation (see (3.6)).

To give a simple example, in this section we shall bear in mind an isotropic universe, then the parameterization function $v(\mu, Q)$, as well as the gauge-fixing function $f(Q)$, will depend only on a scale factor, i. e.

$$\frac{a^3}{N} = v(\mu, a), \quad \mu = f(a) + k\,. \tag{4.3}$$

The quasi-energy-momentum tensor of the gauge subsystem reads:

$$T^{v}_{\mu(obs)} = \mathrm{diag}\left(\varepsilon_{(obs)}, -p_{(obs)}, -p_{(obs)}, -p_{(obs)}\right); \tag{4.4}$$

$$\varepsilon_{(obs)} = -\frac{\dot{\pi}_0}{2\pi^2} \frac{v^2(\mu, a)}{a^6 v_{,\mu}}\bigg|_{\mu = f(a)+k}; \tag{4.5}$$

$$p_{(obs)} = \varepsilon_{(obs)}\left[1 - \frac{a}{3v(\mu, a)}\left(v_{,\mu} f_{,a} + v_{,a}\right)\right]\bigg|_{\mu = f(a)+k}. \tag{4.6}$$

The last formula gives the equation of state for the gauge subsystem depending on parameterization and gauge condition. Note that, again, the equation of state after substitution $\mu = f(Q) + k$ depends on the function $v(f(Q) + k, Q)$, but not on the functions $v(\mu, Q)$, $f(Q)$ separately.

Let us choose

$$v(\mu, a) = \frac{a^2}{\mu}, \quad \mu = 1 + \frac{1}{a^4}\,. \tag{4.7}$$

(Here and below we shall assume that the classical value $\bar{k} = 0$). As follows from (4.3), it corresponds to the condition for the lapse function N:

$$N = a + \frac{1}{a^3}\,. \tag{4.8}$$

This gauge condition is rather interesting in some respects. For large a we have $N = a$ (conformal time gauge), while in the limit of small a the condition (4.8) can be rewritten as $Na^3=1$. The latter corresponds to the constraint on metric components $\sqrt{-g} = \mathrm{const}$. This constraint is known to lead to the appearance of Λ-term in the Einstein equations [12, 8]. For $v(\mu, a)$ and $f(a)$ determined by (4.7) the equation of state (4.6) reduces to

$$p_{(obs)} = \varepsilon_{(obs)}\left[1 - \frac{1}{3}\left(\frac{4}{1 + a^4} + 2\right)\right] \tag{4.9}$$

In the course of cosmological evolution the equation of state changes from $p_{(obs)} = -\varepsilon_{(obs)}$ in the limit of small a to $p_{(obs)} = \frac{1}{3}\varepsilon_{(obs)}$ in the limit of large a. The former corresponds a medium with negative pressure typical for an exponentially expanded early universe with Λ-term, the latter is an ultrarelativistic equation of state, and the Einstein equations in the limit of large a have a solution describing a Friedmann universe in the conformal time gauge $N = a$. Therefore, we can see that the gauge subsystem appears to be a factor of cosmological evolution; its state changing over the history of the Universe determining a cosmological scenario.

5. Discussion

If we adopt the ADM parameterization, namely, regard the lapse function N as a gauge variable and impose the condition (4.8), according the above consideration, we shall come to the conclusion that at large a, when $N = a$ in a good approximation, the density matrix will have a Gaussian peak (3.13) with q representing a. In other words, this simple model confirms that the scale factor becomes a classical variable in the region of large a.

On the other hand, we are free to choose another gauge variable, μ, giving the function $v(\mu, a)$. Then, we shall change properties of environment and the character of its interaction with the physical subsystem. Remind that the density matrix (3.9) is obtained by integrating out a gauge variable defined by (2.2). In particular, as was said above, for a given reference frame one can choose μ to satisfy the condition $\mu = a$. So, instead of (4.7) one can put

$$v(\mu, a) = \frac{a^2}{1 + \frac{1}{\mu^4}}, \quad \mu = a.. \tag{5.1}$$

It leads to the same condition (4.8) for N and the same equation of state (4.9). The density matrix will have a Gaussian peak (3.13) in the whole region where the condition $\mu = a$ can be imposed, or, where the reference frame determined by (4.8) can be chosen. (One could make the simplest choice $N = a$, but in this case in the quasiclassical limit one would get a Friedmann universe without Λ-term and, correspondingly, without a stage of inflation.) The choice (5.1) means that we choose in (3.1) the basis, which is the set of eigenstates of the gauge operator $(\mu - a)$:

$$(\mu - a)|k\rangle = k|k\rangle; \quad |k\rangle = \delta(\mu - a - k). \tag{5.2}$$

The basis (5.2) plays the role of a preferred basis in the sense that the density matrix is about diagonal. In this case we choose an environment causing the density matrix to diagonalize. However, the variable μ, which represents the environment, may not have a clear physical meaning like the lapse function N.

The gauge subsystem has also other features, which are usually implied for environment. In particular, in any region where some gauge condition is fixed, a state of the Universe is not disturbed by interaction with the gauge system. Indeed, since one of canonical equations in extended phase space is a gauge condition in a differential form (2.5), a gauge operator commutes with the Hamiltonian (2.7) [10]. For example, when the state of the Universe is a specific state discussed above with $E = 0$, described by a quasiclassical wave function $\Psi_{k0}(Q)$, it will not be disturbed by interaction with the environment.

To summarize, under a suitable definition of the parameterization function $v(\mu, a)$, one can get that the density matrix would have a Gaussian peak in some variable in any region where a certain reference frame is chosen. An appropriate gauge condition may be rather complicated, as the condition (4.8), which includes different regimes in limiting cases. (In fact, Eq. (4.8) describes a transition from the condition $Na^3 = 1$ to $N = a$). In simple cosmological models it is possible to introduce a single reference frame in the whole spacetime, and the behavior of the Universe can be regarded as classical almost over the whole its history. The situation is different if one admits nontrivial topology of spacetime. In general, spacetime manifold may consist of regions covered by different coordinate charts, so that one should introduce different reference frames in these regions. A transition from one reference frame to another cannot be described by a single gauge condition like (4.8), and the Universe cannot behave in a classical manner nearby borders of regions where different reference frames are introduced. It especially concerns spacetime manifolds with horizons. Let us note

that the quasiclassical approximation is not valid nearby the borders of these regions as well, so that the two requirements – the quasiclassical character of a wave function and fixing a certain reference frame – completely coincide.

In [11] we have considered a small variation of the gauge-fixing function $f(Q)$ while the parameterization function $v(\mu, Q)$ being fixed. This variation corresponds to a transition to another reference frame and another basis

$$(\mu - f(Q) - \delta f(Q))|k\rangle = k|k\rangle; \quad |k\rangle = \delta(\mu - f(Q) - \delta f(Q) - k). \tag{5.3}$$

Then the Hamiltonian in physical subspace (3.4) acquires additional terms, which contain anti-Hermitian part in the original subspace defined by the basis (3.2). Accordingly, a measure in the physical subspace $M(f(Q)+k, Q)$ (see (3.7)) changes to $M(f(Q)+\delta f(Q)+k, Q)$. The change of the measure and the appearance of an anti-Hermitian part of the Hamiltonian (3.4) show that a transition to another reference frame has an irreversible character. As a consequence of interaction with the subsystem of gauge degrees of freedom, the Universe may not stay in one of states of the superposition (3.5), in particular, in a special state with $E = 0$ for which a classical limit could be obtain. Its state would change in some irreversible way.

References

[1] J. J. Halliwell, *Phys. Rev. D,* **39**, 2912 – 2923 (1989).

[2] W. H. Zurek, *Phys. Rev. D,* **24**, 1516 – 1525 (1981).

[3] W. G. Unruh, W. H. Zurek, *Phys. Rev. D,* **40**, 1071 – 1094 (1989).

[4] T. Fukuyama, M. Morikawa, *Phys. Rev. D,* **39**, 462 (1989).

[5] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, *Int. J. Mod. Phys. A*, **14**, 4473 – 4490 (1999).

[6] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, *Int. J. Mod. Phys. A*, **15**, 3207 – 3220 (2000).

[7] T. P. Shestakova, *Gravitation & Cosmology*, **5**, 297 – 300 (1999).

[8] T. P. Shestakova, in *Proceedings of the IV International Conference "COSMION-99"* (*Moscow, 1999*), *Gravitation & Cosmology,* **6**, Supplement, 47 – 50 (2000).

[9] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, *Gravitation & Cosmology*, **7**, 18 – 28 (2001).

[10] V. A. Savchenko, T. P. Shestakova and G. M. Vereshkov, *Gravitation & Cosmology*, **7**, 102 – 116 (2001).

[11] T. P. Shestakova, in *Proceedings of the V International Conference on Gravitation and Astrophysics of Asian-Pacific countries, Gravitation & Cosmology*, **8**, Supplement II, 140 – 142 (2002).

[12] S. Weinberg, *Rev. Mod. Phys.*, **61**, 1 – 23 (1989).

Gravity as a projection of the cosmological force

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The six-dimensional geometrical treatment of gravitation based on the principle of similarity of the basic properties of substance and light is given. To the principle corresponds movement of particles only with the speed of light in a multidimensional space in Compton vicinity of usual three-dimensional space (X) that is subspace of a multidimensional space. The total space is supposed to be six-dimensional Euclidean one (R_6), as for it a simple interpretation of spin of electron and other particles is possible. From the fact of existence of macroscopic three-dimensional bodies it follows, that the particles are kept in a microscopic vicinity of the subspace X by forces (F) of cosmological nature. Particles are moving in R_6 on a geodesic satisfying Fermat's principle, from what the law of conservation of energy of a particle in R_6 is follows, and the potential energy appears by the reserved energy of movement in a subspace, which is additional one to the subspace X. The geodesic passes on a pipe surface in R_6 with varying of radius and the speed of light along this pipe. The axis of the pipe is located in subspace X. The curvature of a trajectory is determined by normal component of force F to the trajectory and the pipe. Metric coefficients with neglect of quantum corrections are determined by unique function of co-ordinates and, in the case of satisfaction of Einstein's equation $R_{00} = 0$, differ from respective coefficients of known spherically symmetric solutions in Einstein's theory of gravitation and the relativistic theory of gravitation only in postpost-Newtonian approximation. The found metrics is obtained as well from a hypothesis about a superposition of local gravitational potentials of partial infinitesimal masses composing a complete gravitational mass. The given treatment is the external geometry of this pipe which is not requires of use of tensor calculation and not supposes of curvatures of space (not space but pipe is deformed) as distinct from the metric theory of gravitation which can be treated as internal geometry of this pipe.

Use of the global principle of simplicity [1] has resulted in of six-dimensional geometrical treatment of Lorentz-transformations, of the interval of relativity, of the relativistic mechanics, of spin and isospin, of intrinsic magnetic moment, of the thin structure formula, of the distinction between particles and antiparticles, of de Broglie waves, of Klein – Gordon equation, of CPT-theorem, of quark model of particles composed from u- and d-quarks, [2,3] and of cosmological expansion [4,5].

The treatment is based on the principle of similarity of the basic properties of substance and light, examples of that are diffraction of electrons and photoeffect. To it corresponds the assumption that particles of substance move with speed of light in multidimensional space. This assumption goes back to Einstein's statement "the nature saves on principles" and an idea of F. Klein [6-8] that particles move with speed of light in multidimensional space. It also entered in the principle of simplicity. The first substantiation of six-dimensionality of space was given in [9], where fundamental physical constants are calculated. The six-dimensional treatment of gravitation is given below.

The basic property of light is that in absence of gravitation it propagates with identical speed in any frame of reference. Then the particles of substance should move with the same speed. It is possible only in multidimensional space if positions of particles are recording in projection on homogeneous and isotropic three-dimensional subspace (X). If the formulas of the Newtonian mechan-

ics to refer to six-dimensional Euclidean space R_6, then in projection of events on X these formulas give relativistic results.

The total space is supposed six-dimensional ones, as only for it a simple interpretation of spin of electron and other particles is possible. The particles should be kept in a small vicinity of the three-dimensional Universe by forces orthogonal to it (of a cosmological nature), differently would there are not any macroscopic bodies. We consider a small site of the Universe representing interest at the description of a field of gravitation, as Euclidean subspace X with neglecting of curvature of the Universe on this site. Let us assume that for particles moving in R_6 and considered as material points, formulas of the Newtonian mechanics are applicable at suitable choice of time, which has been mentioned below, and that the positions of particles is fixed by observer in projection on X.

The particle, which is at rest in a projection on X in the inertial frame of reference K of the observer " at rest ", moves with speed of light c in the simplest case on a circle in three-dimensional subspace Y adding up X to R_6, with the center of the circle in X. In any other inertial frame of reference this particle is moving on a helical line located on a cylindrical surface (a motion pipe) in R_6 with an axis in X. We assume that the proper time of a particle is proportional the number of its revolutions in Y around axis of a pipe of movement. This number is proportional to $|\cos\theta|$, where θ is the angle of an inclination of a helical line to directrix of the pipe. If the particle makes one revolution per a proper time τ by clock of the observer "at rest", relatively which the particle moves along the pipe with a speed $v = c\sin\theta$, it will take place per time $t = \tau / |\cos\theta|$, where

$$\sin\theta = v/c\,, \qquad \cos\theta = \pm\sqrt{1-(v/c)^2}\,. \tag{1}$$

In (1) and further positive sign refers to a particle rotating around an axis of a pipe in a positive direction, negative sign concerns to an antiparticle rotating in an opposite direction.

The lapses of proper time of a particle (or antiparticle) $d\tau$ and of time of the observer at rest dt are connected by a ratio

$$dt = \pm d\tau / \cos\theta = d\tau \Big/ \sqrt{1-(v/c)^2}\,. \tag{2}$$

In the frame of reference at rest K the particle has a component of speed on directrix equal to $c\cdot\cos\theta$. According to (2), the proper time of a particle from the point of view of the observer at rest is proportional to $\cos\theta$ as well, so that the particle in the proper frame of reference K' moves with speed c also.

The displacement of a particle on an interval ds on the directrix of a motion pipe and respective turn on the central angle around of the axis of the pipe, where a is radius of the pipe, are identical in any frame of reference. Having designated through dx in system K a projection of a displacement $d\varsigma$ of a particle on the surface of the pipe on its generatrix and having applied the Pithagorian theorem, one obtains $ds^2 = (cdt)^2 - dx^2$. If to consider this ratio as initial one, then from it follows $d\varsigma = cdt$, i.e. that the particle moves in R_6 with the speed c.

The particle at rest in X is moving in Y with speed c and consequently has a rest momentum $p_y = mc$ and rest energy $E = p_y \cdot c = mc^2$.

By virtue of a principle of similarity of the basic properties of substance and light being a concrete definition of a principle of simplicity, the rest energy mc^2 should also be equal to $h\nu$,

where ν is the frequency of rotation of a particle around an axis of a motion pipe. From here radius of the tube equal to $a = \hbar / mc$, and the length of directrix equal to Compton length that corresponds to the period h of the coordinate of action in the 5-optics [8].

In a field of gravitation the radius a of the motion pipe and speed c_ς of moving over pipe depend on coordinates of subspace X, i. e. from a position of a particle concerning massive bodies. Thus metric coefficients in expression for ds^2 are dependent on a form of functions a and c_ς. The connection between a, c_ς and θ is imposed by a condition, what the particle moves over the pipe on a geodesic according to Fermat's principle. By definition $c_\varsigma = d\varsigma / dt$, where $d\varsigma$ is a trajectory segment, which the particle passes over pipe in a time dt by clock of a distant observer. Father, c_ς and a are supposed not depending from the angle θ. Let's show that along a geodesic onto the pipe

$$\left(a/c_\varsigma\right)\cos\theta = const. \tag{3}$$

At $c_\varsigma = const$ the geodesic is describing by Clairaut's law $a\cos\theta = const$ [10], and at $a = const$ by Snell's law. In a general case

$$\frac{d\theta}{d\varsigma} = \frac{\partial\theta}{\partial a}\frac{da}{d\varsigma} + \frac{\partial\theta}{\partial c_\varsigma}\frac{dc_\varsigma}{d\varsigma} \quad . \tag{4}$$

From laws of Clairaut and Snell, by differentiation, one finds $\frac{\partial\theta}{\partial a} = \frac{1}{a}\cot\theta$, $\frac{\partial\theta}{\partial c_\varsigma} = -\frac{1}{c_\varsigma}\cot\theta$, respectively. Inserting these expressions into (4) results in $\frac{d\theta}{d\varsigma} = \cot\theta\,\frac{c_\varsigma}{a}\frac{d}{d\varsigma}\left(\frac{a}{c_\varsigma}\right)$, which on integration gives equation (3).

The given treatment of gravitation turns out by external geometry of a motion pipe of a particle, while the metric theory of gravitation by internal geometry of the motion pipe.
It should be noted that in each normal cross-section of a motion pipe all radial directions, being orthogonal to the subspace X, are equal in rights even in a case of curved pipe axis. Hence the metrics on a pipe surface does not depend on polar angular coordinate in any normal section, and the internal geometry [10] on a pipe surface is the same as on a respective surface of rotation in three-dimensional space.

The projection of speed c_ς on tangent to a meridian is equal $v_\varsigma = c_\varsigma \sin\theta$. The coordinate speed v of the particle registered by the removed observer equals

$$v = \frac{d\sigma}{dt} = v_\varsigma \frac{d\sigma}{d\xi} = c_\varsigma \sin\theta \frac{d\sigma}{d\xi} = c_\varsigma \sin\theta\left[1 + \left(\nabla a \cdot \cos\beta\right)^2\right]^{-1/2} \tag{5}$$

where ξ and σ are lengths of arches along a meridian and axis of the pipe, respectively, β is an angle between ∇a and tangent to an axis. Co-ordinate speed of light one obtains by means of tending in (5) θ to $\pi / 2$:

$$c_k = c_\varsigma\, d\sigma / d\xi = c_\varsigma \left[1 + \left(\nabla a \cdot \cos\beta\right)^2\right]^{-1/2} \tag{6}$$

On displacement through distance $d\varsigma$ on the pipe a particle rotates about pipe axis through the angle $d\alpha = d\eta/a$ where $d\eta = \cos\theta d\varsigma$ is projection of this displacement on the directrix of the pipe. The angle $d\alpha$ is the same for any observer, i.e. is invariant, as number of revolutions around of the axis of the pipe is identical for any observer. The quantity $ds = a_\infty d\alpha$, where a_∞ is the radius of the pipe at infinite distance from the center of gravitation, also is invariant. It is an interval of the metric theory of gravitation. Under Pithagorian theorem one has $d\eta^2 = d\varsigma^2 - d\xi^2$. Substituting here $d\varsigma = c_\varsigma dt$, multiplying both parts of the equality on $(a_\infty/a)^2$ and taking into account that $d\alpha = d\eta/a$, one can find

$$(a_\infty d\alpha)^2 = ds^2 = \left(\frac{c_\varsigma a_\infty}{a} dt\right)^2 - \left(\frac{a_\infty}{a} d\xi\right)^2 . \tag{7}$$

This with account of (6) it is possible rewrite as

$$ds^2 = \gamma(cdt)^2 - \gamma[(c/c_k)d\sigma]^2 , \tag{8}$$

where c is the limiting value of the speed c_ς on infinity,

$$\gamma = (c_\varsigma a_\infty / ca)^2 . \tag{9}$$

It follows from (8) that the proper time τ of a particle is connected with time t of an observer removed at infinity, by the relation

$$d\tau/dt = \sqrt{\gamma} , \tag{10}$$

and the elements of spatial distances dl and $d\sigma$, relatively for local and distant observers, by the relation

$$dl = \sqrt{\gamma}(c/c_k)d\sigma , \tag{11}$$

and for the local observer

$$ds^2 = (cd\tau)^2 - dl^2 . \tag{12}$$

The relations (10) and (11) can be obtained as well so. For the local observer the radius of pipe a (or equal to it Compton length $2\pi a$, which can be measured) stands duty as a scale of length. As a result the lengths are measuring along the meridian, and consequently

$$dl = \frac{a_\infty}{a} d\xi = \frac{a_\infty}{a}\frac{d\xi}{d\sigma} d\sigma = \frac{a_\infty c_\varsigma}{ac_k} d\sigma . \tag{13}$$

Whence taking into account a designation (9), one obtains the relation (11). A period of rotation in Y of a particle situated near this observer stands duty as a scale of time for the local observer. This period is proportional to c_ς/a, whence formula (10) follows. The speed v_{loc} of a particle for the local observer according to (9), (10) and (13) is equal to $v_{loc} = \frac{dl}{d\tau} = \frac{a_\infty}{a}\frac{d\xi}{dt}\frac{dt}{d\tau} = \frac{a_\infty}{a} c_\varsigma \sin\theta \frac{1}{\sqrt{\gamma}} = c\sin\theta$. By this

$$v/c_k = v_\varsigma/c_\varsigma = v_{loc}/c = \sin\theta . \tag{14}$$

Whence it is seen that upper limit of local speed for the local observer (at $\sin\theta \to 1$) is equal to speed of light at infinity. The formulas (3) and (10) in view of a designation (9) can be presented as

$$\left(1/\sqrt{\gamma}\right)\cos\theta = const., \quad \left(dt/d\tau\right)\cos\theta = const. \tag{15}$$

The relations (14) and (15) allow to express speed of a particle through γ:

$$\left(\frac{v}{c_k}\right)^2 = \left(\frac{v_\varsigma}{c_\varsigma}\right)^2 = \left(\frac{v_{loc}}{c}\right)^2 = 1 - \frac{\gamma}{\gamma_0}\cos^2\theta_0 = 1 - \frac{\gamma}{\gamma_0}\left[1 - \left(\frac{v_{loc}}{c}\right)_0^2\right], \tag{16}$$

where zero in the index marks the values at the initial moment of time.

As the proper time of a particle is measured by number of its revolutions around an axis of a motion pipe, the difference of clock- readings in the end and in the beginning of a journey of an arbitrarily moving observer is proportional to the integral from the interval. In fact, (12) can be represented according to (14) as $ds^2 = (cd\tau)^2\left[1 - (v_{loc}/c)^2\right] = (\cos\theta \cdot cd\tau)^2 = (cd\tau')^2$, where $d\tau' = \cos\theta \cdot d\tau$ is an increasing in the proper time for this observer. Whence integrating $ds = cd\tau'$ along the trajectory between points A and B, one finds $\tau'_B - \tau'_A = \frac{1}{c}\int_A^B ds$.

For the local observer, the acceleration of a particle is equal $dv_{loc}/d\tau$. Taking into account (16), one finds $\frac{dv_{loc}}{d\tau} = \frac{dv_{loc}}{dl}\frac{dl}{d\tau} = \frac{1}{2}\frac{dv_{loc}^2}{dl} = -\frac{c^2}{2}\frac{\cos^2\theta_0}{\gamma_0}\frac{d\gamma}{dl}$. Whence the acceleration of gravity force for the local observer will be $g_{loc} = \frac{c^2}{2\gamma}\frac{d\gamma}{dl_{||}}$, where $dl_{||}$ is an element of spatial distance in the direction of gradient of function γ from the point of view of the local observer. Introducing a gravitational potential Φ_{loc} by equality $g_{loc} = d\Phi_{loc}/dl_{||}$ and integrating, one finds

$$\sqrt{\gamma} = \exp\left[-\frac{1}{c^2}\int_{l_{||}}^{\infty} g_\Lambda dl_{||}\right] = \exp\left(\frac{1}{c^2}\Phi_\Lambda\right) = \frac{d\tau}{dt}. \tag{17}$$

The formula (17) describes a slow down of time in a field of gravitation. Elimination of $\sqrt{\gamma}$ between (15) and (17) and tacking into account (14) one gets that along a geodesic

$$\left[1 - (v_\Lambda/c)^2\right]\exp\left[\left(2/c^2\right)\Phi_\Lambda\right] = const. \tag{18}$$

In the domain of weak fields the formulas (17) and (18) are reduced to forms $d\tau/dt = 1 - \left(\Phi_\Lambda/c^2\right)$, $\left(v_\Lambda^2/2\right) - \Phi_\Lambda = const.$ Last formula expresses the law of conservation of energy in the mechanics of Newton. Similarly we shall find magnitude of acceleration from the point of view of the removed observer:

$\frac{dv}{dl} = \left\{\frac{1}{2}\left(\frac{v}{c_k}\right)^2 \frac{d}{d\sigma_{||}}c_k^2 - \left(\frac{c_k}{c_{||}}\right)^2\left[1 - \left(\frac{v}{c_k}\right)^2\right]g\right\}\cos\beta$, where $g = \frac{c_{||}^{\ 2}}{2\gamma}\frac{d\gamma}{d\sigma_{||}}$ is the acceleration of force of gravity, $d/d\sigma_{||}$ means differentiation in a direction of a gradient of γ, $c_{||}$ is value of c_k in this direction.

The particle at rest in X rotates in Y with frequency $\nu_0 = c_\varsigma/(2\pi a)$, having energy of rest $E_0 = h\nu_0 = \hbar c_\varsigma/a = \hbar\sqrt{\gamma}\, c/a_\infty = m_\infty c^2\sqrt{\gamma}$. For a moving particle the total energy will be equal to $E = E_0/|\cos\theta|$. The Lagrangian formalism yields also the same results.

The action S is determined within accuracy up to a constant factor as integral from a scalar. Scalar here is the angle of turn of a particle around of an axis of motion pipe. A constant multiplier one chooses by such, that in absence of gravitation Lagrange function to be $L = -mc^2\cos\theta$, as in the relativistic mechanics. Then one has $S = -\hbar\int_{\alpha_1}^{\alpha_2} d\alpha$. The Lagrange function L is defined by the formula $S = \int_{t_1}^{t_2} Ldt$. From here, one finds $L = -\hbar\dot{\alpha}$. From (7) one has $\dot{\alpha} = (1/a)\sqrt{c_\varsigma^2 - v_\varsigma^2}$, so that $L = -(\hbar/a)\sqrt{c_\varsigma^2 - v_\varsigma^2} = -(\hbar c/a_\infty)\sqrt{\gamma}\cos\theta$. Thus a projection of the momentum of a particle on a meridian of pipe is $p_\xi = \dfrac{\partial L}{\partial v_\varsigma} = \dfrac{\hbar v_\varsigma}{a\sqrt{c_\varsigma^2 - v_\varsigma^2}} = \dfrac{\hbar}{a}\tan\theta$, and energy equals $E = p_\xi v_\varsigma - L = \dfrac{\hbar c_\varsigma}{a\cdot\cos\theta} = \dfrac{\hbar c}{a_\infty}\dfrac{\sqrt{\gamma}}{\cos\theta} = \dfrac{m_\infty c^2\sqrt{\gamma}}{\cos\theta}$, so $p_\xi = Ev_\varsigma/c_\varsigma^2$ and the total momentum of a particle $p = \partial L/\partial c_\varsigma$ becomes $p = E/c_\varsigma = \hbar/(a\cdot\cos\theta)$. From here and from (15) it is seen that at moving along a geodesic $E = const$. Thus occurs only flow of energy of movement from the latent form in subspace Y into the evident form in subspace X or on the contrary. The potential energy is the reserved energy of motion in extra dimension space Y.

In absence of gravitation a particle at rest in X rotates in Y on a circumference of radius a_∞ with speed of light c. Appropriate to such rotation centripetal force is equal to $F = p_y c/a_\infty = \hbar c/a_\infty^2 = m_\infty c^2/a_\infty$, in $c^2/ag = 2.38\cdot 10^{28}$ time exceeding weight of the particle at the terrestrial surface. This force can have only cosmological nature. The same result turns out and at movement of a particle on a helical line: $F = pcK/\cos\theta$, where $K = \cos^2\theta/a_\infty$ is the curvature of helical line. From this it is seen that $F = \hbar c/a_\infty^2$ at any θ.

In a field of gravitation the angle of an inclination of a meridian to pipe axis is determined by relations: $\sin\chi = da/d\xi$, $\cos\chi = \sqrt{1-(da/d\xi)^2} = 1/\sqrt{1+(da/d\sigma)^2}$ $\tan\chi = da/d\sigma$. A component of the cosmological force perpendicular to the geodesic, in the osculating plane, is equal to the centripetal force proportional to the curvature K of a trajectory:

$$pc_\varsigma K/\cos\theta = F\cos\chi, \tag{19}$$

where $K = \sqrt{K_\perp^2 + (\sigma'')^2}$, $K_\perp^2 = (y_1'')^2 + (y_2'')^2$; y_1 and y_2 are coordinates of the particle in two mutually perpendicular directions in a section of the pipe, the prime means derivative along a trajectory. It is possible to write these coordinates as $y_1 = a\cdot\cos\alpha$, $y_2 = a\cdot\sin\alpha$. Then, taking into account that $ds = ad\alpha = \cos\theta\, d\varsigma$, $d\xi = \sin\theta\, d\varsigma$, (15) and $\sigma' = \cos\chi\sin\theta$, one finds:

$$K_\perp^2 = \left(a\alpha'^2 - a''\right)^2 + \left(a\alpha'' + 2a'\alpha'\right)^2 = \left(1 - \gamma\frac{\cos^2\theta_0}{\gamma_0}\right)\frac{\cos^2\theta_0}{\gamma_0}\left[\frac{d\sqrt{\gamma}}{d\xi} + \frac{\sqrt{\gamma}}{a}\frac{da}{d\xi}\right]^2 +$$

$$+\left[\frac{\cos^2\theta_0}{\gamma_0}\sqrt{\gamma}\left(\frac{\sqrt{\gamma}}{a} + \frac{d\gamma}{d\xi}\frac{da}{d\xi}\right) - \left(1 - \gamma\frac{\cos^2\theta_0}{\gamma_0}\right)\frac{d^2a}{d\xi^2}\right]^2, \tag{20}$$

$\sigma'' = -\cos\chi\frac{\cos^2\theta_0}{2\gamma_0}\frac{d\gamma}{d\xi} - \frac{1}{\cos\chi}\left(1 - \frac{\cos^2\theta_0}{\gamma_0}\gamma\right)\frac{da}{d\xi}\frac{d^2a}{d\xi^2}$. Substituting the found expressions for p and F in (19) yields: $a_\infty\sqrt{\gamma}\,K/\cos^2\theta = \cos\chi$. Whence and from (20) to an accuracy of $a_\infty^2\left[\left(d\sqrt{\gamma}/d\xi\right)^2 + \left|d^2\sqrt{\gamma}/d\xi^2\right|\right]$ one obtains $\sqrt{\gamma}\,a_\infty/a \approx 1$. Thus on the basis of (9) one has:

$$a/a_\infty = \sqrt{\gamma}\,, \quad c_\varsigma/c = c_\perp/c = \gamma\,, \tag{21}$$

where $c_\perp$ is speed of light in a direction, perpendicular to the gradient of the field.

In the metric theory of gravitation, it is considered that the field of gravitation is generated only by massive bodies and is accompanied by decrease of speed of light and by a slowing down of proper time in vicinity of massive bodies. In six-dimensional treatment of gravitation, massive bodies themselves gravitation does not create, they only decrease speed of light in bodies' vicinities. It results in reduction of radius of an orbit of movement in Y at preservation of equality of values of centrifugal force and cosmological one. But then a motion pipe of a particle is not a cylindrical surface, its meridians have an inclination to the pipe axis, therefore the projection of cosmological force onto a meridian becomes distinct from zero. This projection both represents force of gravitation and is equal to $F_\xi = -F\sin\chi = -\left(\hbar c/a_\infty^2\right)da/d\xi$, whence in approximation (21)

$$F_\xi = -\left(\hbar c/a_\infty\right)d\sqrt{\gamma}/d\xi = -mc^2\,d\sqrt{\gamma}/d\xi\,. \tag{22}$$

In spherically symmetric field the asymptotic decomposition of γ into a power series in $1/r$, where r is the radial co-ordinate (distance from the centre of gravitation from the point of view of the distant observer), has a form

$$\gamma = 1 - \left(r_g/r\right) + b_2\left(r_g/r\right)^2 + b_3\left(r_g/r\right)^3 + \cdots, \tag{23}$$

where $r_g = 2GM/c^2$ represents gravitational radius, G gravitational constant, M mass of an attractive body.

In (23) coefficient at the first power, as well as in the metric theory of gravitation, is chosen equal -1, in order, far away from the center of gravitation, gravitational potential was Newtonian one [11,12]. Substituting (23) in (22) yields

$$F_\xi = -\frac{GMm_\infty}{r^2}\left[1 - \left(2b_2 - \frac{1}{2}\right)\frac{r_g}{r} - \frac{3}{2}\left(b_3 + \frac{b_2}{2} - \frac{1}{8}\right)\left(\frac{r_g}{r}\right)^2 + \cdots\right].$$

The gravitation has the same effect onto rays of light how appropriate anisotropic medium, and the speed of light is described by the formula for ray velocity [13]:

$$\frac{1}{c_k^2} = \left(\frac{\sin\beta}{c_\perp}\right)^2 + \left(\frac{\cos\beta}{c_\parallel}\right)^2, \tag{24}$$

where β is an angle between a direction of propagation of light and gradient of a field. Denoting projections of an element $d\sigma$ of a trajectory in X onto the direction of gradient of a field and onto perpendicular to it direction through $d\sigma_{||}$ and $d\sigma_{\perp}$, respectively, and substituting (24) in (8), one obtains

$$ds^2 = \gamma(cdt)^2 - \gamma\left(\frac{c}{c_{||}}d\sigma_{||}\right)^2 - \gamma\left(\frac{c}{c_{\perp}}d\sigma_{\perp}\right)^2.$$

Whence in neglect by the quantum corrections under conditions (21) one finds

$$ds^2 = \gamma(cdt)^2 - (1/\gamma)d\sigma_{||}^2 - (1/\gamma)d\sigma_{\perp}^2. \tag{25}$$

The metric (25) is described only by one function of co-ordinates – function γ. The centrifugal force $p_{\xi}v_{\varsigma}/R\cos\theta$, where R is the radius of curvature of a trajectory in X, is counterbalanced by component of force of gravity $-F_{\xi}\sin\beta$. From here one obtains

$$\tan^2\theta = \left(R/\sqrt{\gamma}\right)\left(d\sqrt{\gamma}/d\xi\right)\sin\beta. \tag{26}$$

The Lagrange function $L = -\hbar\sqrt{1-(\dot{r}/c_{||})^2 - (r\dot{\varphi}/c_{\perp})^2}$ in polar co-ordinates r, φ does not depend explicitly on φ, so that $\partial L/\partial\dot{\varphi} = const.$, whence one obtains the law of conservation of angular momentum $(c/c_{\perp})^2 rv\sin\beta = const.$ Substituting (21) yields $v(r/\gamma)\sin\theta\sin\beta = const.$ By means of this formula with due account of (15) it is possible to eliminate $\sin\beta$ or $\sin\theta$ in (26).

For introduction of coordinates with reference to (25) we may use an Einstein's equation for components of the Riccian tensor. In vacuum, this is $R_{00} = 0$. In spherically symmetric field, this equation for $\gamma = \exp(v)$ is reduced to $v'' + v'(2/r) = 0$ [11]. Its solution, satisfying the asymptotics (23), has a form $v = -r_g/r$, $b_2 = 1/2$. By this the metrics (25) in spherically symmetric field coincides in the post-Newtonian approximation with Shwarzshild's metrics in isotropic coordinates [11,12] and with the metrics of the relativistic theory of gravitation [14], but differs from both these metrics in the next approximation. This solution is obtained as well from a hypothesis, that the superposition of partial fields v_j (i.e. of local gravitational potentials) takes place for any components M_j of the total mass M, including infinitesimal ones, so that $v = \sum_j v_j$. Really, for $M_j = M/n$ $(r_{gj} = r_g/n)$ one has $v_j = -r_{gj}/r$, $v = \lim_{n\to\infty}(n v_j) = -r_g/r$. Given superposition corresponds to the principle of simplicity also and at any spatial distribution of masses. Anyway, for such static distribution of masses the replacement of exponential function $\exp\left[-\sum_j (r_{gj}/r_j)\right]$ by the first three terms of its power series expansion gives the metrics coincident with post-Newtonian metrics given in [12].

It is essential, that as distinct from others of multidimensional theories of gravitation, in the given approach based on the principle of simplicity, a compactification of space of extra dimensions is not a necessary. It is replaced here by presence of cosmological force, confining particles in Compton vicinity of three-dimensional subspace (X). Here compactificated not extra dimensions, but trajectories of elementary particles in space of extra dimensions. Existence of this force is not postulated. It follows from the principle of simplicity (specifically - from the principle of similarity

of the basic properties of substance and light, according to which $mc^2 = h\nu$, and the value of force is equal to $p_y c / a_\infty = m^2 c^3 / \hbar$) and from fact of existence of three-dimensional bodies. If such force would not exist, the particles would not be kept in a vicinity of three-dimensional subspace. Then for an explanation of existence of three-dimensional bodies it should with necessity involve a compactification of space of extra dimensions, despite of the problem of an explanation of its occurrence.

The problem of compactification in multidimensional theories of gravitation arises because of impossibility of an explanation of existence of three-dimensional bodies in multidimensional space, without the consideration of mechanism of confining of particles in a small vicinity of three-dimensional subspace. In the given approach this problem does not arise.

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References

1. Margolin A. A. The principle of simplicity. //Khimia i zhizn (Chemistry and life). 1981. № 9. P. 79. 1981 (in Russian).
2. Urusovskii I. A. Six-dimensional treatment of the relativistic mechanics and spin, metric gravitation theory and the expanding Universe. //Zarubezhnaja radioelectronika. Uspekhi soremennoi radioelectroniki. 1996. № 3. P. 3-21. (in Russian).
3. Urusovskii I. A. Six-dimensional treatment of the quark model of nucleons. // *ibid*., 1999. № 6. P. 64-74. (in Russian).
4. Urusovskii I. A. Six-dimensional treatment of the expanding Universe. //*ibid*., 2000. № 6. P. 66-77. (in Russian).
5. Urusovskii I. A. Cosmological consequences of inconstancy of speed of light in the time. /Russian acoustical society. Annual collection of Works of School -seminar of Prof. S. A. Rybak. Moscow: 2001. P. 176-192. (in Russian).
6. Klein F. Uber neuere englische Arbeiten zur Gesammelte matematishe Abhandlungen, B.2, Springer, Berlin, 1922, 601 S.// Zeit. f. Math. u. Phys. 1901. S. 375.
7. Klein F., Vorlezungen über die höhere Geometrie, 3. Aufl. Berlin,1926.
8. Rumer Yu. B. Investigations on the 5-optics. M.: Gostekhizdat, 1956. 192 p. (in Russian).
9. di Bartini Robert Oros, Doklady of Academy of science of USSR. 1965. V. 163. № 4. P. 861-865 (in Russian).
10. Kagan V. F., Principle of the theory of surfaces, Chap. 1. M.-L.: Gostekhizdat, 1947. 512 p. (in Russian).
11. Eddington A. S. The mathematical theory of relativity. Cambridge University Press, 1923. 247p.
12. Brumberg V. A., Relativistic celestrial mechanics. M.: Nauka, 1972, Cap. 5. 382 p. (in Russian).
13. M. Born, E. Wolf. Principle of optics. Pergamon Press, Oxford, London, etc., 1964.
14. Logunov A. A., Mestvirishvily M. A. Foundations of relativistic theory of gravitation. Publishing house of the Moscow University, 1986. 307 p. (in Russian).

Толкование парадокса дальностей корабля “Pioneer-10” на основе аномальных эффектов гравитации

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Обработка результатов измерения траектории корабля “Pioneer-10” выявила парадокс – доплеровская дальность через тридцать лет полета корабля существенно превысила расчетную, определенную по закономерностям небесной механики. Анализ возможных причин возникновения парадокса в рамках существующих представлений о физике космоса, природе гравитации и технике космических аппаратов дал отрицательный ответ – ни один из рассмотренных факторов не способен сформировать полученную разницу дальностей [1].

Парадокс положительно разрешается введением в рассмотрение аномального сдвига длины волны, формируемого гравитационным полем Земли. Этот сдвиг обусловил превышение соответствующей дальности над действительной. Для получения истинной дальности на основе доплеровских измерений необходимо вычесть из полученного сдвига длины волны аномальную составляющую.

Ошибка в определении радиальной скорости $\Delta\dot{R}$ корабля “Pioneer-10” за счет неучета аномального сдвига длины волны в гравитационном поле Земли при интерпретации результатов доплеровских измерений составляет [2]

$$\Delta\dot{R} = \Delta\lambda\lambda_0^{-1}\cdot C_0 = \ln\frac{R_1^2(R_2^2+r_{bh}r)}{R_2^2(R_1^2+r_{bh}r)}\cdot C_0 = 1{,}391\cdot10^{-9}\cdot3\cdot10^{8} = 4{,}172\cdot10^{-1}\ \text{мс}^{-1},$$

где $\Delta\lambda\lambda_0^{-1}$ - относительное увеличение длины волны сигнала, R_1 – расстояние от центра масс Земли до источника излучения, R_2 – такое же расстояние до приемника излучения, r – радиус Земли, $r_{bh} = \frac{GM}{C_0^2}$ - радиус Земли в состоянии Черной дыры, M – масса Земли, C_0 = $3\cdot10^8$ мс$^{-1}$ – скорость света.

За тридцать лет полета корабля ($t_к$ = $9{,}4608\cdot10^8$ с) эта ошибка в радиальной скорости $\Delta\dot{R}$ обусловила ошибку в определении дальности до корабля

$$\Delta R = \Delta\dot{R}\cdot t_к = 4{,}173\cdot10^{-1}\cdot9{,}4608\cdot10^{8} = 3{,}948\cdot10^{8}\ \text{м},$$

что эквивалентно действию в течение всего времени $t_к$ полета корабля виртуального ускорения $\Delta\ddot{R} = \frac{2\Delta R}{t_k^2}$ = $0{,}882\cdot10^{-9}$ мс$^{-2}$, которое с точностью до второго знака совпадает с величиной $\Delta\ddot{R}$ = $0{,}809\div0{,}865\cdot10^{-9}$ мс$^{-2}$, полученной специалистами NASA [1].

Литература

1. Slava G.Turyshev, John D.Anderson, Philip A.Laing, Eunice L.Lau, Anthony S.Liu, Michael Martin Nieto “The apparent anomalous, weak, long-range acceleration of Pioneer 10 and 11”. Proceedings of the XXXIV The Rencontres de Moriond Series: Moriond workshops january 23-30, 1999.
2. Г.Р.Успенский. Гравитационная оптика. Космонавтика и ракетостроение №16, 138-151 с., ЦНИИМАШ, Королев, 1999 г.

О понятии массы в курсе физики

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On the Mass Concept in the Physics Study Course

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The indefiniteness of the mass concept makes some difficulties for students studying and understanding physics. The reason of this indefiniteness is that one and the same term “a mass” is used to characterize substance quantities, inertial and gravitational properties, there are such terms as “an effective mass”, “longitudinal mass” and so on. It is suggested to reduce the content of the mass concept up to a measure of proper energy only. In this case the mass becomes the energy intrinsic parameter of the material system and determines potential possibilities of the system to participate in any interaction. The inertness and “gravitationness” are properties of a system, which reveal themselves under interactions and their magnitudes are determined by the proper energy quantity (the energy reserve) as well as by conditions of interactions.

Неоднозначность понятия массы создает у учащихся определенные трудности при изучении и осмыслении законов физики. Причина неоднозначности в том, что с помощью одного и того же термина «масса» характеризуют количество вещества, инертные и гравитационные свойства тел, используются понятия эффективной массы, продольной и т.п. Споры относительно понятия массы не утихают и до настоящего времени: «Какую из двух масс, массу покоя или релятивистскую массу, назвать простым словом масса, обозначить буквой m без индексов и тем самым признать «главной» массой» [1]. По мнению одних, «релятивистская масса имеет естественное операционное определение, основанное на формуле $\mathbf{P} = m\,\mathbf{v}$. Она подчиняется закону сохранения и аддитивна. Она эквивалентна и энергии, и гравитационной массе. Ее следует называть массой и обозначать m» [1]. По мнению других: «Согласно современной терминологии оба термина «релятивистская масса» и «масса покоя» являются устаревшими, пользоваться ими не стоит и «рационально отдать предпочтение» просто массе m без всяких прилагательных или иных дополнительных слов…» [2,3]. БСЭ, том 15: масса есть коэффициент пропорциональности между импульсом и скоростью, (тогда масса – это импульс при единичной скорости?); или масса – коэффициент пропорциональности между силой и ускорением (тогда масса – это сила, вызывающая единичное ускорение?). В разных теориях гравитации при установлении соответствия между активной, пассивной гравитационными и инертной массами содержание понятия масса становится более неопределенным и запутанным. Например, инертная масса может зависеть от выбора системы трехмерных координат. В ньютоновской механике из третьего закона Ньютона следует равенство активной и пассивной гравитационных масс независимо от размера и состава тела; равенство инертной массы с двумя остальными принимается как эмпирический факт. В теории Эйнштейна для точечных тел имеет место равенство инертной и пассивной гравитационных масс, но подобное равенство для массивных и протяженных тел – пока под вопросом. В некоторых теориях гравитации все три массы одного и того же тела могут различаться [4].

Так каково же содержание понятия массы: это - мера количества вещества, мера инертности, мера гравитационных свойств, мера энергии покоя,.? Таким образом, не только современный курс физики, но и развитие науки требуют уточнения содержания понятия массы.

Предлагаемый подход сужает содержание понятия массы до меры внутренней энергии или, как можно было бы назвать, до меры собственной энергии материальной системы. Собственная энергия системы материальных объектов складывается из суммы собственных энергий каждого объекта в отдельности, суммы кинетических энергий всех объектов в Ц-системе отсчета и плюс энергии взаимодействия этих объектов между собой. В инерциальной системе отсчета, в которой центр масс покоится, собственная энергия численно равна энергии покоя E_0, и поэтому согласно соотношению $E^2 - p^2$

$c^2=E_0^2$– является лоренцевым инвариантом. Собственная энергия «привязана» к одной системе отсчета, Ц-системе, а энергия покоя – к двум системам: Ц-системе и той, где центр масс покоится.

Величину $m=E_0/c^2$– назовем собственной массой системы. Собственная масса не аддитивна, сохраняется в изолированной системе, не зависит от относительной скорости центра масс, не преобразуется и является лоренцевым инвариантом. Таким образом, собственная масса в соответствии с данным определением является внутренней энергетической характеристикой системы, мерой «энергосодержания» системы [5]. Важность выделения понятий собственной энергии и собственной массы подчеркнуто в [6]: « Любой объект обладает энергией уже потому, что он *существует*».

Большинство физиков и авторов учебных пособий по физике вкладывают в содержание понятия массы или массы покоя тот же смысл, что и в предлагаемом подходе (например,[2,3,5,7]), но – неуверенно, так как за понятием «просто» массы появляются понятия релятивистской массы, гравитационной массы и т.д. Как же определить место этих понятий?

Исходные положения наших рассуждений: Свойства материальных объектов обнаруживаются только при взаимодействии с другими материальными объектами и, очевидно, зависят от характера взаимодействия, в частности, от характера взаимодействия с наблюдателем; закономерности проявления разных свойств могут быть различными.

Результат взаимодействия зависит, в частности, от свойств объекта, определяющих его участие в этом взаимодействии, и от запаса собственной энергии. Если, например, происходит изменение скорости относительного движения центра масс материального объекта (системы объектов), то при этом проявляется свойство инертности, которая количественно учитывается в законах динамики через величину $m_{ин}$,, называемую инертной массой. Правильнее было бы эту величину $m_{ин}$ называть «инертностью», хотя она и выражена в единицах массы. Инертность зависит от величины собственной энергии или от собственной массы (разница лишь в единицах измерения, но не в сути): $m_{ин}=f(E_0)$. Если, например, сила, вызывающая изменение скорости, перпендикулярна к направлению скорости, то инертность $m_{ин}=m\gamma$, а если **F** II **v** то $m_{ин}=m\gamma^3$, т.е. инертность зависит как от запаса собственной энергии, так и от скорости и угла между **F** и **v**, а в других случаях $f(E_0)$.может быть и иной. Таким образом, инертность в СТО, в отличие от классической теории, проявляется как относительное свойство и зависит от способа изменения скорости относительного движения.

Проявляемые при гравитационных взаимодействиях свойства активной и пассивной «гравитационности» и инертности также имеют свои закономерности и определенным образом должны быть связаны с собственной массой.

Итак, сужение содержания понятия массы до меры собственной энергии увеличивает объем этого понятия: запас собственной энергии определяет потенциальную возможность материального объекта к участию его в любом взаимодействии. Выделенные же из понятия массы свойства «инертность», «гравитационность», которые обнаруживаются в уже имеющем место конкретном взаимодействии объектов, количественно зависят от «энергосодержания» этих объектов и от характера, условий взаимодействия, что вполне закономерно.

В заключение подчеркнем необходимость представления в курсе физики еще не решенных проблем науки в целях развития у учащихся активного интереса к процессу познания.

Литература

1. Храпко Р.И. Что есть масса? УФН, т.170, №12,2000.
2. Окунь Л.Б. Понятие массы. УФН, т.158, вып.3, 1989.
3. Окунь Л.Б. О письме Р.И. Храпко «Что есть масса?». УФН, т.170, №12, 2000.
4. Логунов А.А., Мествиришвили М.А. Основы релятивистской теории гравитации. ЭЧАЯ, том17, вып.1. ОИЯИ, Дубна.
5. Иродов И.Е. Механика. Основные законы. – М.: «Наука». 2001.
6. Фейнман Р., Лейтон Р., Сэндс М. Фейнмановские лекции по физике. – М. «Мир».1965.
7. Ландау Л.Д., Лившиц Е.М. Теория поля. – М. «Наука». 1988.