ISSN 2309-7604

BAUMAN MOSCOW STATE TECHNICAL UNIVERSITY
Department of Physics
&
Education Committee, Russian Federation State Duma
Department of Physics, University of Liverpool, Great Britain
S.C.&T., University of Sunderland, Great Britain
Moscow Physical Society of Russian Federation
Russian Gravitational Society

Physical Interpretations of Relativity Theory

Proceedings of International Scientific Meeting PIRT-2013

Moscow: 1 – 4 July, 2013

Moscow, Liverpool, Sunderland

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Edited by M.C. Duffy, V.O. Gladyshev, A.N. Morozov, V. Pustovoit, P. Rowlands

Moscow, 2013

The conference is supported by the
Education Committee, Russian Federation State Duma
Department of Physics, University of Liverpool, Great Britain
University of Sunderland, Great Britain
School of Computing, and Technology
Russian Academy of Science
Russian Gravitational Society
Moscow Physical Society

Physical Interpretation of Relativity Theory : Proceedings of International Meeting. Moscow, 1 – 4 July 2013. / Ed. by M. C. Duffy, V. O. Gladyshev, A. N. Morozov, V. Pustovoit, P. Rowlands. – Moscow: BMSTU, 2013. – 470 p.

This volume contains papers which accepted for inclusion in the program of lectures of meeting “Physical Interpretation of Relativity Theory” which is organized by the Bauman Moscow State Technical University, School of Computing and Technology, University of Sunderland, Liverpool University and British Society for Philosophy of Science.

The conference is called to examine the various interpretations of the (mathematical) formal structure of Relativity Theory, and the several kinds of physical and mathematical models which accompany these interpretations.

The Proceedings of the PIRT-2013 includes papers dealing with the following major themes: Cosmology, Gravitation and Space-Time Structure; Time, Reference Frames and the Fundamentals of Relativity; Experimental Aspects of Relativity; Formal Structures and Physical Interpretations of Relativity Theory; Nature and Models of the Physical Vacuum; Information Theoretical Aspects of Space-Time Structure; The Poincare-Lorentz and the Einstein-Minkowski expositions of the Relativity Principle; Analogues of Relativity and Quantum Mechanics; Historical and Philosophical Aspects of Relativity;

The meeting is intended to be of interest to physicists, mathematicians, engineers, philosophers and historians of science, post-graduate students.

Introduction

The given collection of articles on the conference "Physical interpretations of relativity theory", held in Bauman University, is dedicated to many aspects of modern theory of gravity, general relativity and the problems that are still far from its final decision and which are of great interest to researchers.

Difficulties faced by the theory of gravity, general relativity, quantum gravity, stimulate the search for new approaches and new ideas. Some of these issues are reflected in the works included in this collection.

General theory of relativity is one of the most developed theories that claims to describe the Universe.However, due to the huge astronomical observations, the development of space research, the launch of space vehicles beyond the solar system, a lot of questions arise which do not yet have a clear and explicit answer. These observations raised many questions: is the gravitational constant in Einsten's equations of general relativity constant in time and whether it retains its value over long distan es to the "outskirts" of the Universe, what is the value of the cosmological constant, is modern quantum gravity capable to describe the physical processes inside the black holes, what is the speed of propagation of gravitational waves, etc.

In addition to these fundamental questions, challenging the foundations of physics, there are issues of an applied nature, relating to the methods and difficulties of creating unique installations for the direct detection of gravitational waves.

Modern physics stands at the threshold of major discoveries related to identification of how the Universe works. There are many questions which attract researchers and, therefore, it is clear that the reader will be Interested in this collection of works.

The collection is considered for theoretical physicists, experimentalists and radiophysicists who started gravitational research or are just going to conduct them.It will also be of interest to teachers and students.

Member of RAS, Prof. V.I. Pustovoit

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Time Entropy (A System Approach)

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1. Problems of kinematic geodesy

In most problems of physics space coordinates (for example X, Y, Z) and a time coordinate (external, astronomical tine are considered as arguments (initial data) and characteristics of physical fields (for example, temperature, potentials and others) at this appear as function of known coordinates.

Such approach is largely realized in local (laboratory) conditions when the task accuracy of coordinates allows no need to consider witch an indication kinematic system relative to which coordinates are defined. But even in such local task as water efflux forming the funnel under the action of the Coriolis force it is necessary to take into account earthly in common reference system.

Let's agree to understand the reference system with a given algorithm of coor**dinate production under coordinates system.**

Under the Inertial reference system in kinematic geodesy let's agree to understand a set of **points of geodetic network between which distance changes at repeated measurements on the bounded interval of standard** (astronomical) time on the haven't been detected (IRS).

Correspondingly, by non-inertial reference system let's call a set of points of geodetic network **with changing from epoch to epoch of repeated distance measurements (NRS).**

Historic predestination of development of the kinematic geodesy theory is due to by the fact that repeated geodetic measurements are done in the conditions of relative mutual mobilities of ear they surface elements (planned and high altitude) including non-tidal movements of water mass in the oceans, mass inside the Earth and in outer space.

Changes of planned and high altitude marks and gravity from one side rise a theoretical problem of time difference of geodetic measurements in the conditions of final velocity of repeated measurements (geodetic network renewval) in the changing reference system from the other side having assumed and built the frame of reference to the repeated epoch which is not connected witch the previous one we completely lose the possibility to define what has happened; whether the reference system has changed or the point displacement has taken place [1, 2, 3].

Practically earthly in common the reference system is materialized by the set of points of geodetic network on the terrestrial surface and when used as the coordinates carrier of the Earth artificial satellites it is materialized by points in the near Space (Glonas, Novstar GPS system).

The task of the body location determination relative to the reference system which presents a set of bodies was formulated by E. Mah: "Instead of referring, a moving body to the space (to some coordinates system) we'll directly consider its relation to world space bodies by which this coordinates system can be determined" [4, p. 198].

Real measured quantity in geodesy is distance between points. Angular measurements largely depend on the vertical line deviation which in turn depends on the Earth gravitational field therefore later on we'll use only measured distances as primary initial data as well as take into **account development velocity of geodetic network and distance measurement.**

O. Yu. Petrova paid attention to the specific of coordinate obtaining in the conditions of the measurer velocity. [7, p. 57]

Simultaneous geodetic event let's assume to consider the o**btaining of the same (IRS) bench mark.**

At the development of geodetic network the most important is propagation velocity of geodetic data in networks but not the rate of the signal transmission between two points. Under the geodetic data propagation we understand coordinates transmission to one or other points of earthly surface in earthly in common coordinates system.

2. Axiomatic elements of kinematic geodesy.

Let's define the space as the system in the following manner:

Distance is a set of bodies (geodetic network points) witch relations (communications, distances) between them. The reference system is a distance part destined for determination of objects coordinates.

Then time (internal, geodetic) let's define as, the **change of the distance state (reference system) detected by observers (meters) in the process of its notion (observation).**

Internal geodetic time can be: 1) discrete (disappearance of one and appearance of the other space bodies); 2) continuous (distance change between bodies). Continuous time in turn can be periodic and non-periodic.

Thus, the elements of kinematic geodesy are space points (bench mark), distances, the measurer with its properties, as well as the calculator witch its properties (science of finances of measurement treatment), clock (standard time). The measurer properties will be velocity of its travel V_M (propagation velocity of geodetic data in the network which is to be known a priori (before experiments) and considered as initial data. When needed it is necessary to take into account the final time of coordinates calculation t_c.

Let's formulate the first Euclids postulate as follows: "From each point to each point it is possible to measure or calculate the distance".

Let's complete the first postulate with one more: "it is possible to calculate relative travel **between two spaces points (velocity) based on** two distance reference system".

We'll analyze directly measured (or calculated) values without resorting to any coordinate **system.**

Standard (external in the respect of the given experiment) is period's count of the **periodic process (the Earth rotation, pulse, quartz generator, clock and so on).**

The space point (bench mark) is underdetermined concept understood from the experience either intuitively or somebody's experience (on examples by training).

Measurement is the measurer action done with the help of measuring apparatuses (for example, clock) by certain rules (algorithms, laws).

Calculation is the calculator action performed with the help of measuring instruments (computers) by certain rules (formulas) which must be also worked out.

Measurement algorithms (measurement ways) or calculation are given intuitively either they are found or obtained by training.

The algorithm should allow obtaining the value with required accuracy. The required accuracy is defined by society possibilities and requests.

Distance, travel is the measurement or calculation result.

3. Time entropy.

There exist two different parameters in a physical sense, having the same dimension - a second: 1) the t_c time moment (periodic fluctuation period number of a reference (external) time); 2) the time duration (time interval) $\Delta t_c = t_{2c} - t_{1c}$ as a difference of final t_{2c} and initial t_{1c} of event time moments (condition). In non-inertial reference frame (NRF) [2,3] the moments of inner (geodesic) time represent a T_{ij} matrix of mutual distances between the benchmarks of the reference system – the matrix of ties (relations).

The quantity

$$g_i = \frac{N_i}{N_0} \quad \textbf{(1)}$$

Where N_i - is a number of fixed NRF condition changes and N_0 - is a general number of changes is defined as the NRF change underdetermination (as opposed to the event probability).

The quantity $\sum_{ij} T_{ij}$ can be determined as dlenie (in difference from duration) characterizing itself degree of variability of NSO for an interval Δt_K to (measured by reference time). Then, following Planck-Boltsmanu and Shannon, it is possible to accept entropy of dlenie of the equal

$$S_T = -\sum_{k=1}^{n} g_i \ln \frac{\Delta t_c}{\|T_{ij}\|} \tag{2}$$

Where $\|T_{ij}\|$ – norm of a matrix.

The quantity 2characterizes by itself (NRS) variability extent and can be used for procedure analysis of togo-plans comparison (including the NRS too), relating to different measurement epochs. Such investigation is possible on the basis of the principle of the least entropy performance. [15]

$$dS_T \Rightarrow min \tag{3}$$

4. Kinematic metrics for the NRS.

Now let's turn to kinematic metric obtaining which aspect naturally depends on measurement **way of the distance in kinematics at the finite velocity of measurement realizing (and calculation). There are two such ways:**

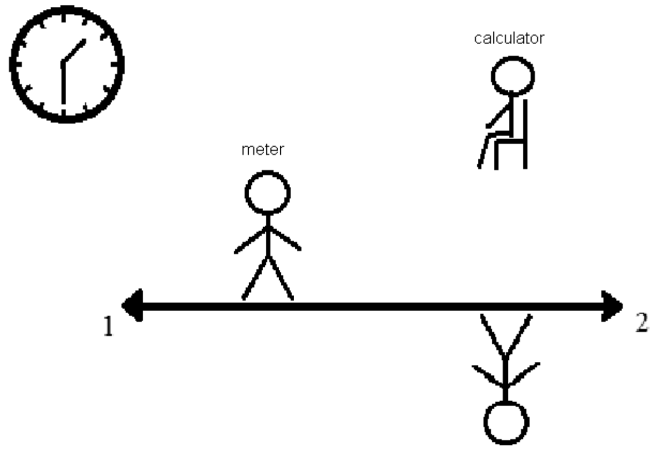

Fig. 1. The Wording of the Minkovsk'y metric.

1. **The measurer with the clock moves from the point 1 to the point 2 with the velocity V_M (for example, a car in a traffic flow);**
2. **The measurer (sound, light wave) moves from the point 1to the point 2, where it is reflected and comes back.**

At this the clock are either in the point 1 or the point 2. Depending on the considered case: 1) the measurer starts from the immobile space point 1; 2) the measurer starts from the mobile point 2. For simplicity let's call the **first way as the measurer there and the second one as the measurer there and back.**

Is the measurer starts from the point 1 at the moment of time $\tau_1^{(0}$ and velocity V_2 is more or equal to the measurer velocity there will be no any distance value at the experiment result (no geometry). Besides it is important to underline that the value V_2 must be given as initial data (kinematic hypothesis). Otherwise the only value that can be obtained as the measurement result is V_M $(\tau_1^{(1)} - \tau_1^{(0)})$. Given the velocity V_2 it is possible to write:

$$x(\tau_1^{(0)}) = V_M\ (\tau_1^{(1)} - \tau_1^{(0)}) - V_2\ (\tau_1^{(1)} - \tau_1^{(0)}) \tag{4}$$

where $x(\tau_1^{(0)})$ is instantaneous (euclid) distance between space points 1 and 2 at the time $\tau_1^{(0)}$ (coordinate underdo-finiteness). Let us consider a reverse case when the measurer starts from the moving point 2 at the time $\tau_2^{(0)}$ and reaches the point 1 at the time $\tau_2^{(1)}$

$$x(\tau_2^{(1)}) = V_{M}\ (\tau_2^{(1)} - \tau_2^{(0)}) + V_2\ (\tau_2^{(1)} - \tau_2^{(0)}) \qquad (5)$$

Metrics (4) and (5) aren't forminvariant though is we discuss task symmetry it would be **natural to require performance of the metric form invariance condition.**

Let us consider the metric as geometric average distance between two measurements cases

$$x(\tau_1^{(0)})\ x(\tau_2^{(1)}) = X^2(\tau) = V_{M}^2(\tau_1^{(1)} - \tau_1^{(0)})\ (\tau_2^{(1)} - \tau_2^{(0)}) - V_2(\tau_1^{(1)} - \tau_1^{(0)})\ (\tau_2^{(1)} - \tau_2^{(0)}) =$$
$$= V_{M}^2(\tau_1^{(1)} - \tau_1^{(0)})^2 - V_2^2(\tau_1^{(1)} - \tau_1^{(0)})^2 \qquad (6)$$

Thus we have obtained the metric coinciding in form to Minkovsky's interval and meeting **the requirement of forminvariance simultaneously for both measurement cases.**

Is mutual 1 and 2 space points displacement occurs for the measurement time $\tau_1^{(1)}$. Then in respect of metrics (1), (2), (3) there can be a number of kinematic hypotheses, for example: uniformly accelerated motion of the point 2; oscillatory motion of the point 2.

To define the travel let's make use of the metric (4). The second metric (4) reference system is **made in time span to the epoch**

$$\Delta\tau_э = \tau_M + \tau_c + \tau_n, \qquad (7)$$

Where τ_M is measurement time, τ_c is the time of formula and calculations derivation, τ_n is the time of underdetermined spacing interval between frames of reference (the account of finite development velocity of the geodetic network).

Then the travel is

$$\Delta x_э = x_2(\tau_1^{(0)}) - x_2(\tau_1^{(0)}) = (V_{изм} - V_2)[\Delta(\tau_2^{(1)} - \tau_2^{(0)})] \qquad (8)$$

And velocity is $V = \frac{\Delta x_э}{\Delta\tau_э}$

$$V = \Delta X_э / \Delta\tau_Э \qquad (9)$$

Setting the first frame of reference at zero then $\Delta\tau_э = \tau_э$. The time of the spacing interval $\tau_П$ really takes place in cartography at the map renewal to the epoch including in connection with impossibility of simultaneous coordinates transfer to all points of geodetic network.

Before definition let's set a question how many measurers with clock there **must be to measure** the whole distance more or less simultaneously within one epoch. Let's assume that the **measurers number is equal to the communication number in the system 1 as the task of coordinate definition (for example, Cartesia coordinates) requires as a minimum 4 bodies to** enter the system. Let's call the matric of mutual distances by the moment of geodetic time **(statistic time, time potentials).**

By using a recital model of geodetic space-time [6] let's introduce the concert of Mah's **kinematic metric.**

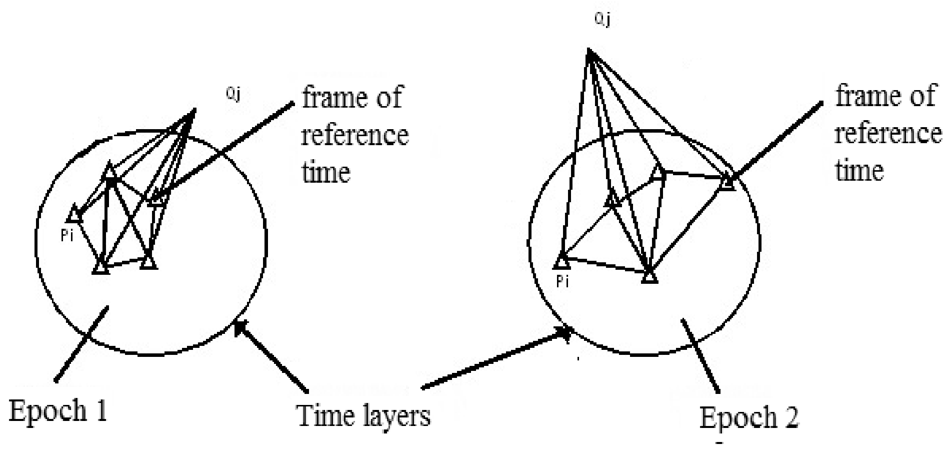

Fig. 2. To the Mah's metric definition.

$$\mathbf{M}= |\bar{R}| + iS_{\text{T}} \qquad \mathbf{(10)}$$

Where: $|\bar{R}|$-is the modulus of a traditional vector radius, S_{T}-is time entropy.
It is possible to define velocity on the basis of Mah's metric both in relation to the standard time **and internal (geodetic) time S_{T}:**

$$V_0 = \frac{dM}{dS_T} \qquad \mathbf{(11)}$$

Thus the following results have bun obtained during the work:

1. **The problems of kinematic geodesy have bun formulated;**
2. **System space and time definitions have been given;**
3. **Time entropy for discrete internal time has been introduced;**
4. **Axiomatic elements of kinematic geodesy have been formulated;**
5. Minkovsky's and E. Mah's metrics have been obtained for different ways of space **measurement.;**
6. **Suggested metrics have been formulated in the absence of the coordinate system;**
7. **Formulas of travel and velocity for corresponding metrics have been found;**
8. The formula of Mah's metric for object motion on the background of NRS has been **suggested;**

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Consequences of the existence of the Deser–Dirac scalar field in Nature: dark energy and the black holes problems

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The solutions of the field equations of the conformal theory of gravitation with Deser–Dirac scalar field in Cartan–Weyl spacetime at the very early Universe and in the static spherically symmetric case for a central compact mass are obtained. In this theory dark energy (describing by an effective cosmological constant) is a function of the Deser–Dirac scalar field β. The first solution describes the exponential decreasing of β at the inflation stage and has a limit to a constant value of the dark energy at large time. This can give a way to solving the fundamental cosmological constant problem as a consequence of the fields dynamics in the early Universe. The second solution coincides with the Papapetrou–Yilmaz–Rosen metric, which gives the same results as the Schwarzschild solution at large r, but has only one singularity at $r = 0$. This solution eliminates the existence of the black hole solution in gravitational physics.

Introduction

In [1, 2, 3] the Poincaré–Weyl gauge theory of gravitation (PWTG) has been developed. The gauge field introduced by the subgroup of dilatations is named as dilatation field, its vector-potential is the Weyl 1-form, and quanta of this field can have nonzero rest masses. As it has been shown in [1, 2, 3], an additional scalar field $\beta(x)$ is introduced in PWTG as an essential geometrical addendum to the metric tensor:

$$g^{CW}_{\mu\nu} = \beta^2(x) g_{\mu\nu}\,, \qquad g_{\mu\nu} = g_{ab} h^a{}_\mu h^b{}_\nu\,, \tag{1}$$

where $g^{CW}_{\mu\nu}$ is a metric tensor of a Cartan–Weyl space, and $g_{ab} = g^M_{ab}$ is the Minkowski metric tensor. In what follows, we shall use the Minkowski metric tensor for manipulation with indices of a tangent space, as it commonly accepted, and therefore we shall use in (1) the scalar field β in the explicit form.

The properties of the field β coincide with those of the scalar field introduced by Dirac [4] and earlier by S. Deser [5]. Some members of the Deser–Dirac scalar field Lagrangian have structure of the Higgs Lagrangian and can cause an appearance of nonzero rest masses of particles.

We shall use the exterior form machinary that is the modern generalization of the well-known tetrad machinary [6]. Let us consider a connected 4D oriented differentiable manifold $\mathcal{M}$ equipped with a metric $\hat{g}$ of the index 3, a linear connection $\Gamma^a{}_b$ and a volume 4-form η. Then a Cartan–Weyl space $\mathcal{C}W_4$ is defined as the such manifold equipped with a curvature 2-form $\mathcal{R}^a{}_b$, a torsion 2-form $\mathcal{T}^a$ and a nonmetricity 1-form $\mathcal{Q}_{ab}$ obeing the Weyl condition

$$\mathcal{Q}_{ab} = \frac{1}{4} g_{ab} \mathcal{Q} \,. \tag{2}$$

Here $\mathcal{Q}_{ab} = -\mathcal{D} g_{ab}$, and $\mathcal{D} = \mathrm{d} + \Gamma \wedge \ldots$ is the exterior covariant differential.

Lagrangian density and variational equations

As a consequence of PWTG, the theory should be invariant under the following conformal transformations with the parameter $\varepsilon(x)$ (d is the exterior differential),

$$\begin{aligned}
&\delta\beta = \varepsilon(x)\beta \,, \quad \delta h^a{}_\mu = -\varepsilon h^a{}_\mu \,, \quad \delta g_{ab} = 0 \,, \quad \delta g_{\mu\nu} = -2\varepsilon g_{\mu\nu} \,, \quad \delta\Gamma^a{}_b = \delta^a_b \mathrm{d}\varepsilon \,, \\
&\delta\mathcal{Q}^{ab} = 2g^{ab}\mathrm{d}\varepsilon \,, \quad \delta\mathcal{Q} = 8\mathrm{d}\varepsilon \,, \quad \delta\mathcal{R}^a{}_\mathrm{b} = 0 \,, \quad \delta\mathcal{T}^a = -\varepsilon\mathcal{T}^a \,.
\end{aligned} \tag{3}$$

On the basis of PWTG, a conformal theory of gravitation in Cartan–Weyl spacetime with the Deser–Dirac scalar field has been developed [6, 7, 8, 9, 10]. In exterior form formalism 4-form Lagrangian density (invariant under (3)) is the following [9, 10],

$$\mathcal{L} = \mathcal{L}_G + \mathcal{L}_M + \beta^4 \Lambda^{ab} \wedge (\mathcal{Q}_{ab} - \frac{1}{4} g_{ab} \mathcal{Q}) \,, \tag{4}$$

$$\begin{aligned}
\mathcal{L}_G = 2f_0 \Bigg[&\frac{1}{2}\beta^2 \mathcal{R}^a{}_b \wedge \eta_a{}^b - \beta^4 \Lambda \eta + \frac{1}{4}\lambda \mathcal{R}^a{}_a \wedge *\mathcal{R}^b{}_b + \\
&+ \tau_1 \mathcal{R}^{[a}{}_{b]} \wedge *\mathcal{R}^b{}_a + \tau_2 (\mathcal{R}^{[ab]} \wedge \theta_a) \wedge *(\mathcal{R}^{[c}{}_{b]} \wedge \theta_c) + \\
&+ \tau_3 (\mathcal{R}^{[ab]} \wedge \theta_c) \wedge *(\mathcal{R}^{[c}{}_{b]} \wedge \theta_a) + \tau_4 (\mathcal{R}^{[a}{}_{b]} \wedge \theta_a \wedge \theta^b) \wedge *(\mathcal{R}^{[c}{}_{d]} \wedge \theta_c \wedge \theta^d) + \\
&+ \tau_5 (\mathcal{R}^{[a}{}_{b]} \wedge \theta_a \wedge \theta^d) \wedge *(\mathcal{R}^{[c}{}_{d]} \wedge \theta_c \wedge \theta^b) + \\
&+ \tau_6 (\mathcal{R}^a{}_b \wedge \theta_c \wedge \theta^d) \wedge *(\mathcal{R}^c{}_d \wedge \theta_a \wedge \theta^b) + \\
&+ \rho_1 \beta^2 \mathcal{T}^a \wedge *\mathcal{T}_a + \rho_2 \beta^2 (\mathcal{T}^a \wedge \theta_b) \wedge *(\mathcal{T}^b \wedge \theta_a) + \\
&+ \rho_3 \beta^2 (\mathcal{T}^a \wedge \theta_a) \wedge *(\mathcal{T}^b \wedge \theta_b) + \xi \beta^2 \mathcal{Q} \wedge *\mathcal{Q} + \zeta \beta^2 \mathcal{Q} \wedge \theta^a \wedge *\mathcal{T}_a + \\
&+ l_1 \mathrm{d}\beta \wedge *\mathrm{d}\beta + l_2 \beta \mathrm{d}\beta \wedge \theta^a \wedge *\mathcal{T}_a + l_3 \beta \mathrm{d}\beta \wedge *\mathcal{Q} \Bigg] \,.
\end{aligned} \tag{5}$$

Here $\mathcal{L}_G$ is the gravitational field Lagrange density, $\mathcal{L}_M$ is the matter Lagrange density. The first term in $\mathcal{L}_G$ is the Gilbert–Einstein Lagrangian density generalized to the Cartan–Weyl space, the second term is a generalized cosmological term describing a vacuum energy (Λ is the Einstein cosmological constant). The last term in (4) contains the Weyl's condition (2) with the Lagrange multiplier Λ^{ab}.

We use the exterior form variational formalism on the base of the Lemma on the commutation rule between variation and Hodge star dualization [11]. The independent variables are the basis 1-form θ^a, the nonholonomic connection 1-form $\Gamma^a{}_b$ and the scalar field $\beta(x)$.

The variational field equations are the following [6, 9, 10].

Γ-equation:

$$
\begin{aligned}
&2f_0\Bigg[\beta^2\left(-\frac{1}{4}\mathcal{Q}\wedge\eta_a{}^b+\frac{1}{2}\mathcal{T}^c\wedge\eta_a{}^b{}_c+\frac{1}{2}\eta_{ac}\wedge\mathcal{Q}^{bc}+\mathrm{d}\ln\beta\wedge\eta_a{}^{\mathrm{b}}\right)+\\
&+\lambda\frac{1}{2}\mathcal{D}(\delta^b_a*\mathcal{R}^c{}_c)+\tau_1 2\mathcal{D}\left(*\mathcal{R}^{[b}{}_{a]}\right)+\tau_2\mathcal{D}\left(2\delta^{[d}_a\delta^{b}_{c]}\theta_d\wedge*(\mathcal{R}^{[fc]}\wedge\theta_f)\right)+\\
&+\tau_3\mathcal{D}\left(2\delta^{[d}_a\delta^{b}_{c]}\theta_f\wedge*(\mathcal{R}^{[fc]}\wedge\theta_d)\right)+\tau_4 2\mathcal{D}\left(*(\mathcal{R}^c{}_d\wedge\theta_c\wedge\theta^d)\theta_a\wedge\theta^b\right)+\\
&+\tau_5 2\mathcal{D}\left(*(\mathcal{R}^{[c}{}_{d]}\wedge\theta_c\wedge\theta^{[b})\theta_{a]}\wedge\theta^d\right)+\tau_6 2\mathcal{D}\left(*(\mathcal{R}^c{}_d\wedge\theta_a\wedge\theta^b)\theta_c\wedge\theta^d\right)+\\
&+\rho_1 2\beta^2\theta^b\wedge*\mathcal{T}_a+\rho_2 2\beta^2\theta^b\wedge\theta_c\wedge*(\mathcal{T}^c\wedge\theta_a)+\rho_3 2\beta^2\theta^b\wedge\theta_a\wedge*(\mathcal{T}^c\wedge\theta_c)+\\
&+\xi 4\beta^2\delta^b_a*\mathcal{Q}+\zeta\beta^2\left(2\delta^b_a\theta^c\wedge*\mathcal{T}_c+\theta^b\wedge*(\mathcal{Q}\wedge\theta_a)\right)+l_2\beta\theta^b\wedge*(\mathrm{d}\beta\wedge\theta_{\mathrm{a}})+\\
&+l_3 2\beta\delta^b_a*\mathrm{d}\beta\Bigg]-2\beta^4\Lambda_a{}^b=\beta^4\mathcal{J}^b{}_a\,.
\end{aligned}
\tag{6}
$$

Here $\beta^4\mathcal{J}^b{}_a=-\frac{\delta\mathcal{L}_M}{\delta\Gamma^a{}_b}$ is a hypercharge current,

$$
\mathcal{J}^b{}_a=\frac{1}{2}n\left(S^b{}_a+\frac{1}{4}\delta^b_a J\right)u\,,
\tag{7}
$$

where $S^b{}_a$ is a spin tensor, J is a dilatonic charge, n is a concentration, 3-form u is a flow, $u=\vec{u}\rfloor\eta$, of matter particles.

θ–equation:

$$
\begin{aligned}
&2f_0\Bigg[\beta^2\left(\frac{1}{2}\mathcal{R}^b{}_c\wedge\eta_b{}^c{}_a\right)+\beta^4\Lambda\eta_a+\\
&+\lambda\left(\frac{1}{4}\mathcal{R}^c{}_c\wedge*(\mathcal{R}^b{}_b\wedge\theta_a)+\frac{1}{4}*(*\mathcal{R}^b{}_b\wedge\theta_a)\wedge*\mathcal{R}^c{}_c\right)+\\
&+\tau_1\left(\mathcal{R}^{[c}{}_{b]}\wedge*\left(\mathcal{R}^b{}_c\wedge\theta_a\right)+*\left(*\mathcal{R}^b{}_c\wedge\theta_a\right)\wedge*\mathcal{R}^{[c}{}_{b]}\right)+\\
&+\tau_2\left(2\mathcal{R}_{[ab]}\wedge*(\mathcal{R}^{[cb]}\wedge\theta_c)-*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_b\wedge\theta_a)\mathcal{R}^{[dc]}\wedge\theta_d-\right.\\
&\left.-*(*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_b)\wedge\theta_a)\wedge*(\mathcal{R}^{[dc]}\wedge\theta_d)\right)+\\
&+\tau_3\left(2\mathcal{R}^{[b}{}_{c]}\wedge*(\mathcal{R}_{[a}{}^{c]}\wedge\theta_b)-*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_d\wedge\theta_a)\wedge\mathcal{R}^{[dc]}\wedge\theta_b-\right.\\
&\left.-*(*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_d)\wedge\theta_a)\wedge*(\mathcal{R}^{[dc]}\wedge\theta_b)\right)+\\
&+\tau_4\left(*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_b\wedge\theta^c)(4\mathcal{R}_{[af]}\wedge\theta^f+*(\mathcal{R}^{[e}{}_{f]}\wedge\theta_e\wedge\theta^f)\eta_a)\right)+\\
&+\tau_5\left(*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_b\wedge\theta^d)(2\mathcal{R}_{[ad]}\wedge\theta^c-2\delta^c_a\mathcal{R}_{[fd]}\wedge\theta^f+*(\mathcal{R}^{[f}{}_{d]}\wedge\theta_f\wedge\theta^c)\eta_a)\right)+\\
&+\tau_6\left(*(\mathcal{R}^{[b}{}_{c]}\wedge\theta_e\wedge\theta^f)(*(\mathcal{R}^e{}_f\wedge\theta_b\wedge\theta^c)\eta_a+2g_{ab}\mathcal{R}^e{}_f\wedge\theta^c-2\delta^c_a\mathcal{R}^e{}_f\wedge\theta_b\,)\right)+
\end{aligned}
$$

$$
\begin{aligned}
&+\rho_1\beta^2\Big(2\mathcal{D}(*\mathcal{T}_a)+\mathcal{T}^c\wedge*(\mathcal{T}_c\wedge\theta_a)+*(*\mathcal{T}_c\wedge\theta_a)\wedge*\mathcal{T}^c+4\mathrm{d}\ln\beta\wedge*\mathcal{T}_{\mathrm{a}}\Big)+\\
&+\rho_2\beta^2\Big(2\mathcal{T}^d\wedge*(\mathcal{T}_a\wedge\theta_d)+2\mathcal{D}(\theta_b\wedge*(\mathcal{T}^b\wedge\theta_a))+4\mathrm{d}\ln\beta\wedge\theta_{\mathrm{b}}\wedge*(\mathcal{T}^{\mathrm{b}}\wedge\theta_{\mathrm{a}})-\\
&-*(*(\mathcal{T}^c\wedge\theta_d)\wedge\theta_a)\wedge*(\mathcal{T}^d\wedge\theta_c)-*(\mathcal{T}^b\wedge\theta_c\wedge\theta_a)(\mathcal{T}^c\wedge\theta_b)\Big)+\\
&+\rho_3\beta^2\Big(2\mathcal{D}(\theta_a\wedge*(\mathcal{T}^b\wedge\theta_b))+2\mathcal{T}_a\wedge*(\mathcal{T}^b\wedge\theta_b)-*(\mathcal{T}^b\wedge\theta_b\wedge\theta_a)(\mathcal{T}^c\wedge\theta_c)-\\
&-*(*(\mathcal{T}^b\wedge\theta_b)\wedge\theta_a)\wedge*(\mathcal{T}^c\wedge\theta_c)+4\mathrm{d}\ln\beta\wedge\theta_{\mathrm{a}}\wedge*(\mathcal{T}^{\mathrm{b}}\wedge\theta_{\mathrm{b}})\Big)+\\
&+\xi\beta^2\Big(-\mathcal{Q}\wedge*(\mathcal{Q}\wedge\theta_a)-*(*\mathcal{Q}\wedge\theta_a)*\mathcal{Q}\Big)+\zeta\beta^2\Big(\mathcal{D}*(\mathcal{Q}\wedge\theta_a)-\mathcal{Q}\wedge*\mathcal{T}_a+\\
&+\mathcal{Q}\wedge\theta^b\wedge*(\mathcal{T}_b\wedge\theta_a)+2\mathrm{d}\ln\beta\wedge*(\mathcal{Q}\wedge\theta_{\mathrm{a}})+*(*\mathcal{T}_{\mathrm{b}}\wedge\theta_{\mathrm{a}})\wedge*(\mathcal{Q}\wedge\theta^{\mathrm{b}})\Big)+\\
&+l_1\Big(-\mathrm{d}\beta\wedge*(\mathrm{d}\beta\wedge\theta_{\mathrm{a}})-*(*\mathrm{d}\beta\wedge\theta_{\mathrm{a}})\wedge*\mathrm{d}\beta\Big)+l_2\Big(\beta(\mathcal{D}*(\mathrm{d}\beta\wedge\theta_{\mathrm{a}})+\\
&+\mathrm{d}\beta\wedge\theta^{\mathrm{b}}\wedge*(\mathcal{T}_{\mathrm{b}}\wedge\theta_{\mathrm{a}})-\mathrm{d}\beta\wedge*\mathcal{T}_{\mathrm{a}}+*(*\mathcal{T}_{\mathrm{b}}\wedge\theta_{\mathrm{a}})\wedge*(\mathrm{d}\beta\wedge\theta^{\mathrm{b}}))+\\
&+\mathrm{d}\beta\wedge*(\mathrm{d}\beta\wedge\theta_{\mathrm{a}})\Big)+l_3\beta\Big(-\mathrm{d}\beta\wedge*(\mathcal{Q}\wedge\theta_{\mathrm{a}})-*(*\mathcal{Q}\wedge\theta_{\mathrm{a}})*\mathrm{d}\beta\Big)\Big]=-\beta^4t_a\,,
\end{aligned}
\tag{8}
$$

where $\beta^4 t_a = \frac{\delta\mathcal{L}_M}{\delta\theta^a}$ is a matter canonical energy-momentum 3-form.

β–equation:

$$
\begin{aligned}
&2f_0\Big[\beta\mathcal{R}^a{}_b\wedge\eta_a{}^b-4\beta^3\Lambda\eta+2\rho_1\beta\mathcal{T}^a\wedge*\mathcal{T}_a+\\
&+2\rho_2\beta(\mathcal{T}^a\wedge\theta_b)\wedge*(\mathcal{T}^b\wedge\theta_a)+2\rho_3\beta(\mathcal{T}^a\wedge\theta_a)\wedge*(\mathcal{T}^b\wedge\theta_b)+\\
&+2\xi\beta\mathcal{Q}\wedge*\mathcal{Q}+2\zeta\beta\mathcal{Q}\wedge\theta^a\wedge*\mathcal{T}_a+l_1(-2\mathrm{d}*\mathrm{d}\beta)+\mathrm{l}_2(-\beta\mathrm{d}(\theta^{\mathrm{a}}\wedge*\mathcal{T}_{\mathrm{a}}))+\\
&+l_3(-\beta\mathrm{d}*\mathcal{Q})\Big]+4\beta^3\Lambda^{ab}\wedge(\mathcal{Q}_{ab}-\frac{1}{4}g_{ab}\mathcal{Q})=-\beta^4t^{(\beta)}\,,
\end{aligned}
\tag{9}
$$

where $\beta^4 t^{(\beta)} = \frac{\delta\mathcal{L}_M}{\delta\beta}$ is a dilatonic current. Q–equation:

$$\mathcal{Q}_{ab}-\frac{1}{4}g_{ab}\mathcal{Q}=0. \tag{10}$$

We need to obtain a consequence of the Γ-equation (6). After anti-symmetrization on indices a, b and exterior multiplication by θ_b, the condition results [10],

$$2(1-\rho_1+2\rho_2)\mathcal{T}-3(\frac{1}{4}+\zeta)\mathcal{Q}+3(2-l_2)\mathrm{d}\ln\beta=0. \tag{11}$$

Here $\mathcal{T}$ is a torsion trace 1-form, $\mathcal{T}=*(\theta_a\wedge*\mathcal{T}^a)$, and we have used the Frenkel condition,

$$S_{ab}u\wedge\theta^b=-S_{ab}u^b\eta=0\,, \tag{12}$$

which means a space-like spin nature.

Cosmological solutions for early universe

We solve the field equations at the very early universe for the scale factor $a(t)$ and the field $\beta(t)$, when a matter density has been very small. In this case for the Friedman–Robertson–Waker (FRW) metric one can obtain the second consequence of the equation (6) by means of contraction this equation on indices a, b:

$$(2\rho_1 - 4\rho_2 + 8\zeta)\mathcal{T} + (16\xi + 3\zeta)\mathcal{Q} + (3l_2 + 8l_3)\mathrm{d}\ln\beta = 0\,. \tag{13}$$

In the case of the FRW metric the torsion satisfies to the condition, $\mathcal{T}^a = (1/3)\mathcal{T}\wedge\theta^a$. With the help of the equations (11), (13) one can find the Weyl 1-form $\mathcal{Q}$, the torsion trace 1-form $\mathcal{T}$, and therefore the torsion 2-form $\mathcal{T}^a$ via the derivative of the scalar field β.

Then the field equations (6)–(10) for the FRW metric give only three equations [6, 10],

$$(0,0):\qquad 3\left(\frac{\dot a}{a}\right)^2 + 6\frac{\dot a}{a}\frac{\dot\beta}{\beta} + 3B_3\left(\frac{\dot\beta}{\beta}\right)^2 = \Lambda\beta^2\,, \tag{14}$$

$$(1,1):\qquad 2\frac{\ddot a}{a} + 2\frac{\ddot\beta}{\beta} + 4\frac{\dot a}{a}\frac{\dot\beta}{\beta} + \left(\frac{\dot a}{a}\right)^2 + (B_2-2)\left(\frac{\dot\beta}{\beta}\right)^2 = \Lambda\beta^2\,, \tag{15}$$

$$\beta\quad:\qquad A\left(\frac{\ddot\beta}{\beta} + 3\frac{\dot a}{a}\frac{\dot\beta}{\beta}\right) + (B-A)\left(\frac{\dot\beta}{\beta}\right)^2 = 0\,. \tag{16}$$

Here the constants B_1, B_2 and $B_3 = (1/3)(2B_1 + B_2)$ are determined by the coupling constants of the Lagrangian density (5).

The system of equations (14)–(16) is not consistent because we have three equations and only two unknown functions $a(t)$, $\beta(t)$. Let us put

$$B_1 = B_2 = B_3 = 1\,, \tag{17}$$

and introduce new variables,

$$u = \ln a(t)\,,\quad v = \ln\beta(t)\,. \tag{18}$$

Then the system of equations (14)–(16) with (17), (18) is equivalent to the system of equation,

$$(\dot u)^2 + 2\dot u\dot v + (\dot v)^2 = (\Lambda/3)\exp(2v)\,, \tag{19}$$

$$\ddot u + \ddot v - \dot u\dot v - (\dot v)^2 = 0\,, \tag{20}$$

$$\ddot v + 3\dot u\dot v + (B/A)\,(\dot v)^2 = 0\,. \tag{21}$$

which is consistent, because the equation (19) is equivalent to the equation,

$$\dot{u} + \dot{v} = \pm \left(\frac{\lambda}{3}\right) \exp(v)\,, \qquad \lambda = \sqrt{3\Lambda}\,, \tag{22}$$

and the equation (20) is the consequence of the equation (22) (in the following we shall choose in the equation (22) the sign "+").

Then the equation (21) with the equation (22) is equivalent to the equation,

$$\ddot{v} + \lambda \dot{v} \exp(v) + \omega (\dot{v})^2 = 0\,, \qquad \omega = (B/A) - 3\,, \tag{23}$$

which has the first integral (λ_0 is the constant of integration),

$$\dot{v} = \lambda_0 e^{(-\omega v)} - \frac{\lambda}{\omega + 1} e^{v}\,. \tag{24}$$

Therefore for two unknown functions, $a(t)$ and $\beta(t)$, we have two equations, (22) and (24), with the parameter ω, determinate by the coupling constants of the Lagrangian density (5), and the arbitrary parameter λ_0. This system of equation generalizes the well-known Friedman–Lemaitre equaton and has a wide set of solutions for the ultra-early universe, when the contribution of the field β much more exceeds the contribution of other matter.

One of the solutions has been obtained for the case, when $\omega = 0$, $\lambda_0 = \lambda$ [6] :

$$\beta(t) = \frac{1}{1 - \exp(-\lambda(t + t_0))}\,, \quad a(t) = a_1 \exp((\lambda/3)(t + t_0))(1 - \exp(-\lambda(t + t_0)))^{4/3}\,, \tag{25}$$

where t_0 is a constant of integration.

The second solution (for the case $\omega = 1$, $\lambda_0 = \lambda$) is the following [12], [13],

$$\beta(t) = \frac{1 + \exp(-\lambda(t + t_0))}{1 - \exp(-\lambda(t + t_0))}\,, \quad a(t) = a_1 \exp((\lambda/3)(t + t_0)) \frac{(1 - \exp(-\lambda(t + t_0)))^{5/3}}{1 + \exp(-\lambda(t + t_0))}\,. \tag{26}$$

These solutions realizes exponential diminution of a field β (see Fig. 1) for the function (25)), and thus sharp exponential decrease of physical vacuum energy (dark energy) by many orders. We have $\Lambda_{eff} = \beta^2 \Lambda \to \Lambda$ in a limit at $t \to \infty$. Thus, the effective cosmological constant can slightly differ already by the end of inflation from the limiting value equal to its modern size Λ that provides the subsequent transition from the Friedman epoch to the epoch of the accelerated expansion in accordance with the modern observant cosmological data.

Spherically symmetric solution

Then we find a static spherically symmetric solution for a central compact mass m in the case $\Lambda = 0$ [6, 14, 15, 16, 17]. In this case (as in the previous case) the torsion also satisfies to the condition, $\mathcal{T}^a = (1/3)\mathcal{T} \wedge \theta^a$. As a consequence of (11)) we can put $\mathcal{T} = s\mathrm{d}\ln\beta$, $\mathcal{Q} = q\mathrm{d}\ln\beta$, where s and q are constants.

Then we obtain the solution of the field equations (6)–(10) for the metric ($s = 0, \quad q = 4$),

$$ds^2 = e^{-\frac{r_0}{r}} dt^2 - e^{\frac{r_0}{r}} (dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\phi^2))\,, \tag{27}$$

and the solution for the scalar field,

$$u = \ln\beta = \frac{kr_0}{r} + \ln\beta_\infty\,, \qquad \beta(r) = \beta_\infty\, e^{\frac{kr_0}{r}}\,, \qquad k = \sqrt{\frac{-3}{64\xi}}\,, \tag{28}$$

where β_∞ is the value of the Deser–Dirac scalar field at infinity.

These solutions are realized under the following conditions on the coupling constants of the Lagrangian density (5):

$$3l_1 + 64\xi + 16\zeta = 0\,, \qquad l_2 + 4\zeta = 0\,, \qquad 3l_3 + 4\zeta = 0\,. \tag{29}$$

In the simplest case, if one put

$$l_2 = 0\,, \qquad l_3 = 0\,, \qquad \zeta = 0\,, \qquad 3l_1 + 64\xi = 0\,, \tag{30}$$

then one finds,

$$k = \frac{1}{\sqrt{l_1}}\,, \qquad \beta(r) = \beta_\infty\, e^{\frac{r_0}{\sqrt{l_1} r}}\,, \tag{31}$$

The metric (27) at large values of r will give the same results as the Schwarzschild metric, if the quantity r_0 coincides with the gravitational radius, $r_0 = r_g = 2Gm/c^2$.

The metric (27) has been first obtained by A. Papapetrou [18], and next by H. Yilmaz [19] and N. Rosen [20] (see also [21, 22, 23, 24]). The interest to this metric has emerged from the fact that this metric has no singularity, characteristic for the Schwarzschild metric, and therefore does not describe a black hole type solution. We can conclude that the presence of the Deser–Dirac scalar field eliminates the existence of the black hole solution. The solution (27), (28) can become apparent, for instance, on the final stage of the collapse of massive stars.

Conclusion

In conclusion we give some final remarks. We have proposed a new approach to modern theory of gravity based on the hypothesis of the necessary existence of the Deser–Dirac scalar field in Nature, which realizes itself both in cosmological and local new phenomenons.

On the basis of the observational data, it is accepted in modern cosmology that the dark energy (described by the cosmological constant) is of dominant importance in dynamics of the universe. In this connection the major unsolved problem of modern fundamental physics is very large difference of around 120 orders of magnitude between a very small value of Einstein cosmological constant Λ, which can be estimated on the basis of modern observations in cosmology, and the value of the cosmological constant in the early Universe, which has been estimated by theoretical calculations in quantum field theory of quantum fluctuation contributions to the vacuum energy [25, 26, 27, 28, 29]. In the present work we try to understand the cosmological constant problem as the effect of the Deser–Dirac scalar field dynamics in the early Universe.

The new cosmological solutions (25), (26) realize exponential diminution of the Deser–Dirac scalar field, and thus sharp exponential decrease of physical vacuum energy (dark energy) by many orders. Thus this result can explain the exponential decrease in time at very early Universe of the dark energy describing by the effective cosmological constant.

These solutions could be realized at the very beginning of the Universe evolution, when the cosmological constant Λ_0 estimated by quantum field theory was equal $\Lambda_0/\Lambda = \beta_0^2 \sim 10^{120}$, and the number $\beta_0 \sim 10^{60}$ was very large.

This can give a way to solving the one of the fundamental problems of the modern theoretical physics – the problem of cosmological constant [25, 26, 27, 28, 29] as a consequence of fields dynamics at the early Universe.

We point out that the ultra-rapid decrease of the energy of physical vacuum according to the laws (25) or (26) occurs only prior to the Friedman era evolution of the Universe. Further evolution of the Universe is determined not by a scalar field, but mainly by the born ultra relativistic matter and the radiation interacting with it.

On the local scale one can conclude that the presence of the Deser–Dirac scalar field, which always exists, because it has an equally fundamental status as the metric, eliminates the existence of the black hole solution. In the Yilmaz original theory of gravitation [19, 21], it has been postulated that the metric of a Riemann space is a function of the scalar field only. In contrast to the Yilmaz theory, in the conformal theory of gravitational field developed here, the metric and the scalar field are the different geometrical structures of the Cartan–Weyl space.

As a consequence of the solution obtained, the Deser–Dirac scalar field intensive concentrates near massive objects. In [6] the hypothesis has been formulated that *the Deser–Dirac scalar field is realized itself not only as the 'dark energy', but also as one of the components of the 'dark matter'.* The Higgs particle, which has been recently found at Large Hadron Collider (CERN), maybe in reality is a quantum of the Deser–Dirac scalar field [6].

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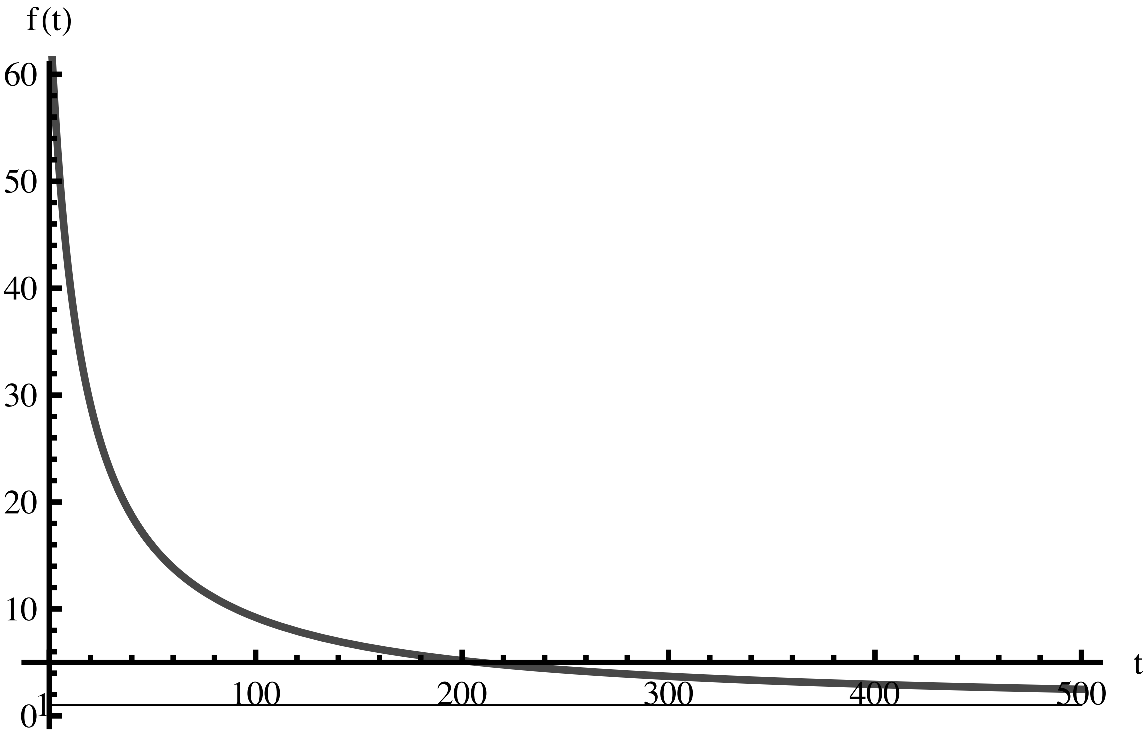

Fig. 1. The solution (25) for the Deser–Dirac scalar field in the early Universe.

Cosmological solution for the early Universe in a Cartan–Weyl space with the Deser–Dirac scalar field

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Post-Rimannian spaces allocated such geometrical structures as curvature, torsion and nonmetricity have now the increasing value in the classical theory of gravitation. On the basis of the Poincaré–Weyl gauge theory of gravitation [1] it has been shown, that a geometrical spacetime background is the geometrical structure of Cartan–Weyl space $\mathcal{CW}_4$ with a curvature 2-form $\mathcal{R}^a{}_b$, a torsion 2-form $\mathcal{T}^a$, and also the nonmetricity 1-form of the Weyl's type $\mathcal{Q}_{ab} = \frac{1}{4} g_{ab}\mathcal{Q}$. Also it has been shown, that in the framework of the gauge procedure developed in [1], in a natural way, as an additional geometrical variable there emerges a scalar field β with the properties of the scalar field entered by Dirac [2] and earlier by Deser [3]. Developing the given approach, the conformal theory of gravitational field has been constructed in [4]–[7] and the field equations of the theory have been deduced in a formalism of external forms [6], [7] (Λ is the Einstein cosmological constant).

The new solution of these field equations for the ultra-early universe, when the contribution of the field β much more exceeds the contribution of other matter, is found ($\lambda = \sqrt{3\Lambda}$):

$$\beta(t) = \frac{1+\exp(-\lambda(t+t_0))}{1-\exp(-\lambda(t+t_0))}\,, \quad a(t) = a_1 \exp((\lambda/3)(t+t_0))\frac{(1-\exp(-\lambda(t+t_0)))^{5/3}}{1+\exp(-\lambda(t+t_0))}\,.$$

This solution realizes exponential diminution of a field β, and thus sharp exponential decrease of physical vacuum energy (dark energy) by many orders. We have $\Lambda_{eff} = \beta^2\Lambda \to \Lambda$ in a limit at $t \to \infty$. Thus, the effective cosmological constant can slightly differ already by the end of inflation from the limiting value equal to its modern size Λ that provides the subsequent transition from the Friedman epoch to the epoch of the accelerated expansion in accordance with the modern observant cosmological data.

The result received can give an opportunity for the decision (as consequence of fields dynamics in the ultra-early universe) the cosmological constant problem [8], consisting in the fact that effective cosmological "constant" Λ_{eff} (according to estimations of the quantum field theory in the ultra-early universe) surpasses on 120 orders the value of the Einstein cosmological constant Λ in present period [8].

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New opto-acouctical gravitational detector in BNO INR RAS

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Introduction

Detection of gravitational waves predicted by general relativity is the purpose of the gravitational-wave detectors. The modern gravitational wave detectors are of two types. The first type is a resonace-bar detector [1], this detector is a solid acoustic resonator with a hight quality factor. Gravitational wave induces a force response which detects by high-sensitivity sensor. To reduce the thermal noise and functioning of SQUIDs in a displacement sensor these antennas operate at cryogenic temperatures. The most modern antennas of this type are: EXPLORER, NAUTILUS. The highest sensitivity of such antenna is near their acoustic resonant frequency of about 1 kHz.

The second type is the large interferometer antennas [2, 3]. The core element of such antennas is a Michelson interferometer. A gravitational wave changes the optical length of the interferometer that leads to the appearance of the signal at the output. The main feature of these antennas is their complexity, the huge size and high cost. The highest sensitivity is achieved in the wide frequency range from tens of hertz to kilohertz.

We proposed a combined antenna type - OGRAN - Optical and acoustic GRavitational ANtenna [4], combining a principle of resonant antennas and interferometry. The main OGRANs element is solid acoustic resonator - aluminum alloy cylinder with an axial tunnel. At the both sides of the axial tunnel are fixed mirrors which make a Fabry-Perot interferometer. Thus gravitational wave interacts with an acoustic resonator and changes the optical length of the Fabry-Perot interferometer. Using an optical displacement sensor can achieve sensitivity at room temperature comparable with cryogenic resonance antennas. The estimated sensitivity of the installation is sufficient for rare events in our galaxy and its immediate neighborhood. Because these events are accompanied by the release of large amounts of neutrinos, it was proposed joint work OGRAN with BNO - Baksan Neutrino Observatory - in a coincidence-anticoincidence scheme. The project started in the Sternberg Astronomical Institute in the 2000s in collaboration with ILP of SB RAS and INR of RAS. **OGRAN**

The experimental setup is shown in Fig. The main sensitive element - detector - is an alu-

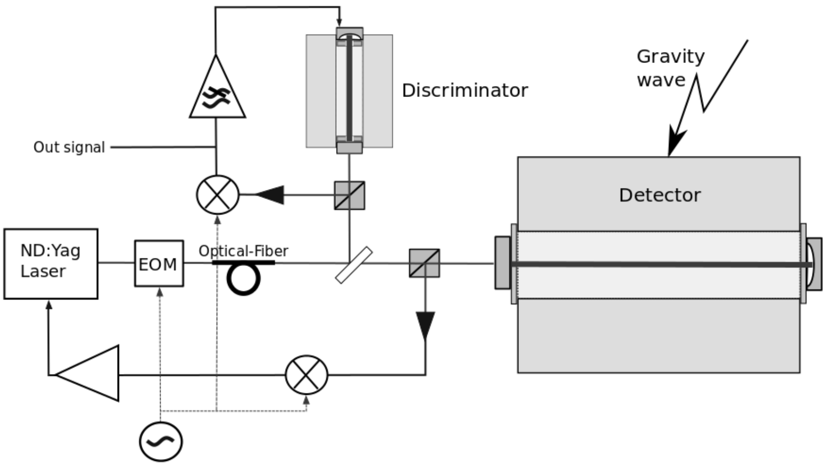


Fig. 1. (a) setup noise before modification of mirrors mounts and Lorentz curves fitted for some spectral peaks; (b) setup noise after modification of mirrors mounts and corresponding fitted curve

minum alloy cylinder with a length of 2 meter and mass about 2 tons with axial tunnel. To reduce the influence of ambient noise detector is positioned in a vacuum chamber at seismic filter mappings. Mirrors mounted on the sides of the axial tunnel. The mounts of mirrors is an important technical part of the installation. Main purpose of mounts is to transfer of vibrations from the end of the detector to the mirror. High frequency of acoustic vibrations mounts system is a necessary condition of fulfillment this function. Another important factor is the high Q-quality of the detector with fixed mirrors. We went through several mounting options and stood on the mount based on the U-section. Measured transmission coefficient of vibrations is better than 0.9, and the Q-quality of the detector is around 10^5 . The optical displacement system based on a comparison of the resonance frequency of Fabry-Perot interferometer on the detector relative to the frequency of the reference Fabry-Perot interferometer. Reference optical interferometer - discriminator - is interferometer Fabry-Perot mirrors which are fixed to the cylinder made of glass with low thermal expansion – Sitall. Ceramic Sitall necessary to lower the thermal drift of the resonant frequency. The cylinder is such that its frequency acoustic oscillations is higher than the acoustic frequency detector, so that the frequency range where the detector is most sensitive discriminator is calm.

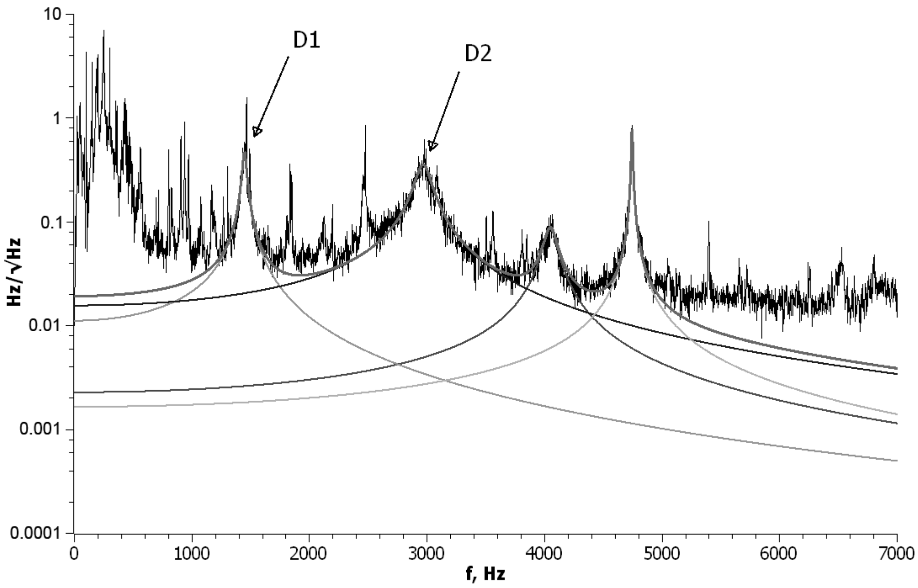
The main element of the optical system is tunable ND:YAG laser. Using the Pound-Drever technique laser frequency is kept at the top of the detector interferometer optical resonance. The frequency of the laser is controlled by 2 piezoelectric ceramics: slow (up to 100Hz) and fast (up to 50kHz) and electro optical crystal (up to 500kHz), which change the optical length of the laser cavity. To keep the operating point of the reference interferometer, one of its mirrors is fixed to the piezoelectric crystal. In the discriminator channel the Pound-Drever technique also has been used to operate with the optical length of Fabri-Perrot reference cavity, but only at low frequencies (up to 100 Hz). On the upper frequencies corresponding to the acoustic resonance frequency of the detector the signal remain at rest. Relative displacement of the laser frequency and the resonance frequency of the reference interferometer carries information about the movement of detector's mirror and it was used as the useful signal.

In 2010, the operation of the optical detection system on a pilot model [5] was demonstrated. Currently, full-size antenna have been installed. During the configuration process we faced a number of difficulties.

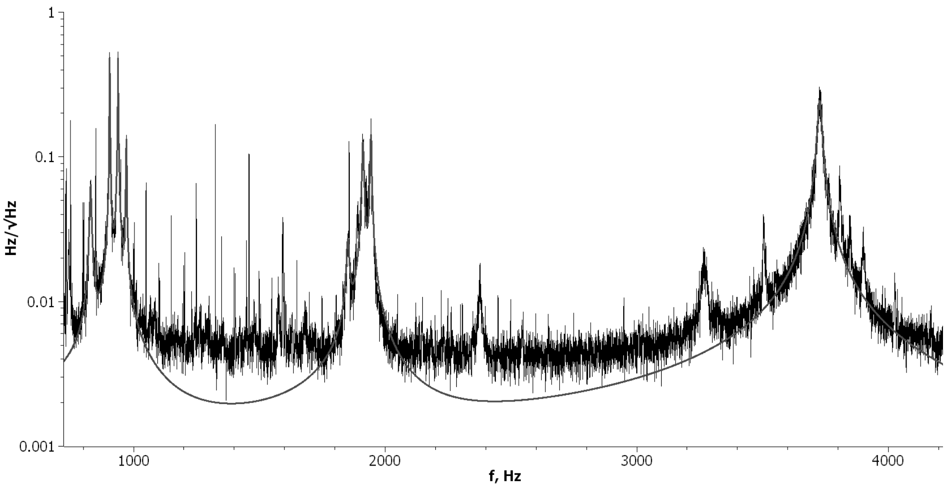
Phase modulation is an important part of the Pound-Drever technique. In the case of phase modulation of the laser radiation consists of three components, the central band at the optical frequency ω_0 and two spaced to the modulation frequency $\omega_0 \pm \Omega$ sidebands. At reflection from the interferometer the central component get the phase shift which is proportional to the difference between the laser frequency and the resonance frequency of the interferometer. Sidebands don't get resonance and reflect from the interferometer as from mirrors, thus violat the phase relationship between the central band and sidebands, that gives rise to amplitude modulation at the modulation frequency. This amplitude modulation carries information about the detuning of the laser and interferometer, and it's used as a control signal. Nonpoissonian laser noises are in the area up to 1 MHz, so the use of high-frequency modulation of about 10 MHz reduce their impact on the control signal. However, we are faced with the fact that in addition to the phase modulation, modulator themselfs produce residual amplitude modulation at the modulation frequency. This modulation makes nonpoissonian low frequency noise in the frequency domain, on the same spectral range corresponding to the setup signal. That reduces the accuracy of the Pound-Drever technique and increase total noise. We have found that a residual amplitude modulation had the complex spatial structure. Different parts of the wavefront was modulated with different amplitude and phase. To solve this problem we used single-mode polarisation maitanance fiber as a filter of spatial modes. After passing through the fiber, parasitic amplitude modulation becomes homogeneous. Optical fiber can be tuned that the value of the residual amplitude modulation is negligible.

Lowering the thermal noise of the mirrors mounts

Experimental setup output noise spectrum before the reference cavity's (discriminator's) mirrors mounts design change is shown on fig. 2 (a). Let us represent this spectrum as a sum of spectra of some harmonic oscillators with dissipation excited by some forces with white spectrum. According to this model, each of these oscillators' vibrations spectrum looks as follows.



(a)



(b)

Fig. 2. (a) setup noise before modification of mirrors mounts and Lorentz curves fitted for some spectral peaks; (b) setup noise after modification of mirrors mounts and corresponding fitted curve

$$x_i = \frac{F_i}{\sqrt{m_i^2 \left(\omega_i^2 - \omega^2\right)^2 + H_i^2 \omega^2}} \tag{1}$$

Where F_i is a spectrum of force for i-th oscillator, m_i is an effective mass, $\omega_i = 2\pi f_i$ is a resonant angular frequency of i-th oscillator, H_i is a damping coefficient. Sum of this spectra is described by the formula:

$$x = \sqrt{\sum_{i=1}^{n} x_i^2}$$

Smooth curves on fig. 2 (a) depict spectra of some oscillators corresponding to some peaks of the real signal spectrum. F_i, m_i, H_i and ω_i parameters' values were determined with least-squares method. However, the model (1) has only three independent parameters. Thus, selecting the values of F_i, m_i, H_i and ω_i is an ambiguous problem.

As one can see from the fig. 2 (a), in the frequency range of interest (around 1300 Hz) mean level of the real signal spectrum noise is well described by our model.

Let us suppose that the nature of the peaks we consider is the thermal noise. Then according to the fluctuation-dissipation theorem one have to substitute F_i in (1) with the following expression:

$$F_i = \sqrt{4 k_B T H_i} \tag{2}$$

Where k_B is a Boltzmann constant, T is a temperature. Considering (2), there are now only three independent parameters m_i, H_i and ω_i in (1). So selection of these parameters values become an unambiguous problem. Effective masses for peaks D1 and D2 at fig. 2 (a) are 157 g and 18 g correspondingly. Also, according to experiments carried out, it was determined that the peaks D1 and D2 are the acoustic resonances of the discriminator assembly. Since the values of the effective masses of these resonances are typical for discriminator's mirrors' mounts parts weights, the hypothesis was offered: D1 and D2 peaks are the discriminator's mirrors mounts thermal noise and this thermal noise is the sensitivity limit of the whole setup for the moment.

New mirror mounts were designed and produced to eliminate these peaks. New design was mainly based on two following considerations. 1-st, resonant frequencies of new design should be high. Thus parts of the mounts should be light and stiff. 2-nd, Q-factors of resonances should be as high as possible. This why it was decided to do not use glue bondings which was widely used in the old design. The new mirror mount design is shown on fig. 3. The most original feature of this design is the expanding lobes for fixing the mount in the inner aperture of the discriminator's body.

Setup output noise after the new mirrors mounts installation (and also after EOM residual

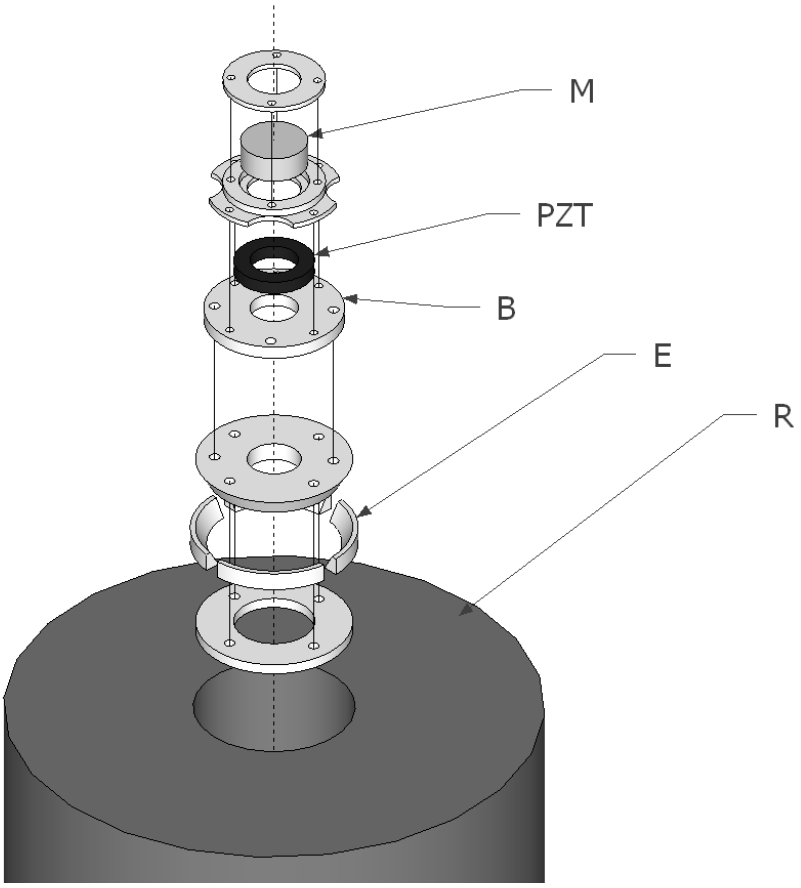


Fig. 3. New mirror mount design. R — reference cavity body, E — expanding lobes, B — angular adjusted base, PZT — piezoelectric driver, M — mirror

amplitude modulation suppression) is shown on fig. 2 (b). By comparing fig. 2 (a) and (b) one can see that D1 and D2 peaks have disappeared. There are some new peaks revealed for which apparently the new mounts are responsible. These new peaks are more narrow and at higher frequencies then old ones. Fitting them with the Lorentz curves reveals that we even have an opportunity to improve the sensitivity of the setup by factor ~ 3.

Thus changing the discriminator's mirrors mounts gave us the potential to improve the setup sensitivity in more than order of magnitude. This potential is partially realized for now.

*

Conclusion

With all the improvements we made the main OGRAN experimental setup has reached it's design sensitivity. However it shows relatively high sensitivity for environmental noises. The site where the setup to be finally installed and operated in duty cycle is Baksan neutrino observatory (BNO). According to measurements made by Geophysics service of RAS, BNO site have low seismic background. Also the OGRAN chamber is protected with more then kilometer of mountain rock above from cosmic rays which possibly may cause false signal appearance.

Transfer of the OGRAN experimental setup to BNO site is scheduled on autumn 2013. This will be the first GW bar detector operated underground.

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On relativistic symmetry of Finsler spaces with mutually opposite preferred directions

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It is shown that in Minkowski space there exist transformations of the coordinates of events alternative to the 3-parameter Lorentz boosts. However, in contrast to the boosts, they constitute a 3-parameter noncompact group which, in turn, is a subgroup of the homogeneous 6-parameter Lorentz group. Moreover, in the same space, there exists another 3-parameter noncompact group isomorphic to above-mentioned one. As we shall see, these two 3-parameter noncompact groups are rudiments of the 3-parameter groups of relativistic symmetry of the axially symmetric Finslerian spaces with the preferred directions $\boldsymbol{\nu}$ and $-\boldsymbol{\nu}$, respectively. Finally, it will be also demonstrated that inversion of the preferred direction $\boldsymbol{\nu}$ in the axially symmetric Finslerian space-time does not change the Lobachevski geometry of 3-velocity space. However, this leads to an inversion of the corresponding family of horospheres of the space.

1. Introduction

As it is known, space-time is Riemannian within the framework of GR, and the distribution and motion of matter only determines the local curvature of space-time without affecting the geometry of the tangent spaces. In other words, regardless of the properties of the material medium which fills the Riemannian space-time, any flat tangent space-time remains the space of events of SR, i.e. the Minkowski space with its Lorentz symmetry, which is usually identified with the relativistic symmetry.

However, in recent literature there is an increasing interest in the problem of violation of Lorentz symmetry (see [1] and the references cited therein). Particularly, the string-motivated approach to this problem is widely discussed.

The point is that even if the original unified theory of interactions possesses Lorentz symmetry up to the most fundamental level, this symmetry can be spontaneously broken due to the emergence of the condensate of vector or tensor field. The appearance of such a condensate, or of a constant classical field on the background of Minkowski space, implies that it can affect the dynamics of the fundamental fields and thereby modify the Standard Model of strong, weak and electromagnetic interactions. Since the constant classical field is transformed by the passive Lorentz transformations as a Lorentz vector or tensor, its influence on the dynamics of fundamental fields of the Standard Model is described by the introduction of the additional terms representing all possible Lorentz-covariant convolutions of the condensate with the Standard fundamental fields into the Standard Lagrangian. The phenomenological theory, based on such a Lorentz-covariant modification of the Standard model is called the Standard Model Extension (SME) [2].

By design, the phenomenological SME theory is not Lorentz-invariant, since its Lagrangian is not invariant under active Lorentz transformations of the fundamental fields against the background of fixed condensate. In addition, in the context of SME, a violation of Lorentz symmetry also involves the violation of relativistic symmetry, since the presence of non-invariant condensate breaks the physical equivalence of the different inertial reference systems.

It should be noted that in the low-energy limit of gravitation theories with broken Lorentz and relativistic symmetries, there appears an unlimited number of possibilities to build a variety of effective field theories, each of which being potentially able to explain at least some of the recently discovered astrophysical phenomena. At the same time, the very existence of the Finsler geometric models of space-time [3], [4] within which a violation of Lorentz symmetry occurs without the violation of relativistic symmetry strongly constrains the possible effective field theories with broken Lorentz symmetry: in order to be viable, such theories, in spite of the presence of Lorentz violation, should have the property of relativistic invariance.

Note also that, as shown in [4], the Ridge/CMS-effect revealed at the Large Hadronic Collider, directly demonstrates that in the early Universe there spontaneously emerged the axially symmetric local anisotropy of space-time with a group $\mathrm{DISIM}_b(2)$ as an inhomogeneous group of local relativistic symmetry and the corresponding Finsler metric

$$ds^2 = \left[\frac{(dx_0 - \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)\,. \tag{1}$$

This metric, proposed for the first time in [5], depends on two constant parameters r and $\boldsymbol{\nu}$, and generalizes the Minkowski metric. Here r determines the magnitude of spatial anisotropy, characterizing, thus, the degree of deviation of (1) from the isotropic Minkowski metric. Instead of the 3-parametric group of rotations of Minkowski space, Finsler spaces (1) allow only one 1-parameter group of rotations around the unit vector $\boldsymbol{\nu}$, which represents a physically preferred direction in 3D space.

One of the most important distinguishing features of Finsler spaces (1) consists in their noninvariance under the discrete improper transformations: $x_0 \to -x_0$ or $\boldsymbol{x} \to -\boldsymbol{x}$. This suggests that the emergence of the axially symmetric local anisotropy of space-time in the early Universe should be accompanied by violation of the CPT invariance (in this connection see [6]). In order to scrutinize such a problem, we shall take here the first steps towards the objective, namely consider inhomogeneous groups of local relativistic symmetry of Finsler spaces (1) and of

$$ds^2 = \left[\frac{(dx_0 + \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)\,. \tag{2}$$

Obviously, the Finsler space (2) is obtained from (1) by replacing $x_0 \to -x_0$ or $\boldsymbol{x} \to -\boldsymbol{x}$.

2 . Axially symmetric Finsler spaces and their isometry groups as inhomogeneous groups of local relativistic symmetry

For a start let us consider the flat Finsler space-time (1). As to the isometry group of (1) and to its Lie algebra, for the first time they were found (in an explicit form) in [7], [8], [9]. The respective group turned out to be 8-parametric: four parameters correspond to space-time translations, one parameter, to rotations about preferred direction $\boldsymbol{\nu}$, and three parameters, to the generalized Lorentz boosts. One should notice that at present, after the works [10], [11], this 8-parameter group is increasingly referred to as DISIM$_b$(2), where b is the new designation of the above-mentioned parameter r (for more details concerning what has been said, see, in particular, [12], [13]). As to the abbreviation DISIM$_b$(2), this stands for Deformed Inhomogeneous SIMilitude group that includes a 2-parameter Abelian homogeneous noncompact subgroup. Nevertheless, hereafter we shall hold on to our original designations.

Now let us consider infinitesimal transformations of the 8-parameter isometry group of the axially symmetric Finsler space-time (1). Originally (see [7]), the corresponding transformations of its 3-parameter homogeneous noncompact subgroup, i.e. infinitesimal transformations of relativistic symmetry of space-time (1), were obtained in the form

$$\begin{aligned} dx_0 &= (\,-\,r\,(\boldsymbol{\nu n})x_0 - \boldsymbol{nx}\,)\,d\alpha\,, \\ d\boldsymbol{x} &= (\,-\,r\,(\boldsymbol{\nu n})\boldsymbol{x} - \boldsymbol{n}x_0 - [\boldsymbol{x}[\boldsymbol{\nu n}]]\,)\,d\alpha\,, \end{aligned} \tag{3}$$

where the unit vector $\boldsymbol{n}$ and α are the group parameters. As to the infinitesimal transformations of the above-mentioned 1-parameter group of rotations and of the 4-parameter group of space-time translations, they have the form

$$d\boldsymbol{x} = [\boldsymbol{x\nu}]\,d\omega\,; \quad dx^i = da^i\,, \quad i = 0,\,1,\,2,\,3\,. \tag{4}$$

Using all these infinitesimal transformations with the condition that the third space axis is directed along $\boldsymbol{\nu}$ and three successive directions (along the first-, the second- and the third axis) are chosen for $\boldsymbol{n}$, we arrive at the simplest representation of generators of the 8-parameter isometry group of the Finsler space-time (1). As a result,

$$\begin{aligned} X_1 &= -(x^1p_0 + x^0p_1) - (x^1p_3 - x^3p_1)\,, \\ X_2 &= -(x^2p_0 + x^0p_2) + (x^3p_2 - x^2p_3)\,, \\ X_3 &= -rx^ip_i - (x^3p_0 + x^0p_3)\,, \\ R_3 &= x^2p_1 - x^1p_2\,; \qquad\qquad p_i = \partial/\partial x^i\,. \end{aligned} \tag{5}$$

These generators satisfy the following commutation relations:

$$\begin{aligned} &[X_1X_2] = 0\,, & &[R_3X_3] = 0\,, \\ &[X_3X_1] = X_1\,, & &[R_3X_1] = X_2\,, \\ &[X_3X_2] = X_2\,, & &[R_3X_2] = -X_1\,; \end{aligned} \tag{6}$$

$$\begin{aligned} &[p_ip_j] = 0\,; \\ &[X_1p_0] = p_1\,, & &[X_2p_0] = p_2\,, & &[X_3p_0] = rp_0 + p_3\,, & &[R_3p_0] = 0\,, \\ &[X_1p_1] = p_0 + p_3\,, & &[X_2p_1] = 0\,, & &[X_3p_1] = rp_1\,, & &[R_3p_1] = p_2\,, \\ &[X_1p_2] = 0\,, & &[X_2p_2] = p_0 + p_3\,, & &[X_3p_2] = rp_2\,, & &[R_3p_2] = -p_1\,, \\ &[X_1p_3] = -p_1\,, & &[X_2p_3] = -p_2\,, & &[X_3p_3] = rp_3 + p_0\,, & &[R_3p_3] = 0\,. \end{aligned} \tag{7}$$

The operators X_1, X_2, X_3 and their Lie algebra correspond to the special case where the third spatial axis is directed along $\boldsymbol{\nu}$. In the general case where spatial axes are oriented arbitrarily with respect to the preferred direction, the corresponding operators generate the following finite homogeneous transformations (the generalized Lorentz boosts making up the 3-parameter group of relativistic symmetry of the flat axially symmetric Finslerian event space (1)):

$$x'^i = D(\boldsymbol{v},\boldsymbol{\nu})R^i_j(\boldsymbol{v},\boldsymbol{\nu})L^j_k(\boldsymbol{v})x^k\,, \tag{8}$$

where $\boldsymbol{v}$ denotes the velocities of moving (primed) inertial reference frames, the matrices $L^j_k(\boldsymbol{v})$ represent the ordinary Lorentz boosts, the matrices $R^i_j(\boldsymbol{v},\boldsymbol{\nu})$ represent additional rotations of the spatial axes of the moving frames around the vectors $[\boldsymbol{v}\boldsymbol{\nu}]$ through the angles

$$\varphi = \arccos\left\{1 - \frac{(1-\sqrt{1-\boldsymbol{v}^2/c^2})[\boldsymbol{v}\boldsymbol{\nu}]^2}{(1-\boldsymbol{v}\boldsymbol{\nu}/c)\boldsymbol{v}^2}\right\} \tag{9}$$

of the relativistic aberration of $\boldsymbol{\nu}$, and the diagonal matrices

$$D(\boldsymbol{v},\boldsymbol{\nu}) = \left.\frac{1-\boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1-\boldsymbol{v}^2/c^2}}\right)^r I \tag{10}$$

stand for the additional dilatational transformations of the event coordinates.

Note that the structure of the generalized Lorentz boosts (8) ensures the fact that, in spite of new (Finsler) geometry of the flat event space (1), the 3-velocity space remains to be a Lobachevski space (see, for instance [14]) with metric

$$dl_v^2 = \frac{(d\boldsymbol{v})^2 - [\boldsymbol{v}d\boldsymbol{v}]^2}{(1-\boldsymbol{v}^2)^2}\,. \tag{11}$$

Thus, the transition from the Minkowski event space to the flat axially symmetric Finsler event space (1) leaves the relativistic 3-velocity space unchanged. Therefore it is clear that the 3-parameter group of the generalized Lorentz boosts (8) induces an isomorphic 3-parameter group of the corresponding motions of the Lobachevski space. In particular, the Abelian (see (6)) 2-parameter subgroup with the generators $X_1\,, X_2$ (see (5)) induces a 2-parameter subgroup of such motions of the Lobachevski space which leave invariant a family of the horospheres $(1-\boldsymbol{v}\boldsymbol{\nu})/\sqrt{1-\boldsymbol{v}^2} = const$, i.e., of surfaces perpendicular (see Fig. 1) to the congruence of geodesics parallel to $\boldsymbol{\nu}$ and possessing Euclidean inner geometry.

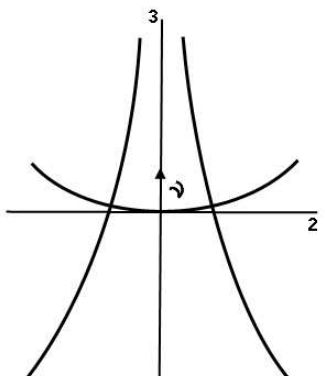

Figure 1: Horosphere 2D image in the Lobachevski space. The horosphere belongs to the family $(1-\boldsymbol{v}\boldsymbol{\nu})/\sqrt{1-\boldsymbol{v}^2}=const.$

Now, along with initial Finsler space-time (1), let us consider the Finsler space-time (2). Since its metric can be obtained from (1) by replacing $\boldsymbol{\nu} \to -\boldsymbol{\nu}$, we shall treat (2) as axially symmetric Finsler space-time with the opposite direction of $\boldsymbol{\nu}$.

For easier comparison of corresponding equations peculiar to the spaces (1) and (2), we represent, for example, infinitesimal transformations of 3-parameter groups of relativistic symmetry of these spaces in the following form

$$
\begin{aligned}
dx_0^{\substack{(1)\\(2)}} &= (\mp r(\boldsymbol{\nu n})x_0 - \boldsymbol{n x})\, d\alpha\,, \\
d\boldsymbol{x}^{\substack{(1)\\(2)}} &= (\mp r(\boldsymbol{\nu n})\boldsymbol{x} - \boldsymbol{n}x_0 \mp [\boldsymbol{x}[\boldsymbol{\nu n}]])\, d\alpha\,,
\end{aligned}
\tag{12}
$$

where the unit vector $\boldsymbol{n}$ and α are the group parameters. Here and below, two-level index $\substack{(1)\\(2)}$ in the left side of each equation indicates that the equation relates to space (1) and space (2). In accordance with the architecture of this index, the upper signs in the right side of each equation correspond to the case of space (1), whereas the lower signs correspond to the case of space (2).
As for the infinitesimal transformations of the above-mentioned 1-parameter group of rotations and 4-parameter group of space-time translations, they have the form

$$
d\boldsymbol{x}^{\substack{(1)\\(2)}} = \pm [\boldsymbol{x\nu}]\, d\omega\,; \qquad (dx^i)^{\substack{(1)\\(2)}} = da^i\,, \quad i = 0,\,1,\,2,\,3\,. \tag{13}
$$

If, as before, the spatial axes are chosen so that $\boldsymbol{\nu} = (0,\,0,\,1)$, and three successive directions (along the first-, the second- and the third axis) are chosen for $\boldsymbol{n}$, then the generators and the corresponding Lie algebras of the 8-parameter isometry groups of Finsler spaces (1) and (2) appear as

$$
\begin{aligned}
X_1^{\substack{(1)\\(2)}} &= -(x^1p_0 + x^0p_1) \mp (x^1p_3 - x^3p_1)\,, \\
X_2^{\substack{(1)\\(2)}} &= -(x^2p_0 + x^0p_2) \pm (x^3p_2 - x^2p_3)\,, \\
X_3^{\substack{(1)\\(2)}} &= \mp r(x^0p_0 + \boldsymbol{xp}) - (x^3p_0 + x^0p_3)\,, \\
R_3^{\substack{(1)\\(2)}} &= \pm (x^2p_1 - x^1p_2)\,; \qquad\qquad p_i = \partial/\partial x^i\,.
\end{aligned}
\tag{14}
$$

$$
\begin{aligned}
&[X_1^{\substack{(1)\\(2)}} X_2^{\substack{(1)\\(2)}}] = 0\,, && [R_3^{\substack{(1)\\(2)}} X_3^{\substack{(1)\\(2)}}] = 0\,, \\
&[X_3^{\substack{(1)\\(2)}} X_1^{\substack{(1)\\(2)}}] = \pm X_1^{\substack{(1)\\(2)}}\,, && [R_3^{\substack{(1)\\(2)}} X_1^{\substack{(1)\\(2)}}] = \pm X_2^{\substack{(1)\\(2)}}\,, \\
&[X_3^{\substack{(1)\\(2)}} X_2^{\substack{(1)\\(2)}}] = \pm X_2^{\substack{(1)\\(2)}}\,, && [R_3^{\substack{(1)\\(2)}} X_2^{\substack{(1)\\(2)}}] = \mp X_1^{\substack{(1)\\(2)}}\,;
\end{aligned}
\tag{15}
$$

$$
\begin{aligned}
&[p_i p_j] = 0\,; && \\
&[X_1^{\overset{(1)}{(2)}} p_0] = p_1\,, && [X_2^{\overset{(1)}{(2)}} p_0] = p_2\,, \\
&[X_1^{\overset{(1)}{(2)}} p_1] = p_0 \pm p_3\,, && [X_2^{\overset{(1)}{(2)}} p_1] = 0\,, \\
&[X_1^{\overset{(1)}{(2)}} p_2] = 0\,, && [X_2^{\overset{(1)}{(2)}} p_2] = p_0 \pm p_3\,, \\
&[X_1^{\overset{(1)}{(2)}} p_3] = \mp p_1\,, && [X_2^{\overset{(1)}{(2)}} p_3] = \mp p_2\,, \\
&[X_3^{\overset{(1)}{(2)}} p_0] = \pm\, r p_0 + p_3\,, && [R_3^{\overset{(1)}{(2)}} p_0] = 0\,, \\
&[X_3^{\overset{(1)}{(2)}} p_1] = \pm\, r p_1\,, && [R_3^{\overset{(1)}{(2)}} p_1] = \pm\, p_2\,, \\
&[X_3^{\overset{(1)}{(2)}} p_2] = \pm\, r p_2\,, && [R_3^{\overset{(1)}{(2)}} p_2] = \mp\, p_1\,, \\
&[X_3^{\overset{(1)}{(2)}} p_3] = \pm\, r p_3 + p_0\,, && [R_3^{\overset{(1)}{(2)}} p_3] = 0\,.
\end{aligned}
\tag{16}
$$

Now compare the 3-parameter noncompact homogeneous group of relativistic symmetry of space (1) (the generators $X_1^{(1)}, X_2^{(1)}, X_3^{(1)}$) with the corresponding group of space (2) (the generators $X_1^{(2)}, X_2^{(2)}, X_3^{(2)}$). According to their Lie algebras (see (15)), these groups are isomorphic to the corresponding 3-parameter subgroups (with generators $X_1^{(1)}, X_2^{(1)}, X_3^{(1)}|_{r=0}$ and $X_1^{(2)}, X_2^{(2)}, X_3^{(2)}|_{r=0}$, respectively) of the homogeneous Lorentz group. Similarly to the case of space (1), the Abelian 2-parameter subgroup (with the generators $X_1^{(2)}, X_2^{(2)}$) induces a 2-parameter subgroup of such motions of the Lobachevski space which leave invariant a family of the horospheres $(1+\boldsymbol{v\nu})/\sqrt{1-\boldsymbol{v}^2} = const$, i.e. of surfaces perpendicular (see Fig. 2) to the congruence of geodesics parallel to $-\boldsymbol{\nu}$ and possessing Euclidean inner geometry.

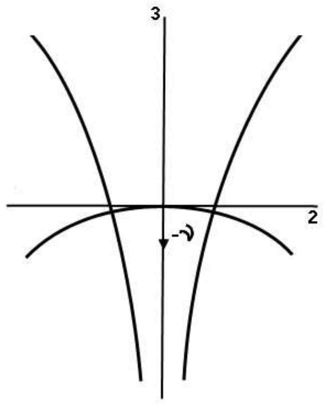

Figure 2: Horosphere 2D image in the Lobachevski space. The horosphere belongs to the family $(1+\boldsymbol{v\nu})/\sqrt{1-\boldsymbol{v}^2} = const$.

3. Two 3-parametric noncompact subgroups of the homogeneous Lorentz group as rudiments of the 3-parametric groups of relativistic symmetry of the axially symmetric Finsler spaces with mutually opposite preferred directions

If $r = 0$, the metrics of axially symmetric Finsler spaces (1) and (2), i.e.

$$\left(ds^2\right)^{(1)}_{(2)} = \left[\frac{(dx_0 \mp \boldsymbol{\nu} d\boldsymbol{x})^2}{dx_0^2 - d\boldsymbol{x}^2}\right]^r (dx_0^2 - d\boldsymbol{x}^2)\,, \tag{17}$$

reduce to the Minkowski one $ds^2 = dx_0^2 - d\boldsymbol{x}^2$. However transformations of relativisic symmetry of these spaces, i.e. transformations

$$\left(x'^{\,i}\right)^{(1)}_{(2)} = D(\boldsymbol{v}, \pm\boldsymbol{\nu}) R^i_j(\boldsymbol{v}, \pm\boldsymbol{\nu}) L^j_k(\boldsymbol{v}) x^k\,, \tag{18}$$

in which

$$D(\boldsymbol{v}, \pm\boldsymbol{\nu}) = \left. \frac{1 \mp \boldsymbol{v}\boldsymbol{\nu}/c}{\sqrt{1 - \boldsymbol{v}^2/c^2}} \right)^r I\,,$$

do not reduce to the ordinary Lorentz boosts

$$x'^{\,i} = L^i_k(\boldsymbol{v}) x^k\,. \tag{19}$$

Incidentally, these boosts can be represented in the following explicit form

$$x_0' = \frac{x_0 - (\boldsymbol{v}\boldsymbol{x})}{\sqrt{1 - \boldsymbol{v}^2}}\,,$$

$$\boldsymbol{x}' = \boldsymbol{x} - \frac{\boldsymbol{v}}{\sqrt{1 - \boldsymbol{v}^2}}\left[x_0 - \left(1 - \sqrt{1 - \boldsymbol{v}^2}\right)(\boldsymbol{v}\boldsymbol{x})/\boldsymbol{v}^2\right]. \tag{20}$$

At $r = 0$, i.e. in the case of Minkowski space where all directions in 3D space are equivalent, $\boldsymbol{\nu}$ has no physical meaning. In this case, each of the two rudimentary transformations

$$x'^{\,i} = R^i_j(\boldsymbol{v}, \pm\boldsymbol{\nu}) L^j_k(\boldsymbol{v}) x^k \tag{21}$$

differs from the Lorentz boost (19) by the corresponding additional rotation

$$x'^{\,i} = R^i_k(\boldsymbol{v}, \pm\boldsymbol{\nu}) x^k \tag{22}$$

of the spatial axes. This additional rotation is adjusted in such a way that if a ray of light has the direction $\boldsymbol{\nu}$ or $-\boldsymbol{\nu}$ in one frame, then it will have respectively the same direction in all the frames.

In order to find an explicit form of the additional rotation (22) we should use the following general formula

$$\boldsymbol{x}' = \boldsymbol{x} + [\boldsymbol{N}[\boldsymbol{N}\boldsymbol{x}]](1 - \cos\varphi) - [\boldsymbol{N}\boldsymbol{x}]\sin\varphi\,. \tag{23}$$

This formula determines the transformed components $\boldsymbol{x}'$ of radius vector $\boldsymbol{x}$ after rotation of the spatial axes around arbitrary unit vector $\boldsymbol{N}$ through an angle φ.

In our case the respective $\boldsymbol{N}$ and φ can be obtained by means of solving hyperbolic triangles in the Lobachevski 3-velocity space (see Fig. 3).

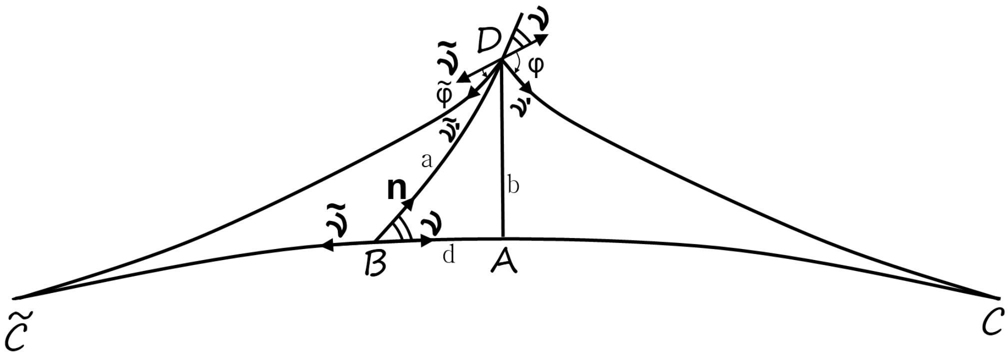

Figure 3: Hyperbolic triangles in the relativistic 3-velocity space.

In Fig. 3, the point $\mathcal{B}$ depicts the initial reference frame, $\mathcal{D}$ the reference frame moving at velocity $\boldsymbol{v}$ (the unit vector $\boldsymbol{n}$ indicates the direction of $\boldsymbol{v}$, i.e. $\boldsymbol{n} = \boldsymbol{v}/v$). In the reference frame $\mathcal{B}$ the ray of light has the direction $\boldsymbol{\nu}$ or $\tilde{\boldsymbol{\nu}} = -\boldsymbol{\nu}$, and in the reference frame $\mathcal{D}$, the direction $\boldsymbol{\nu'}$ or $\tilde{\boldsymbol{\nu}}'$, respectively. The $\mathcal{DC\tilde{C}}$ angle is zero (the straight lines $\mathcal{DC}$ and $\tilde{\mathcal{C}}\mathcal{C}$ are parallel). The $\mathcal{D\tilde{C}C}$ angle is zero (the straight lines $\mathcal{D\tilde{C}}$ and $\mathcal{C\tilde{C}}$ are also parallel). In addition, $\angle\mathcal{DAC} = \angle\mathcal{DA\tilde{C}} = \pi/2$ and $\angle\mathcal{ADC} = \angle\mathcal{AD\tilde{C}} = \Pi(b)$. Here $\Pi(b)$ is the Lobachevski angle for parallelism. Its dependence on the distance b between two points $\mathcal{D}$ and $\mathcal{A}$ is determined by the formula $\Pi(b) = 2\arctan e^{-b}$.

In order to find the angle φ of relativistic aberration of $\boldsymbol{\nu}$, the vector $\boldsymbol{\nu}$ should be carried in parallel from $\mathcal{B}$ to $\mathcal{D}$ along the straight line $\mathcal{BD}$ and then one should make use of the formulae of hyperbolic geometry, taking into account that $\tanh a = v/c$. As a result, we arrive at (9). From the same Fig. 3 one can see that the rotation is performed around the vector $[\boldsymbol{v\nu}]$. Now, with the help of (23), we are able to write down the transformation which corresponds to the additional rotation $x'^{\,i} = R^i_k(\boldsymbol{v},\boldsymbol{\nu})x^k$ of the spatial axes. It has the form

$$\boldsymbol{x}' = \boldsymbol{x} + \frac{(\sqrt{1-\boldsymbol{v}^2}-1)(\boldsymbol{vx}) + \boldsymbol{v}^2(\boldsymbol{\nu x})}{\boldsymbol{v}^2(1-\boldsymbol{v\nu})}\boldsymbol{v} + \\ + \frac{(1-\sqrt{1-\boldsymbol{v}^2})[2(\boldsymbol{v\nu})(\boldsymbol{vx}) - \boldsymbol{v}^2(\boldsymbol{\nu x})] - \boldsymbol{v}^2(\boldsymbol{vx})}{\boldsymbol{v}^2(1-\boldsymbol{v\nu})}\boldsymbol{\nu}\,. \tag{24}$$

Note that in (24) we put $c = 1$.

Similarly, in order to find the angle $\tilde{\varphi}$ of relativistic aberration of $\tilde{\boldsymbol{\nu}}$, the vector $\tilde{\boldsymbol{\nu}}$ should be carried in parallel from $\mathcal{B}$ to $\mathcal{D}$ along the straight line $\mathcal{BD}$. As a result, we get

$$\tilde{\varphi} = \arccos\left\{1 - \frac{(1-\sqrt{1-\boldsymbol{v}^2/c^2})[\boldsymbol{\nu v}]^2}{(1+\boldsymbol{v\nu}/c)\boldsymbol{v}^2}\right\} \tag{25}$$

From Fig. 3 one can see that such a rotation is performed around the vector $[\boldsymbol{\nu v}]$. Now, with the help of (23) we are able to write down the transformation which corresponds to another additional rotation $x'^{\,i} = R^i_k(\boldsymbol{v}, \tilde{\boldsymbol{\nu}})x^k = R^i_k(\boldsymbol{v}, -\boldsymbol{\nu})x^k$ of the spatial axes. It has the form

$$\boldsymbol{x}' = \boldsymbol{x} + \frac{(\sqrt{1-\boldsymbol{v}^2}-1)(\boldsymbol{vx}) - \boldsymbol{v}^2(\boldsymbol{\nu x})}{\boldsymbol{v}^2(1+\boldsymbol{v\nu})}\,\boldsymbol{v} + \\ + \frac{(1-\sqrt{1-\boldsymbol{v}^2})[2(\boldsymbol{v\nu})(\boldsymbol{vx}) - \boldsymbol{v}^2(\boldsymbol{\nu x})] + \boldsymbol{v}^2(\boldsymbol{vx})}{\boldsymbol{v}^2(1+\boldsymbol{v\nu})}\,\boldsymbol{\nu}\,. \tag{26}$$

Here, as before, we put $c = 1$.

4. Conclusion

Having studied the axially symmetric Finsler spaces with mutually opposite preferred directions and their isometry groups, we gave particular attention to the limiting case $r = 0$. As it turned out, if $r = 0$, the respective Finsler metrics $ds^2 = [(dx_0 \mp \boldsymbol{\nu} d\boldsymbol{x})^2/(dx_0^2 - d\boldsymbol{x}^2)]^r\,(dx_0^2 - d\boldsymbol{x}^2)$ reduce to the Minkowski one $ds^2 = dx_0^2 - d\boldsymbol{x}^2$. However, transformations of relativistic symmetry of the above-mentioned Finsler spaces do not reduce to the ordinary Lorentz boosts. At $r = 0$, i.e. in the case of Minkowski space where all directions in 3D space are equivalent, $\boldsymbol{\nu}$ has no physical meaning. In this case, each of the transformations of the pair of rudimentary transformations $x'^{\,i} = R^i_j(\boldsymbol{v}, \pm\boldsymbol{\nu})L^j_k(\boldsymbol{v})x^k$ differs from the Lorentz boost $x'^{\,i} = L^i_k(\boldsymbol{v})x^k$ by the corresponding additional rotation $x'^{\,i} = R^i_k(\boldsymbol{v}, \pm\boldsymbol{\nu})x^k$ of the spatial axes. This additional rotation is adjusted in such a way that if a ray of light has the direction $\boldsymbol{\nu}$ or $-\boldsymbol{\nu}$ in one frame, then it will have respectively the same direction in all the frames. Thus, at $r = 0$, the two sets of rudimentary transformations represent two alternatives to the Lorentz boosts, however, in contrast to the boosts, they constitute two different but isomorphic 3-parameter noncompact groups (see the two horospheres in Fig. 4 illustrating this fact). Physically, such noncompact transformations are realized as follows. First choose as $\boldsymbol{\nu}$ a direction towards a preselected star (or opposite direction) and then perform an arbitrary Lorentz boost by complementing it with such a turn of the spatial axes that in a new reference frame the direction towards the star (or opposite direction, respectively) remains unchanged. These two sets consisting of the described transformations form two different 3-parameter noncompact groups.

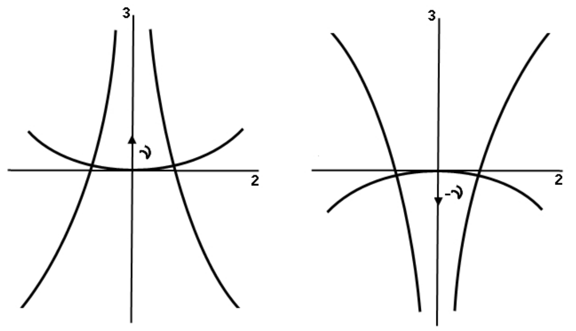

Figure 4: Two different horospheres in Lobachevski space as examples of two different surfaces of transitivity arising from the two rudimentary groups.

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Variation of the fine structure constant from extra dimensions

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Modern observations indicate a variability of the fine structure constant α in space and time. We present a construction method, where variations of α and other fundamental physical constants (FPCs) follow from the dynamics of extra space-time dimensions in the framework of curvature-nonlinear multidimensional theories of gravity. An advantage of this method is a unified approach to variations of different FPCs. A particular model is constructed, explaining the observable variations of α in space and time.

1 Introduction

The problem of possible variations of the fundamental physical constants (FPCs) in time and space is one of the most challenging problems of modern physics, directly related to the central problem of unification of all interations. It traces back to Dirac's and Eddington's famous papers of the 1930s and since then gains much attention in both theoretical and experimental studies.

However, to date, a variability of only one FPC has been revealed by observations more or less confidently, it is the fine structure constant α. The analysis of absorption spectra of various ions in the radiation of distant quasars, performed in the recent years (above all, from the data obtained at the Keck telescope on the Hawayian islands), has led to a conclusion that α is changing with time, so that in the past it was slightly smaller than now (the relative change $\delta\alpha/\alpha$ is about 10^{-5} [1]). In 2010, an analysis of new data obtained at the VLT (Very Large Telescope), located in Chile, and their comparison with the Keck data led to a conclusion on spatial variations of α, i.e., on its dependence on the direction of observations. The VLT observations in the Southern part of the celestial sphere gave values of the parameter α in the past slightly larger than now. This anisotropy has a dipole nature [2,3] and has been termed "the Australian dipole" [4]. The dipole axis is located at a declination of $-61 \pm 9^\circ$ and at a right ascention of $17.3 \pm 0,6$ hours. The deflection of α value at an arbitrary point r of space from its modern value α_0, measured on Earth, is

$$\delta\alpha/\alpha_0 = (1.10 \pm 0.25) \times 10^{-6}\, r \cos\psi, \tag{1}$$

where ψ is the angle between the direction of observation and the dipole axis, while the distance r is measured in billions of light years. The confidence level of this result (as compared with a "monopole" model where values of α are the same in all directions) has been estimated as 4.1σ. A more detailed discussion of the observational data can be found, e.g., in [3].

The tightest laboratory constraints on today's α variations have been obtained by comparison of readings of different atomic clocks [5]: $(d\alpha/dt)/\alpha = (-1.6 \pm 2.3) \times 10^{-17}$ per year. This result

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is of the same order of magnitude as the tightest constraints obtained previously from an isotopic composition analysis of the decay products in the natural nuclear reactor that operated in the Oklo region (Gabon) about 2 billion years ago [6, 7],

$$-3.7 \times 10^{-17}/\text{yr} < d(\ln\alpha)/dt < 3.1 \times 10^{-17}/\text{yr}. \tag{2}$$

Thus in the modern epoch, at least on Earth since the Oklo times, the parameter α did not change more rapidly than by approximately 10^{-17} per year.

It should be noted that most of the theoretical approaches [8–23] to explain variations of α, introduce scalar fields whose existence and manner of interaction with the electromagnetic field are postulated from the outset and are not explained in any way.

In this paper we are trying to explain the observed variations of α by the dynamics of extra dimensions using the approach formulated in [24], where a methodology was suggested allowing for a transition from multidimensional gravity with higher derivatives to Einstein-Hilbert gravity with effective scalar fields.

We here do not pretend to develop a new and complete cosmological model and restrict ourselves to a description of the present epoch of accelerated expansion of the Universe. This simplified model can be included into a more complete model that accounts for all epochs, beginning with primordial inflation, described in detail in our previous paper [25].

The paper is organized as follows. Sec. 2 briefly describes the general formalism used. In this framework, in Sec. 3 we build a homogeneous and isotropic cosmological model able to describe the present accelerated Universe along with a time dependence of the fine structure constant α. In Sec. 4, this cosmological model is slightly perturbed on large scale, which enables us to explain spatial variations of α. Sec. 5 is a brief conclusion.

2 Multidimensional gravity and its reduction

Consider a $(D = 4 + d_1)$-dimensional manifold with the metric

$$ds^2 = g_{\mu\nu}dx^\mu dx^\nu + \mathrm{e}^{2\beta(x)} b_{ab} dx^a dx^b \tag{3}$$

where the extra-dimensional metric components b_{ab} are independent of x^μ, the observable four space-time coordinates.

We suppose that b_{ab} describes a compact d_1-dimensional space of nonzero constant curvature, i.e., a sphere $(K = 1)$ or a compact d_1-dimensional hyperbolic space $(K = -1)$ with a fixed curvature radius r_0 normalized to the D-dimensional analogue m_D of the Planck mass, i.e., $r_0 = 1/m_D$ (we use the natural units, with the speed of light c and Planck's constant $\hbar$ equal to unity).

Consider, in the above geometry, a sufficiently general curvature-nonlinear theory of gravity with the action

$$S = \frac{1}{2} m_D^{D-2} \int \sqrt{{}^D g}\, d^D x\, (L_g + L_m), \qquad L_g = F(R) + c_1 R^{AB} R_{AB} + c_2 \mathcal{K}, \tag{4}$$

where $F(R)$ is an arbitrary smooth function, c_1 and c_2 are constants, L_m is a matter Lagrangian and ${}^D g = |\det(g_{MN})|$.

To obtain specific models, we simplify the theory (4) in the following way: (a) express everything in terms of 4D variables and $\beta(x)$; (b) Suppose that all quantities are slowly varying, i.e., neglect all terms of orders higher than two in the derivative operators ∂_μ; (c) in the 4D theory, initially looking as a kind of scalar-tensor theory of gravity, pass over to the Einstein conformal frame. More detailed calculations can be found in [26].

In the Einstein frame we have

$$
\begin{aligned}
S &= \frac{1}{2}\mathcal{V}[d_1]\int \sqrt{\widetilde{g}}\,(\operatorname{sign} F')\Big\{\widetilde{R}_4 + K_{\rm E}(\phi)(\partial\phi)^2 - 2V_{\rm E}(\phi) + \widetilde{L}_m\Big\}, && (5)\\
\widetilde{L}_m &= (\operatorname{sign} F')\frac{{\rm e}^{-d_1\beta}}{F'(\phi)^2}L_m; && (6)\\
K_{\rm E}(\phi) &= \frac{1}{4\phi^2}\Big[6\phi^2\Big(\frac{F''}{F'}\Big)^2 - 2d_1\phi\frac{F''}{F'} + \frac{1}{2}d_1(d_1{+}2) + \frac{4(c_1+c_2)\phi}{F'}\Big], && (7)\\
-2V_{\rm E}(\phi) &= (\operatorname{sign} F')\frac{{\rm e}^{-d_1\beta}}{F'(\phi)^2}[F(\phi) + c_*\phi^2], && (8)
\end{aligned}
$$

where the tilde marks quantities obtained from or with $\widetilde{g}_{\mu\nu}$ (metric in the Einstein frame); the indices are raised and lowered with $\widetilde{g}_{\mu\nu}$; $F = F(\phi)$ and $F' = dF/d\phi$; ϕ is related to ${\rm e}^{\beta}$ by

$$
\phi(x) = R_b\,{\rm e}^{-2\beta(x)} = Kd_1(d_1-1)\ {\rm e}^{-2\beta(x)} \tag{9}
$$

Let us consider the electromagnetic field $F_{\mu\nu}$ as matter in the initial Lagrangian, putting

$$
L_m = \alpha_1^{-1}F_{\mu\nu}F^{\mu\nu}, \tag{10}
$$

where α_1 is a constant. After reduction to four dimensions this expression acquires the factor ${\rm e}^{d_1\beta}$ arising from the metric determinant: $\sqrt{{}^Dg} = \sqrt{{}^4g}\,{\rm e}^{d_1\beta}$. In the subsequent transition to the Einstein picture the expression $\sqrt{{}^4g}F_{\mu\nu}F^{\mu\nu}$ remains the same (it is the well-known conformal invariance of the electromagnetic field), hence the Lagrangian (6) takes the form

$$
\widetilde{L}_m = \alpha_1^{-1}\,{\rm e}^{d_1\beta}F_{\mu\nu}F^{\mu\nu}, \tag{11}
$$

and for the effective fine structure constant α we obtain

$$
\frac{\alpha}{\alpha_0} = {\rm e}^{d_1(\beta_0-\beta)}, \tag{12}
$$

where α_0 and β_0 are values of the respective quantities at a fixed space-time point, for instance, where and when the observation is taking place.

3 The cosmological model

Depending on the choice of $F(R)$, the parameter c_1 and c_2 and the matter Lagrangian in the action (4), the theory under consideration can lead to a great variety of cosmological models. Some of them were discussed in [24], mostly those related to minima of the effective potential (8) at nonzero values of ϕ.

Here, we would like to focus on another minimum of the potential $V_{\rm Ein}$, existing for generic choices of the function $F(R)$ with $F' > 0$ and located at the point $\phi = 0$. The asymptotic $\phi \to 0$ corresponds to growing rather than stabilized extra dimensions: $b = {\rm e}^{\beta} \sim 1/\sqrt{|\phi|} \to \infty$. A model with such a late-time behavior may still be of interest if the growth is sufficiently slow and the size b does not reach detectable values by now. Recall that the admissible range of such growth comprises as many as 16 orders of magnitudes if the D-dimensional Planck length $1/m_D$ coincides with the 4D one, i.e., about 10^{-33} cm: the upper bound corresponds to lengths about 10^{-17} cm or energies of the order of a few TeV. This estimate certainly changes if there is no such coincidence.

One should note that small values of ϕ to be considered here are still very large as compared to 4D quantities, and so our general assumptions are well justified. Indeed, according to (9),

$|\phi| = d_1(d_1-1)/b^2$, where $b \lesssim 10^{16}$, hence $|\phi| \gtrsim d_1^2 \cdot 10^{-32}$, while the quantity $\widetilde{R}_4$, if identified with the curvature of the modern Universe, is of the order 10^{-122} in Planck units (that is, close to the Hubble parameter squared, or (the Hubble time)$^{-2}$, see also Eq. (23) below).

Let us check whether it is possible to describe the modern state of the Universe by an asymptotic form of the solution for $\phi \to 0$ as a spatially flat cosmology with the 4D Einstein-frame metric

$$d\tilde{s}_4^2 = dt^2 - a^2(t)d\vec{x}^2, \tag{13}$$

where $a(t)$ is the Einstein-frame scale factor. We shall be working in the framework of quadratic gravity with a cosmological constant, i.e.,[5]

$$F(\phi) = -2\Lambda_D + F_2\phi^2, \tag{14}$$

where Λ_D is the initial cosmological constant. Then, substituting $F' = 2\phi$ and $F'' = 2$, we obtain for the kinetic and potential terms in the Lagrangian (5) in the first approximation in ϕ:

$$\begin{aligned} &K_{\rm E} \approx K_0/(2\phi^2), \qquad K_0 = \frac{1}{2}\Big[\frac{1}{2}d_1^2 - d_1 + 6 + 2(c_1+c_2)\Big]; \\ &V_{\rm E} \approx V_0\,{\rm e}^{-2\bar{d}\beta}, \qquad V_0 = \frac{\Lambda_D}{4d_1^2(d_1-1)^2}, \qquad 2\bar{d} = d_1 - 4. \end{aligned} \tag{15}$$

It is clear that this model can work only if $d_1 > 4$. In terms of β instead of ϕ, the Lagrangian takes the form

$$L = \widetilde{R}_4 + 2K_0(\partial\beta)^2 - 2V_0\,{\rm e}^{-2\bar{d}\beta} + \widetilde{L}_m, \tag{16}$$

Neglecting the gravitational influence of the electromagnetic field (that is, considering only vacuum models), one can write down the independent components of the Einstein and scalar field equations with the unknowns $\beta(t)$ and $a(t)$ in the form

$$3\frac{\dot{a}^2}{a^2} = K_0\dot{\beta}^2 + V_0\,{\rm e}^{-2\bar{d}\beta}, \tag{17}$$

$$\ddot{\beta} + 3\frac{\dot{a}}{a}\dot{\beta} = \frac{V_0\bar{d}}{K_0}\,{\rm e}^{-2\bar{d}\beta}. \tag{18}$$

To solve these equations, let us use the slow-rolling approximation (for more details see [25]). Then we obtain

$$\dot{\beta} = \frac{\bar{d}\sqrt{V_0}}{K_0\sqrt{3}}\,{\rm e}^{-\bar{d}\beta}, \tag{19}$$

whence

$${\rm e}^{\bar{d}\beta} = \frac{\bar{d}^2}{K_0}\sqrt{\frac{V_0}{3}}(t+t_1), \tag{20}$$

where t_1 is an integration constant. For the scale factor $a(t)$ we have

$$\frac{\dot{a}}{a} = \frac{p}{t+t_1} \qquad \Rightarrow \qquad a = a_1(t+t_1)^p, \qquad a_1 = \text{const}, \qquad p = \frac{K_0}{\bar{d}^2}. \tag{21}$$

[5]We assume for certainty $\phi > 0$, or, which is the same according to (9), $K = +1$, but everything can be easily reformulated for $\phi < 0$.

Substituting the solution to the slow-rolling conditions, we make sure that they hold as long as $p \gg 1$, or in terms of the input parameters of the theory,

$$p = \frac{d_1^2 - 2d_1 + 12 + 4(c_1 + c_2)}{(d_1 - 4)^2} \gg 1. \tag{22}$$

We will assume that this condition holds. Recall that in this paper we do not consider the Universe dynamics during the radiation and matter dominated epochs, so that the expressions (21) are related to the modern epoch only.

A further interpretation of the results depends on which conformal frame is regarded physical (observational) [27, 28], and this in turn depends on the manner in which fermions appear in the (so far unknown) underlying unification theory involving all interactions.

Let us adopt the simplest hypothesis that the observational picture coincides with the Einstein picture and make some estimates. (It thus immedeiately follows that the gravitational constant does not vary.) Then, the inverse of the modern value of the Hubble parameter (the Hubble time) is estimated as

$$t_H = 1/H_0 = a_0/\dot{a}_0 \approx 4,4 \times 10^{17}\text{c} \approx 8 \times 10^{60}\, t_{\rm pl}, \tag{23}$$

where $t_{\rm pl}$ is the Planck time and the index "0" marks quantities belonging to the present time, which is a usual notation in cosmology. From (21) it follows that $H_0 = p/(t_0 + t_1)$, whence

$$t_* := t_0 + t_1 = pt_H \gg t_H. \tag{24}$$

With $p \gg 1$, the model satisfies the observational constraints on the factor w in the effective equation of state $p = w\rho$ of dark energy that causes the accelerated expansion of the Universe: at $w = \text{const}$ we have $a \sim t^{2/(3+3w)}$, consequently, $w = -1 + 2/(3p)$ is a number close to -1: for example, to have $w \approx -0.99$, one should put only $p = 66$. Meanwhile, the recent observational data allow for a comparatively large range of w [29–32] but anyway admitting $w = -1$ corresponding to a cosmological constant. This follows from combining the recent measurements of cosmic microwave background anisotropies, Supernovae luminosity distances, baryonic acoustic oscillations, and $H(z)$ measurements, though different tests lead to different confidence intervals.

Furthermore, the "internal" scale factor $b(t) = \text{e}^{\beta}$ grows much slower than $a(t)$:

$$b(t) = b_0\Big(\frac{t + t_1}{t_*}\Big)^{1/\bar{d}}, \qquad b_0 = \Big(\frac{1}{H_0}\sqrt{\frac{V_0}{3}}\Big)^{1/\bar{d}}. \tag{25}$$

Using the expression for V_0 from (15), one can estimate the initial parameter Λ_D, connecting it with the size of the extra factor space b_0: in Planck units,

$$\Lambda_D = 12H_0^2 d_1^2 (d_1 - 1)^2 b_0^{d_1 - 4} \approx \frac{3}{16} d_1^2 (d_1 - 1)^2 b_0^{d_1 - 4} \times 10^{-120}. \tag{26}$$

As already mentioned, the "internal" scale factor $b = \text{e}^{\beta}$ should be in the range $1 \ll b_0 \lesssim 10^{16}$ in Planck units. The estimate (26) shows that the present model makes much easier the well-known "cosmological constant problem" (the difficulty of explaining why in standard cosmology $\Lambda_{\rm standard} \sim 10^{-122}$ in Planck units). For instance, if (in the admissible range) $b_0 = 10^{15}$ and $d_1 = 12$, it follows $\Lambda_D = 3267$ without any indication of fine tuning.

4 Spatial variations of α

In the previous section we discussed the properties of a homogeneous model which does not contain any spatial variation of α (and any other physical quantity). Let us try to describe variations of α by taking into account spatial perturbations of the scalar field and the metric. Only long-wave perturbations will be of interest for us, with characteristic lengths larger than the horizon size.

The observed statistically isotropic sky means that there is no preferred axis. Nevertheless, super-horizon components were produced by quantum fluctuations at the beginning of inflation in the same way as fluctuations of smaller scale. It means that the dipole component must exist, though hard to observe due to its contamination by the Doppler effect caused by the motion of our Local Group with respect to the CMB. Hence there must exist a weakly expressed distinguished direction along which the metric and scalar field inhomogeneity is most clearly pronounced.

Accordingly, we now choose a metric more general than (13),

$$ds_{\rm E}^2 = e^{2\delta\gamma} dt^2 - a(t)^2 e^{2\delta\lambda} dx^2 - a(t)^2 e^{2\delta\eta}(dy^2 + dz^2), \tag{27}$$

where x is the distinguished direction and $\delta\gamma, \delta\lambda, \delta\eta \ll 1$ are functions of x and t. In addition, we replace the effective scalar field $\beta(t)$ with $\beta(t) + \delta\beta(x,t)$.

Then the relevant Einstein-scalar equations corresponding to the Lagrangian (16) can be written as follows (preserving only terms linear in the "deltas"):

$$\delta\ddot\beta + \frac{3\dot a}{a}\delta\dot\beta + \dot\beta(\delta\dot\lambda - \delta\dot\gamma) - \frac{1}{a^2}\delta\beta'' + \frac{1}{2K_0}\delta(V_\beta\, e^{2\gamma}) = 0, \tag{28}$$

$$\frac{\dot a}{a}(\delta\dot\lambda - \delta\dot\gamma) = \delta(V\, e^{2\gamma}), \tag{29}$$

$$\frac{\dot a}{a}\delta\gamma' = K_0 \dot\beta\, \delta\beta', \tag{30}$$

where we have chosen the gauge (in other words, the reference frame in perturbed space-time) $\delta\eta \equiv 0$, the dot and the prime stand for $\partial/\partial t$ and $\partial/\partial x$, respectively. We have also denoted $V = V_{\rm E} = V_0\, e^{-2\bar d\beta}$ and $V_\beta = dV/d\beta$.

Integration of (30), without loss of generality, leads to

$$\delta\gamma = \frac{K_0}{H}\dot\beta\delta\beta, \tag{31}$$

where, as before, $H = \dot a/a$. This equation enables us to estimate the quantity $\delta\beta$. Indeed, according to the CMB data [33], we can take $\delta\gamma \sim 10^{-5}$, while the coefficient before $\delta\beta$ is of the order of unity at the present epoch (according to Eq. (21) we have $(K_0/H_0)\dot\beta(t = t_0) = K_0/(\bar d \cdot p) = \bar d \sim 1$), so we obtain $\delta\beta \sim 10^{-5}$. This restriction concerns scales smaller than the horizon, whereas it is the superhorizon fluctuations that are responsible for the dipole component. Nevertheless, it is well known that slow motion during the inflationary stage allows for considering the Hubble parameter H to be an (almost) constant. The same can be said about the fluctuation magnitude equal to $H/(2\pi)$. Hence our restiction $\delta\beta \sim 10^{-5}$ applies to superhorison fluctuations as well.

Substituting $\delta\gamma$ (31) to (28) and taking the difference $\delta\dot\lambda - \delta\dot\gamma$ from (29), we finally arrive at the following single wave equation for $\delta\beta$:

$$\delta\ddot\beta + \frac{3\dot a}{a}\delta\dot\beta - \frac{1}{a^2}\delta\beta'' + \delta\beta\Big[\frac{2\dot\beta^2}{H^2}VK_0 + \frac{2\dot\beta}{H}V_\beta + \frac{1}{2K_0}V_{\beta\beta}\Big] = 0. \tag{32}$$

with an arbitrary constant K_0 and an arbitrary potential $V(\beta)$. In our case, with $V = V_0\,\mathrm{e}^{-2\bar{d}\beta}$ and K_0 given in (15), we obtain

$$\delta\ddot{\beta} + \frac{3\dot{a}}{a}\delta\dot{\beta} - \frac{1}{a^2}\delta\beta'' + \frac{2V_0\,\mathrm{e}^{-2\bar{d}\beta}}{p}\delta\beta = 0, \tag{33}$$

while the background quantities $a(t)$ and $\beta(t)$ are determined by the solution (20), (21). It remains to find a solution for $\delta\beta$ which, being added to the background $\beta(t)$, would be able to account for the observed picture of variations of α.

Since the background is x-independent, we can separate the variables and assume

$$\delta\beta = y(t)\sin k(x + x_0)$$

where k has the meaning of a wave number, of order of the cosmological horizon scale, and $y(t)$ must be as small as 10^{-5}. Then $y(t)$ obeys the equation

$$\ddot{y} + \frac{3p}{t + t_1}\dot{y} + \Big[\frac{k^2}{a_1^2(t + t_1)^{2p}} + \frac{6p}{(t + t_1)^2}\Big]y = 0. \tag{34}$$

Since Eq. (34) was derived in a certain approximation and describes only a restricted period of time close to the present epoch, it is reasonable to seek the solution in the form of a Taylor series:

$$y(t) = y_0 + y_1(t - t_0) + \frac{1}{2}y_2(t - t_0)^2 + \ldots, \qquad y_i = \text{const}. \tag{35}$$

Then y_0 and y_1 can be fixed at will as initial conditions, and Eq. (34) leads to expressions of $y_2, y_3, \ldots$ in terms of y_0 and y_1. Even more than that, for a certain neighborhood of $t = t_0$ we can simply suppose $y = y_0 + y_1(t - t_0)$. Actually, this approximation is good enough for $t - t_0 \ll t_* = t_0 - t_1$.

In this approximation we obtain the following expression for variations of α:

$$\frac{\alpha}{\alpha_0} \approx 1 - \frac{d_1}{\bar{d}}\,\frac{t - t_0}{t_*} - d_1 \sin[k(x + x_0)]\,[y_0 + y_1(t - t_0)] + O(\epsilon^2), \tag{36}$$

where $O(\epsilon^2)$ means $O((t - t_0)^2/t_*^2)$. Assuming that the observer is located at $x = 0$ and requiring $\alpha/\alpha_0 = 1 + O(\epsilon^2)$ at $x = 0$, we obtain the condition

$$y_1 \sin(kx_0) = -1/(\bar{d}\,t_*). \tag{37}$$

This explains very small, if any, variations of α on Earth at present and since the Oklo times. Indeed, since 2×10^9 yr $\approx \frac{1}{7}t_H$ while $t_* = pt_H$, the addition $O(\epsilon^2)$ is of the order of $1/(50p^2)$, where $p \gg 1$. If we take, for instance, $p = 1000$, then at the Oklo time (2×10^9 years ago) we obtain a relative α variation of the order 0.5×10^{-8}, which makes about 0.25×10^{-17} per year.

A substitution of (37) and (37) into (36) at $t - t_0 = -x$ for $x > 0$ gives

$$\alpha/\alpha_0 \approx 1 - d_1 y_0 \sin(kx_0) + d_1 y_0\, kx \cos(kx_0) + O(\epsilon^2) \tag{38}$$

at $x \ll t_*$. The same result is obtained if we substitute $t - t_0 = x$ for $x < 0$.

Fig. 1 compares the observational data and the predictions of our model with the parameters indicated. Recall that we are considering long-wave fluctuations, such that $k \le 1/r_H \sim 0.1$ (billion years)$^{-1}$ (where r_H is the modern horizon size), with a small magnitude $y_0 \le 10^{-5}$. The relations obtained are in good agreement with these estimates. We are using the conventional normalization $a_0 = 1$.

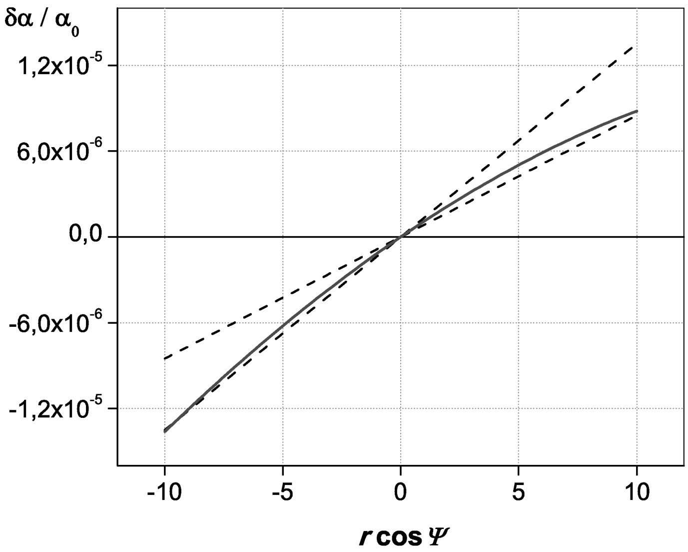

Figure 1: The r dependence of $\delta\alpha/\alpha_0$ (the distance r is measured in billions of light years). The dashed lines correspond to Eq. (1), the solid red line to Eq. (36) at the parameter values $d_1 = 12$, $p = 10^7$, $y_0 = -4.7\times10^{-6}$, $y_1 = -10^{-7}$ (bill. years)$^{-1}$, $k = 0.02$ (bill. light years)$^{-1}$, $x_0 = 1$ billion of light years.

Evidently, our model, in addition to the input theoretical parameters like d_1, c_1, c_2, contains the parameters k, x_0, y_0, y_1, depending on the initial form of the extra space metric.

In the framework of chaotic inflation, these parameters vary in different regions of the visible part of the Universe. Their choice enables us to explain the spatial variations of α in agreement with the observations [2]. Actually, there are only two conditions imposed on them: (37) and the relationship identifying (38) with the expression (1) at $r = x$ and $\cos\psi = 1$, i.e., on the dipole axis. We obtain (in Planck units)

$$d_1 y_0 k \cos(k x_0) \approx -2 \times 10^{-66}. \tag{39}$$

(This numerical value is used for obtaining the solid line in Fig. 1.) The small constant shift of the α value at $x = 0$ against the background does not change the interpretation of the results. Noteworthy, Eq. (39) is not a fine-tuning relation but simply fitting of the model parameters to observational data. In fact, the very small number in the r.h.s. of (39) results from the natural scale of $k \sim 1/r_H$, where $r_H \sim 10^{28}$ cm $\sim 10^{61}\,\ell_{\rm pl}$ is the Hubble radius in terms of the Planck length $\ell_{\rm pl}$; five more orders of magnitude in (39) are related to the smallness of α variations.

5 Conclusion

We have studied the possible effect of extra dimensions on large-scale variations of the fine structure constant α in space and time. In the multidimensional paradigm under consideration, the observable values of α and probably other physical quantities, including fundamental constants, depend on the size of the extra factor space. Variations of the dark energy density can be mentioned as an example. Indeed, the space-time variations of the energy density are dominated by those of the potential $V = V_\mathrm{E}$ given in (15). The relative variation $\delta V/V = -2\overline{d}\delta\beta$ is of the same order of magnitude as the space-time variations of α according to (12). They are too small to be observed in the near future.

We have focused on the behavior of α because it is the only fundamental constant for which there are more or less reliable data indicating its variations. We are also planning to analyze the behavior of other constants, above all, the gravitational constant and the particle masses.

The agreement with observations is provided in our model by the choice of initial data, which can be interpreted as random values of the extra-dimensional metric at the inflationary stage of the Universe. Thus spatial and temporal variations of α can be manifestations of the multidimensional space-time geometry.

The model described here is very simple, tentative and approximate and works fairly well for times not too far from the present epoch. It does not consistently include other kinds of matter than dark energy (represented by a scalar field of multidimensional origin)

An advantage of the present model of α variation against many others (see, e.g., [8–23]) is that it assumes a common origin of dark energy and FPC variations. Our model also predicts the existence

Acknowledgments

The authors wish to thank A. Panov for his interest in our work. The work of S.R. and I.S. was supported in part by the Ministry of Education and Science of the Russian Federation, project 14.A18.21.0789.

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About the problem of understanding in physics

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The question of how we understand something lies on the intersection of philosophy, epistemology and science. This question is extremely important, in the meantime, he rarely even given explicitly, and the responses to it, or at times vague, or just ignorant. We pose, for example, the question: why was this or that event? The answer is usually states that because of this there was some reason. And in life and in work, we met with the cause-and-effect relationship immense amount of time, and so used to it that we consider it self-evident, does not need any explanation. Meanwhile, with the penetration of human microcosm, with the advent of quantum mechanics, this provision is already being questioned. We will discuss this issue below.

Let us have something, the concept of a phenomenon, object, and we want to understand what it is. Consider an example. Suppose we are interested in the question: what is an absolutely solid body? We can give the following answer: it is – a solid, in which the deformation of these conditions can be ignored. This answer is quite correct, and gives us an understanding of what is solid.

But in this response involves other concepts, objects: a solid, deformation. We can give the definition of these concepts (objects), which, in turn, will be featured some other objects.

And so it builds up a chain of definitions: to identify some objects that we attract other objects, which, in turn, define a third, etc.

This chain by very definition cannot be infinite – it breaks somewhere (or, if we go deeper into the matter, it is better to say, it begins somewhere).

So, all the concepts (objects) in physics are divided into two classes: primary and secondary. Secondary objects are understood, defined in terms of other objects that are at a deep step. The primary objects are defined in terms of something else, they will at least for the moment in the development of science, are the original, fundamental, primary.

They are, in a sense, like the axioms of mathematics. However, the widely held view that the primary objects of physics are not defined at all, is fundamentally wrong. They are determined, but it is different than the secondary objects.

Determination of the primary objects (concepts) is composed of two mutually complementary aspects: the operational and intuitive. Operational definition is the job of pilot operations, the methods by which we measure the basic parameters of the object and thus make up an idea of it, understand it.

The intuitive aspect of the definition is related to the fact that the very concept of the primary objects is developed gradually on the basis of our experience (both individual and universal). Among the most striking examples of primary, original, fundamental concepts (objects) that are space and time. We live in a time and space, we are able to measure their parameters, we know their laws.

Lack of understanding of the fact that the primary physical objects, in principle, can not be determined as well as secondary, are often leads to fantastic results. For example, in the book "Philosophy" Ph.D. Professor V.A. Kanke writes:

"The totality of the spatial characteristics of things such as the length, area, volume, angular dimensions, relationships, "left", "right" is called a space. The space is an expression of coexistence of things (phenomena, processes). The set of temporal characteristics of the type of things durations relations "earlier" and "later", "time" is called time. Expresses its removability of things (phenomena, processes)" [1].

Ignorance of these definitions is just incredible! And this book is recommended to the Ministry of Education as a textbook for university students has withstood a variety of publications, was awarded the diploma of the Ministry, and the professor V.A. Kanke continued good health and to publish many more books!

We emphasize once again very important for the further point: our understanding of the primary concepts, objects produced on the basis of our macroscopic experience. However, it does not follow that the concept of primary facilities established on the basis of macroscopic experience, remain in force during the transition to the world of completely different scales, the transition to the microcosm or megaworld.

And at the beginning of the twentieth century, with the emergence and development of quantum mechanics, it gradually became clear that many of the primary concepts taken from macroscopic experience, generally lose their meaning in the transition to a microcosm. These concepts are not converted, and completely lose its meaning, it just does not correspond to the real nature. For example, in microcosm no such thing as a trajectory, it would seem so obvious in the macrocosm.

This realization did not come immediately, gradually, and caused a real shock to the world. This, along with the deep mathematical techniques, and is the main reason why a person who has not studied physics seriously, can not understand the laws of the microcosm.

How is created, produced primary concepts of the microworld? Our macroscopic experience here can do nothing to help us, he is powerless. Perceptions of primary concepts, objects are constructed microcosm, created by our mind. But to call it a free construction of the mind can not. These structures should be such that obtained on the basis of their results confirmed our macroscopic experience. But even if such confirmation is held, there remains the question of whether this is the only possible configuration or there are others?

There is one very important fact: the dimension in microcosm – it is always a microparticle interaction with macro – device, because the answer is always given on the classical macroscopic language. We have, apparently, there is no realistic chance to get at least some kind of measurement in quantum, microscopic language. In view of this, quantum mechanics tells the classical language that is not quite adequate to the microworld.

And here's the apparent contradiction arises, the combination of incongruous - the so-called wave-particle duality. Accepted argued that in some experiments the microparticle manifests itself as a corpuscle, in others - like a wave, ie and has the properties of both. In our view, there is no wave-particle duality does not exist.

Just microparticle – is the original, primary, fundamental object for which we have no visual representations, and that the interaction with macroinstrument speaks classical corpuscular and wave language, that is how to translate their language into another.

As you know, today is almost universally accepted is the so-called "Copenhagen" interpretation of the wave function, proposed by Max Born. This interpretation is also called statistical or probabilistic. About how painfully difficult this interpretation is working its way into practice, the following fact: Max Born put forward his idea in 1926, and received the Nobel Prize for it in 1954 – after 28 years!

It should be emphasized the fundamental difference between the probabilistic description that existed in the classics, from the probabilistic description of quantum mechanics. In classical, at least in principle, the probability for more detailed examination could not be administered at all, there is the probability – is just a description of the method. In quantum mechanics, the probability – is fundamental, original, ultimate source of nature, the probability of each individual characteristic of an isolated particle. This was brilliantly demonstrated in experiments Fabrikant-Biberman-Sushkina 1949 [2], which investigated the diffraction of electrons flying alone.

The wave function in quantum mechanics describes the probability wave. Here, both aspects are equally important – and probabilistic, and wave. Moreover, if the wave aspect quickly gained general acceptance in the first place, due to the diffraction experiments of Davisson-Germer, the probabilistic aspect was part of the physics of long and hard, and the discussion on this topic is still going on. Here is the chronology on the topic related to the three main protagonists.

Louis de Broglie in 1923 associated with the movement of the particle wave propagation, received the Nobel Prize in 1929. Erwin Schrödinger in 1926, introduced his equation, received the Nobel Prize in 1933. Max Born, proposed the idea of a probabilistic interpretation of the wave function in 1926, won the Nobel Prize in 1954. So, the time intervals between the opening and the Nobel Prize: Louis de Broglie – 6 years, Erwin Schrödinger – 7 years, Max Born – 28 years!

So, what is the probability wave, and what is its nature? First of all, it is – not the material, not a physical wave in the sense that it does not make any tangible medium, does not carry any energy. This is – a purely mathematical wave, which varies scalar complex value – the wave function.

However, this wave is very real, just as real, our thoughts, our information, our consciousness, all the spiritual sphere of human life. The reality is all of the above, in particular, that all of this has an impact on our material world, in particular, the probability wave determines the behavior of microscopic particles, interference and diffraction effects, the tunneling effect and much more.

Thus, primary, fundamental, underlying basis in microcosm, at least to date, are the likelihood and determinism. They blend in rather peculiar microcosm. It was described in GJ Myakisheva [3] and in the paper [4].

Determinism, cause-and-effect relationship has not canceled in quantum mechanics, but only significantly complemented by a probabilistic element. In our opinion, in microcosm causal is not a specific numerical value of a parameter (or, otherwise, a dynamic variable) quantum-mechanical object at a time, and the probability of this value, and we have a whole set of values, and each of which - with their probability. This view is most clearly set out, perhaps, in the work of GJ Myakisheva [3], in which he calls her a "probabilistic causation," and we'll add: "probabilistic determinism".

According to this view the state of microparticles system at the moment is uniquely determined by the initial conditions (that is, the state at the previous instant or interval of time) and external conditions, but a way to describe the state becomes a probability. In line with the objective point of view, the classical picture is simple to the extreme: $A \rightarrow B$, where A-reason, B-investigation.

A quantum-mechanical cause-and-effect relationship becomes more complex, it is symbolically depicted in Figure 1. Here, A, B – is still a cause and effect, and the P – probability of a generator, which assigns the same reason A whole set, the set of consequences of B. This set can be either discrete or continuous, as limited, and the infinite. The generator of the probability P to be understood symbolically, he apparently does not exist as a separate specific element in our world. So, one reason *A* can cause a lot of implications *B*, each – with its probability.

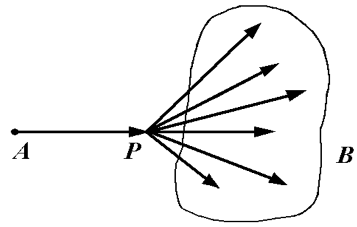

Fig. 1: The symbolic image of a causal due to quantum mechanics

Of course, this picture should go over the limit in the classical picture. To understand this transition is simple: from a macroscopic point of view, the set *B* becomes negligible, almost shrinks to a point, and we return to the classical causal link $A \rightarrow B$. Thus, all the arrows directed from *P* to *B*, naturally merge into one.

It should be emphasized in order to avoid misunderstandings, we understand the non-determinism as a picture of the world in which things happen for no reason at all, absolutely unpredictable. In quantum mechanics, there is a cause and predictability of the probability of an event, and, therefore, to talk about the abolition of determinism is not necessary.

By the way, determinism leads to fatalism of absolute predestination of the future, and non-determinism - the complete unpredictability of events. Thus, determinism takes away the freedom of human will, and non-determinism makes it impossible to plan anything, to act expediently. At full indeterminism, for example, when you turn the car's steering to the right of the car itself can turn in any direction or continue to go straight.

In this sense we can say that nature is wise to chose something in between, somewhere in between, and still has a place in the life and freedom of the will, and the predictability of events.

Planning for your future and, within limits, the impact on it, is an essential element of human activity, the human brain, consciousness. This plan is only possible in the simultaneous presence of two elements: on the one hand, the freedom of the will, and, on the other hand, certain laws that govern nature. Knowing these patterns, using them and having to certain limits free will, and we plan for the future.

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On the question of limit for signal velocity

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The situation leading to restriction of signalling velocity by velocity of light in vacuum is considered in detail. It is noted that realistic specifically atomic clock measures the number of periods of some space-time periodical process. These number of periods can be considered as value of phase function that is relativistic invariant. It is show that the known broken causality paradox for superluminal signalling does not appear in the case when we consider the real measured time by atomic clock for example. From this consideration follows the conclusion that the special relativity theory does not forbid the superluminal signalling.

Introduction

Major problems of modern theoretical physics are the problem of unification of all interactions and the problem of theoretical obtaining of all observable parameters for elementary particles. In particular the unification of electromagnetic and gravitational interactions is remaining one of major problems of physics.

These problems can be resolved within the limits of the unified field theory. We can consider the gauge-invariant nonlinear electrodynamics with singularities as the corresponding field model [1, 4]. The known pattern of similar models is Born – Infeld electrodynamics [2].

Among other important problems of theoretical and experimental physics it is necessary to consider the question of possibility or impossibility for superluminal velocities of particles and for superluminal signalling. In particular the finiteness of velocity of light is considered as one of factors limiting possible superhigh speed of future computers. In this connection the question about the limit value for signalling rate is rather important from the practical party.

The point of view according to which the special relativity theory forbids signalling with velocity exceeding the velocity of light in vacuum is well known.

But there are opinions in literature (see for example [6]) that the special relativity theory does not forbid in reality the superluminal motions and the superluminal signalling.

It must be noted also that the wave theory considered in the framework of some field model does not contain the fundamental limitation for velocity of signalling. This limitation can appear in the scope of special relativity theory and it is not obligatory as shown below.

Abolition of unjustified fundamental limitation for maximal velocity of signal transferring is one of requirements for future progress of science.

The approach based on field representation of the substance for resolving the seeming paradox of special relativity theory appearing in this connection is presented here (see also [4]).

Superluminal wave solution of field equations

As it is shown in [3] there are the wave solutions of linear electrodynamics in the form of nondeliquescent cylindrical waves the group velocity of which is greater than the velocity of light in vacuum (see also [4]).

These solutions appear when we consider so-called Bessel beams. The electromagnetic field components of the appropriate solutions have the following general form in cylindrical coordinates $\{\rho, z, \varphi\}$:

$$\boldsymbol{\mathcal{F}}(k_\rho\,\rho)\,\exp\bigl(\boldsymbol{\imath}(k\,z-\omega\,x^0)\bigr)\ , \tag{1}$$

where $\boldsymbol{\mathcal{F}}(\tilde{\rho})$ is some linear combination of Bessel function for dimensionless argument $\tilde{\rho} = k_\rho\,\rho$.

Here we use the hypercomplex representation for electromagnetic field $\mathbf{F} = \mathbf{E} + \boldsymbol{\imath}\,\mathbf{B}$, where $\boldsymbol{\imath}$ is hyperimaginary unit [4, 3].

The following general dispersion relation for Bessel beam is obtained from the field equations:

$$k_\rho^2 = \omega^2 - k^2\ . \tag{2}$$

When the square of longitudinal wavenumber exceeds the square of circular frequency $k^2 > \omega^2$ we have the case with hyperimaginary value for transverse wavenumber

$$k_\rho = \pm\boldsymbol{\imath}\sqrt{k^2-\omega^2}\ . \tag{3}$$

The functions $\boldsymbol{\mathcal{F}}$ for hyperimaginary argument $k_\rho\,\rho$ is connected with known Hankel functions for imaginary argument.

The appropriate to (3) expression for circular frequency

$$\omega = \pm\sqrt{k^2-|k_\rho|^2} \tag{4}$$

just gives the wave group velocity exceeding the velocity of light in vacuum:

$$1 < \left|\frac{\partial\omega}{\partial k}\right| = \frac{|k|}{\sqrt{k^2-|k_\rho|^2}} < \infty\ . \tag{5}$$

It is known that a local modulation of wave phase propagates with the group velocity for the wave (see for example [4, 10]). Thus we can consider this phase modulation as a signal.

Superluminal localized solution of linear and nonlinear electrodynamics was considered also in the paper [8].

Existence of such solutions conflicts with the aforesaid thesis about the limitation of signalling velocity by the velocity of light in vacuum.

Conservation of causality for arbitrary signal velocity

It is often considered that the superluminal signalling breaks causality of events so that the future can influence on the past.

Generally speaking we must accept that the conception of causality depends on the concrete model of reality. We must always take into account also this relativity of causality. The concept of signalling from space point A to space point B is connected with causality that can be called causality of local realism.

Seeming violation of this causality of local realism which we have because of existence of superluminal velocities needs a special consideration.

The specified statement about infringement of causality is closely connected with so-called relativity of simultaneity for spatially divided events in the different inertial reference frames connected by Lorentz transformation.

The relativity of simultaneity was considered in known classical work by Einstein of 1905 [5]. The books [9, 7] are interesting also from the point of view for discussion of paradoxes of relativity theory.

Let us consider in details the situation when a signal is transferred with superluminal velocity.

Let there are two inertial systems of coordinates $\{x^\mu\}$ and $\{x'^\mu\}$ which directions of axes coincide. The system $\{x'^\mu\}$ moves relatively the system $\{x^\mu\}$ with a constant velocity $\vec{V}$ directed on axes x^3, x'^3. Here we use the system of physical units in which the velocity of light in vacuum equals 1. Then we have the following coordinate transformation for two space-time points A (x^μ_A) and B (x^μ_B):

$$x'^0_A = \frac{x^0_A - \vec{V} x^3_A}{\sqrt{1 - \vec{V}^2}}\ , \qquad x'^0_B = \frac{x^0_B - \vec{V} x^3_B}{\sqrt{1 - \vec{V}^2}}\ , \tag{6a}$$

$$x'^1_A = x^1_A\ , \qquad x'^1_B = x^1_B\ , \tag{6b}$$

$$x'^2_A = x^2_A\ , \qquad x'^2_B = x^2_B\ , \tag{6c}$$

$$x'^3_A = \frac{x^3_A - \vec{V} x^0_A}{\sqrt{1 - \vec{V}^2}}\ , \qquad x'^3_B = \frac{x^3_B - \vec{V} x^0_B}{\sqrt{1 - \vec{V}^2}}\ . \tag{6d}$$

From (6a) we obtain

$$\left(x'^0_B - x'^0_A\right) = \frac{(x^0_B - x^0_A) - \vec{V}(x^3_B - x^3_A)}{\sqrt{1-\vec{V}^2}} \tag{7a}$$

$$= \left(x^0_B - x^0_A\right)\frac{1 - \vec{V}\,\mathcal{V}_{AB}}{\sqrt{1-\vec{V}^2}}\ , \tag{7b}$$

where

$$\mathcal{V}_{AB} \doteqdot \frac{x^3_B - x^3_A}{x^0_B - x^0_A}\ . \tag{7c}$$

Now if a signal is emitted in space-time point A and received in space-time point B then its velocity $\mathcal{V}_{AB}$ obviously is given by formula (7c). Let

$$|\mathcal{V}_{AB}| > 1\ , \tag{8}$$

that is the signal is transferred with the velocity exceeding of the constant velocity of light in vacuum. Then we can select the velocity of reference frame $|\vec{V}| < 1$ such that $\vec{V}\,\mathcal{V}_{AB} > 1$.

According to formula (7b) in this case we have that the relation $x'^0_B < x'^0_A$ follows from relation $x^0_B > x^0_A$. Thus the event B happens after A in one coordinate system and the event A happens after B in other coordinate system. In last case we have that the signal is received before it is emitted.

It should be noted that this result looks no more paradoxically than the relativity of simultaneity which follows from formula (7a): space divided events $(x^3_B \neq x^3_A)$ are simultaneous in one system $(x^0_B = x^0_A)$ and they are not simultaneous in other system $(x'^0_B \neq x'^0_A)$.

The paradox is resolved if we take into account that space-time coordinates x^μ are actually the abstract independent variables of some field functions which present essentially the real effects. The field functions depend on space-time coordinates invariantly such that the transition to another coordinates of course does not change the essence of the effects.

Such point of view is fully natural for most of possible approaches to description for effects of the material world.

The clock which we can practically realize does not measure the time coordinate x^0 of space-time point directly! Real clock always measures the number of periods for some space-time process that is relativistic invariant.

In other words when we measure time in any coordinate system the value of some phase function is really measured. This phase is relativistic invariant:

$$\Theta = k_\mu\, x^\mu = k'_\mu\, x'^\mu\ . \tag{9}$$

Let us explain the aforesaid by the example. Let the time be measured by means of counting for the number of periods of some space-time function (field configuration) which is a standing wave in coordinate system $\{x^\mu\}$. For simplicity we consider a harmonic standing wave of the following form:

$$\Psi(x^i)\, e^{\imath\Theta} \;, \tag{10a}$$

where $\Psi(x^i)$ is some space distribution for amplitude of wave, the phase function Θ has the form

$$\Theta = -\omega\, x^0 \;. \tag{10b}$$

Let us take two copy of the clock (10) and synchronize it such that the phase shift between their oscillations is absent. Let us put these clocks at two space divided points A and B. Then we will have the following wave functions near these points:

$$\Psi(x^i - x^i_A)\, e^{\imath\Theta} \quad \text{and} \quad \Psi(x^i - x^i_B)\, e^{\imath\Theta} \;. \tag{11}$$

Assume that some event at the point A happens after n periods of the clock A and some event at the point B happens after $m > n$ periods of the clock B. We can consider emitting and receiving of some signal as these events.

In moving coordinate system $\{x'^\mu\}$ the clock wave functions have the form

$$\Psi\big(x^i(x'^\mu) - x^i_A(x'^\mu_A)\big)\, e^{\imath\Theta} \quad \text{and} \quad \Psi\big(x^i(x'^\mu) - x^i_B(x'^\mu_B)\big)\, e^{\imath\Theta} \;, \tag{12a}$$

where the functions $x^i(x'^\mu)$ are determined by Lorentz transformation and Lorentz invariant phase function has the form

$$\Theta = k'_\mu\, x'^\mu \;. \tag{12b}$$

Here k'_μ are the components of wave four-vector which are obtained from the components $\{k_0, 0, 0, 0\}$ ($k_0 = -\omega$) also by Lorentz transformation.

It seems to be fully evident that if we have the relation $m > n$ in coordinate system $\{x^\mu\}$ then this relation is remaining in moving coordinate system $\{x'^\mu\}$. The velocity of transferring for considered hypothetical signal does not matter here. The value of this velocity can be taken arbitrary big by increase of distance between the points A and B when the difference $(m - n)$ is constant.

Observable relativistic effects

It should be noted that stated consideration fully conforms with known observable effect of time dilation for moving objects. This effect is proved with high precision in particular for decay time of muons moving with near-light velocity.

To illustrate the time dilation let us consider the same clock (10) but we use here the designation $\{\underset{\cdot}{x}^\mu\}$ for a proper coordinate system of the particle-soliton:

$$\Psi(\underset{\cdot}{x}^i)\,\mathrm{e}^{\imath\,\Theta}\ . \tag{13a}$$

Let the clock (13a) be decayed after n periods of the phase function

$$\Theta = -\underset{\cdot}{\omega}\,\underset{\cdot}{x}^0\ , \tag{13b}$$

where $\underset{\cdot}{\omega}$ is some constant of the particle-soliton.

Let us consider also the appropriate moving clock. To obtain their description we must use so-called active Lorentz transformation for the field configuration (13).

This transformation gives a new solution of Lorentz-invariant field equations. Obtained solution corresponds to the moving particle-soliton. This solution moving along x^3 axis has the form (13) in coordinate system $\{x^\mu\}$ with the following substitution:

$$\underset{\cdot}{x}^0 = \frac{x^0 - \not{V}\,x^3}{\sqrt{1-\not{V}^2}}\ , \tag{14a}$$

$$\underset{\cdot}{x}^1 = x^1\ , \tag{14b}$$

$$\underset{\cdot}{x}^2 = x^2\ , \tag{14c}$$

$$\underset{\cdot}{x}^3 = \frac{x^3 - \not{V}\,x^0}{\sqrt{1-\not{V}^2}}\ . \tag{14d}$$

Thus we have the moving clock (13) with substitution (14) in coordinate system $\{x^\mu\}$ Besides, we consider the clock resting in coordinate system $\{x^\mu\}$:

$$\Psi(x^i)\,\mathrm{e}^{-\imath\,\underset{\cdot}{\omega}\,x^0}\ . \tag{15}$$

The decay happens when the phase functions amount to the value $2\pi\,n$ for both resting clock and moving one. But in the case of the moving clock we have the following expression for phase function at the center of the soliton:

$$\Theta = -\underset{\cdot}{\omega}\,\underset{\cdot}{x}^0 \tag{16a}$$

$$= -\underset{\cdot}{\omega}\,x^0\sqrt{1-\not{V}^2}\ . \tag{16b}$$

This expression (16b) is obtained by substitution of (14a) to (13b) with the following condition for the soliton center:

$$x^3 = \not{V}\, x^0\ . \tag{17}$$

Let us substitute for the phase function of moving soliton Θ in (16) with its value at the moment of decay that is $\Theta = 2\pi\, n$. Also we substitute for the phase function of resting soliton $(-\underset{\cdot}{\omega}\, x^0)$ in (16b) with the appropriate value at the moment of decay for moving soliton that is $-\underset{\cdot}{\omega}\, x^0 = 2\pi\, m$. Here $2\pi\, m$ is the value for the phase function of the resting particle at the moment of decay of the moving particle.

Then we have from (16) the following:

$$m = \frac{n}{\sqrt{1 - \not{V}^2}} \tag{18}$$

that is the life time of the moving particle measured by the clock connected with the resting particle turn out to be more. Exactly this formula is proved experimentally.

Atomic clock and future experiments

At the present time we have the time standard which determines that the second equals to 9 192 631 770 periods of radiation corresponding to energy hyperfine transition between two levels of ground quantum state of caesium-133 atom. The time standard is realized in so-called atomic clock accuracy movement of which is very high.

It is important here that the atomic clock is a real clock in which the completion for the full period of some space-time process corresponds to each time reading.

Thus the atomic clock can be described by space-time function (10) the specification of which is sufficient for the present consideration. Of course in this case the function $\Psi(x^i)$ in (10) can be extremely complicated.

So the space-time periodical process is used as time standard. Thus the really measured time which can be called physical time is the value of phase function that is relativistic invariant.

Extraordinarily accuracy movement of the atomic clock can allow to realize the experiments for a registration of the superluminal signalling velocity. In this case according to considered pattern the synchronization of two clocks must occur by direct contact with subsequent placement of it to the given space points A and B. The maintenance of synchronization must be provided by the accuracy movement but not light rays.

Conclusions

The stated here consideration is based on assumption of invariance of substance theory equations relatively Lorentz transformation. This invariance is actual essence of Einstein special relativity theory.

So if we consider the really measured time or physical time then the exceeding of signalling velocity over the velocity of light in vacuum is presented as possible.

This statement has high practical importance from the various points of view.

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Quantum-Hydrodynamical Accretion onto Miniholes

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A particle accretion onto miniholes has been considered in the framework of
E. Madelung's quantum hydrodynamics. A classical model of disc accretion onto black holes at the final stage of close binary evolution is served as a starting point.
The total luminosity of the accretion disc and the energy of quanta have been calculated.

1. Introduction

A quantum accretion of particles onto miniholes has been considered in the framework of E. Madelung's quantum hydrodynamics [1,2].
. The total luminosity of the accretion disc and the energy of quanta being emitted have been calculated. The results obtained may be of importance for interpreting the electromagnetic radiation of graviatoms comprising particles captured by miniholes [3].

2. Classical Accretion onto Black Holes

A classical model of disc accretion onto black holes at the final stage of close binary evolution may serve as the starting point.

Due to a differential rotation of the disc there arise shear stresses between adjacent layers resulting in angular momentum transfer outward. The matter in the interior approaches the disc centre. The gas is being heated up to elevated temperatures because of friction emitting thermal radiation.

The total luminosity of the accretion disc

$$L_d = \frac{GM\dot{m}}{2R_i}, \tag{1}$$

where M is the black hole mass, $\dot{m}$ is the accretion rate, R_i is the last stable orbit radius. Since

$$R_i = \frac{6GM}{c^2}, \tag{2}$$

we obtain

$$L_d = \frac{1}{12}\dot{m}c^2. \tag{3}$$

3. Quantum Hydrodynamics

Calculation of a quantum accretion of baryons onto the miniholes may be considered in the framework of E. Madelung's quantum hydrodynamics [1,2].

The wave function is written in the form

$$\Psi = \sqrt{\rho} e^{i\frac{S}{\hbar}}, \tag{4}$$

where ρ is the probability density, S is the action. From Schrödinger's equation

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + U\Psi \tag{5}$$

follows the equations of quantum hydrodynamics:

$$\frac{\partial \rho}{\partial t} + \frac{1}{m} \nabla\left(\rho \nabla S\right) = 0, \tag{6}$$

$$\frac{\partial S}{\partial t} + \frac{1}{2m} (\nabla S)^2 + U - \frac{\hbar^2}{2m} \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} = 0, \tag{7}$$

where the last term

$$Q = -\frac{\hbar^2}{2m} \frac{\Delta\sqrt{\rho}}{\sqrt{\rho}} \tag{8}$$

is Bohm's potential, m is the particle mass.
From (7) we have

$$\frac{\partial S}{\partial t} = -E, \ \nabla S = \sqrt{2m\left(E - U - Q\right)} . \tag{9}$$

4. Quantum Accretion on Miniholes

The stationary quantum accretion rate defined in terms of a substantial derivative reads

$$\dot{m} = \int \nabla S \nabla \rho dV, \tag{10}$$

which reduces to

$$\dot{m} = 4\pi \sum_{n=2}^{\infty} \int_{3r_g}^{\infty} \frac{dS_n}{dr} \frac{d\rho_n}{dr} r^2 dr, \tag{11}$$

where

$$\frac{dS_n}{dr} = \sqrt{2m[E_n - U(r) - Q_n]} \tag{12}$$

$$U(r) = -\frac{GMm}{r}, \tag{13}$$

$$r_g = \frac{2GM}{c^2} \tag{14}$$

$$E_n = Q_n, \tag{15}$$

$$\rho_n = \frac{\exp\left(-\frac{2r}{na_B^g}\right)}{2\pi n a_B^g r^2}, \tag{16}$$

providing

$$r \gg na_B^g,\ r \gg l^2 a_B^g, \tag{17}$$

where

$$a_B^g = \frac{\hbar^2}{GMm^2}. \tag{18}$$

Thus we consider asymptotic values of the baryon location probabilities on higher hydrogen-like levels. The quantum accretion intensity

$$I = \frac{\sqrt{\pi}}{3}\alpha_g^2 A(\alpha_g)\frac{m^2c^4}{\hbar}, \tag{19}$$

where

$$A(\alpha_g) = \sum_{n=2}^{\infty}\frac{1}{n^{3/2}}\left\lfloor 1-\Phi\left(\alpha_g\sqrt{\frac{12}{n}}\right)\right\rfloor, \tag{20}$$

$$\alpha_g = \frac{GMm}{\hbar c} \tag{21}$$

$$\Phi(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}dt. \tag{22}$$

The thermal radiation energy

$$\hbar\omega = 0.8424 A^{1/4}(\alpha_g)mc^2. \tag{23}$$

So, we have

$$I = 5.75\cdot 10^{19}\, erg\, s^{-1} \text{ and } \hbar\omega = 515\ MeV$$

for $\alpha_g = \frac{1}{2},\ m = m_p.$

5. Conclusion

For the particles not satisfying the geometrical condition of graviatom existence, since their sizes exceed the gravitational radii of miniholes, there occurs a quantum accretion onto mini-holes which we have considered in the framework of E. Madelung's quantum hydrodynamics. The energies and intensities of the thermal radiation being due to the accretion of particles onto mini-holes exceed the corresponding values for the dipole radiation of the graviatoms providing the particles' sizes to be neglected.

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SPHERICALLY SYMMETRIC SOLUTION OF GRAVITATION THEORY WITH DESER–DIRAC SCALAR FIELD IN RIEMANN–WEYL SPACE

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According to the observational data of modern cosmology the dark energy (described by the cosmological constant) is of dominant importance in dynamics of the Universe. Therefore the major unsolved problem of modern fundamental physics is very large difference of around 120 orders of magnitude between a very small value of Einstein cosmological constant Λ, which can be estimated on the basis of modern observations in cosmology, and the value of the cosmological constant in the early Universe, which has been estimated by theoretical calculations in quantum field theory of quantum fluctuation contributions to the vacuum energy. The presence of a scalar field in the early Universe can change the default view on inflation and provide solution to the problem of the cosmological constant. The purpose of this paper is to obtain external spherically symmetric solutions with a scalar field to the central mass has no singularity, which is characteristic for the Schwarzschild metric but at large distances coincides with it, and so giving a prediction experimentally indistinguishable from the metric Schwarzschild.

1.Introduction

As a result of the Poincar'e-Weyl gauge theory of gravity developed in [1], space-time is endowed with a geometric structure of the Cartan-Weyl space with 2-forms of curvature $\mathcal{R}^a{}_b$ and torsion $\mathcal{T}^a$, 1-form of nonmetricity of the Weyl's type $\mathcal{Q}_{ab} = \frac{1}{4} g_{ab} {-} Q$, as well as an additional geometric structure in the form of the scalar Deser-Dirac field β. If torsion is zero, this space is called the Riemann-Weyl space.

2.General framework:spherically symmetric solution with scalar field for the central mass

Building on this approach, in [2] - [4] conformal theory of the gravitational field was built, in (5) the field equations were derived in the formalism of the outer forms. As a consequence of the equations in the spherically symmetric case [5], [6], we can set $\mathcal{T} = s \, \mathrm{d} \ln \beta$, $\mathcal{Q} = q \, \mathrm{d} \ln \beta$, where s and q –the constant to be determined.For the central mass m solution for the metric and the scalar field has the form (if $q = 4$, $s = 0$):

$$ds^2 = e^{-\frac{r_0}{r}} dt^2 - e^{\frac{r_0}{r}} (dr^2 + r^2 (d\theta^2 + \sin^2\theta \, d\phi^2)) \,, \qquad \beta(r) = \beta_\infty \, e^{\frac{k r_0}{r}} \,, \tag{1}$$

and under certain conditions on the coupling constants of the gravitational Lagrangian.

The metric (1) is known as the Yilmaz-Rosen metric [7],[8], first emerged in the original theory of gravitation of Yilmaz [2], in which it was postulated that the metric of Riemann space is a special function of the scalar field. The interest in the metric (1) aroused by the fact that this metric has no singularity, which is characteristic for the Schwarzschild metric, and does not describe a black hole type solution, but at large distances with $r_0 = r_g = 2Gm/c^2$ coincides with the Schwarzschild metric, hence, it gives the same predictions experimentally indistinguishable from the Schwarzschild metric. Therefore existence of the Deser-Dirac scalar field, which is essential, because it has a fundamental geometrical status, as well as the metric [1], under certain circumstances it can modify at short distances metric of black holes. From here there is a possible, the essential role of the field-Deser Dirac on extremely short distances, which can be manifested,for example, in determining the final stage of the collapse of massive stars. In the found solution scalar field decreases exponentially with distance from the central body and so concentrated near massive bodies. The scalar field Deser-Dirac is also as the "dark energy"the main component of "dark matter the need for the existence of which follows from the cosmological observational data.

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Radiation accelerated magnetic jets

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The model of radiation acceleration of jet in a magnetized channel over a thin accretion disk is constructed. The mathematical model consists of the radiative transfer equation and the system of equations of ideal MHD with the gravitational force of the central object, and the radiation pressure. The high energetic collimated outflow with bullet structure is obtained.

Introduction

The nature of compact objects jets acceleration and collimation is still a controversial area of modern astrophysics. Known a great amount of jets, most of them are observed in radio spectrum. Jet of M87 galaxy contains of fast-moving charged particles, which are concentrated to the bullets of up to 10 light years size. The flow has the form of a cone with an opening angle approx. $6°$. The matter speed in M87 galaxy jet reaches the value of $0.8c$, where c denotes luminal speed. The matter speed in SS433 jet is approx. $0.26c$. The characteristic properties of jets emerging in different star systems [1, 2], — collimation, high energetics, bullet structure — do not find a general explanation and are the consequences of various physical processes in nature.

We explore a mathematical model of the jet acceleration in the channel over the hot gravitating object under the influence of the radiation pressure of a thin disk surrounding the compact object. The model contains an ideal radiation MHD system of equations in a two-dimensional axisymmetric approximation and is based on the MHD model of the accelerating channel formation [6].

This work was partially supported by the RFBR (projects 12-01-00109, 12-02-00687, 12-01-31193) and Scientific school 1434.2012.2.

Model of the jet

To explain the different observed properties of jets different effects are used. The flow collimation is usually explained using MHD models [3, 5, 6]. Acceleration of matter is explained in different ways (e.g. [7]), but most consistent with the use of central object radiation pressure [8, 4].

In [6] it was shown that over gravitating object surrounded by a thin rotating disc and shipped to the cloud of accreting plasma the magnetized channel is formed (Fig. 1). The plasma outflow emerges within this channel, the source of matter for the outflow is the thin disk.

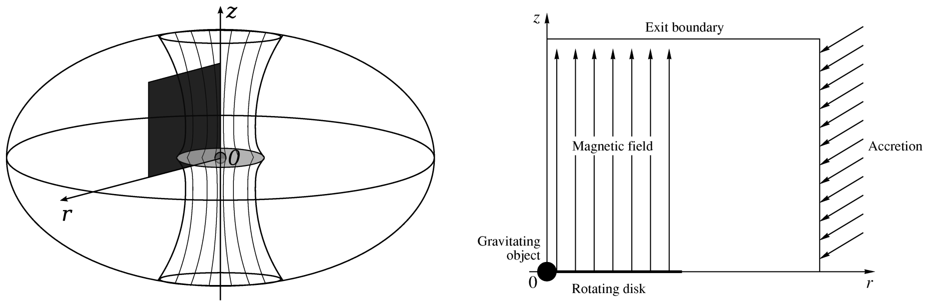

Fig. 1. Model scheme and computational domain

The channel has dense optically thick walls and the matter inside the channel is rarefied. The matter optical thickness (Thompson scattering) is

$$\tau = \sigma_T n L_0 \approx 6.7 \cdot 10^{-4}. \tag{1}$$

Here L_0 — space scale, n — matter concentration, σ_T — Thompson scattering cross section.

In [4] considered a zero-dimensional dynamic model of the solid-state bullet radiation acceleration in the channel with hot bottom. It is shown that the bullet reaches the subluminal speeds, up to $0.9c$.

We will use the model scheme shown on Fig. 1 and the [6] results as initial conditions. We suppose that:

1. the matter is a perfect inviscid perfectly-ionized gas;
2. the medium is perfectly conducting, the magnetic field is frozen into the thin disk and rotates with it, leading to collimation of the flow;
3. the gravitational field is determined by the gravity of the central body (star), self-gravitation of gas is not taken into account;
4. the radiation is considered in the approximation of the gray matter, the photons have single scattering by electrons (Thompson scattering); the source of radiation is only the thin disk, the radiation is focused into the acceleration channel.

Equations and conditions

The complete system of radiation MHD in mono-energetic quasi-stationary approximation has the form

$$\frac{\partial \rho}{\partial t} + \nabla \rho \mathbf{v} = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho \mathbf{v} + \mathbf{G}) + \nabla \cdot \left(\hat{\mathbf{\Pi}} + \hat{\mathbf{T}}\right) = \frac{1}{4\pi}(\nabla \times \mathbf{B}) \times \mathbf{B} + \mathbf{F}_g, \tag{3}$$

$$\frac{\partial}{\partial t}(e + U) + \nabla \cdot (\mathbf{v}(e+p) + \mathbf{W}) = \frac{1}{4\pi}((\nabla \times \mathbf{B}) \times \mathbf{B}) \cdot \mathbf{v} + \mathbf{F}_g \cdot \mathbf{v}, \tag{4}$$

$$\boldsymbol{\omega} \cdot \nabla I(t, \mathbf{x}, \boldsymbol{\omega}) + k(t, \mathbf{x}) I(t, \mathbf{x}, \boldsymbol{\omega}) = \beta(t, \mathbf{x}) \int\limits_{\Omega} \Gamma(t, \mathbf{x}, \boldsymbol{\omega}, \boldsymbol{\omega}') I(t, \mathbf{x}, \boldsymbol{\omega}')\, d\boldsymbol{\omega}', \tag{5}$$

$$\frac{\partial \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}), \tag{6}$$

where ρ is plasma density, $\mathbf{v}$ — speed vector, $\hat{\mathbf{\Pi}}$ — momentum flux density tensor, $\Pi_{ij} = p\delta_{ij} + \rho v_i v_j$, δ_{ij} — Kronecker delta, p — gas pressure, e — energy density, $\mathbf{B}$ — magnetic field, $\mathbf{F}_g$ — bulk density of the gravitational force, I — radiation intensity, $\Gamma(t, \mathbf{x}, \boldsymbol{\omega} \cdot \boldsymbol{\omega}')$ — the scattering function, $k(t, \mathbf{x})$ — extinction coefficient, $k = \alpha + \beta$, $\alpha(t, \mathbf{x})$ — absorption coefficient, $\beta(t, \mathbf{x})$ — scattering coefficient,

$$\mathbf{W} = \int\limits_{\Omega} \boldsymbol{\omega} I\, d\boldsymbol{\omega},\ \mathbf{G} = \mathbf{W}/c^2,\ U = \frac{1}{c} \int\limits_{\Omega} I\, d\boldsymbol{\omega},\ T_{ik} = \frac{1}{c} \int\limits_{\Omega} \omega_i \omega_k I\, d\boldsymbol{\omega}.$$

The gas is perfect, so $p = \rho\varepsilon(\gamma - 1)$, where ε is the specific internal energy of the matter,

$$e = \frac{\rho |\mathbf{v}|^2}{2} + \frac{p}{\gamma - 1}.$$

The scattering cross section is $\sigma_T = 6.652 \times 10^{-29}\mathrm{cm}^2$ ($\beta = n\sigma_T$).

The star with mass M is located in the axis origin, the gravitational field is $\boldsymbol{F} = -\mathcal{G}\frac{M\rho}{R^2}\frac{\mathbf{x}}{R}$, where $\mathcal{G}$ is the gravitational constant, $R = \sqrt{z^2 + r^2}$.

The boundary conditions are the following.

- On the outside cylindrical boundary the condition of the spherical supersonic nonmagnetized interstellar plasma inflow is given.
- The upper boundary models the transition to the regime the flow at infinity [9].
- On the axis of the rotation $r = 0$ the limited solution condition is imposed.
- The bottom boundary is divided into two parts. On the part $z = 0$, $r_d < r < r_M$, where r_d is the thin disk radius, the equatorial symmetry condition is put on. On the part $z = 0$, $0 < r < r_d$ the thin disk is modelled. The disk rotates with angulat speed $\omega(r) = \omega \cdot (1 - (r/r_d)^2)$.

The axial magnetic field $B_{z0}(r)$ is frozen into the disk. The disk is perfectly conducting and it is the source of matter for the outflow.

To obtain the boundary conditions for the radiation we assume that thin disk radiates as a black body with temperature $7 \cdot 10^4$ K, as it is observated in SS433. The radiation is focused into the channel, so $\cos\theta > 0.9$, where θ is the polar angle of light rays, coming from the disk.

Numerical scheme

To solve the system of equations splitting into physical processes is used. Recalculation of the unknown quantities in the difference cells at each time step consists of four steps.

1. Solution of the gas dynamic system of equations by a method of the Godunov type (HLLC) and accounting for the geometry of the problems and action of the gravity field as the terms in the right hand part [10].

2. Approximation of Faraday's law on the difference cell in the cylindrical system of coordinates matched with the problem geometry.

3. Radiation transport equation integration by the discrete ordinates method [11].

4. Recalculation of the gas variables accounting the electrodynamical and radiation forces.

For details see [6, 11].

Computational results

Calculations were made using the cluster K-100, KIAM RAS. Triangular meshes with 8000 cells were used.

The time step in the calculations was automatically selected depending on the speed of the plasma flow in the computational domain. In order to avoid the growth of instabilities Courant number was assumed to be $CFL = 0.05$.

The initial conditions for the calculations were the results of [6], namely the distributions of the variables at time $t = 18.05$. In the investigated area there is a magnetized channel inside which rarefied matter jet outflow moves in the positive direction of the axis Oz, the density of the material decreases with distance from the central region (Fig. 2).

Enabling the radiation field leads to an additional acceleration force acting on a rarefied matter, and the magnitude of this force is greater the closer to the source of radiation. Fig. 3 shows the initial distribution of the radiation intensity and the module radiation pressure force, acting on the outflow matter. The action of this force in the calculation results in that the speed of outflow

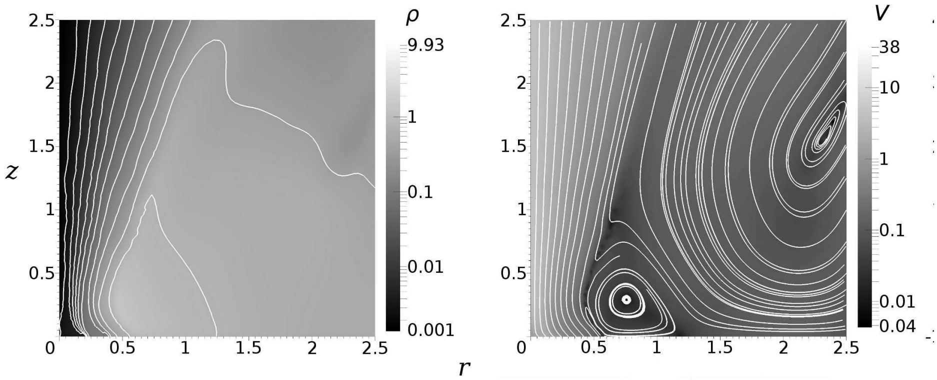

Fig. 2. Accelerating channel in [?]: distributions of density and speed at time $t = 15$ (steady regime)

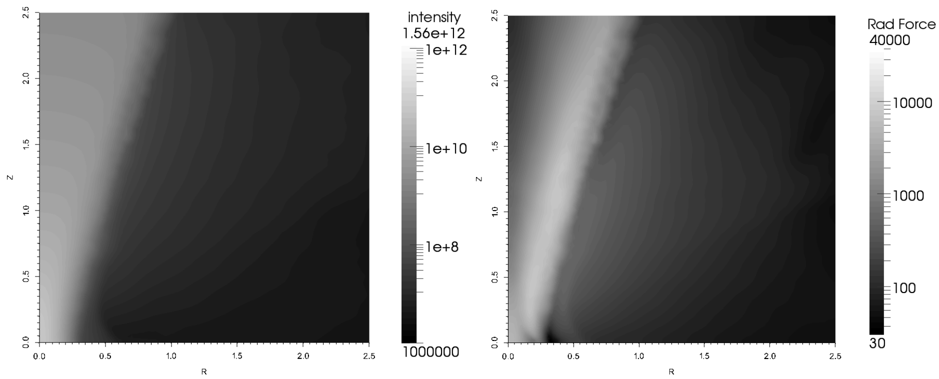

Fig. 3. Starting distribution of the intensity and radiation pressure force.

quickly reaches values of the order 10^3. The acceleration occurs primarily in the immediate vicinity of the central radiating object.

Radiation pressure force decreases with the distance from the radiation source, so that the lower (with respect to the axis Oz) layers are quickly accelerated, which leads to compression and high pressure in the outflow. Finally, at time $t = 18.055$ maximum flow speed reaches a maximum for the time of calculation and not further increased, the initial effect is passed, the outflow comes to the stable regime.

Fig. 4 shows the distributions of the flow variables along the Oz axis at the time $t = 18.075$. Fig. 5 shows distributions of the density and speed module.

The obtained speed of outflow is $1/6\,c$. The outflow is well collimated, the magnetic field in the channel retains its structure. The pressure and the density of the gas starting from some height of z falls with distance from the central object, so the outflow will not slow down out of the computational domain, and reaching the speed limit, will propagate maintaining high energy in a

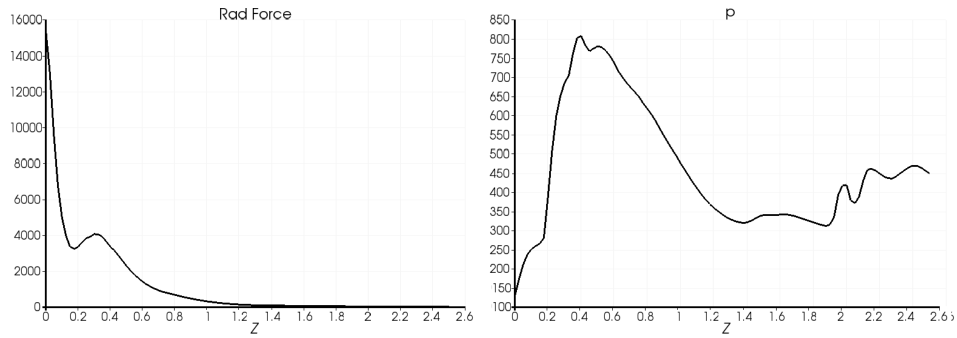

Fig. 4. Distribution of the intensity radiation pressure force and pressure along the Oz axis.

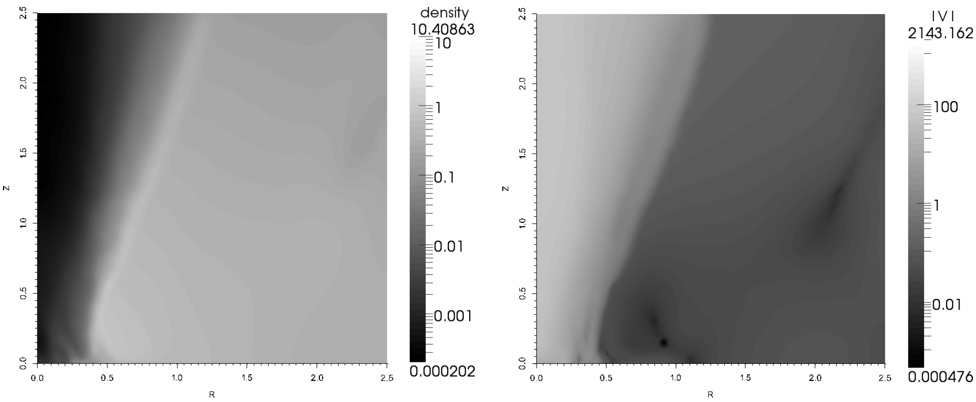

Fig. 5. Distribution of the density and speed module.

rarefied media . At the same time, the maximum pressure falls on the neighborhood of the plane $z = 0.4$, i.e. on top of the channel narrow. Such distribution of pressure gradient along the Oz axis can lead to the effect of matter locking in the vicinity of the central object if the radiation pressure force is insufficient to overcome this barrier.

Particular interest is the steady periodic regime of outflow. In this regime the average high speed of the matter is preserved, but the periodic bursts of flow speed are observed. Bursts period is $T_b = 0.01$, it coincides with the time the plasma needed to get away from the central object and overcome narrowing of the acceleration channel, coming on a steady speed.

The system goes to the oscillation regime immediately after the establishment of flow speed, when the first portion of the accelerated by radiation plasma leaves the computational domain. After the departure of this portion a region of rarefied matter over the thin disk is formed. As it was pointed out, in the channel narrow over the radiating central object the barrier consisting of a high-pressure plasma is produced. It partially locks the matter inside the cavity. The mass flux from the disk is bounded below by maintaining plasma density at least equal to ρ_w, so that the mass flow from the disk is greater than the mass flow through the barrier in the channel narrow.

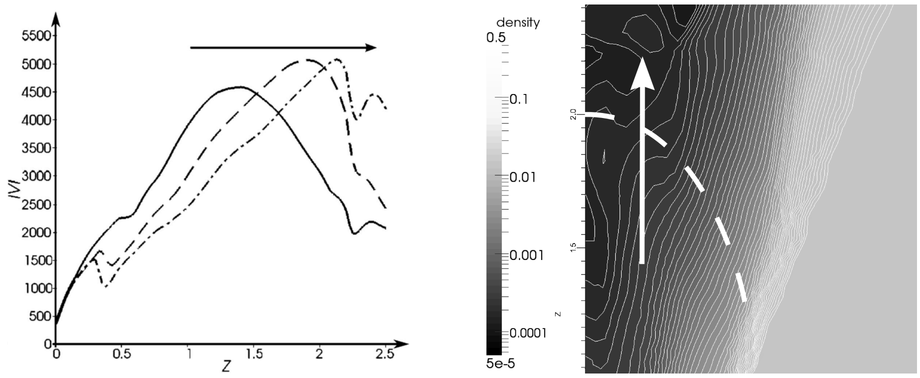

Fig. 6. Formation of bullet — speed $r = 0$ section and density distribution near the front.

This leads to the gradual accumulation of the matter in the cavity and subsequently its launch by increasing the strength of the radiation pressure. Cycle of the resulting mechanism action is as follows.

1. Over the thin disk there exists rarefied cavity, locked on the top by the pressure barrier of the previous burst. Due to the predominance of the mass flux from the disk on the mass flux through the barrier cavity starts to be filled with the plasma and becomes opaque to the radiation of the central object.

2. Increase of the density of the gas in the cavity leads to an increase in the radiation pressure force on the order. Impact of the radiation pressure is greater the closer the substance is to the central object. The matter in the cavity starts to shrink below the radiation pressure, the gas pressure in the cavity grows.

3. The gas pressure in the cavity exceeds the gas pressure in the channel narrow barrier. The matter accumulated in the cavity forms a quickly moving up the Oz axis hot bubble. The burst of speed is produced while it passes the channel narrow and grows further by the bubble expanding.

4. The bubble comes to the expanding part of the channel, its density falls with the expansion of the channel, the speed continues to increase, in the speed distribution front burst is formed, the lower layers of gas move faster, tapering the front (see Fig. 6). Next to the bubble, a region of a dilute gas of a low pressure, pressure barrier in channel narrow is closed, the accumulation of a new portion of matter begins.

5. At the exit of the computational domain bubble front forms a shock wave which rakes over the cold slowly moving gas from the background flow in the channel. Formed bullet moves at speeds of the order of $1/6c$. The bullet continues to expand and leaves the computational

domain. The cavity of the central object comes filled with the plasma flowing from the thin disk.

Discussion

Let us now turn from the computational results in dimensionless form to estimating values of the dimensional variables of the problem. We will assume that the bulk of the matter in the accretion flow is molecular hydrogen. The dimensionless parameters can be written in Gaussian units as follows: concentration $n_0 = 10^8$ cm^{-3}; density $\rho_0 = 3.34 \times 10^{-16} \frac{\text{g}}{\text{cm}^3}$; space scale $L = 10^{15}$ cm; magnetic field $B_0 = 0.06$ Oe; central body mass $M = 3M_\odot = 6\times10^{33}$ g; time scale $t_0 = 1.12\times10^9$ s; speed scale $V_0 = 0.9 \times 10^6$ cm/s.

The calculations model the outflow from forming protostar with a mass $M = 3M_\odot$. Protostar is surrounded by the disk with radius $r_d = 0.6L_0 \approx 40$ AU A supersonic flow of matter is accreted onto the system at the rate $\sim 5\times10^{-5}$ $M_\odot$/year. Central area of accretion system radiates, so that radiation intensity is similar to black body radiation at temperature 7×10^4 K. Above the star with a magnetized disk the sparse channel is formed, source of matter ios the disk. The radiation of the central object is focused within the channel. A collimated outflow is formed perpendicular to the equatorial plane of the disk and is accelerated by the radiation pressure. Outflow speed oscilates and average value is 2×10^4 km/s, jet openning angle is $\sim 10°$.

The flow consists of individual bullets moving with a greater speed than the background flow. Bullets spped maximum is 5×10^4 км/с. Bullets period of formation is 13 days.

Note that in contrast to the model [4], a bullet is not presented inside the channel before accelerating. In our model, like it was in [12], the bullet is formed in a continuous flow inside the accelerating channel. The resulting jet is stable since the lower limit for mass flow from the disk is provided. Collimation of the jet is provided by the axial magnetic field and the presence of the chanel walls of soft spiral swirl magnetic field.

Conclusions

We have constructed a model of radiation acceleration of matter in the channel above a magnetized thin ideally conducting disk. The mathematical model of the system shown in Fig. 1 is based on the radiative transfer equation and the system of equations of ideal MHD with the gravitational force exerted by the central object and the radiation pressure. For the numerical solution of equations applied modification algorithms developed and discussed in [6, 11]. The numerical methods (finite difference scheme for a system of ideal MHD equations and the method of discrete directions for RTE) adapted for the calculations in a cylindrical coordinate system and implemented in a software package for SMP-machines with graphics accelerators.

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Experimental investigations of polarization of laser radiation in a rotating optical disk

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Results of experimental investigations for the dependence of polarization of coherent optical radiation with the wave length $\lambda = 0{,}632991 \mu m$ in a rotating optical disk on rotating frequency are presented in the work.

Introduction

The first investigations for the dependence of light polarization on medium motion were carried out by H.Lorentz [1] and A.Fizeau [2].

The effect of turn of a polarization plane of an electromagnetic wave, which is normally incident to a rotating dielectric, was predicted in the work [3]. The angle of polarization turn is linear dependent on frequency of a rotating dielectric and an optical path.

The estimation of the effect for the normal beam incidence on the surface, where velocity of a rotating dielectric has tangential break, was obtained in the work [4]. The angle of polarization turn is equal to

$$\Delta\varphi = \left(n_g - \frac{1}{n_\varphi} \right) \frac{\omega l}{c}, \tag{1}$$

where n_g and n_φ are refraction indexes for group and phase velocities of light, l is optical path length, ω is frequency of a rotating dielectric cylinder, c is the light velocity in vacuum.

The experimental test of the formula (1) was fulfiled in the work [5], where laser radiation passed through the cylindrical rod with length 100 mm and diameter 20 mm. The rod was manufactured from dense flint glass with the refraction index 1,840 for $\lambda = 632{,}8$ μm and was rotated with the speed up to 6000 revolutions per minute (rpm). The angle of polarization turn was equal to $\Delta\varphi = 2 \times 10^{-6}$ rad, when rotating frequency was 100 Hz.

The articles [6] - [8] was also devoted to researches of the effect of polarization plane turn in a rotating optical medium.

We usually consider that a rotating optical transparent cylinder or a disk are homogeneous and isotropic [9-13]. Hence, anisotropy of optical properties in the disk can appear as a result of medium motion. Our experiments demonstrate the nonlinear dependent on frequency of the rotating disk.

Experimental set

In the work we used the laser radiation with wave length $\lambda = 0{,}632991$ μm in the rotating glass disk with refraction index $n = 1{,}71250$ when the incident angle to a flat surface of the disk was $\vartheta_0 = 60^\circ$. The optical disk (OD) had diameter $D = 62$ mm, flat surfaces of OD was metallic reflecting covering. The projection of geometric path length of a beam in a medium on a flat surface of OD was equal to $l = 41$ mm.

The optical scheme is presented in the figure 2. A horizontal or vertical linear-polarized component of light was selected in dependence on the position of the polarizer $P1$.

The laser radiation passed through the optical system $OS1$ and was divided with the flat beamsplitter BS into 2 beams. The beams reflected from the mirrors $M1$ and $M2$, passed through the optical disk OD. After going out the ring optical system the beams passed through the polarizer $P2$, the optical system $OS2$, and was incident to the photodetector PD.

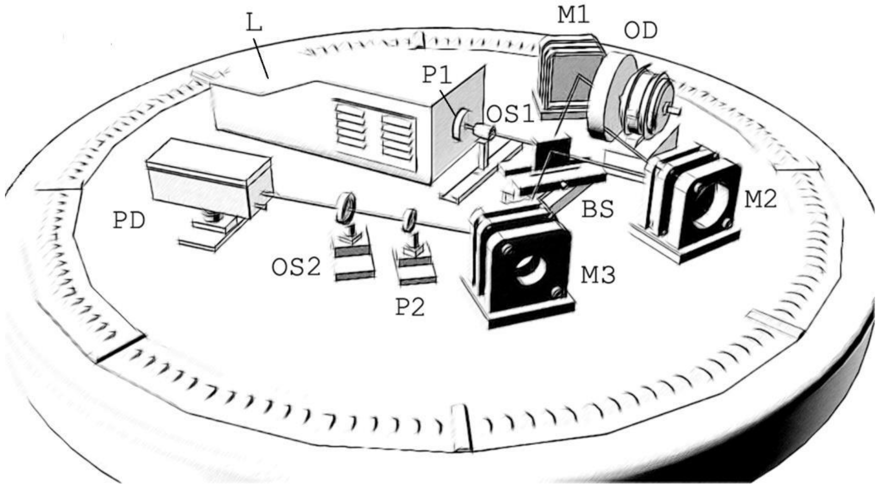

Fig.2. Scheme of experimental set

Intensity of light on PD altered after rotation started. By turning $P2$ we could obtain the dependence of a turn angle of a light polarization plane on frequency of the rotating OD.

Dependence of PD signal amplitude on frequency of the rotating disk OD

In the experiment we measured voltage on PD after turn of a polarization plane in OD for different rotating frequencies of the optical disk and also an angle of turn of a polarization plane. Initially we found the angle of polarization plane turn for the polarizer $P2$, where passing light had minimal intensity.

After beginning rotation of dielectric the signal on the photodetector increased due to the polarization plane turn. To define the angle of polarization plane turn $\Delta\varphi$, the polarizer $P2$ was turned so that light intensity becomes minimal that. Measurements were made for frequencies $f = 100 \ldots 220$ Hz. The results are presented in the figures 3 and 4.

It was very important that time between two neighbour measurements for different frequencies should be 1-2 minutes in order to the optical disk reached the steady state of rotation.

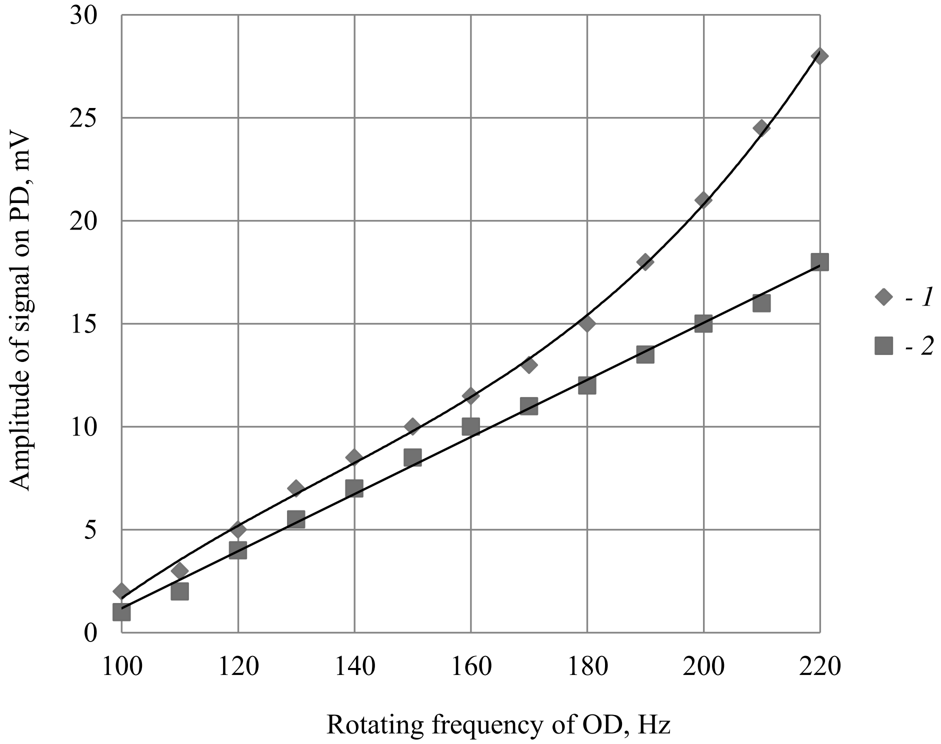

Fig.3. Dependence of voltage of photodetector on rotation frequency of disk for horizontal light polarization (1 – for initial state of polarizer, 2 – after turn of polarizer when signal is minimal).

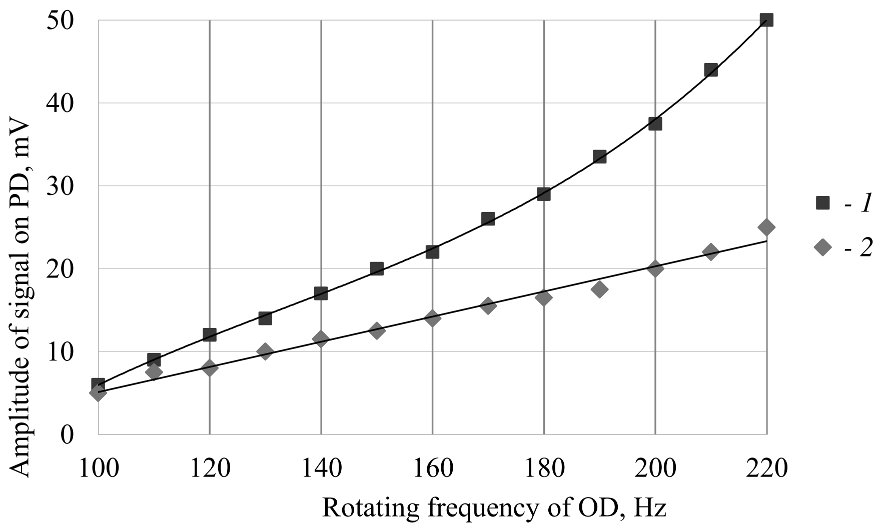

Fig.4. Dependence of voltage of photodetector on rotation frequency of disk for vertical light polarization (1 – for initial state of polarizer, 2 – after turn of polarizer when signal is minimal).

Turn of light polarization in a moving medium

We observed a transitional process of polarization plane turn for time. The process can be revealed due to drift of signal amplitude on PD when a rotating frequency f is fixed.

To illustrate the phenomenon we watched an angle of beam polarization plane turn and ellipticity for time. The experiment was carried out when a rotating frequency was $f = 150$ Hz, and then it was repeated for different frequencies.

Measurements were reiterated in every 5 minutes. Results of measurements for 140 minutes are presented in the figures 5, 6.

From the figure 5 (diagram 1) it follows that in 20 minutes after beginning the measurements the signal amplitude in the top of a time response exceeds the value 30 *mV*. The value exponentially reaches 36-37 *mV* after 2 hours measurements.

From diagrams it follows that after 20 minutes measurements the angle $\Delta\varphi$ riches satiation and then don't alter its value. Further measurements showed that changing the light signal amplitude occurs due to angle drift of a beam.

Also we found non-linear dependence of signal amplitude because of depolarization that is shown in the figure 6.

As it is seen in the figure, the signal, conditioned with the depolarization phenomenon, firstly increased till some value and then decreased. We detected the effect is observed for frequencies less 60 Hz. For the frequency region 70-200 Hz it was only observed exponential growth of voltage on photodetector.

Also we discovered that dielectric has dynamic memory. When rotation of optical disk was stopped light polarization turns to the initial state in 30-40 minutes.

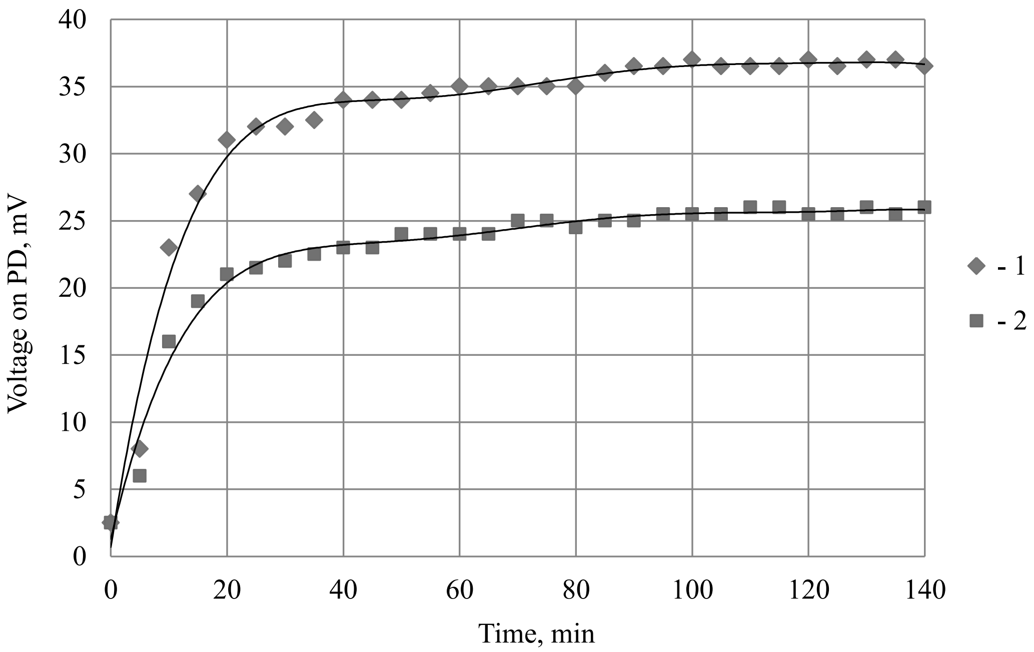

Fig. 5. Dependence of PD voltage amplitude in maximum on time with rotating frequency 150 Hz. (1 – for initial state of polarizer, 2 – after turn of polarizer when signal is minimal).

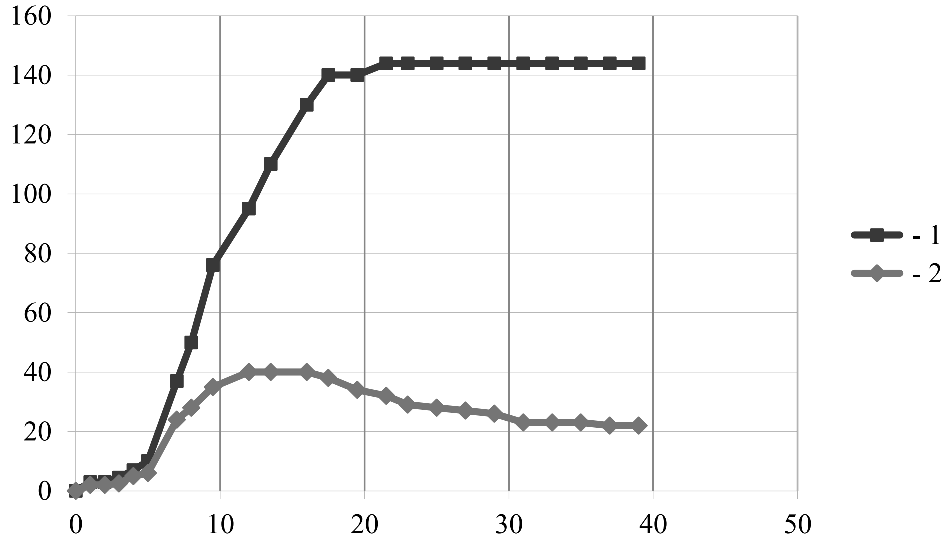

Fig. 6. Dependence of PD voltage amplitude in maximum on time with rotating frequency 50 Hz. (1 – for initial state of polarizer, 2 – after turn of polarizer when signal is minimal).

Dependence of the angle of polarization turn on frequency of the rotating disk OD

In further experiments measurements of intensity carried out for each beam separately. At first we observed transitional process for fixed rotating frequency of OD, during the process the signal amplitude on PD had exponential growth. Then, after the signal amplitude reached its maximum, we turned a polarizer so that the signal had minimum value after them. Then, the disk stopped and the minimum of the amplitude gradually recurred to the starting state.

Results of measurements are shown in the figure 7 for the frequency region $f = 3 - 200$ Гц.

Its follows from the figures that the polarization plane turns occur a larger angle if rotating frequencies are low.

For low rotating frequencies of OD (20-30 Hz) the angle of a polarization plane turn grows up to 90 grad.. For the rotating frequency 3 Hz the angle of a polarization plane turn reaches $\Delta\varphi = 70^{\circ}$ for the vertical component of polarization on a laser exit. If we increase the rotating frequency till 100 Hz the angle decreases to $\Delta\varphi = 10 - 20^{\circ}$. After this we can watch slow increasing the turn angle when the rotating frequency of OD grows.

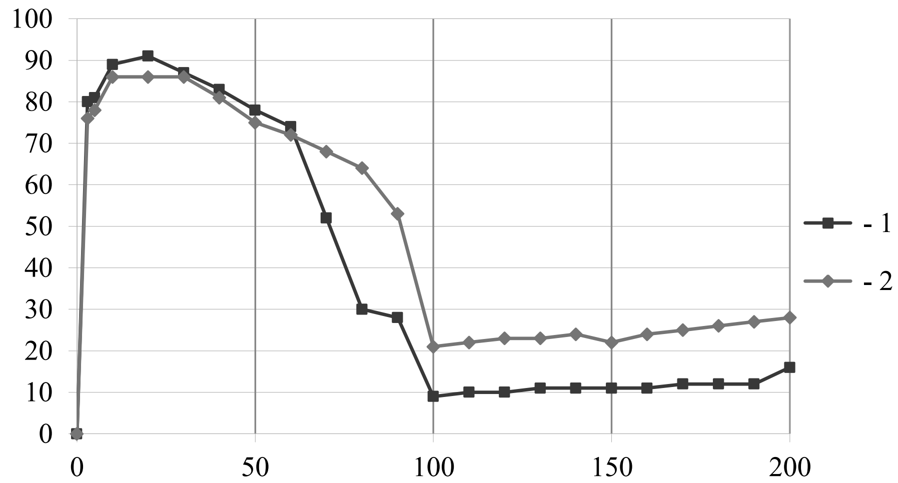

Fig. 7. Dependence of angle of polarization plane turn on rotating frequency of OD for vertical (diagram 1) and horizontal (diagram 3) polarizations on laser exit.

There are angles of a polarization plane turn $\Delta\varphi_i(f_i)$ about 70° for frequencies 2-5 Hz.

Thus, from the figure 7 we can observe two effects. We can see large angles of a polarization plane turn in a low frequency region, essentially nonlinear depending on rotation frequency. The dependence has about linear character in the region (100-200Hz).

The second effect is possible influence of elastic deformation, which is considerable for the rotating frequencies and can leads to the effect close to the photo elasticity phenomenon.

But using coherent sources in the experiments doesn't allow for dispersion to influence to results of interference pattern shift. For values: $\omega = 630$ rad/s, n=1.4766, λ=0.6328 μm, R_0 =0.06 *m*, $\vartheta_0 = 30°$, and for laser with wave length difference $\Delta\lambda = 0.01$ μm altering of a refraction angle doesn't exceed the value $\Delta\vartheta_2 = 10^{-8}$ °.

Conclusion

In the investigations the process of turn of light polarization plane was found out, time of the process lasts 15-20 minutes for region of frequencies from 0 to 200 Hz. Linear polarized light after passing through the rotating optical disk OD becomes as elliptic polarized light, and degree of polarization decreases if rotating frequency of OD has growth. The plane turn of light polarization depends on the rotating frequency.

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About opportunity of dark matter elemental particles generation and detection

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In this paper the modern opportunities for elemental particles (axions and paraphotons) of dark matter detection is analyzed. According to the theory predictions these particles have very small rest mass, corresponding to the energy value 0,001-1,0 meV. The experimental conditions of visible range laser emission conversion into axions and paraphotons reemission and opposite processes under observation of Primakov effect ("Light Shining Through Wall") are analyzed. Experimental schemes for investigation of axions and paraphotons decay processes resulting pairs of entangled photons of microwave region have been proposed..

Introduction

Relativistic field theory is based on continuous model of physical vacuum, possessing of isotropy and homogeneity properties. So relativistic effects are the consequence of space-time homogeneity and isotropy. Euclid space-time is invariant under transitions from one inertial coordinates system to another with t=t' condition. Minkovsky space-time has invariance property under Lorenz coordinate-time (x,t) transformations (under hyperbolic rotations in x,t space-time), satisfying to relation:

$$c_0^2 t^2 - x^2 = Const \tag{1}$$

Modern theory of media [1,2] is based on using of different microscopic models: atomic and molecular structures, crystalline lattice structures and so on. According to the known idea, physical vacuum is also some type of medium. So we have come to conclusion that physical vacuum should also have some microstructure.

Microstructure of physical vacuum is difficult for experimental observations because such microstructure may be revealed only at the very small scale, corresponding to very high energies.

We have proposed [3-6], that physical vacuum has discrete microstructure, constructed from the very small size highly packed particles. In this paper we develop the idea, proposed before in our works [3-6], that physical vacuum is some type of crystal, composed from maximons – particles, having the largest rest mass (10^{-5} g) and extremely small rest size (10^{-33} cm). In this article we analyze the properties of initial physical vacuum microstructure - very hot and compressed crystalline media, emerging as a result of gravitational attractions between maximons. We describe the properties of physical vacuum after ferroelastic phase transition, taking place as a result of initial physical vacuum cooling and superstructure forming with point density deviations. The aim of this work is the description of physical vacuum quasiparticles spectra and searching of experimental ways for scalar bosons (elemental particles of dark matter) observation.

Microstructure of initial physical vacuum

Dimension considerations with using of fundamental constants value ($\hbar$=1,05x10^{-34} kgm^2/s ;

c_0=2,99x10^8m/s; G=6,67$10^{-11}$$m^3$/kgs^2) result in the following crystalline lattice constant value(length of Plank):

$$a_0 \sim \sqrt{\frac{G\hbar}{c_0^3}} \approx 10^{-35} m = 10^{-33} cm. \tag{2a}$$

The value of maximon mass we can estimate from the Compton relation:

$$m_M = \frac{2\pi\hbar}{a_0 c_0} \sim 10^{-5} g. \tag{2b}$$

According to the modern picture of physical vacuum evolution, at the first stage the physical vacuum presented itself very hot and compressed medium. Such property was the result of gravitational attraction between maximons. So the phase state of initial physical vacuum corresponds to the very dense ($\rho \sim \frac{m_M}{a_0^3} \sim 10^{94} \frac{g}{cm^3}$) medium with large dynamic fluctuations of maximon. For the moment of the largest physical vacuum contraction we approximate corresponding microstructure as crystalline lattice with a_0=10^{-33}cm lattice constant. One dimensional model of such type media may be approximated by monoatomic crystalline chain, unstable relating to the longitudinal acoustical mode but stable relating to transversal one, resulting ferroelastic phase transition. In this case the equations of motion with taking into account only nearest neighbors may be presented as:

$$m_M \ddot{w}(l) = -\gamma_T[2w(l) - w(l-1) - w(l+1)], \gamma_T > 0;$$
$$m_M \ddot{w}(l) = -\gamma_L[2w(l) - w(l-1) - w(l+1)], \gamma_L < 0. \tag{3}$$

Here w(l) is the deviation of the particle with number l from its equilibrium position. With using of flat monochromatic transversal waves $w(l) = w_0 \exp i(kla_0 - \omega t)$ as decision of (3), the dispersion law for (3) equation may be presented as:

$$\omega = 2\frac{c_0}{a_0}\sin\frac{ka_0}{2}. \tag{4}$$

where. $c_0^2 = \frac{\gamma_T}{m_M}.$ At small wave vector value we have from (4) the known dispersion law for transversal electromagnetic waves in physical vacuum:

$$\omega = c_0 k. \tag{5}$$

From using of (4) we conclude, that there is the difference between phase V_{ph} and group V_{gr} velocities of electromagnetic waves in physical vacuum:

$$V_{ph} = \frac{2\frac{c_0}{a_0}\sin\frac{ka_0}{2}}{k}, V_{gr.} = c_0 \cos\frac{ka_0}{2}. \tag{6}$$

At large enough k-value V_{gr}–velocity becomes smaller c_0 and go to zero at $k \to \frac{\pi}{a_0}$.

Longitudinal acoustical waves are unstable at linear approximation. Such property is the consequence of very large density fluctuations in initial physical vacuum.

Microstructure of physical vacuum after ferroelastic phase transition

As a result of physical vacuum cooling and its instability relating to longitudinal acoustical mode, the new phase state is realized. Initial physical vacuum microstructure becomes space-ordered superstructure, formed by point defects (density deviations).

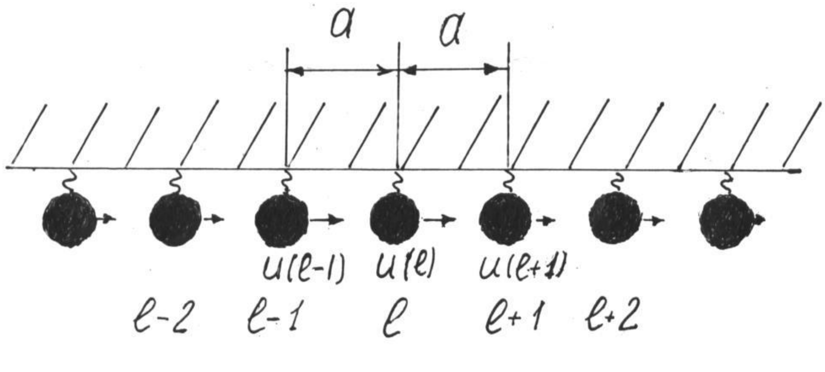


Fig.1. Monoparticle chain with additional bonds.

Lattice constant of such superlattice may be evaluated as a~10^{-15}cm (a>>a_0~ 10^{-33} cm) according to weak interaction distance value. The new phase state may be classified as photonic structure [7-10], (photonic crystal or photonic glass) , constructed from quantum dots with size about D~10^{-15}cm. Accordingly the average distance between density defects in such photonic structure should be about 10^{-15}cm.

At the first approximation the dynamics of the new phase state of physical vacuum may be described with the help of monoparticle crystalline chain[3-6] with additional bonds model using (see Fig.1).

In Fig.1 the dark circles correspond to the density defects and u(l) is the deviation of this point defect with l-number from the equilibrium position. Additional bonds correspond to the interaction of defect with the rest part of initial lattice; M is the effective mass of the defect. Taking into account only nearest neighbor interaction between defects we have the following law of motion:

$$M\ddot{u}(l) = -\chi_{0T}u(l) - \chi_T[2u(l) - u(l-1) - u(l+1)] \qquad (7a)$$

$$M\ddot{u}(l) = -\chi_{0L}u(l) - \chi_L[2u(l) - u(l-1) - u(l+1)]. \qquad (7a)$$

Here u(l) is the translational deviation of the particle (defect) from the equilibrium position. In three-dimensional case such deviations correspond to vectors objects.

For flat monochromatic waves $u(l) = u_0 \exp i(kla - \omega t)$ we have the following dispersion law for transversal and longitudinal waves :

$$\omega^2(k) = \frac{\chi_{0T}}{m} + 4\frac{\chi_T}{m}\sin^2\frac{ka}{2}; \chi_{0T} = 0; \chi_T > 0; \qquad (8a)$$

$$\omega^2(k) = \frac{\chi_{0L}}{m} + 4\frac{\chi_L}{m}\sin^2\frac{ka}{2}; \chi_{0L} < 0; \chi_L = \chi_L = \chi > 0. \qquad (8b)$$

In relation (8b) we takes into account the instability of lattice according to longitudinal waves. Thus, equations 8(a,b) may be presented as :

$$\omega^2(k) = 4\frac{c_0^2}{a^2}\sin^2\frac{ka}{2}; \quad \frac{c_0^2}{a^2} = \frac{\gamma_T}{m}; \tag{9a}$$

$$\omega^2(k) = -\omega_0^2 + 4\frac{c_0^2}{a^2}\sin^2\frac{ka}{2}; \quad \frac{c_0^2}{a^2} = \frac{\gamma_L}{m}; \; \omega_0^2 = -\frac{\chi_{0L}}{m}. \tag{9b}$$

At small wave vector k-value instead of 9(a,b) we have dispersion laws for transversal and longitudinal waves (continuum approximation):

$$\omega = c_0 k; \tag{10a}$$

$$\omega^2 = -\omega_0^2 + c_0^2 k^2. \tag{10b}$$

Relation 10(a) coincides with the known equation of relativity theory for dispersion law of transversal electromagnetic waves in physical vacuum. Relation 10(b) is also known in relativity theory and corresponds to so call tachyon waves, group velocity of which is always greater than velocity of light c_0 –value:

$$\frac{d\omega}{dk} = \frac{c_0^2 k}{\sqrt{-\omega_0^2 + c_0^2 k^2}} > c_0 \; . \tag{11}$$

Note, that quantum dots with finite diameter ($D=10^{-15}$ cm) have additional degrees of freedom s(l), corresponding to scalar (breathing type) mode, pseudoscalar mode and others. Scalar mode may be described by analyzing of one-dimensional monoparticle crystalline chain dynamics. Corresponding equation of particle moving may be presented as following:

$$\mu\ddot{s}(l) = -\gamma_{0s}s(l) - \gamma_s[2s(l) - s(l-1) - s(l+1)], \tag{12}$$

By using the decision of (12) in the shape of flat monochromatic waves we obtain dispersion law for scalar waves:

$$\omega^2(k) = \frac{\gamma_{0S}}{\mu} + 4\frac{\gamma_s}{\mu}\sin^2\frac{ka}{2} = \omega_{0S}^2 + 4\frac{c_0^2}{a^2}\sin^2\frac{ka}{2} \tag{13}$$

At small wave vector k-value instead of (13) we have the following dispersion laws for scalar waves (breathing mode in continuum approximation:

$$\omega^2(k) = \omega_{0S}^2 + c_0^2 k^2. \tag{14}$$

For three-dimensional microstructure of real crystal there are three acoustical branches, corresponding to transversal and longitudinal waves, and optical branches due to additional degrees of freedom. In the case of physical vacuum media we also should wait for three acoustical branches (transverse and longitudinal waves) and, besides these, additional branches, corresponding to scalar and vector (high energies) bosons (see Fig.2).

Under quantum description of lattice vibration every point of dispersion branches $\omega_j(\vec{k})$ with number j corresponds to some type of quasiparticles: acoustical or optical phonons. Energy E and momentum $\vec{p}$ of such quasiparticles satisfies to the known relations:

$$E = \hbar\omega; \vec{p} = \hbar\vec{k}. \tag{15}$$

On the other hand, quasiparticles may be considered also as some classical particles. For one dimensional case the corresponding Hamilton equations for such particles are:

$$\frac{dx}{dt}=\frac{\partial E}{\partial p};\ \frac{d^2x}{dt^2}=\frac{d}{dp}\frac{\partial E}{\partial p}\frac{dp}{dt}=\frac{\partial^2 E}{\partial p^2}\frac{dp}{dt}=\frac{1}{m_{ef}}\frac{dp}{dt} \qquad (16a)$$

$$\frac{dx}{dt}=-\frac{\partial E}{\partial x};\ m_{ef.}=\frac{1}{\frac{\partial^2 E}{\partial p^2}}=\frac{\hbar}{\frac{\partial^2 \omega}{\partial k^2}}. \qquad (16b)$$

So if the dispersion law ω(k) is known, we may calculate velocity $V=\frac{dx}{dt}=\frac{d\omega}{dk}$ and effective mass [11-13] of such quasiparticles:

$$m_{ef}=\frac{1}{\frac{d^2E}{dp^2}}=\frac{\hbar}{\frac{d^2\omega}{dk^2}}. \qquad (17).$$

Accordingly, the equations (9,10; 13,14) may be presented as:

$$E^2(p)=4\frac{c_0^2}{\hbar^2a^2}\sin^2\frac{pa}{2\hbar};\ \frac{c_0^2}{a^2}=\frac{\gamma_T}{M}; \qquad (18)$$

$$E^2(p)=-E_0^2+4\frac{c_0^2}{\hbar^2a^2}\sin^2\frac{pa}{2\hbar};\ -E_0^2=\frac{\hbar^2\gamma_{0T}}{M}; \qquad (19)$$

$$E^2(p)=E_{0S}^2+4\frac{c_0^2}{\hbar^2a^2}\sin^2\frac{pa}{2\hbar};\ E_{0S}^2=\frac{\hbar^2\chi_{0S}}{\mu}; \qquad (20a)$$

$$E^2(p)=E_{0S}^2+c_0^2p^2. \qquad (20b)$$

Fig.2 illustrates the spectra of discussed quasipartickles. Dotted lines correspond to the photons, predicted by the known field theory (continuum approximation). Solid curves corresponds to quasipartickles of superlattice structure of physical vacuum, appeared after ferroelastic phase transition and Brillouin zone folding.

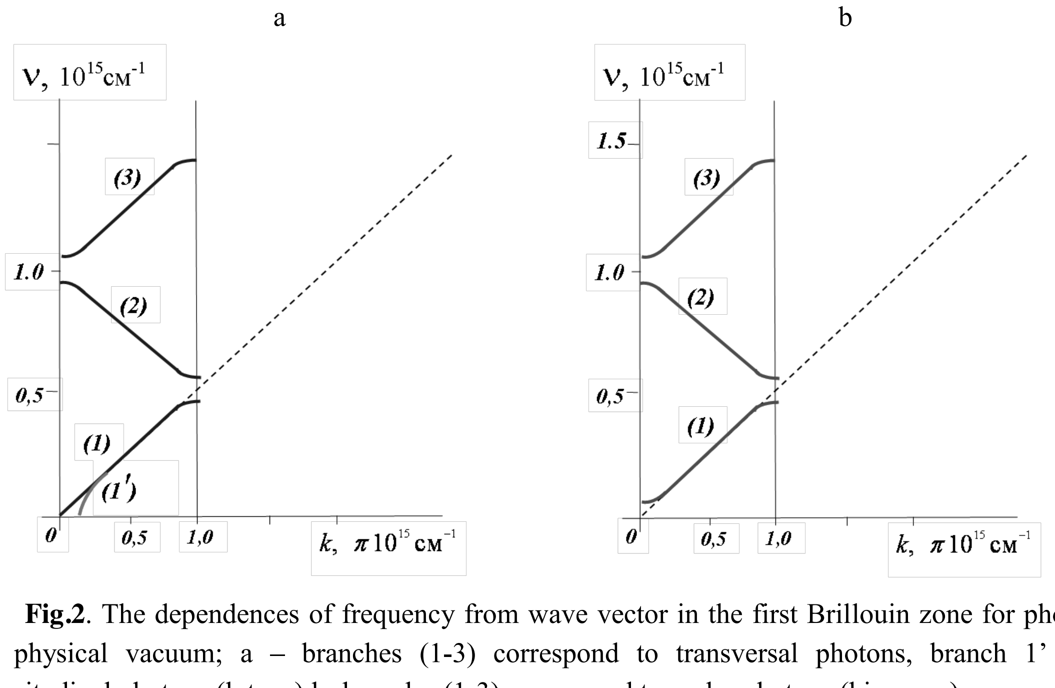


Fig.2. The dependences of frequency from wave vector in the first Brillouin zone for photons in physical vacuum; a – branches (1-3) correspond to transversal photons, branch 1' - to longitudinal photons (lotons);b- branches(1-3) correspond to scalar photons (higgsons).

Fig2a corresponds to transversal and longitudinal photons (“lotons”) and vector bosons (curves 2 and 3 at high energies); Fig2b illustrates the dispersion branches, corresponding to scalar photons (bosons), named also as: paraphotons, boundtons, hidden photons, hitons, or darktons. Scalar bosons have nonzero rest mass and are candidates on elemental particles of dark matter.

Laser excited photon-boson conversion in media and in physical vacuum

The examples of photon - boson conversion in media are well known Raman scattering (RS) processes in crystals [14,15]. If the intensity of excited emission is no very large, so called Stokes and anti – Stokes spontaneous *RS (SSRS, ASRS)* - processes on optical phonons take place. Fig. 3 illustrates SSRS (3a) and *ASRS* (3b) elementary processes diagrams .

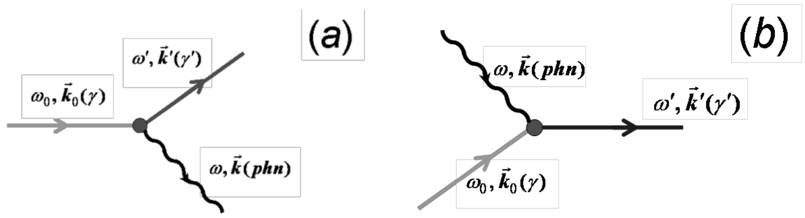


Fig. 3. Elemental Raman processes with taking part of optical phonon (photon-optical phonon conversion) ; a -Stokes, b –anti-Stokes processes.

Stokes *RS* - process corresponds to the decay of exciting photon with energy $\hbar\omega_0$ into another photons(scattered one) with energy $\hbar\omega'$ and quasiparticle - optical phonon with energy $\hbar\omega$. So in this case the energy of photon, resulting from elemental Raman process, decreases with comparing to the initial photon energy. For Stokes (Fig.3a) and anti-Stokes (Fig.3b) *RS* elemental processes the energy and momentum conservation laws should take place. For example, for Stokes elemental RS process we have :

$$\begin{aligned} \hbar\omega_0 &= \hbar\omega' + \hbar\omega \\ \hbar\vec{k}_0 &= \hbar\vec{k}' + \hbar\vec{k} \end{aligned} \tag{21}$$

Here $\hbar\omega_0$, $\hbar\omega'$, $\hbar\omega$ are initial and scattered photon and appeared phonon energies; $\hbar\vec{k}_0, \hbar\vec{k}', \hbar\vec{k}$ - the corresponding moments. If anti-Stokes *RS* takes place (Fig. 3b), the corresponding conservation laws are :

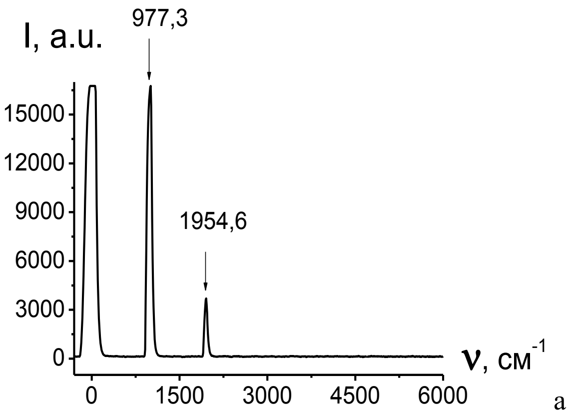
$$\begin{aligned} \hbar\omega_0 + \hbar\omega &= \hbar\omega' \\ \hbar\vec{k}_0 + \hbar\vec{k} &= \hbar\vec{k}' \end{aligned} \quad . \tag{22}$$

To now a lot of experiments on spontaneous *RS (SRS)* on optical phonons in different crystals has been fulfilled: in sulfur, barium nitrate, calcite, organic crystals and so on. The main problem of SRS is the very small intensity with comparing to excited emission (as a rule I_{SRS}~10^{-6} I_0). So for recording of SRS spectra it is needed to use unique spectrometer with very sensitive detector. As a rule in SRS the largest RS intensity corresponds to total symmetric optical mode, i.e. to scalar boson (optical phonon). When the intensity of excited emission is very large (about 10^8 W/cm^2) the intensity of Raman line, corresponding to" photon- scalar phonon" conversion process, very sharply increases and becomes comparable to the intensity of exciting line emission. Such effect is known as Induced (Stimulated) Raman Scattering (IRS).

In common case the total probability of RS in crystals for Stokes RS $W^{(S)}_{n_s+1;m_i+1}$ (rate of the scattering process, measured in 1/s) can be written as [14,15]:

$$W^{(S)}_{n_s+1,m_i+1} = (n_s+1)(m_i+1)W_i^{(S)} = (n_s+1)W^{(SRS)}. \quad (23)$$

Here we use the value $W^{SRS} = (m_i+1)W_i^{(S)}$, describing the probability of SRS; n_s is the figure of photons per one mode of Stokes electromagnetic field; m_i - is the figure of optical phonons per one mode, taking part in *RS* . With pumping intensity increasing, *IRS* may be realized for one (as a rule) optical Raman active mode. The examples of IRS-spectra, obtained from organic aromatic powder POPOP and $CaCO_3$ - monocrystal under powerful picoseconds' green (532 nm) laser excitation, are shown in Fig. 4.



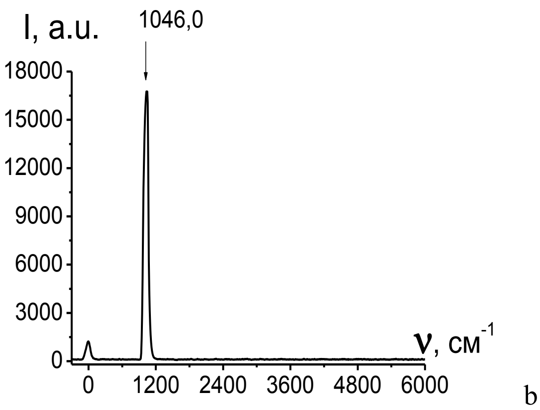


Fig.4. IRS-spectrum of POPOP(a)and $CaCO_3$ (b)crystals; at left there are peaks due to exciting line(532 nm); satellite 977.3cm^{-1} corresponds to the first Stokes line and 1954.6 cm^{-1} – the second one.

The relation between probabilities (1/c) for *IRS* ($W^{(IRS)}$)and SSRS($W^{(SSRS)}$) according to (23) may be written as:

$$W^{(IRS)}) = n_s W^{(SRS)} \quad (24)$$

So IRS intensity is [10]:

$$I^{(IRS)} = n_s I^{(SRS)} \quad (25)$$

When the sufficiently high pumping intensity exceeds the threshold of *IRS* (n_s>>1) for *IRS* intensity takes place:

$$I^{(IRS)} = I^{(SRS)}(0)\exp(gI_0 z). \quad (26)$$

Typical value of gain coefficient g in real Raman active media is: $g \sim 10^{-2}$ cm/MW. Accordingly for the samples length z=1cm under pumping laser intensity I0~1 GW/cm2 we obtain:

$$I^{(IRS)} = I^{(SRS)}(0)\exp(gI_0 z) \approx 10^{-6} I_0 \exp(gI_0 z) \approx 10^{-2} I_0 \qquad (27)$$

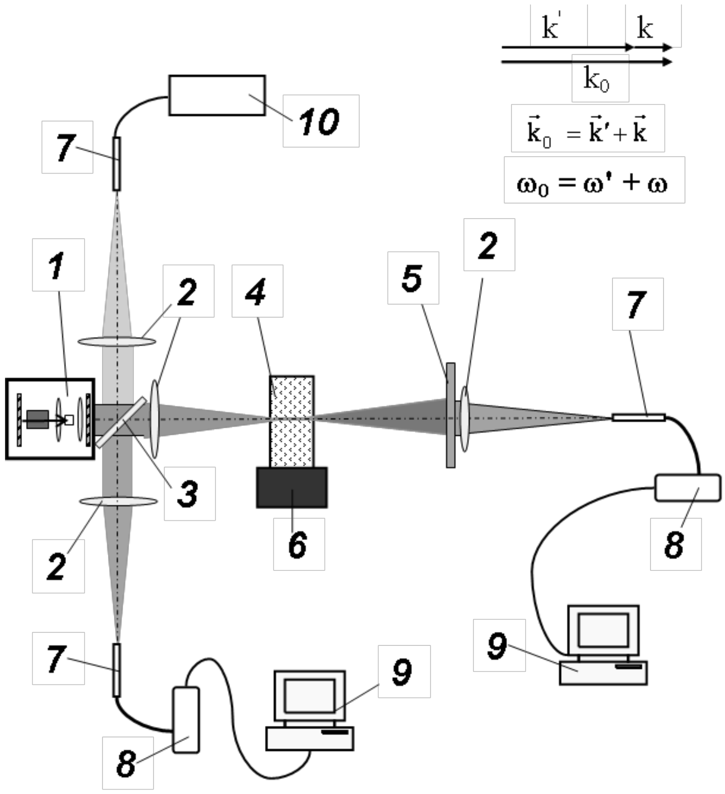


Fig.5. Experimental setup[16-18] for IRS-spectra recording; 1-powerfull pulsed laser; 2-lenses; 3-semitransparent plate; 4 - Raman active medium; 5-absorbance filter; 6-holder; 7-fiber tip; 8-fiber minispectrometer; 9-computer, 10 - detector of laser emission.

Thus the efficiency of photon-scalar boson conversion in media as a result of IRS-effect is large enough for experimental observations of the secondary emission spectra.

Typical experimental scheme [16-18] for IRS-spectra recording is presented at Fig. 5. The exciting emission (1) falls onto the sample 4 (liquid, powder, photonic crystal, photonic glass, monocrystal) and collects at forward or backward geometry by fiber minispectrometer (8).

The possibility of the existence in the physical vacuum of low energy scalar and pseudoscalar bosons (paraphotons and axions) is widely discussing in numerous [19-37] works. Low-energy ("cold") scalar and pseudoscalar bosons have very small rest effective mass, small velocity of moving and are predicted to be dark matter elemental particles. Such prediction follows from the theory of elementary particles at high energy and also from astrophysical observations. "Hot" scalar and pseudoscalar bosons are weakly interacting with environment particles and propagate in space with velocity, close to c_0.

In our works [9,10,12,13] there was predicted the existence of scalar bosons (paraphotons) in media and have proposed the schemes of laboratory experiments for paraphotons detection. One type of scalar bosons in media is so called bound scalar states of two photons (“boundtons”). Such particles may appear as a result of attraction between photons with opposite polarizations due to exchange of phonons. Strong photon-photon attraction should appear during IRS in Raman active media, having low threshold of IRS. The examples of such type media are different types of photonic traps: powdered Raman active substances; photonic crystals or photonic glass, filled by IRS-medium (aromatic compounds, inorganic substances) and others.

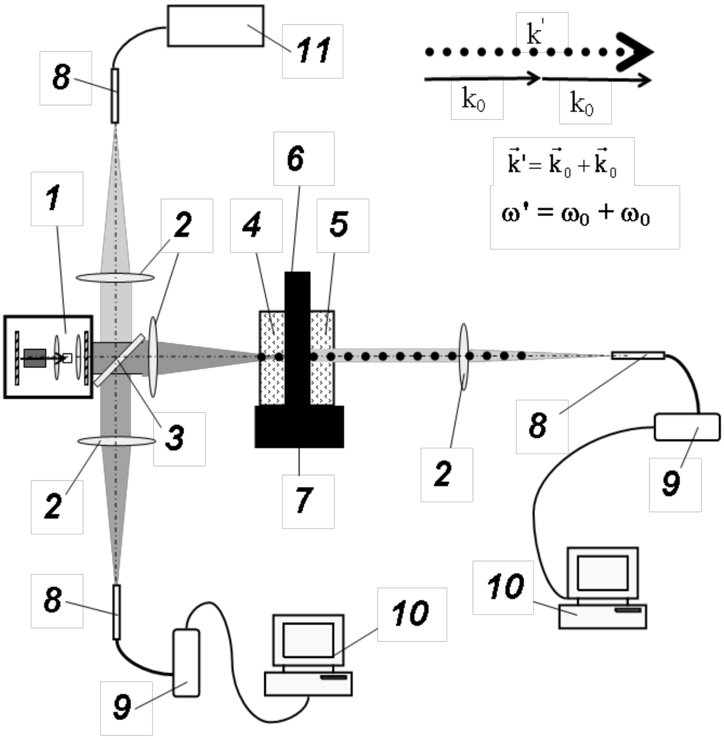


Fig.6. Experimental scheme for generation and detection of scalar photons; 1-powerfull pulsed laser; 2-lenses; 3-semitransparent plate; 4,5- Raman active media; 6-opaque wall; 7-holder; 8-fiber tip; 9-fiber minispectrometer; 10-computer, 11 - detector of laser emission.

The generation of scalar bosons should take place due to two photon - scalar boson conversion processes at powerful laser excitation. The detection of appeared scalar photons should be in another photonic trap as a result of destruction of bound photonic states onto pairs of initial photons. Thus, two-photon bounding in media should take place as a result of strong photon-phonon interaction, i.e. a result of IRS-processes. Proposed experimental scheme for generation of physical vacuum scalar bosons and regeneration of usual photons is presented at Fig.6. In this setup IRS and two photon-scalar boson conversion processes should take place in photonic trap (4). Emerging in this trap the scalar photons (dotted line) should penetrate through opaque wall (6) into another photonic trap 5. In this photonic trap the destruction of paraphotons onto initial photons gives the opportunity to detect the usual photons with the same frequencies as exciting emission photons. As photonic traps we may use 3D-photonic crystals, photonic glasses, monocrystals with rare earth imperfections and ruby or ZrO_2-balls.

Conclusions

Thus on the base of proposed crystalline model of physical vacuum we have described the spectra of quasiparticles, existing in physical vacuum media: “transverse” photons, “longitudinal” photons and “scalar” bosons (paraphotons). Such quasiparticles may have nonzero (positive or negative) rest mass. The velocity of these quasiparticles (group velocity of corresponding waves) may be close to zero, equal to fundamental value (c_0 – constant) and also may be larger than c_0 (for longitudinal photons).

Scalar bosons may be slowly moving (“cold” particles) or fast moving (“hot” particles) with c_0 - velocity through the media without essential interaction with nontransparent for usual electromagnetic waves substances.

For observation of paraphotons (elemental particles of dark matter) laboratory experimental schemes have been proposed. Such schemes is based on using of "Light Shining Through Wall Effect", predicted before and widely discussed in a number of modern works [32-37].

This work was supported by Russian Foundation for Basic Research; grants numbers - 11-02-00164, 12-02-00491, 12-02-90422, 12-02-90021, 13-02-90420, 13-02-00449.

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Gravitational wave detector based on photonic crystal

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In proposed work the photonic crystal shall be considered as the high-dispersive system, having the similar properties at wavelengths of order of critical. The large dispersion and its the strong dependence in this area from length of polyatomic monostructures, of which adds together the photonic crystal, make it possible use they as a part of optical interferometer as of high-sensitive sensor of elongation. Has been shown (together with sensitivity estimations) how at the base of this kind sensors is constructible gravitational wave detector, high-sensitive registrant of magnetic and electric field.

Ordered nano porosity of opal permits light to propagate in these photonic crystals through channels, the playing the role of lightguides with cross dimensions commensurable with wavelength λ_0 in free space. In case of lookalike commensurability λ in waveguide have to elongate $\lambda = \lambda_0 \Big/ \sqrt{1-(\lambda_0/\lambda_C)^2}$ in degree of proximity λ_0 to critical wavelength λ_C, which, as is known, is determined by the transversal channel dimensions. It is also well-known, that near critical length group wave velocity decreases, as $c_g = c_0\sqrt{1-(\lambda_0/\lambda_C)^2}$. Besides the waveguide, the light guide or adequate spatially the periodic structure plays the role of highly dispersion retarded system. In experiments with light propagation into opals [1] electromagnetic wave has managed to brake up to speed of $c_g \approx 15$m/sec, so that the factor of c_0/c_g reached 2×10^7. The growth of wave phase of $\delta\varphi$, responding the change of λ_C when unchanged physical path length of $\ell = const$, passable light there is $\delta\varphi = \frac{d}{d\lambda_C}\left(\frac{2\pi\ell}{\lambda}\right)\delta\lambda_C$. Will express the relative increment of critical wavelength through the relative change of cross dimensions of sensing element, performed from opal $\delta\lambda_C/\lambda_C = \delta h/h$: $\delta\varphi = \frac{2\pi\ell}{\lambda_0}\frac{d}{d\lambda_C}\left(\sqrt{1-(\lambda_0/\lambda_C)^2}\right)\delta\lambda_C = \frac{2\pi\ell}{\lambda_0}\frac{\lambda_0^2/\lambda_C^3}{\sqrt{1-(\lambda_0/\lambda_C)^2}}\lambda_C\delta h/h$. Eventually with factor account of wave retardation the phase growth will make $\delta\varphi \approx \frac{2\pi\ell}{\lambda_0}\times\frac{c_0}{c_g}\times\frac{\delta h}{h}$. According to fig.1 cross dimensions sensing element of h are changed under the action of pressure, arising from elongation of cylindrical gravitational antenna of $\delta H = H|\delta g_{ij}|$, responding the dynamic variations of metrics, notifiable. The continuity of mechanical constraint [2] antenna with photonic crystal of $\delta H=\delta h$ in the conditions of action 3rd of Newton's law also Hooke law is guaranteed by the relation maintenance of $H/h=S_H/S_h\times E_H/E_h$, where S - section area of antenna and crystal, E - appropriate the Young's modulus.

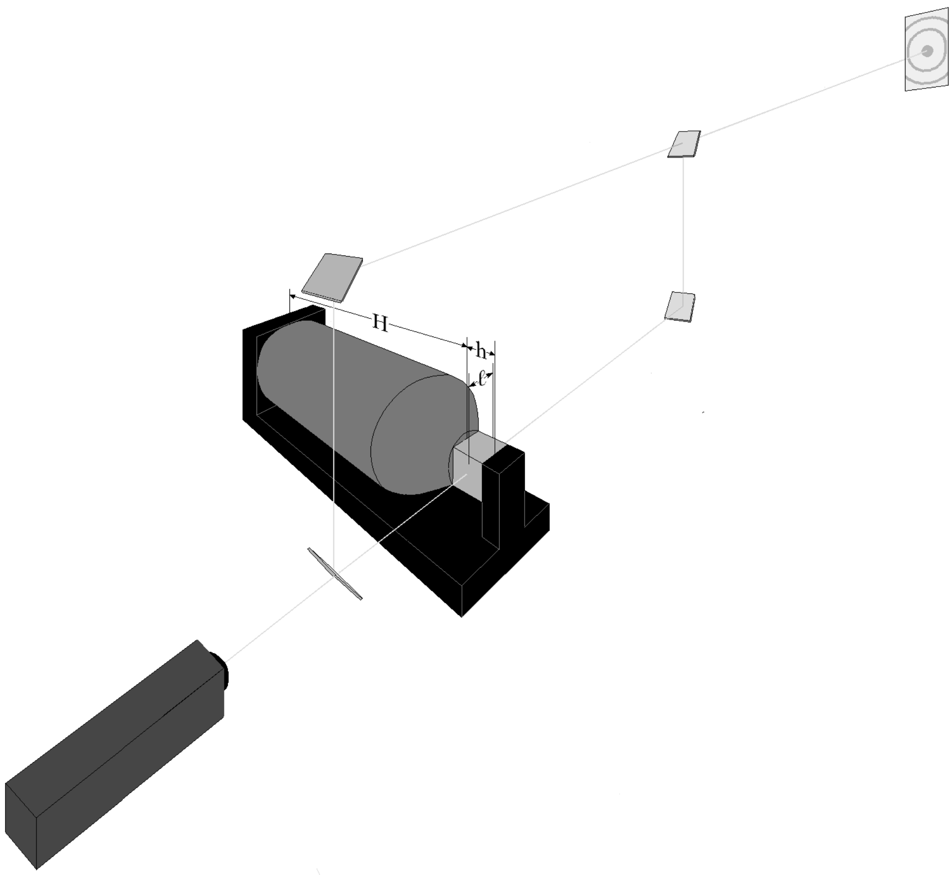

Fig.1: Scheme of gravitational wave detector based on photonic crystal, included in down interferometer arm of Mach/Zehnder.

As a result without factor account of mechanical quality (which creates the extra stock of sensitivity from 3 up to 5 orders) $\delta\varphi \approx \frac{2\pi\ell}{\lambda_0}\times\frac{c_0}{c_g}\times\frac{H}{h}\times\left|\delta g_{ij}\right|$. In detector (fig.1) with parameters of $\ell\approx$1cm, $\lambda_0\approx$500nm, $c_0/c_g\approx 2\times10^7$, $H\approx$1m, $h\approx$1cm «the interferometric response» on the gravitational wave, having the amplitude of $|\delta g_{ij}|\approx 10^{-21}$, will make $\delta\varphi\approx(1{,}25\times10^5)\times(2\times10^7)\times100\times10^{-21}\approx2{,}5\times10^7$rad, that appears to be availability for registration helped by multi-pixels CCD the force of high redundancy of displayable signal, and so demonstrates the competitiveness suggested system.

Another opportunity of a still more radical increase in the effective length of the working arm of a compact interferometric detector of extremely small mechanical vibrations offers itself when TEM typewaves are not used. If the light wave is directed to a waveguide formed by the gap with the lower wall made of a mobile mirror fixed on the end of the deformed working substance and the gap width turns out to be comparable with the wavelength λ_0 in free space, then the wavelength $\lambda = \dfrac{\lambda_0}{\sqrt{1-(\lambda_0/\lambda_C)^2}}$ in the waveguide must extend as λ_0 approaches the critical value λ_C.

For waves of lower non-transverse type λ_C is equal to a double gap thickness. The wave phase increment may be due on the one hand to the physical extension of the light path $\delta\ell$ and on the other hand for ℓ=*const* to the change in λ_C: $\dfrac{2\pi d\ell_{opt}}{\lambda} = d\varphi = \dfrac{d}{d\lambda_C}\left(\dfrac{2\pi\ell}{\lambda}\right)d\lambda_C$. We shall rewrite the latter equation expressing the change in the critical wavelength in terms of the working substance extension $d\lambda_C = 2dh = 2h|\delta g_{ij}|$ and representing the change in the optical path as $d\ell_{opt} = \ell_{eff}|\delta g_{ij}|$, (here we introduced the effective value of the optical path ℓ_{eff}). Then $\dfrac{2\pi\ell_{eff}|dg_{ij}|}{\lambda} = d\varphi = \dfrac{2\pi\ell}{\lambda_0}\dfrac{d}{d\lambda_C}\left(\sqrt{1-(\lambda_0/\lambda_C)^2}\right)d\lambda_C = \dfrac{2\pi\ell}{\lambda_0}\dfrac{\lambda_0^2/\lambda_C^3}{\sqrt{1-(\lambda_0/\lambda_C)^2}}2h|dg_{ij}|$. As a result, the effective value of the optical path can be approximately represented as $\ell_{eff} \approx \dfrac{\ell h\sqrt{2}}{\sqrt{\lambda_C\Delta\lambda}}$, where $\Delta\lambda = \lambda_C - \lambda_0$ is turning-out of the employed wavelength from the critical value. Thus, for ℓ=1*м* h=1*м* λ_C=0,5*мкм* and $\Delta\lambda$=100A° ℓ_{eff} will be 20000km. The optical path in the gap can be composed of smaller regions ℓ_{opt}/N as they are passed N times with reflection from the end mirrors.

Along with gravitational wave detection the alike interferometric systems may prove useful in solution «more earth-bound» and besides not less than actual problems of applied character. Let us consider the application possibilities of interferometric systems, able, as shown above to register ultrasmall (much are smaller than size of nucleus of atom) the elongations of probe bodies, to create compact ultrahigh sensitive magnitometric instrument, as alternatives to of quantum magnetometer based on SQUID. It is clear that by being able produce the ultrahigh precise measurements of elongation, it is possible would be register the weak variations of magnetic field, if as of probe body use a magnetostrictor. In work [3] in magnetometer construction was suggested to comprise the fiber-optic variant of Mach/Zander interferometer (fig.2).

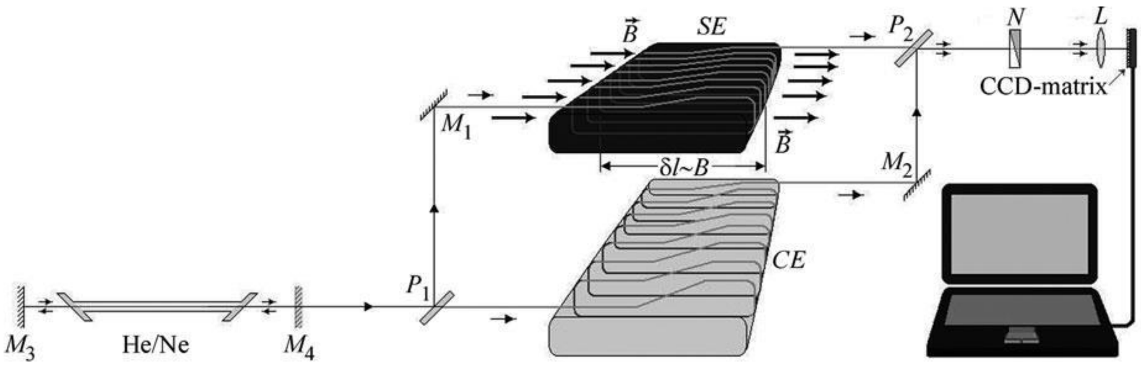

Fig.2 Fiber-optic Mach–Zehnder interferometer. He/Ne is the He-Ne laser, $M1$ and $M2$ are the interferometer mirrors, $P1$ and $P2$ are the beam splitters, SE is the magnetosensitive element (the coil with light guide fiber densely reeled up on a magnetostrictive frame), CE is the coil of the reference wave playing the role of a passive compensator of the magnetically independent contribution to the phase shift, L is the lens, and N is the polarizer.

In order to interferometer has sensed the elongation of magnetostrictive working body, caused by registered change of magnetic field, the frame of multiturn coil in one of its arms it is necessary to fabricate from material with high magnetostrictive parameters. In case when as of ultrasensitive sensor of elongation it is proposed to use the photonic crystal (that is fully considered in the first part of paper) suitably role working body (grey cylinder on fig.1) has to accomplish the magnetostrictor.

The «direct» magnetostrictive effect, i.e., «ordinary» magnetostriction (discovered by J.Joule in 1842; the «inverse» effect was discovered by E.Villari in 1865) manifests itself as a change in sizes of the magnetostrictive sample during its magnetization [4]. Quantitatively, in the simplest case (not going deep into «tensorial jungle» of widespread phenomenological theories of magnetostriction), the "direct" effect can be described by the elementary formula $(\Delta l/l) = \aleph B$, where $(\Delta l/l)$ is the relative elongation of a magnetostrictor under the applied field with induction B and $\aleph$ is the proportionality factor playing the role of the main characteristic of magnetostrictive properties of a concrete material. Modern microscopic theories of magnetostriction [5–7], based on quantum-mechanical concepts, consider the direct relation of the ferromagnetic crystal strain to the change in the intensity of the spin/spin interaction and, from this standpoint, make it possible to approximately estimate the magnetostrictive characteristics similar to $\aleph$. In [2,8], with the purpose of improving the sensitivity of gravitational antennas and sensors of ultralow pressure variations, which are proposed to be developed on the basis of the SQUID/magnetostrictor system, proceeding from the most general quantum-mechanical concepts on the exchange interaction, the conditions are considered, which promote amplification of the «inverse» magnetostrictive effect. In [2,8], it was shown that, although within the phenomenological model of ferromagnetism the high sensitivity of the magnetostrictive sensor should be searched near the Curie temperature, from the standpoint of the microscopic theory the highest sensitivity can be achieved when the Stoner factor [9,10] comes close to unity. A simple phenomenological calculation using thermodynamic potentials [11] in principle makes it possible to relate the parameters of the direct and inverse magnetostrictive effects for a given material. Similar reciprocity theorems [11] show that the amplification of the direct magnetostrictive effect corresponds to the amplification of the inverse effect and vice versa. This situation makes it possible to extend the conclusions on the possibility of increasing the magnetostrictive characteristics, made in [2,8] for the inverse effect, to the direct effect.

Returning to the magnetometer design (Fig.2, [3]), we will assume that N turns of a flexible light guide fiber are densely reeled up on the frame of length l, made of a magnetostrictive material characterized by a sufficiently large constant ℵ; this coil forms one of the arms of theMach–Zehnder interferometer with a known resolution ($\Delta\lambda/\lambda$). Then, equating the magnetostrictive tension of the optical waveguide to the elongation which can be detected by the interferometer, we obtain $N\Delta l_{\{N=1\}}=N(l\aleph B)=\Delta\ell_{\{N\}}=\Delta\lambda$, from which the magnetic field resolution is $B_{\min}=\frac{\lambda}{\aleph N\ell}(\Delta\lambda/\lambda)$. Substituting the value of ℵ taken for a 54%Pt 46%Fe alloy having far from record magnetostrictive parameters $\aleph(54\%Pt:46\%Fe)\approx6\times10^{-3}T^{-1}$, and taking into account the far from record resolution of the Mach–Zehnder interferometer with arm length $Nl\approx100$m $(\Delta\lambda/\lambda)\approx10^{-6}$ (having about three order of magnitude "in reserve") into this formula, we obtain the minimum detectable magnetic field $B_{\min}\approx10^{-12}$T for the helium–neon laser wavelength $\lambda_{\text{He-Ne}}=0.633\ \mu$m.

The sensitivity on the order of several picoTesla, achievable according to the above estimates for magnetometers the fiber-optic interferometer with the magnetostrictive transducer and analogous characteristics at interferometric system, registering the elongation of magnetostrictor by photonic crystal, appears quite comparable to the «magnetic» sensitivity of the modern SQUID, if the latter is taken with consideration of the transfer coefficient of the superconducting transformer of the flux whose value is required to recalculate the «intrinsic» sensitivity of the quantum interferometer ($\delta\Phi=10^{-6}\div10^{-7}\Phi_0/\sqrt{\Gamma ц}$, $\Phi_0=\pi\hbar/e\approx2{,}07\times10^{-15}Вб$) into the sensitivity at the input turn of the transformer (the magnetic signal loss in the transfer coefficient from one to three orders of magnitude is the «price» to be paid for the necessity of matching the macroscopic sizes of the signal source with the microscopic sizes of the SQUID ring containing Josephson junctions).

Certainly, the above estimates of the sensitivity of the magnetometers with the magnetomechanical transducer based on the magnetostrictive effect were made without taking into account of thermal instability of the used physical effects. However, the disregarded thermal drift can be eliminated by corresponding signal processing algorithms, at least in the cases, when the experimental conditions imply the synchronism of the recorded response «in reply to a dosed perturbation». In such cases, the response with drift suppression can be fixed by its accumulation. In essence, such algorithms (digital and/or analog) are close to the synchronous detection principle well known in radio engineering [12,13]. In this case, the experimental conditions allowing their application are implementable in both bio- and geomagnetic studies. As example of a neurophysiologic experiment for such a scheme can be the study of the dynamics of the magnetoencephalographic activity response to a pulsed light flash and others. An experiment in which the current logging scheme will be implemented with the proposed magnetometers used under field conditions as non-contact recorders of the geomagnetic response can be very important in practice of geologicalworks and, probably, be in high demand in economy. The logging results can be used to plotmaps reflecting the three-dimensional pattern of the distribution of the terrestrial rock conductivity; their contrast curves, in turn, reflect possible boundaries of deposits (oil, ores, and others). The current in the form of a short intense pulse (perturbation) is introduced into rock through a grounding electrode; its spread over the ground is noncontactly recorded by the induced magnetic field pattern (synchronous response). In this case, the practically required sensitivity of the detecting magnetometer is at the level of one picoTesla.

Presented above sensitivity estimation of interferometric systems involving the photonic crystal, which was suggested to use as a detector of disturbances of gravitational and magnetic fields looked to some extent incomplete, if for frames of reference remained the analogous methodic the registration of low reduced electric field. By such methodic can be realized, if nano pore the opal to fill in nitrobenzene i. e. substance with great factor of Kerr`s nonlinearity β, and besides to ensure the high dispersion of spatially periodic retarded system of photonic crystal, selecting the operating wavelength of order of critical $\lambda_0 \lesssim \lambda_C$. The refractive index changes of δn under the action of electric field E in square-law Kerr-effect is described known by the formula $\frac{\delta n}{\lambda_0} = \beta E^2$ [14]. The phase raid, responding the light propagation in spatially periodic retarded system centered on the index of refraction n, is $\varphi = \frac{2\pi\ell}{\lambda_0/n}\sqrt{1-\left(\frac{\lambda_0}{n}\Big/\lambda_C\right)^2}$, where ℓ - the distance «elapsed» wave in photonic crystal. The interferometer records the phase growth of $\delta\varphi$, caused by refractive index change $\delta\varphi = \frac{d}{dn}\left(\frac{2\pi\ell}{\lambda_0/n}\sqrt{1-\left(\frac{\lambda_0}{n}\Big/\lambda_C\right)^2}\right)\delta n = \frac{2\pi\ell}{\lambda_0}\left(\sqrt{1-\left(\frac{\lambda_0}{n}\Big/\lambda_C\right)^2} + \left(\frac{\lambda_0}{n}\Big/\lambda_C\right)^2 \Big/ \sqrt{1-\left(\frac{\lambda_0}{n}\Big/\lambda_C\right)^2}\right)\delta n =$ $= \frac{2\pi\ell}{\lambda_0}\left(\frac{c_g}{c_0} + \frac{c_0}{c_g}\left(\frac{\lambda_0}{n}\Big/\lambda_C\right)^2\right)\delta n \underset{c_g \ll c_0}{\approx} \frac{2\pi\ell}{\lambda_0} \times \frac{c_0}{c_g} \times \left(\frac{\lambda_0}{\lambda_c}\right)^2 \times \frac{\delta n}{n^2}$. If the index of refraction changes according to Kerr-effect, then the growth the phases, responding the action of electric field, is $\delta\varphi \underset{c_g \ll c_0}{\approx} 2\pi\ell \times \frac{c_0}{c_g} \times \left(\frac{\lambda_0}{\lambda_c}\right)^2 \times \frac{\beta\delta(E^2)}{n^2}$, where c_0/c_g - slowdown factor of electromagnetic wave propagation velocity, many times intensifying the dispersion characteristics spatially periodic system in the area $\lambda_0 \lesssim \lambda_C$. Based on the opportunity of interferometric the registration in unit frequency band the changes of phase difference at the level of of $\delta\varphi \approx 10^{-6}$, based on last formula one can obtain the estimate of limiting resolution of system on measured the electric field $\sqrt{\delta(E^2)} \approx 600 mV/m$ (for photonic crystal by the length $\ell \approx 1$cm, filled by the nitrobenzene with $\beta_{C6H5NO2} \approx 2{,}3\times10^{-12} m^{-1}V^{-2}$). On the view that such itself not so really high sensitivity, must be achieved at voltmeter with infinite input resistance, it can be suggested that the devices of this kind will find the application in research of air electric fields and missions of short-range weather forecasting. If one use of opals filled by the nitrobenzene as of dielectric sheet of capacitor by the thickness of $w \approx 100 \mu m$ and to displace him stable voltage $V_0 = 3V$, then the interferometric system with infinite internally resistance on direct current in principle will are able to fix a change of potential difference at level of $\delta V = 600pV$.

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Baryon asymmetry as a result of symmetrization of extra dimensions

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Origin of baryon asymmetry is studied in the framework of extra dimensional approach. Baryon excess production and the symmetrization of extra-space are performed simultaneously. Baryon number is conserved long after the inflationary stage when the U(1) symmetry is achieved.

Introduction

Origin of the baryon asymmetry is one of the key problem in cosmology. The question is whether the Universe was born asymmetric or was the asymmetry formed during the evolution of the Universe? Why the baryon charge is conserved today with high accuracy whereas the baryon excess was created in the past at some (also unclear) stage of the Universe evolution [1], [2]?

First attempt to explain the phenomenon of baryon asymmetry was done by Sakharov in 1967. According to his approach there are three known conditions that had to be fulfilled in order to originate baryon asymmetry at some stage.

1. Non-coservation of baryon number.
2. C and CP violation.
3. Deviation from thermal equilibrium.

Nowadays there are a great variety of different approaches [3], [4], [5], [6]. Up to now there is no unique preferable model.

On the other hand, the idea of extra space almost inevitably leads to charge nonconservation. Indeed in the framework of multidimensional gravity observed low energy symmetries are the consequences of isometries of extra space [7].

In the present paper we consider a mechanism of the baryon asymmetry generation accompanied by symmetrization of extra space. It is assumed that the baryon symmetry (asymmetry) is the consequence of symmetry (asymmetry) of extra space. The corresponding symmetry of the theory is restored during the process of evolution at later stages.

Setup

Consider a $D = 8$-dimensional Riemannian manifold $V_8 = M_4 \times V_2 \times H_2$ with a metric G.

The FRW metric with the scale factor $a(t)$ of our 4-dim space M_4 is denoted as $g_{\mu\nu}(x)$ where $x_\mu(\mu = 1,2,3,4)$ are its coordinates. The subspace V_2 with the topology $T_1 \times T_1$ possesses a metric $G^{(V)}_{ab}$ and is described by coordinates $y_a, a = 5,6$ in the interval $0 \leq y_a < 2\pi r_c$. Hyperbolic subspace H_2 with a radius r_d plays a subsidiary role.

Nucleation of manifolds containing a symmetry has zero probability [8]. In this connection consider the metric of the subspace V_2 being slightly deviated from symmetrical one. More definitely, suppose the following form of the metric

$$G^{(V)}_{ab} = G^{(V,stat)}_{ab} + h_{ab}(t, y_1, y_2) \tag{1}$$

Its stationary part $G^{(V,stat)}_{ab} = diag(r_c, r_c)$ is invariant under the $SO(2)$ transformations. The extra space V_2 acquires this symmetry at late times when the fluctuations $h_{ab}(t, y_1, y_2)$ decay into lighter particles.

Restoration of the symmetry of our metric gives rise to the baryon charge conservation.

We suppose that the corresponding charge $Q = Q_B$ is the baryonic one and hence the field $e_a \equiv G_{a7}$ could form a baryonic condensate.

As will be shown later the baryon asymmetry takes place if $h_{ab}(t, y_1, y_2) \neq h_{ab}(t, y_2, y_1)$. We choose the simplest form - first term in the Fourier series

$$h_{ab}(t, y_1, y_2) = \delta_{ab} h(t) \cos(y_1) \tag{2}$$

to perform analytical estimation.

The model is specified by the nonlinear action

$$S = \frac{m_D^{D-2}}{2} \int d^D X \sqrt{G} \left[R + cR^2 \right] \tag{3}$$

There are two parameters in the model - m_D and c while the metric tensor contains another two - r_c and r_d. According to modern experiment these extra space sizes must be smaller than $\sim 10^{-18}$ cm. Another restriction followed from the fact that quantum fluctuations of a metric become important at m_D scale. The inequalities

$$1/m_D \ll r_c, r_d < 0.1\,\mathrm{TeV}^{-1} \tag{4}$$

permit us to deal with classical behavior of the metric of small enough extra space.

Effective Lagrangian

After integrating out the internal coordinates in expression (3) one obtains the effective Lagrangian

$$\begin{aligned}\mathcal{L}_{eff} &= \frac{1}{2}(\partial_t\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 + \partial_t\phi\partial_t\phi^* - m_\phi^2\phi\phi^* \\ &+ 2\lambda\chi^2\phi^2\phi^{*2}\left[\phi+\phi^*\right]^2\end{aligned} \tag{5}$$

where field normalization

$$\begin{aligned}\chi &= \tfrac{r_d}{r_c}\sqrt{24\pi^3 m_D^6 V_\theta}\, h \\ \varphi_a &= \tfrac{r_d}{r_c}\sqrt{8\pi^3 m_D^6 V_\theta}\, e_a\end{aligned} \tag{6}$$

$$\phi = \frac{\varphi_1 + i\varphi_2}{\sqrt{2}}$$

was performed to obtain the standard form of the Lagrangian. Here $\lambda = c \cdot \lambda^* > 0$ and $c > 0$. The last term breaks global $U(1)$ symmetry and therefore is responsible for asymmetrical baryosynthesis. In the modern epoch $\chi(t \to \infty) \to 0$ so that the $U(1)$ is restored. We omitted terms describing the Einstein-Hilbert action and $\Lambda-$ term because they do not play significant role in the model. To obtain them accurately, one should take into account all effects what is far from our purpose. Besides we deleted all interaction terms except the last one which is responsible for the baryogenesis. It can be done if the metric fluctuations are small

$$h(t, y_1, y_2) \ll r_c^2, \quad e_a \ll r_c^2, r_d^2. \tag{7}$$

The masses of the new fields are expressed in terms of initial parameters

$$m_\chi^2 = \frac{1}{3r_c^2}, \quad m_\phi^2 = \frac{4}{r_d^2} \tag{8}$$

The expression for coupling constant

$$\lambda^* = \frac{W_\theta/V_\theta^4}{3072\pi^9 m_D^{18} r_c^8 (r_d^2 - 4c)^4} \tag{9}$$

contains the integrals

$$V_\theta = \int_{\phi_-}^{\phi_+}\int_{\theta_-}^{\theta_+} \sinh(\theta)\mathrm{d}\theta\mathrm{d}\phi \approx 1.57 \tag{10}$$

$$W_\theta = \int_{\phi_-}^{\phi_+}\int_{\theta_-}^{\theta_+} \frac{cosh^2(\theta)}{sinh(\theta)}\mathrm{d}\theta\mathrm{d}\phi \simeq ln(m_D r_d) \tag{11}$$

over the space H_2.

The final form of the effective action

$$S_{\chi,r,\vartheta} = \int dta^3(t)\left[\frac{1}{2}(\partial_t\chi)^2 - \frac{1}{2}m_\chi^2\chi^2 + \frac{1}{2}\dot{r}^2 + \frac{1}{2}r^2\dot{\vartheta}^2 - V(r,\vartheta,\chi)\right] \quad (12)$$

contains the potential

$$V(r,\vartheta,\chi) = \frac{m_\phi^2 r^2}{2} - \lambda\chi^2 r^6\cos^2(\vartheta)$$

Here the field ϕ is represented in the form $\phi(t) = r(t)e^{i\vartheta(t)}/\sqrt{2}$

Effective lagrangian (5) is similar to those considered in the framework of Affleck-Dine model [14]. The only essential difference is the presence of additional field χ. The baryosynthesis is terminated when this field reaches zero value.

Baryon excess

In the FRW space the equation of motion for the field ϑ

$$r^2\ddot{\vartheta} + 3Hr^2\dot{\vartheta} + 2r\dot{\vartheta}\dot{r} = -\lambda\chi^2 r^6\sin(2\vartheta) \quad (13)$$

follow from action (12).

Oscillations of the field ϑ are responsible for the generation of the baryon excess. Indeed the dynamical equation for the field ϑ can be written in more suitable form

$$\frac{1}{a^3}\frac{\partial}{\partial t}\left(a^3r^2\dot{\vartheta}\right) = -\frac{\partial V}{\partial\vartheta} \quad (14)$$

The field ϕ is transformed under the fundamental representation of the group $U(1)$. The baryon charge connected to this group is calculated in standard manner $n_B = j_0 = i(\phi^*\partial_0\phi - \phi\partial_0\phi^*) = r^2\dot{\vartheta}$. Substituting this into equation (14) we obtain formal solution

$$n_B(t) = -a(t)^{-3}\lambda\int_{t_{in}}^{t} a(t')^3 r(t')^6\chi(t')^2 sin(2\vartheta(t'))dt'. \quad (15)$$

Our aim is to study the ability of the model to explain the observable baryon density. According to [9] it can be done in quite simple and elegant way. Suppose that the dynamic of the field ϑ ruled by equation (14) elaborates baryon excess during one e-fold. Then the estimation of integral (15) gives

$$n_B(t_B) \simeq -e^{-3}\lambda\chi^2 sin(2\vartheta)r^6H^{-1}. \quad (16)$$

A baryon charge at the moment $t_B = H^{-1}$ of its creation is connected to the modern baryon axcess $n_B(t_0)$

$$n_B(t_B) \simeq (a(t_B)/a(t_0))^3 n_B(t_0) = (t_0H)^2 n_B(t_0). \quad (17)$$

Keeping in mind expression (17) we obtain the connection between the parameters

$$n_B(t_0) = -t_0^{-2} H^{-3} e^{-3} \lambda \langle \chi^2 \rangle \langle r^6 \rangle sin(2\vartheta) \tag{18}$$

The observed parameters included in this equation are $t_0 = 14 \cdot 10^{16} sec = 6.3 \cdot 10^{41}\,\mathrm{GeV}^{-1}$, $n_B = 2.46 \cdot 10^{-7} cm^{-3} = 1.9 \cdot 10^{-50}\,\mathrm{GeV}^3$. Finally, the relation between unknown parameters acquires the form (we suppose $sin(2\vartheta) \approx -1$)

$$\frac{\lambda \langle \chi^2 \rangle \langle r^6 \rangle}{H^3} \approx 3.8 \cdot 10^{36}\,\mathrm{GeV} \tag{19}$$

Effective baryon production starts when a slow rolling of the field ϑ is terminated what leads to an additional connection

$$3H \simeq \sqrt{\lambda \langle \chi^2 \rangle \langle r^4 \rangle} \tag{20}$$

followed from the equation in (13). Expressions (19) and (20) impose main restrictions on the parameters of the model.

Let us specify parameters of the model and choose $m_D = 10^{14}$ GeV,$c = 42 m_D^{-2}$, $r_c = r_d = 10^2 m_D^{-1}$. Initial values of the fields h and e_a should be small compared to the size of extra space (7) and we assume (6) $\langle |\chi| \rangle \sim \langle |\phi| \rangle \sim 10^6 m_D$. This set of parameters satisfies main equations (19) and (20) if the Hubble parameter $H \sim 2.4 \cdot 10^4$ GeV.

According to (8), the masses of quanta of the fields χ and φ are rather large. The role of such massive particles at post inflationary stage is discussed in e.g. [10], [11], [12] . The charge stored by the field ϕ should be transferred to matter fields. The mechanism is described in detail in [13] and we refer readers to this article.

Conclusion

The picture described above fits very well with the baryon problem. Nonconservation of the baryon charge results in the baryon excess in the beginning. In the modern epoch an extra space is supposed to be stable and possesses $U(1)$ symmetry which relates to conservation of baryon charge. In this paper we show that this mechanism is able to explain the observable baryon excess. For more detailes please see [15].

Acknowledgment

S.R. was supported by the Ministry of education and science of Russian Federation, project 14.A18.21.0789. The work of A.G. was supported by The Ministry of education and science of Russian Federation, project 14.132.21.1446. The authors are grateful to A. A. Korotkevich for clarifying the problem concerning a boundary of compact hyperbolic space.

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Local quantum logics in the topos approach to branching space-time

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1 Introduction

In the work [1] there was proposed a categorial arrangement of Belnapian branching space-times [2]. Definite sets of Belnapian worlds (i.e. the sets of global scenarios in our random World) and their maps were associated with some objects and morphisms of the category (topos) $Set^{\mathcal{C}^{op}}$ of contravariant functors from the category $\mathcal{C}$ of all events to the category of sets Set. In the framework of the topos approach nonclassical intuitionistic logic is naturally embedded into the model of branching space-times. The truth values of propositions in this logics appear to be multivalued with respect to a localized observer.

In the topos approach one may also search for logical structures associated with quantum properties of Reality. This would let one to compare the notion of branching with its quantum counterpart from Everettian construction. In the present work a first step is made to discover structures related to those from quantum formalism.

Let us outline the origin of the peculiar quantum logic, confronting the logic of classical physics. The following reasoning is used by Doring and Isham [3] for slightly another purpose. The classical (Boolean) logic is known to have the simple model – the classical set theory. With any proposition a subset of some parent set is associated. The logical connectives of conjunction and disjunction are modeled by intersection and union respectively. Set complement corresponds to logical negation. Distributivity of classical logic is in parallel with distributivity of intersection and union. This model finds its natural origin in classical physics. Any observable A of classical physical system is a function from the set $\mathcal{S}$ of all states of the system to real numbers:

$$A : \mathcal{S} \to \mathbb{R}. \tag{1}$$

To the assertion "$A \in \Delta$"("The value of observable A lies in Δ"), where Δ

is a Borel subset of $\mathbb{R}$, there corresponds a *subset* $P_{\Delta}^{(A)}$ in $\mathcal{S}$:

$$P_{\Delta}^{(A)} = \{s \in \mathcal{S} : A(s) \in \Delta\}. \tag{2}$$

A completely different type of situation occurs in quantum physics.

The observable is a selfadjoint operator

$$\hat{A} : \mathcal{H}_e \to \mathcal{H}_S$$

of system's states.

The spectral representation

$$\hat{A} = \int_{-\infty}^{\infty} \lambda d\hat{E}_{\lambda}^{(A)} \tag{3}$$

of the operator associates to the observable A the family of projectors $\hat{E}_{\lambda}^{(A)}$ which depend on a real parameter λ. To the proposition "$A \in \Delta$" the closed *linear subspace* $\mathcal{H}_{\Delta}^{(A)}$ in $\mathcal{H}_S$ corresponds. The projector $\hat{P}_{\Delta}^{(A)}$ to the subspace reads

$$\hat{P}_{\Delta}^{(A)} = \int_{\lambda \in \Delta} d\hat{E}_{\lambda}^{(A)} \tag{4}$$

The subset of some set have been told to model classical logic. In a similar manner, the set of subspaces of some Hilbert space act as a basis of various types of quantum logic.

$$\hat{P}_{\Delta}^{(A)} : \mathcal{H}_S \to \mathcal{H}_{\Delta}^{(A)}$$

Quantum logic appeared due to Birkhoff and von Neumann [4]. They associated propositions on a quantum system with closed subspaces of its Hilbert space $\mathcal{H}_S$. The set of subspaces may be endowed with models of logical connectives: conjunction

$$\mathcal{H} \wedge \mathcal{H}' =_{df} \mathcal{H} \cap \mathcal{H}', \tag{5}$$

disjunction

$$\mathcal{H} \vee \mathcal{H}' =_{df} \bigcap \{\mathcal{H}'' : \mathcal{H} + \mathcal{H}' \subseteq \mathcal{H}''\}, \tag{6}$$

are identical to exact lower and upper bounds with respect to the partial order based on inclusion; negation

$$\neg\mathcal{H} =_{df} \mathcal{H}^{\perp}. \tag{7}$$

Here $\mathcal{H}^{\perp}$ is the subspace of vectors from $\mathcal{H}_S$ which are orthogonal to any vector from $\mathcal{H}$. The following equality takes place

$$\mathcal{H}_S = \mathcal{H} \oplus \mathcal{H}^{\perp}, \tag{8}$$

where $\oplus$ stands for direct sum of subspaces (i.e. $\mathcal{H}+\mathcal{H}^{\perp} = \mathcal{H}_S$ and $\mathcal{H}\cap\mathcal{H}^{\perp} = 0$). It follows from (8) that

$$\mathcal{H}^{\perp\perp} = \mathcal{H}. \tag{9}$$

In the set of subspaces ordered by inclusion there are the minimal one (spanned by zero vector) and the maximal subspace which coincides with $\mathcal{H}_S$. It is worth to note that in quantum logical models distributivity is lost of conjunction and disjunction over one another.

Abstracting the main properties of subspaces of $\mathcal{H}_S$, one arrives at the notion of *orthologic* [5] which is also refered to as *minimal quantum logic* in some earlier works. The language of orthologic consists of propositions and two connectives of conjunction, $\wedge$, and negation, $\neg$, associated with (5) and (7) respectively. Double negation leaves any proposition unchanged in accordance with (9). The connective of disjunction $\vee$ for propositions α and β is defined by means of $\wedge$ and $\neg$:

$$\alpha \vee \beta = \neg(\neg\alpha \wedge \neg\beta). \tag{10}$$

This is the so-called de Morgan law. It is easy to show that when applied to the set of subspaces of в $\mathcal{H}_S$ this definition is equivalent to (6).

There is an algebraic representation of orthologic called *orthocomplemented lattice:*

Definition 1.1

Orthocomplemented lattice $\mathcal{OL}$ *is a bounded lattice* $\mathcal{L} = \langle L, \vee, \wedge, \mathbf{1}, \mathbf{0}\rangle$, *which is endowed also with orthocomplement operation* $\{\}^{\perp} : L \to L$

(i) $a^{\perp\perp} = a$;

(ii) if $a \leq b$, then $b^{\perp} \leq a^{\perp}$;

(iii) $a \wedge a^{\perp} = \mathbf{0}$.

The lack of distributivity is a generic feature of orthocomplemented lattices. Only the following conditions take place:

$$(a \wedge b) \vee (a \wedge c) \leq a \wedge (b \vee c) \tag{11}$$

and

$$a \vee (b \wedge c) \leq (a \vee b) \wedge (a \vee c). \tag{12}$$

This fact distinguish orthologic from classical logic and its realization in Boolean lattices. Resuming, one may say that nonclassical character of orthologic and intuitionistic logic in topoi consists in abandoning distributivity in the first case and the rule of excluded middle in the second one.

The search for natural realization of orthologic as the simplest quantum logic is of great interest in the framework of topos approach to branching space-time. Let us take the following concept as a leading one. Besides the algebraic realization of orthologic by orthocomplemented lattices, there is an equivalent alternative of its realization in terms of Kripke semantics of possible worlds [6]. Namely, to every proposition α one can associate the set of worlds where this proposition is true. The Belnapian worlds provide natural material for Kripke semantics. In the following section this will be realized and a structure of local orthologics (attached to every event from $\mathcal{C}$) will be shown to emerge naturally in the topos approach.

2 Local orthologics

The following pair of presheaves (contravariant functors from $\mathcal{C}$ to Set) will act as a starting point: $\langle \mathbf{Loc}, \mathbf{Acc} \rangle$. The functor $\mathbf{Loc}$ was introduced in [1]. For any event e the set $\mathbf{Loc}_e$ consists of all Belnapian worlds which incorporate e. To define the functor $\mathbf{Acc}$, we set

$$\mathbf{Acc}_e = \{\langle w_1, w_2 \rangle \in \mathbf{Loc}_e \times \mathbf{Loc}_e : \exists e \rightsquigarrow e' \, (e' \neq e, \, e' \in w_1, \, e' \in w_2)\} \tag{13}$$

For any causal relation $e_1 \rightsquigarrow e_2$, there is a map of inclusion $\mathbf{Acc}_{e_1 e_2} : \mathbf{Acc}_{e_2} \to \mathbf{Acc}_{e_1}$, so that $\mathbf{Acc}$ is a contravariant functor from $\mathcal{C}$ to Set. The set $\mathbf{Acc}_e$ introduces an *accessibility relation* for pairs of worlds from $\mathbf{Loc}_e$ [2]. In accordance with (13) the worlds w_1 and w_2 are accessible to one another in e iff the event e is not the last common event in these worlds. This relation is reflexive and symmetric but not transitive generally. Using the relation $\mathbf{Acc}_e$ one can introduce the operation $\{\}^\perp$ in the power-set $\mathcal{P}(\mathbf{Loc}_e)$:

$$X \mapsto X^\perp =_{df} \{w \in \mathbf{Loc}_e : \, \forall w' (w' \in X \Rightarrow \langle w, \, w' \rangle \notin \mathbf{Acc}_e)\}, \tag{14}$$

where $X \subseteq \mathbf{Loc}_e$. One gets

$$Y \subseteq X \Rightarrow X^\perp \subseteq Y^\perp \tag{15}$$

for all $X, Y \subseteq \mathbf{Loc}_e$.

It is easy to verify that the result of double implementation of (14) may be written as

$$X^{\perp\perp} = \cup\{Y \in \mathcal{P}(\mathbf{Loc}_e) : \, Y \circ \mathbf{Acc}_e \subseteq X \circ \mathbf{Acc}_e). \tag{16}$$

Here the product $\circ$ of relations appears:

$$X' \circ \mathbf{Acc}_e =_{df} \{w \in \mathbf{Loc}_e : \exists w' (w' \in X', \, \langle w', w \rangle \in \mathbf{Acc}_e)\}.$$

It follows from (16) that

$$X \subseteq X^{\perp\perp} \tag{17}$$

for all $X \in \mathcal{P}(\mathbf{Loc}_e)$. Combining (15) and (17), we arrive at

$$X^{\perp} = X^{\perp\perp\perp}, \tag{18}$$

which (due to evident equality $X^{\perp\perp} = X^{\perp\perp\perp\perp}$) licence us to conclude that this double of (14) introduces the *closure operation* [7] in $\mathcal{P}(\mathbf{Loc}_e)$. The system

$$L_e = \{X \subseteq \mathbf{Loc}_e : X = X^{\perp\perp}\} \tag{19}$$

of closed subsets is known to sustain arbitrary intersection:

$$\forall i \in I (X_i \in L_e) \Rightarrow \bigcap_{i \in I} X_i \in L_e.$$

The system L_e is partly ordered with respect to inclusions, contains minimal (empty) subset $\mathbf{0}$ as well as maximal subset $\mathbf{1}$ equal to $\mathbf{Loc}_e$. The operation

$$\bigwedge_{i \in I} X_i = \bigcap_{i \in I} X_i \tag{20}$$

and

$$\bigvee_{i \in I} X_i = \Big(\bigcup_{i \in I} X_i\Big)^{\perp\perp} = \Big(\bigcap_{i \in I} X_i^{\perp}\Big)^{\perp} \tag{21}$$

turn L_e into complete bounded lattice. The operation $\{\}^{\perp}$ is an orthocomplementation on L_e.

In Kripke semantics for every event e from $\mathcal{C}$ there is an orthocomplemented lattice $\mathcal{OL}_e = \langle L_e, \wedge, \vee, {}^{\perp}, \mathbf{0}, \mathbf{1}\rangle$ which represents some orthologic.

3 Natural topology in $\mathbf{Loc}_e$

We are going to introduce natural topology in $\mathbf{Loc}_e$ by means of some auxiliary constructions. Using the contravariant functor $\mathcal{P} : Set \rightarrow Set$ of power-sets we create the composition $\mathcal{P} \cdot \mathbf{Loc} : \mathcal{C}^{op} \rightarrow Set$. The set $(\mathcal{P} \cdot \mathbf{Loc})_e$ was previously refered to as $\mathcal{P}(\mathbf{Loc}_e)$. We need also the presheaf $\bar{\mathbf{\Omega}} : \mathcal{C}^{op} \rightarrow Set$ which associates to event e the set $\bar{\mathbf{\Omega}}_e$ of all cosieves on e. Any causal arrow $e_1 \rightsquigarrow e_2$ generates the map $\bar{\mathbf{\Omega}}_{e_1 e_2} : \bar{\mathbf{\Omega}}_{e_2} \rightarrow \bar{\mathbf{\Omega}}_{e_1}$ so that

$$\bar{\mathbf{\Omega}}_{e_1 e_2}(\bar{S}_2) = \{e_1 \rightsquigarrow e : e_2 \rightsquigarrow e \in \bar{S}_2\},$$

where $\bar{S}_2$ is some cosieve on e_2.

For any event e we introduce the map

$$\nu_e : \bar{\mathbf{\Omega}}_e \to (\mathcal{P} \cdot \mathbf{Loc})_e, \quad \text{where} \quad \nu_e(S) = \cup\{\mathbf{Loc}_{e'} : \exists\, e \rightsquigarrow e' \in S\}. \tag{22}$$

The maps ν_e for various e may be considered as components of a natural transformation $\nu : \bar{\mathbf{\Omega}} \to \mathcal{P} \cdot \mathbf{Loc}$, which follows from the commutativity of the diagram

$$\begin{array}{ccc} \bar{\mathbf{\Omega}}_{e_2} & \xrightarrow{\nu_{e_2}} & (\mathcal{P} \cdot \mathbf{Loc})_{e_2} \\ {\scriptstyle \bar{\mathbf{\Omega}}_{e_1 e_2}}\downarrow & & \downarrow{\scriptstyle (\mathcal{P}\cdot\mathbf{Loc})_{e_1 e_2}} \\ \bar{\mathbf{\Omega}}_{e_1} & \xrightarrow{\nu_{e_1}} & (\mathcal{P} \cdot \mathbf{Loc})_{e_1} \end{array} \tag{23}$$

for any causal arrow $e_1 \rightsquigarrow e_2$.

Theorem

(a) *there is a topology* $\boldsymbol{\tau_e}$ in $\mathbf{Loc}_e$ *with open sets* $\nu_e(S)$, *where* S is some cosieve;

(b) *the topology* $\boldsymbol{\tau_e}$ *is Hausdorff iff any pair of different Belnapian worlds from* $\mathbf{Loc}_e$ *contains a pair of inconsistent events;*

(c) *the map* $\boldsymbol{\tau : \mathcal{C}^{op} \longrightarrow Set}$ (*so that* $\boldsymbol{\tau : e \mapsto \tau_e}$) *is a presheaf.*

Proof: To prove (a), one has to check the properties of open sets in the collection $\{\nu_e(\bar{S}) : \bar{S} \in \bar{\mathbf{\Omega}}_e\}$. It is evident that $\nu_e(\emptyset) = \emptyset$ and $\nu_e(\bar{S}_{max}) = \mathbf{Loc}_e$, so that the empty subspace in $\mathbf{Loc}_e$ and $\mathbf{Loc}_e$ itself are open. Then one can verify that

$$\bigcup_{i\in I} \nu_e(\bar{S}_i) = \nu_e(\bigcup_{i\in I} \bar{S}_i). \tag{24}$$

The openness of the set in the left follows from the closure of the collection of cosieves on e with respect to intersections. Also the strait check gives

$$\nu_e(\bar{S}_1 \cap \bar{S}_2) \subseteq \nu_e(\bar{S}_1) \bigcap \nu_e(\bar{S}_2). \tag{25}$$

To prove the inverse inclusion, one has to consider the world

$$w \in \nu_e(\bar{S}_1) \bigcap \nu_e(\bar{S}_2).$$

There must exist a pair of causal arrows: $e \rightsquigarrow e_1 \in \bar{S}_1$ and $e \rightsquigarrow e_2 \in \bar{S}_2$ such that $e_1 \in w$ and $e_2 \in w$. The directedness of the world w licences us to infer the existence of an event $e' \in w$ which is a common effect of e_1 and e_2: $e_1 \rightsquigarrow e'$ и $e_2 \rightsquigarrow e'$. Due to closure of cosieves with respect to prolongation of causal arrows we have $e \rightsquigarrow e' \in \bar{S}_1$ and $e \rightsquigarrow e' \in \bar{S}_2$. Therefore $e \rightsquigarrow e' \in \bar{S}_1 \cap \bar{S}_2$ and $w \in \nu_e(\bar{S}_1 \cap \bar{S}_2)$. Combining this fact with (25), we get

$$\nu_e(\bar{S}_1 \cap \bar{S}_2) = \nu_e(\bar{S}_1) \bigcap \nu_e(\bar{S}_2). \tag{26}$$

To prove (b), we first assume $\boldsymbol{\tau_e}$ to be Hausdorff topology. For the given pair of worlds $w_1 \neq w_2$ from $\mathbf{Loc}_e$ there exist a pair of cosieves $\bar{S}_1$ and $\bar{S}_2$ on e so that

$$w_i \in \nu_e(\bar{S}_i)\ (i = 1, 2); \quad \nu_e(\bar{S}_1) \cap \nu_e(\bar{S}_2) = \emptyset.$$

Consequently, there is a pair of causal arrows $e \rightsquigarrow e_i \in \bar{S}_i;\ e_i \in w_i\ (i = 1, 2)$. The events e_1 and e_2 are inconsistent. Now let any pair of worlds $w_1 \neq w_2$ из $\mathbf{Loc}_e$ to include inconsistent events $e'_i \in w_i\ (i = 1, 2)$. Due to directedness of Belnapian worlds, for e and e'_i in w_i there is a common effect e_i, i.e. $e \rightsquigarrow e_i$ and $e_i \in w_i\ (i = 1, 2)$. The events e_1 and e_2 are also inconsistent due to the back closure of Belnapian worlds. One may construct the cosieves $\bar{S}_i = \{e \rightsquigarrow e'_i : \exists e_i \rightsquigarrow e'_i\}$. It is easy to note that any pair of arrows from $\bar{S}_1$ and $\bar{S}_2$ points at inconsistent events. Therefore, $\nu_e(\bar{S}_1) \cap \nu_e(\bar{S}_2) = \emptyset$, i.e. w_1 и w_2 have disjoint neighborhoods. The assertion (b) is proven.

The proof of (c) follows directly from commutativity of (23). $\square$

In (24) and (26) the structure of cosieve set $\bar{\boldsymbol{\Omega}}_e$ is explicitly related to the structure of open sets in topology $\boldsymbol{\tau_e}$. It is worth to consider some further properties of this relation.

Let us introduce the maps

$$\mu_e : (\mathcal{P} \cdot \mathbf{Loc})_e \to \bar{\boldsymbol{\Omega}}_e, \quad \text{where} \quad \mu_e(X) = \{e \rightsquigarrow e' : \mathbf{Loc}_{e'} \cap X = \emptyset\}. \quad (27)$$

One may readily check that μ_e are the components of a natural transformation $\mu : \mathcal{P} \cdot \mathbf{Loc} \to \bar{\boldsymbol{\Omega}}$; this is a morphism from $Set^{\mathcal{C}^{op}}$, directed oppositely to (22).

The set $\bar{\boldsymbol{\Omega}}_e$ of cosieves on e is known to be endowed with Heyting algebra structure [8] which possesses by (local) negation operation

$$\neg_e : \bar{\boldsymbol{\Omega}}_e \to \bar{\boldsymbol{\Omega}}_e, \quad \text{where} \quad \neg_e \bar{S} = \bigcup \{\bar{S}' \in \bar{\boldsymbol{\Omega}}_e : \bar{S} \cap \bar{S}' = \emptyset\}.$$

One may readily verify that

$$\neg_e \bar{S} = \{e \rightsquigarrow e' : \forall e \rightsquigarrow e''\, (e \rightsquigarrow e'' \in \bar{S} \Rightarrow \mathbf{Loc}_{e'} \cap \mathbf{Loc}_{e''} = \emptyset)\} = (\mu_e \cdot \nu_e)(\bar{S}). \quad (28)$$

Therefore, the negation $\neg : \bar{\boldsymbol{\Omega}} \to \bar{\boldsymbol{\Omega}}$ appears to be the following composition of natural transformations:

$$\neg = \mu \cdot \nu. \quad (29)$$

The rearranged composition $\nu \cdot \mu : \mathcal{P} \cdot \mathbf{Loc} \to \mathcal{P} \cdot \mathbf{Loc}$ is also worth to be considered. There following is true.

Lemma 3.2

Если $\nu_e(\bar{S}) \cap \nu_e(\bar{S}') = \emptyset$, *то* $\bar{S}' \subseteq \neg_e \bar{S}$.

The proof uses (28) and the fact that from $e \rightsquigarrow e' \in \bar{S}$ and $e \rightsquigarrow e'' \in \bar{S}'$ it follows that $\mathbf{Loc}_{e'} \cap \mathbf{Loc}_{e''} = \emptyset$. $\square$

Note that from the lemma one may conclude the identity of $\nu_e(\neg_e \bar{S})$ and the internal part of exterior of $\nu_e(\bar{S})$. With (29) this can be written as

$$Int_{\boldsymbol{\tau}_e}(\mathbf{Loc}_e \backslash \nu_e(\bar{S})) = \nu_e(\mu_e \cdot \nu_e)(\bar{S}) \equiv \nu_e \cdot \mu_e(\nu_e(\bar{S})) \tag{30}$$

This relation let us to treat the restriction of $\nu_e \cdot \mu_e$ on $\boldsymbol{\tau_e}$ as *pseudocomplement in the topology* $\boldsymbol{\tau_e}$*, which gives the internal part of exterior of an open set.*

The collection of all open subsets any topological space (as well as the set of all cosieves) caries the structure of Heyting algebra with ordinary intersection and union as conjunction and disjunction and with pseudocomplement as negation. The former reasoning let us to state the

Proposition 3.3

The natural transformations

$$\mu \cdot \nu : \bar{\mathbf{\Omega}} \to \bar{\mathbf{\Omega}} \tag{31}$$

and

$$\nu \cdot \mu : \boldsymbol{\tau} \to \boldsymbol{\tau} \tag{32}$$

are pseudocomplements in the presheaves of cosieves and topologies from 3.1 respectively. $\square$

The following simple but important theorem relates the topology $\boldsymbol{\tau_e}$ in $\mathbf{Loc}_e$ with the local orthologic $\mathcal{OL}_e$.

Theorem

The propositions of the orthologic $\mathcal{OL}_e$ are closed in the topology $\boldsymbol{\tau_e}$.

Proof: Every proposition from $\mathcal{OL}_e$ has the form $X^\perp$, where X is a subset of $\mathbf{Loc}_e$. Let the world $w_0 \notin X^\perp$ be given. It follows from (14) the existence of a world $w_1 \in X$ such that $\langle w_1, w_0 \rangle \in \mathbf{Acc}_e$. To put it otherwise, there is an arrow $e \rightsquigarrow e_0 : w_1, w_0 \in \mathbf{Loc}_{e_0}$. We need the cosieve $\bar{S}_0 = \{e \rightsquigarrow e' : \exists e_0 \rightsquigarrow e'\}$. We have $w_0 \in \nu_e(\bar{S}_0)$. One may readily check that $\nu_e(\bar{S}_0) \bigcap X^\perp = \emptyset$. Genuinely, let there exists a world $w' \in \nu_e(\bar{S}_0) \bigcap X^\perp$. It follows from (14) and $w' \in X^\perp$ that $\langle w_1, w' \rangle \notin \mathbf{Acc}_e$. On the other hand, one may infer from $w' \in \nu_e(\bar{S}_0)$ the existence of an arrow $e_0 \rightsquigarrow e'$ so that $w' \in \mathbf{Loc}_{e'}$. The completeness of worlds with respect to causes gives $e_0 \in w'$. Consequently, $e_0 \in w_1 \cap w'$. Now, due to $e \rightsquigarrow e_0$ we have $\langle w_1, w' \rangle \in \mathbf{Acc}_e$, which contradicts the previous result. Therefore, a neighborhood $\nu_e(\bar{S}_0)$ of the world w_0 must exist, which belongs to the exterior of the proposition under consideration. For this conclusion is valid for any world from the exterior of $X^\perp$, the last proposition is closed in the topology $\boldsymbol{\tau_e}$. $\square$

4 Conclusion

Resuming, we see that in the framework of the topos approach one may 'attach' the local orthologic $\mathcal{OL}_e$ to any event e. Our intuition suggests association of the local orthologic with 'semi-quantum' version of some *local* Hilbert space $\mathcal{H}_e$. The last is the space of states of external reality with respect to an observer *located* at e. Following the arguments from Introduction, the proposition "$A \in \Delta$" for the observer should be related to some subspace from $\mathcal{H}_e$ – the linear manifold which is closed in the Hilbert space topology. This is in parallel with the natural topology $\boldsymbol{\tau_e}$ in $\mathbf{Loc}_e$. Further still Theorem 3.4 asserts the closure of propositions from the local orthologic. Therefore, the intuitive interpretation of local orthologics as local 'pre-quantum' logics finds its support.

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Minimal black holes and their stability

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The existence of mini and micro black holes and their role for new physics are discussed. It is shown they are objects which have energetic structures due to their small dimensions. This in turn follows to any certain conclusions such as an explanation of the μBH stability. The schematics for mini and micro black hole's mass and some results of measurement modeling are offered. The needed sensitivities of the measurements are presented.

Key words: Black Holes, mini Black Holes, Plank's mass, Hawking evaporation, stability, structure of black hole

1. Introduction

Black holes (BH) are regarded usually as an astronomical object, which is so massive that cannot let anything out beyond its limits (event horizon). Although the idea of BH was introduced by J. Michell and P. -S. Laplace in the 18th century, it is difficult to understand the existence of matter collapse from the thermodynamic extensivity's point of view [1]. It was only with the discovery of astrophysical evidence of collapsed stars in 60th this idea has become an integral part of physics [2]. Along with it, the existence of black holes tacitly suggests that quantum mechanics - the physics of the micro world - can be applied to the physics of massive objects.

Due to generally accepted modern thoughts based on General Relativity, there are two possibilities for the formation of black holes. BH with masses larger than $1.5M_\odot$ ($M_\odot$ is the mass of the Sun) are resulting from the gravitational collapse of a star once its fuel burned up and the later radiative cooling. They have subsequently served as centers of stellar and galactic system's formation (for example, supermassive BH with $M_\bullet = 4.1\cdot10^6 M_\odot$ is located in the center of the Galaxy). The morphology of BH notes the crucial inherent character of the singularity at the center, which the sphere (in the absence of spin BH) of radius R_S surrounds it. The size of R_S for the supermassive black hole in the Milky Way is four times less than the orbit of Mercury. A BH can be described by three independent properties only: mass, charge, and angular momentum.

Mini black holes (mBH) with $M_\bullet \leq 10^{-18} M_\odot$ can be produced by fluctuations of density in the early Universe, when the density of bulk matter was high and heterogeneity of density was a source for mBH birth. The process is similar to the formation of drops of water in vapor cooling.

Space mBH is often referred to as primordial black holes. These objects have not been observed yet, but their existence is constantly under review. Their numbers greatly exceed the number of massive BH, but are not more than 10^{-7} of dark mass in the Universe.

In the last decade another type of BH discussed. When high energy particles collide the jets of energy emerge. Theoretical analysis points on a possibility for micro Black Holes (μBH) creation at LHC [3, 4]. μBH can be produced at a rate 1 event/Sec. Mass of μBH is ~ 10^{-8} kg, diameter ~ 10^{-35} m (as generally accepted). But for extra dimensions mass of a BH is about 10^{13} eV/c^2,

diameter $\sim 10^{-15}$ m. The behavior of such quantum objects is not the same as classical massive astrophysical BH.
Why mBH and μBH are so interesting?

2.Mini and micro black holes and new physics

There are several reasons for and we limited by topic scope, point to just a few of them. As was recently shown [5], gravity is not only spacetime correlations generator, but also can create new effects. In their paper the modeling of the conditions for neutron star formation was reported. (Its mass is 1.5 - 3 $M_{\odot}$ but its diameter is 20-30 km and the neutron star evolution may lead to the formation of a BH). According to the Heisenberg uncertainty principle there are virtual particles and some background fields of different frequencies in a vacuum is everywhere and at all times. Thus the vacuum has energy. When "normal" (Earth) conditions are realized the influence of quantum fluctuations is negligible.

However, in the vicinity of a sufficiently dense star such as neutron one, the gravitational field becomes intense. Because the size and duration in the gravitational field are changed, then under certain conditions [6] the vacuum energy density becomes so huge that it begins to compete with the mass density of the nearby astrophysical object and even exceed it. Moreover, since the pace of processes is also determined by gravity, then the existence of exponentially growing fluctuation becomes significantly long (few milliseconds). New gravitationally generated effects which influence the evolution of stars are appearing. Thus, the gravitational control of the energy contained in the empty space was indicated. Obviously, this supervise becomes more with the greater heterogeneity of the gravitational field, and the last is bound with the mass density of the object. BH has maximal density. Their suggestions are consistent with contemporary astrophysical observations [7].

The key provision is also the fact that BH produces entangled particles not due to decay, or conversion of one type of particle to another, but by evaporation. This process was proposed by S. Hawking [8]. Under a small radius of BH (Schwarzschild radius $R_S = 2\,GM_{\bullet}/c^2$, where $M_{\bullet}$ is a BH mass) tidal forces near the horizon are so intense that they break a couple of particle emitted from quantum vacuum fluctuations. Single particle falls into a BH, while the other one repels out. The tidal forces near the surface to be inversely proportional to the square of R_S, the radius is proportional to the BH mass. So, the evaporation loss of energy is to be inversely proportional to the square of a BH mass. A BH evaporates and thus loses its energy (mass). The equivalent black body temperature is

$$k_B T_H = \frac{\hbar c^3}{8\pi G M_{\bullet}} = \frac{\hbar g_0}{2\pi c}, \qquad (1)$$

where ($\hbar\, c/8\pi G$) is square of Plank mass, k_B is the Boltzmann constant. The last fraction contains g_0 – acceleration on the sphere of the Schwarzschild radius. In such description the expression of Hawking temperature is similar to Unruh temperature [9], that makes investigation of minimal BH very interesting from an accelerating observer point of view.

Numerically

$$T_H \simeq 1.23 \cdot 10^{23} \left(\frac{1kg}{M_\bullet}\right) K. \qquad (2)$$

Power losses due to evaporation are determined by

$$P = \frac{c^4}{240\hbar} \cdot \left(\frac{\hbar c}{8\pi G M_\bullet}\right)^2 \simeq 0.33 \cdot 10^{33} \left(\frac{1kg}{M_\bullet}\right)^2 W. \qquad (3)$$

As we can see from eq. (3) the reduction of BH mass $M_\bullet$ causes the rapid growth of radiated power so mBH eventually disappears ejecting out a stream of hard γ-rays in final stage. (There is general point of view, but the scrupulous analyses shows halt of evaporation in some case).

Another feature of entangled particle production near BH is possibly the loss of information. BH can be regarded as an information shredder [10, 11].

Astrophysical interest is that mBH may have occurred in the early Universe when the heterogeneity was great in number and intensity. Their residual masses are evidence of the Big Bang as well as the cosmic microwave background radiation (CMB). If the evaporation of MBH stops, they become stable objects. In this case mBH are particles of dark matter. Moreover, if we suppose inside mBH are chameleons - dark matter particles, those change their masses depending on the surrounding mass, then measuring of an mBH mass will give us different values in space and in a terrestrial lab, but its gravitational field will change its potential [12]. In addition, mBH can focus dark energy in the accelerated expansion of the Universe [13].

The mBH and μBH are the objects of attention not only of astrophysics and cosmologists who use alternative theories of gravity, string theory or bran theory [14] to explain the evolution of the Universe, but also high-energy physicists [3, 15].

The prestigious aim of high-energy physicists is Grand Unification Theory - that gravity is combined with other (electromagnetic, weak and nuclear) interactions into a unique interaction with one constant. Towards the goal the obstacle of disproportion arises: incommensurability of interactions (such as gravity and nuclear). It is due to this that gravitation is not considered when physicists are discussing the results of nuclear reactions. mBH and μBH provide the conditions under which quantum and gravitational processes are equally important, thus protecting routes of theory against false directions.

Additional hidden dimensions can also be found during mBH study. According to ADD and RS models [16, 17, 18, 19] at distances less than 10^{-18} m (about the size of a quark) and energy about few TeV the gravitational field strengthens its intensity, becoming comparable with other interactions and revealing an opportunity of its quantization. One can liken decrease the force of attraction at low energy (temperature) of the particles lessening of the magnetic field through the shielding ferromagnetic screen. It leads to another cross-section, time of evaporation and γ-rays signature of mBH and μBH [3, 20, 21]. If the time of mBH and μBH existence will be reasonable, the question about a communication channel may be asked. The newest idea is that mBH (with mass 0.3 – 1000 tonne range) became the 'nuclei' of usual matter capturing nearby particles (including atoms) and forming the gravitational equivalent of atoms [22].

In addition, interest to mBH is supported by several research sounded completely bizarre at first. Is not the structure of BH similar to unitary atoms or even whether mBH are elementary particles [23, 24]? Therefore, the analysis of a particle trajectory within BH [25] is so relevant. The doubts about the charge of mBH and μBH, their spin and related effects remained.

3. Principles of mBH Registration

It should be noted the growing fleet of tools for astrophysical measurements: ground and space telescopes in the radio, infrared, optical, ultraviolet, X-ray and γ-bands; Cerenkov counters of neutrino; long-baseline radio interferometers, calorimeters, RF-hydrophones; superconducting magnets cooled by superfluid helium, semiconductor sensors for WIMPs, high-performance petawatt lasers to record WISPs and axion like particles.

In the most programs, the search of primordial black holes the detection is performed by recording the emission spectra of X-ray and γ-band [26], recording characteristic of a strong magnetic field and a certificate a lensing near these facilities [21] (in accordance with the General Theory of Relativity, the gravitational field bends the light rays that have passed close to the mass, which may lead to a distorted picture of the sky).

These tools include the Space Telescope observation of γ - radiation "Fermi", one of the most important scientific objectives is to inspection γ-radiation from the evaporation of primordial BH. Laser Interferometer Space Antenna (LISA), a joint mission of NASA and ESA [27], and the last project DECIGO / BBO [28] can also be used to register mBH, sweeps over near the interferometer.

mBH register is also possible with the observation of the spectrum of the acoustic Cherenkov radiation generated by mBH penetrating fluid [29] or ice [30].

Registration of Hawking radiation due to it's specific features is possible also.

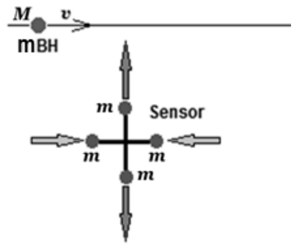
In our previous paper [31], we studied the possibility of mBH register by 3D-sensor network through which a mBH drives. A sensor is the rod of a magnetostrictive material. When mBH moves through the network, its gravitational field creates a momentum of tidal forces. The latter will zoom in and out the magnetostrictive rod, thereby creating additional magnetic fields. Variations of this field are measured by DC SQUID [32, 33].

Here we present results of 1D sensor modeling (Fig.1).

4. Results of modeling of the case, when mBH is passing by the

Parameters of mBH & registration system

M = 10^8 kg
m = 1 kg
l = 0.1 m
L = 1 m
V = 3 * 10^4 m/sec
G = 6.67 * 10^{-11} $m^3/(sec^2\,kg)$



sensor

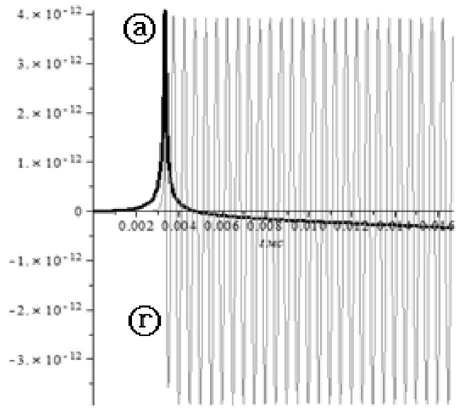


Fig.1. Parameters of mBH and registration system; arrangement of a sensor and trajectory of mBH; results of a sensor response modeling: a) aperiodic case ⓐ, b) resonance case ⓡ.

5. Stability of mBH and μBH

As was noticed above the theory of mBH & μBH must lay on a boundary between classical theory of gravitation and quantum mechanics. A study of their properties can unveil additional hidden dimensions.

Due to their small dimensions it is crucial that μBH are quantum objects [20]. This circumstance leads to what μBH instead of mBH are not thermalized (so they do not undergo Hawking evaporation). Moreover, there are energy levels inside a potential well formed by the gravitational potential. The energy gap between levels is more $m_{\text{Plank}}c^2$ and can be dozen TeV. And it does not matter the exact form of a well. There are gaps their stability is provided by.

6. The conditions of the existence of a steady state

Now let us briefly look at the conditions of stability for micro particle cluster to understand whether microBH be in a stable condition? Atoms can form a stable system – a molecule. From classical and quantum mechanics it is known that in a steady state the system has a minimum total energy.

According to the theory of a covalent bond, for the formation of the molecule in addition to the potential energy of repulsion of like-charged ions the energy of attraction of the ions to the mutual electron, which is constantly moving in the space between them, is needed. This dynamic relationship creates a potential well, in which the system of charges will be.

Now let us see what is inside a minimal BH. The BH catches a particle, her gravitation pulls down particle to the singularity. But the closer the particle to the singularity, the higher its momentum from the Heisenberg uncertainty. With the increase of the momentum the kinetic energy of the particles inside the micro black hole increases also. The total energy became positive and will prevent the approach of the particle to the center of a μBH. On the other words, it may form a potential well depth U_0 and the width R_0 (Fig.2). In the case of microBH a total energy of the system μBH - particle can be written as

$$E_t = -\frac{a}{r} + \frac{b}{r^2} + \frac{d}{r^3},$$

where a / r is Newtonian potential, $b/_{r^2}$ is kinetic energy of localization, $d/_{r^3}$ − is a quadruple interaction center - particle. The last term is much smaller than the first two, as exemplified by the precession of Mercury. Therefore, we will ignore it in a future.

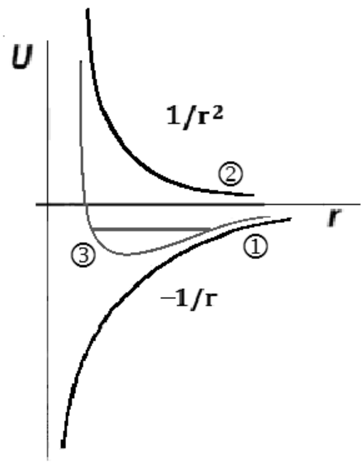


Fig. 2 It is shown the curve of the gravitational energy - ① and the curve of the kinetic energy - ②. After adding them we obtain the curve corresponding to the total energy E_t - ③. The curve has a cavity corresponding to the potential well, which can accommodate an energy level.

The second condition is the existence of a steady state energy level in the well. This condition is expressed in the form

$$U_0 R_0^2 \geq \frac{\pi^2 \hbar^2}{4}. \quad (4)$$

If you take $M_{\bullet} = \sqrt{\hbar c/_G} = 1.22 \cdot 10^{19}\ GeV/c^2$ - a minimum BH mass and assume that it "swallowed" Higgs boson with mass ≐ 125 GeV/c^2, the estimates of the coefficients are $a = 1.47 \cdot 10^{-27}\ J \cdot m$ and $b = 0.62 \cdot 10^{-44}\ J \cdot m^2$.

Now for $r_0 = a/_b = 0.44 \cdot 10^{-17}\ m$ the equivalence of the first two terms is fulfilled, the depth of the well is equal to

$$U_0 = 0.16\,\frac{a^2}{b} = 0.53 \cdot 10^{-10}\,J,$$

with width $R_0 = 3.91 \cdot 10^{-17} m$.

Moreover, the condition (4) is satisfied. Thus, for considering conditions the Higgs boson is trapped in a μBH. So, because the boson cannot fly out of it, the μBH does not lose energy by radiation itself, and at the same time it is at a stable level, All this allows us to conclude the system the minimal BH - particle has a stable, i.e. steady state.

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Causality in quantum teleportation

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Quantum teleportation is a protocol capable of sending an unknown quantum state between two parties (Alice and Bob). It consists of two channels: the quantum one that is a maximally entangled bipartite state, and the classical one – a standard communication channel. It turns out that quantum channel looks as transmitting signal both forward and back in time (e.g. Penrose, 1998). Also it leads to a phenomenon of conditional time travel, which was confirmed experimentally by Laforest et al. in 2003. In our work we examine reversal time process in quantum teleportation with quantum causal analysis, which is a new method giving a formal definition and quantitative measure of causal connection in any bipartite system. We consider a modified protocol of teleportation without an ancillary classical channel. Instead of the unitary transformation, made by Bob after receiving a classical signal from Alice, he measures his particle. We move a moment of Bob's measurement in time and watch how causality between the input state, the outcome of Alice's joint measurement, and Bob's outcome changes. It turns out that Bob's outcome is always the effect relative to the first two values even in the case when it was obtained before the input state for teleportation was prepared. So we obtain time reversal causality, but with cause consisting of absolutely random variable representing Alice's measurement outcome. Therefore we can say that Bob can receive a message from random future. On the other hand, an implementation of causal analysis to time reversal treatment of teleportation, which introduces a proper time frame for teleporting qubit (different from observer's time frame), shows that in this special time frame all the effects appear after corresponding causes. Besides this demonstration of time reversal causality, we have considered teleportation of qubit which is in causal connection with another qubit. As a result the possibility of causality teleportation has been uncovered.

Introduction

Quantum teleportation [1] is a protocol which allows transmitting an unknown quantum state from one spatially separated party (commonly named Alice) to another party (commonly named Bob) without movement of any quantum carriers. To perform this operation Alice and Bob need to share a pair of maximally entangled particles. From the moment of its discovery, entanglement attracts attention by apparent violation of relativity. In the case of teleportation relativity is not violated because Alice and Bob also need a classical channel to complete the protocol. Nevertheless quantum information seems to pass through quantum channel that the entangled pair is. Such suggestion implies the presence of signaling in reverse time considered in Ref. [2] and experimentally tested in Ref. [3].

In this paper we consider the question about causality in quantum teleportation. We use quantum causal analysis [4, 5] – a new method, which proposes formal definitions for terms "cause" and "effect" and also proposes a quantitative measure for strength of causal connection. It helps to validate an implementation of time reversal treatment of teleportation and reveals peculiarities of signaling through reverse time.

We also consider a teleportation of qubit which is causal connection with another qubit. As a result we uncover the possibility of "causality teleportation".

General scheme of quantum teleportation

First, let us describe the general idea of quantum teleportation. Suppose there are two spatially separated parties, commonly named Alice and Bob. One of them (Alice) has a particle *A* in some quantum state $|\psi\rangle$ and wants to transmit this state (but not a particle) to Bob. Quantum teleportation is a protocol which allows Bob to obtain this state on his particle *B*. And with agreement with no cloning theorem, during teleportation particle *A* loses its state. So quantum teleportation is a process of transmitting of quantum state in space without movement of quantum particles.

We will consider the simplest variant of quantum teleportation, where teleporting state is a qubit, that is a superposition of two orthogonal states: $|0\rangle$ and $|1\rangle$. For example, it may be polarization degrees of freedom of photon. For the purposes of convenience we will use modified notation for standard Bell basis vectors:

$$\begin{aligned}
|\Phi^{+}\rangle &\equiv \frac{1}{\sqrt{2}}\left(|00\rangle+|11\rangle\right)\equiv|\Psi_1\rangle,\\
|\Phi^{-}\rangle &\equiv \frac{1}{\sqrt{2}}\left(|00\rangle-|11\rangle\right)\equiv|\Psi_2\rangle,\\
|\Psi^{+}\rangle &\equiv \frac{1}{\sqrt{2}}\left(|01\rangle+|10\rangle\right)\equiv|\Psi_3\rangle,\\
|\Psi^{-}\rangle &\equiv \frac{1}{\sqrt{2}}\left(|01\rangle-|10\rangle\right)\equiv|\Psi_4\rangle.
\end{aligned} \tag{2.1}$$

Initially Alice has a particle *A* in the state $|\psi\rangle=\alpha|0\rangle+\beta|1\rangle$ ($|\alpha|^2+|\beta|^2=1$). Teleportation is based on usage of maximally entangled two-qubit states, for example Bell state $|\Psi_4\rangle$. One particle from entangled pair goes to Alice (*C*), and another one *B* goes to Bob (Fig. 1a).

In the first step Alice makes a join measurement on particles *A* and *C* and gets some state from Bell basis: $|\Psi_?\rangle$. The question mark is sub index indicates that the result it totally random. In Einstein's terms we can say that it is "a result playing dice of God with the Universe".

Alice's measurement causes a collapse according to identity

$$\begin{aligned}
&\left(\alpha|0\rangle+\beta|1\rangle\right)\otimes|\Psi_4\rangle=\\
&\frac{1}{4}\left(|\Psi_1\rangle\otimes\left(\alpha|1\rangle-\beta|0\rangle\right)+|\Psi_2\rangle\otimes\left(\beta|0\rangle+\alpha|1\rangle\right)+|\Psi_3\rangle\otimes\left(\alpha|0\rangle-\beta|1\rangle\right)-|\Psi_4\rangle\otimes\left(\alpha|0\rangle+\beta|1\rangle\right)\right)
\end{aligned} \tag{2.2}$$

The particle *B* turns into one of four pure states, depending on what result Alice has obtained. To get state $|\psi\rangle$ Bob needs to transform his state of *B* but he doesn't know which transformation he needs to apply. But Alice does. She sends the result of her measurement (one of four numbers or 2 bits of classical information) by any classical communication channel. Bob applies proper transformation U and obtains his particle B in state $|\psi\rangle$. In Fig, 1a we use the question mark again to emphasize that this transformation depends on Alice's result.

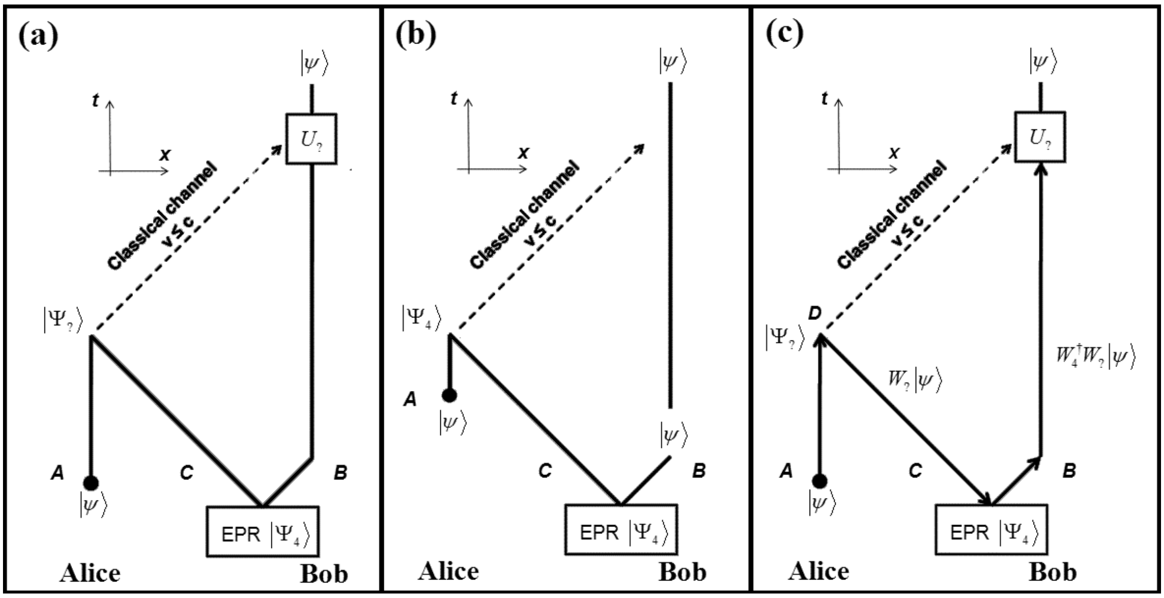

Fig. 1: (a) General scheme of quantum teleportation. (b) Conditional time travel. (c) Time reversal treatment of quantum teleportation.

Conditional time travel and the time reversal treatment

There is an intriguing peculiarity of quantum teleportation, called conditional time travel. If Alice obtains in her measurement the same state as the initial state of *CB* (in our case it is $|\Psi_4\rangle$) then Bob transformation U will be represented by identity matrix. It means that Bob already has his particle in proper state (see Fig. 1b). The question is from which moment Bob already has his particle in proper state. From the viewpoint of standard mathematical approach it seems that Bob's particle collapses in proper state in the moment of Alice's measurement. But it is strange because the problem with instantaneity in space-like interval appears. The last candidate is a moment of EPR pair birth. And really, if we placed a measurement device anywhere on timeline of B, this device would produce statistics like it measures the state $|\psi\rangle$, but only in the case when Alice will get a proper result in her measurement. That is why it is called *conditional* time travel.

In Ref. [3] it was developed an alternative theoretical description for processes in quantum teleportation. The entangled pair has been considered as a channel, which qubit $|\psi\rangle$ follows (see Fig. 1c). Each measurement in basis of entangled states or creation of entangled state has been considered as "time mirror" which changes a direction of qubit propagation in time and makes unitary transformation depending on the corresponding entangled state

$$\left(W_i\right)_{a,b} = \sqrt{2}\langle b,a\,|\,\Psi_i\rangle. \tag{3.1}$$

So after Bell measurement of Alice qubit $|\psi\rangle$ becomes randomly transformed depending on the Alice's result. Then it goes back in time and becomes transformed once again but this transformation is exact. And then goes forward in time to Bob transformation that appears to equal to inverse of all previous transformation: $U_? = \left(W_4^\dagger W_?\right)^{-1} = W_?^\dagger W_4$.

This new time reversal treatment totally confirms with standard tensor product treatment, but its main feature is that it in intuitive way explains the phenomenon of conditional time travel. Next we are going to consider a question about causality which appears in context of time reversal implementation.

Essence of quantum causal analysis

The standard approach to causality is to suppose that effect is something that goes after cause in time order. But retardation is necessary but not efficient condition for causality and moreover in real situations we often do not measure retardation to know that something is a cause and something is an effect of this cause. It indicates that there is some fundamental asymmetry between cause and effect.

The idea of using information theory to define this asymmetry has resulted in an appearance of causal analysis [6], where the cause is defined as subsystem which influences another subsystem (the effect) stronger than vice versa.

Next let us introduce basic principles of quantum causal analysis [4, 5]. Consider some bipartite quantum system *AB*, which is defined by density matrices ρ_{AB}, $\rho_A = \mathrm{Tr}_B \rho_{AB}$ and $\rho_B = \mathrm{Tr}_A \rho_{AB}$. We can use marginal ($S(X) = -\mathrm{Tr}\left[\rho_X \log_2 \rho_X\right]$) and conditional ($S(X\,|\,Y) = S(XY) - S(Y)$) von Neumann entropies to construct a pair of so-called independence functions:

$$i_{A|B} = \frac{S(A\,|\,B)}{S(A)},\ i_{B|A} = \frac{S(B\,|\,A)}{S(B)},\ \text{-}1 \le i \le 1, \tag{4.1}$$

which characterize an influence of *A* on *B* and *B* on *A*.

Causal connection between *A* and *B* corresponds to the inequality $i_{A|B} \ne i_{B|A}$. Then by use of Shannon's theorem about maximal speed of information transmission between *A* and *B* we can obtain minimal times of sending information from *A* to *B* and from *B* to *A*. It turns out that during any period of time effect receives from cause more information than cause receives from effect. Finally we can introduce the velocity of irreversible information flow c_2 (the notation follows the tradition of Ref. [7], where originally, although in less rigorous terms, the course of time pseudoscalar c_2 of the same meaning was introduced):

$$c_2\left(A,B\right) = k\frac{\left(1 - i_{A|B}\right)\left(1 - i_{B|A}\right)}{i_{A|B} - i_{B|A}},\ k = 1. \tag{4.2}$$

Then we can introduce a formal definition for causal connection: A is the cause and B is the effect if $c_2(A,B)>0$. Absence of causal connection corresponds to $i_{A|B}=i_{B|A}$ and $|c_2(A,B)|\to\infty$, so the less $|c_2(A,B)|$ is the stronger causality is.

The main feature of causal analysis is that it does not use a retardation to define causality. For classical causal connection it can be introduced as an axiom:

$$c_2(A,B)>0\Rightarrow\tau_{A\to B}>0,\ \ c_2(A,B)<0\Rightarrow\tau_{A\to B}<0,\ \ c_2(A,B)\to\infty\Rightarrow\tau_{A\to B}\to 0, \qquad (4.3)$$

where $\tau_{A\to B}$ is time delay between embodiments of A and B.

In Ref. [8] Cramer was the first to distinguish the principles of strong and weak causality. The strong (local) causality corresponds to the usual condition for retardation of the effect relative to the cause described by (4.3). Without this axiom we have the weak causality, which corresponds only to nonlocal correlations and implies a possibility of information transmission in reverse time. We will use the violation of (4.3) in quantum teleportation for revealing of such signals and will see that they can carry only random information (hence "the telegraph to the past" is impossible).

There is one interesting property of c_2 which clearly illustrates its meaning. Consider a set of systems A, B, C, D, E (see Fig. 2) which somehow interact with each other.

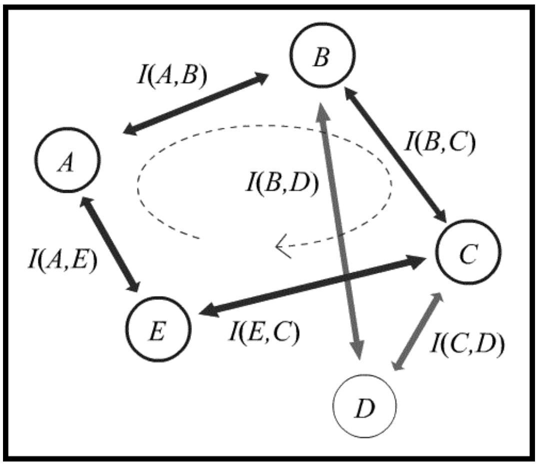

Fig. 2: Illustration of circulation property for c_2.

For any pair of these systems X and Y we can introduce mutual information $I(X,Y)=S(X)+S(Y)-S(XY)$ as a measure of total correlations between them. On the one hand the value of mutual information is symmetric in sense that $I(X,Y)=I(Y,X)$. On the other hand our measure of causality is anti-symmetric: $c_2(X,Y)=-c_2(Y,X)$.

Moreover one can show that (4.2) can be rewritten as

$$c_2(X,Y)=\frac{I(X,Y)}{S(X)-S(Y)}. \tag{4.4}$$

Then, if we choose some closed outline which connects some of these systems and chose a direction in which one can go through this outline we can find that a sum of all values of mutual information divided by corresponding c_2 is equal to zero. E.g. for the outline *A-B-C-E* in Fig. 2 we have $\frac{I(A,B)}{c_2(A,B)}+\frac{I(B,C)}{c_2(B,C)}+\frac{I(C,E)}{c_2(C,E)}+\frac{I(A,E)}{c_2(E,A)}=0$. We can interpret it as the inhibition of causal loops.

Implementation of causal analysis to teleportation

Now we can implement the method of causal analysis to teleportation. First we should consider a standard tensor product treatment. From its point of view teleportation occurs in the moment of Bell measurement. We can consider two configurations of experiment. In case *a* Bob measures his particle *B* before Alice's measurement (Fig. 3a). From the tensor product treatment he just gets some random result. In case *b* Bob measures his particle after Alice's measurement and also gets some random result be this result in encoded version of input state of Alice (Fig. 3b).

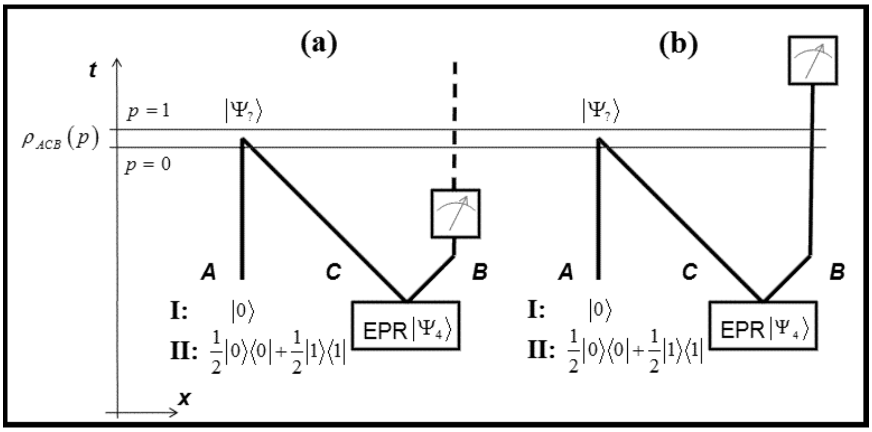

Fig. 3: Two configurations of experiment: (a) Bob measures his particle B before Alice's joint measurement; (b) Bob measures his particle B after Alice's joint measurement.

Also we can introduce two variants of input signal: in variant *I* it is pure state $|0\rangle$ and in variant *II* it is maximally mixed state $\frac{1}{2}|0\rangle\langle 0|+\frac{1}{2}|1\rangle\langle 1|$. So we get four different configurations: *aI*, *aII*, *bI* and *bII*.

To see the behavior of causality we should write a density matrix for the whole system *ACB*. To the purposes of convenience we write it as function of parameter *p*. For $p=0$ we have system just before Bell measurement, for $p=1$ we have system after Bell measurement. Finally we obtain four density matrices during Bell measurement of Alice:

$$
\begin{aligned}
\rho_{ACB}^{aI} &= \rho_{ACB}^{I,\text{out}} + (1-p)\left(\rho_{ACB}^{I,\text{in},\text{mes}B} - \rho_{ACB}^{I,\text{out}}\right),\\
\rho_{ACB}^{bI} &= \rho_{ACB}^{I,\text{out}} + (1-p)\left(\rho_{ACB}^{I,\text{in}} - \rho_{ACB}^{I,\text{out}}\right),\\
\rho_{ACB}^{aII} &= \rho_{ACB}^{II,\text{out}} + (1-p)\left(\rho_{ACB}^{II,\text{in},\text{mes}B} - \rho_{ACB}^{II,\text{out}}\right),\\
\rho_{ACB}^{bII} &= \rho_{ACB}^{II,\text{out}} + (1-p)\left(\rho_{ACB}^{II,\text{in}} - \rho_{ACB}^{II,\text{out}}\right),
\end{aligned}
\tag{5.1}
$$

where

$$
\begin{aligned}
\rho_{ACB}^{I,\text{in}} &= |0\rangle\langle 0| \otimes |\Psi_4\rangle\langle\Psi_4|,\\
\rho_{ACB}^{I,\text{in},\text{mes}B} &= \frac{1}{2}|0\rangle\langle 0| \otimes \left(|01\rangle\langle 01| + |10\rangle\langle 10|\right),\\
\rho_{ACB}^{I,\text{out}} &= \frac{1}{4}\left(|\Psi_1\rangle\langle\Psi_1| + |\Psi_2\rangle\langle\Psi_2|\right)\otimes|1\rangle\langle 1| + \frac{1}{4}\left(|\Psi_3\rangle\langle\Psi_3| + |\Psi_4\rangle\langle\Psi_4|\right)\otimes|0\rangle\langle 0|,\\
\rho_{ACB}^{II,\text{in}} &= \frac{1}{2}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)\otimes|\Psi_4\rangle\langle\Psi_4|,\\
\rho_{ACB}^{II,\text{in},\text{mes}B} &= \frac{1}{4}\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right)\otimes\left(|01\rangle\langle 01| + |10\rangle\langle 10|\right),\\
\rho_{ACB}^{I,\text{out}} &= \frac{1}{8}\left(|\Psi_1\rangle\langle\Psi_1| + |\Psi_2\rangle\langle\Psi_2| + |\Psi_3\rangle\langle\Psi_3| + |\Psi_4\rangle\langle\Psi_4|\right)\otimes\left(|0\rangle\langle 0| + |1\rangle\langle 1|\right).
\end{aligned}
$$

Causality in partition *AC*-*B* for all situations is presented in Fig.4. We see that causality always amplifies with growth of p and the main peculiarity of causality behavior is that for all four configurations $c_2(AC,B)>0$ at $0<p<1$. This is nontrivial result for cases *aI* and *aII*, where Bob performs his measurement before Alice's one. So form the view point of formal causal analysis it is possible to obtain situation when cause happens after effect.

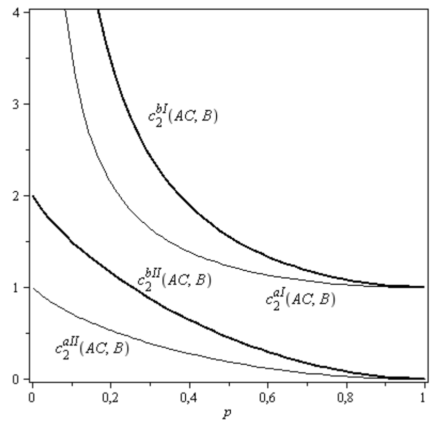

Fig. 4: The behavior of causalities in partition AC-B in different configurations of experiment.

Now let us consider the same four cases with time reversal treatment. It introduces new object *D* which is a result of Bell measurement of *A* and *C*. Moreover in the time reversal treatment there is no differences between the configurations *a* and *b*. Finally we can construct two “density matrices” for the cases *I* and *II* (superscript “tr” emphases that we work in time reversal treatment):

$$\begin{aligned}\rho_{ACBD}^{I,\mathrm{tr}} &= \frac{1}{4}\sum_{j=1}^{4}\left|0,W_j 0,W_4^{\dagger}W_j 0,\Psi_j\right\rangle\left\langle 0,W_j 0,W_4^{\dagger}W_j 0,\Psi_j\right|,\\ \rho_{ACBD}^{II,\mathrm{tr}} &= \frac{1}{8}\sum_{i=0,1}\sum_{j=1}^{4}\left|i,W_j i,W_4^{\dagger}W_j i,\Psi_j\right\rangle\left\langle i,W_j i,W_4^{\dagger}W_j i,\Psi_j\right|.\end{aligned} \tag{5.2}$$

For states (5.2) we obtain the following results: $\left|c_2^{I,\mathrm{tr}}(AC,B)\right| = \infty$, $c_2^{I,\mathrm{tr}}(D,B) = 1$ – these values correspond to the cases *aI* and *bI* at $p=0$ and $p=1$; $c_2^{II,\mathrm{tr}}(AC,B)=1$, $c_2^{II,\mathrm{tr}}(D,B)=0$ – these values correspond to the cases *aII* at $p=0$ and *aII* and *bII* at $p=1$ (see Fig.3). Note that we have obtained $c_2^{bII}(AC,B)=2$ at $p=0$, because of $S(CB)=0$. In time reversal treatment we always have $S(CB)=1$ because the state “knows” that it will be measured.

In time reversal approach we can consider the new partitions: *AD-C* and *AD-B*. From (5.2) we obtain $c_2^{I,\mathrm{tr}}(AD,C)=c_2^{I,\mathrm{tr}}(AD,B)=1$ and $c_2^{II,\mathrm{tr}}(AD,C)=c_2^{II,\mathrm{tr}}(AD,B)=\frac{1}{2}$. We see that these values reveal the propagation of qubit through reverse time. In time reversal treatment all the effects appear after corresponding causes (from the view point of formal causal analysis).

Finally we can reconstruct the full picture of causal connections in quantum teleportation. Entangled pair *CB* is a carrier of two signals: the input state *A* and absolutely random result of Bell measurement *D*. Unitary transformation *U* removes influence of random *D* form *B* and Bob gets initial state of *A*. The most interesting is that by measuring *B* Bob doesn’t just get some random result, this randomness comes through reverse time. If we artificially remove randomness from *D* by corresponding postselection we automatically obtain the conditional time travel.

Teleportation of the causal states

Quantum teleportation has one very interesting modification called *entanglement swapping* [9, 10]. Actually it is teleportation of qubit which is entangled with another qubit. After teleportation it appears to be still entangled. In the Fig. 5a we show entanglement swapping between pair *A-C* and pair *A-B* by teleportation of *C* on *B*.

Entanglement swapping is a particular case of more general situation, when *AC* is described by the arbitrary matrix ρ. And after the same operations we will obtain state *AB* in initial state of *AC*. But the state ρ may be causal in sense of informational asymmetry. For example *A* may be a cause with respect to *C* or vice versa (see Fig. 5b).

In such situations we obtain the teleportation of causality. It is the interesting phenomenon, which can take place in the quantum world. It should be noted that teleportation of causality like standard quantum teleportation is limited by speed of light.

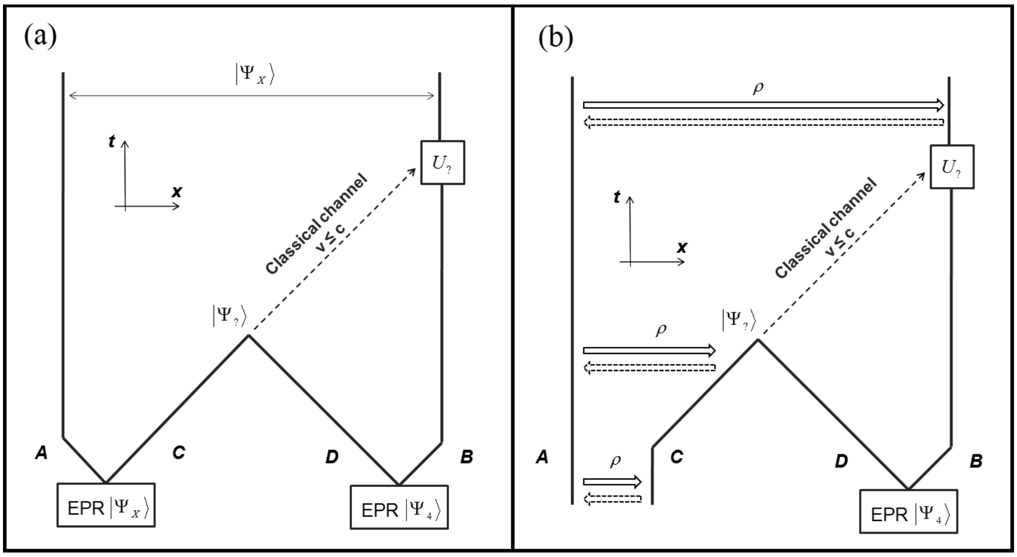

Fig. 5: (a) Scheme of entanglement swapping. (b) Scheme of causality teleportation.

Conclusion

We have considered different treatments of quantum teleportation with quantum causal analysis. Let us make the conclusions.

(1) Causal analysis justifies an implementation of time reversal treatment to teleportation, because exactly in time reversal treatment all the effects appear after the corresponding causes.

(2) Time reversal is an inherent property of quantum entanglement and allows getting information about random future.

(3) Causal analysis shows that "conditional time travel" appears to be a particular case of general signal transmission through reverse time.

(4) Quantum teleportation implies the possibility of causality teleportation, limited by speed of light.

The considered features of time reversal approach may help to understand the experimental results on macroscopic nonlocality (e.g. [11]).

Acknowledgements

This work was supported by RFBR (grants 12-05-00001 and 12-05-98009) and Council for Grants of the President of the Russian Federation (grant SP-961.2013.5).

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Preliminary results of the Baikal experiment on observations of macroscopic nonlocal correlations in reverse time

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Macroscopic quantum entanglement is intriguing phenomenon, the theory of which is still in its infancy. Heuristic consideration of the matter in the framework of action-at-a distance electrodynamics predicts for some processes observability of the advanced nonlocal correlations (time reversal causality). For diffusion entanglement swapping the effective time shifts can be very large. These correlations with greater magnitude than usual retarded ones were really revealed in our previous experiments. Moreover the possibility of the forecast of large-scale heliogeophysical random processes on macroscopic nonlocal correlations had been proven. However the laboratory experiment is difficult because of problem of nonlocal correlation detector shielding against the classical local impacts. Since 2012 a new experiment has been performing on the base of Baikal Deep Water Neutrino Observatory. The thick water layer is an excellent shield against any local impacts on the detectors. A couple of nonlocal correlation detectors, measuring spontaneous variations of self-potential difference between weakly polarized electrode pair, were installed at the depths 52 and 1216 *m*. The bottom detector works under conditions of perfect thermostating and stability of all other environment parameters. Any classical correlations between top and bottom detector signals are impossible. Processing of the first annual time series has revealed rather strong correlation between the signals of bottom, top and spaced at 4200 *km* laboratory detector in Troitsk. The detectors respond to the external (heliogeophysical) processes, and the signal causal connection, revealed by causal analysis turned out directed downwards – from the Earth surface to Baikal floor. But this nonlocal connection proved to be in reverse time – the bottom detector responds earlier than the top one, and top one earlier than surface one. Another result is uncovered nonlocal correlation of the detector signal with a regional source-process – the variation of subsurface (22 *m*) temperature. The temperature is a cause with respect to detector signal, but this nonlocal causal connection is time reversal. The possibility of the temperature forecast with advancement 45 days has been demonstrated.

Introduction

Widely discussed in the past apparent violation of relativity in the entangled states is quite understood now in the framework of quantum nonlocality. Instantaneous and even advanced correlations are possible namely due to absence of any local carriers of interaction. In turn, advanced correlations can occur not only through a space-like interval that could mean usual reversal of time ordering of causally unconnected events. According to the principle of weak causality [1], for the unknown quantum states (or, in other terms, for the random processes) advanced correlations through a time-like interval and hence time reversal causality are possible too. Recently this possibility has been proven experimentally for quantum teleportation [2, 3] and entanglement swapping [4, 5]. Most theoretical efforts in this area are focused on the entanglement of a few microscopic particles. On the other hand, the problem of macroscopic entanglement attracts increasing attention. Macroscopic quantum entanglement is intriguing phenomenon, the theory of which is still in its infancy. But one of the important results of the progress in quantum information theory was discovery of constructive role of dissipation in entanglement generation [6-13]. It bridges the recent research with the early works of Kozyrev, who likely was the first to observe macroscopic entanglement of the dissipative processes with time reversal causality [14, 15].

Background

Our idea was to include dissipation in the framework of Cramer interpretation of quantum nonlocality by Wheeler-Feynman action-at-a-distance electrodynamics [1, 16]. This theory considers the direct particle field as superposition of the retarded and advanced ones. The advanced field is unobservable and manifests itself only via radiation damping, which can be related with the entropy production [17, 18]. Any dissipative process is ultimately related with the radiation damping and therefore the advanced field connects the dissipative processes.

The following heuristic equation of macroscopic entanglement was suggested and tested [17, 23]:

$$\dot{S}_d = \sigma \int \frac{\dot{s}}{x} \delta\left(v^2 t^2 - x^2\right) dV, \tag{2.1}$$

where $\dot{S}_d$ is the entropy production per particle in a probe process (that is a detector), $\dot{s}$ is the density of total entropy production in the sources, the integral is taken over the source volume, σ is cross-section of transaction (it is of an atom order and goes to zero in the classical limit): $\sigma \approx \hbar^4 / m_e^2 e^4$, m_e is the electron mass, e is the elementary charge. The δ-function shows that transaction occurs with symmetrical retardation and advancement. The propagation velocity v for diffusion entanglement swapping can be very small. Accordingly, the retardation and advancement can be very large.

But it should be noted that our equation in its simplest form does not take into account the absorption by the intermediate medium. Its influence, however, is very peculiar. Although the equations of action-at-a-distance electrodynamics are time symmetric, the fundamental time asymmetry is represented by the absorption efficiency asymmetry: the absorption of retarded field is perfect, while the absorption of advanced one must be imperfect [18, 22, 24, 25]. It leads to the fact, that level of advanced correlation through a screening medium may exceed the retarded one.

The experimental problem is to establish correlation between the entropy variations in the probe- and source-processes, according to Eq. (2.1) under condition of suppression of all classical local impacts. The detector based on spontaneous variations of self-potentials of weakly polarized electrodes in an electrolyte proved to be the most reliable one [17-21]. The theory of the electrode detector starts from self-consistent solution of the entropy production in the liquid phase. The entropy of distribution can be expressed in terms of full contact potential. From here one can get the expression of the entropy variation in terms of potential difference between a couple of electrodes, which is the detector signal [17-22].

All known local impacts influencing the detector signal, namely, temperature, pressure, electric field, etc. must be excluded technically and mathematically, which is rather difficult problem.

In our previous works we had conducted a number of the long-term experiments [17-23, 25-29]. Shortly, we revealed macroscopic nonlocal correlations, on the one hand, between the different detectors spaced up to 40 *km*, and, on the other hand, between them and some large-scale astrophysical and geophysical dissipative processes with big random component. Nonlocal nature of correlation had been proven by violation of Bell-like inequality. The most prominent fact was reliable detection of advanced correlations and experimental proof of time reversal causality for the random processes.

The mathematical tool for this proof is causal analysis, which recently plays also important role in theoretical studies of quantum information problems [18, 30-32]. As a matter of fact, although the considered phenomenon is quantum, but as we deal with the classical output of measuring device, we can use simpler classical causal analysis. Recall some points [18-33]. For any variables X and Y several parameters can be defined in terms of Shannon marginal $S(X)$, $S(Y)$ and conditional $S(X/Y)$, $S(Y/X)$ entropies. The most important are the independence functions:

$$i_{Y|X} = \frac{S(Y \mid X)}{S(Y)},\ i_{X|Y} = \frac{S(X \mid Y)}{S(X)},\ 0 \le i \le 1. \tag{2.2}$$

Next the causality function γ is considered:

$$\gamma = \frac{i_{Y|X}}{i_{X|Y}},\ 0 \le \gamma < \infty, \tag{2.3}$$

We can define that X is the cause and Y is the effect if γ<1. And inversely, Y is the cause and X is the effect of γ>1. In the quasiclassical domain that is at positive conditional entropies the measure of causality γ and quantum measure called the course of time (c_2) [18,30-32] are equivalent, in this paper we prefer use γ because of its simplicity. In terms of γ the principle of weak causality is formulated as follows:

$$\gamma < 1 \Rightarrow \tau > 0,\ \gamma > 1 \Rightarrow \tau < 0,\ \gamma \to 1 \Rightarrow \tau \to 0, \tag{2.4}$$

where τ is time shift of Y relative to X.

On theoretical and plenty of experimental examples it had been shown that such a formal approach to causality did not contradict its intuitive understanding e.g. [33-37]. Only in case of nonlocal correlation one can observe violation of this principle. In our previous experiments such time reversal causality had given us even the possibility of successful forecasting of the large-scale heliogeophysical processes [17, 18, 21-29].

Experiment

Since 2012 a new experiment has been performing on the base of Baikal Deep Water Neutrino Observatory. Baikal is the deepest lake in the World and its thick water layer is an excellent shield against the classical local impacts. In particular, the temperature near the floor is constant up to 0.01 *K*.

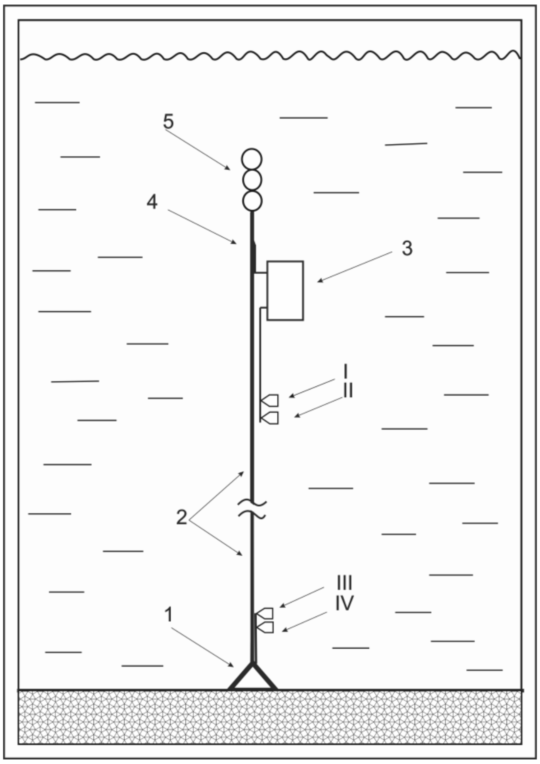

Fig. 1: Baikal Deep Water Setup (1 – anchor; 2 – cable; 3 – electronics unit, acceleration and temperature sensors; 4 – buoy rope; 5 – buoy; I, II – top electrode detector; III, IV – bottom electrode detector).

The experiment aims, first, study of nonlocal correlation between the detectors at different horizons in the lake and spaced at 4200 *km* lab detector in Troitsk, and second, study of correlations of detector signals with the global and regional source-processes.

In Fig.1 the scheme of Baikal Deep Water Setup is shown. The site depth is 1367 *m*. The bottom detector is set at the depth 1216 *m*, the top one is set at the depth 52 *m*. Both the detectors represent a couple of high quality weakly polarized *AgClAg* electrodes HD-5.519.00 with practically zero separation. These electrodes were originally designed for high precision measurements of the weak electric fields in the ocean, and they are best in the World by their self-potential insensitivity to the environmental conditions.

The signal are measured and stored in the electronics unit set at the depth 22 *m*. The sampling rate is 10 *s*. The calibration and zero control are done automatically daily. The relative error of measurements is less than 0.01%. In addition, the electronics unit contains the temperature and acceleration sensors. The setup is fixed by the heavy anchor on the floor and by the drowned buoy at he depth 15 *m*.

The setup is designed to be operated autonomically for a year. It was installed from the ice in March, 2012. In March, 2013 the setup was lifted on the ice for data reading and battery changing and then it was installed again for the next year.

It is known that the strongest macroscopic nonlocal correlations are observed at extremely low frequencies, that is at periods of several months. Therefore our experiment is planed for several years.

Preliminary results of the first annual series

So from classical point of view the detector signals must be uncorrelated random noises. But it is not the case. In Fig. 2 the normalized amplitude spectra of the bottom detector U_b , top one U_t and far distant Troitsk lab one U_l are presented. The period range is from 10 to 220 days. It is seen that at the longest periods the spectra are similar. We observe the semiannual variation, about 100-days solar intermittent variation [38] and its second harmonic, the split maxima around period of solar rotation and its second and third harmonics. The longer period, the better spectra similarity. It is also seen that spectrum of U_b more exactly corresponds to close U_t one than to distant U_b one.

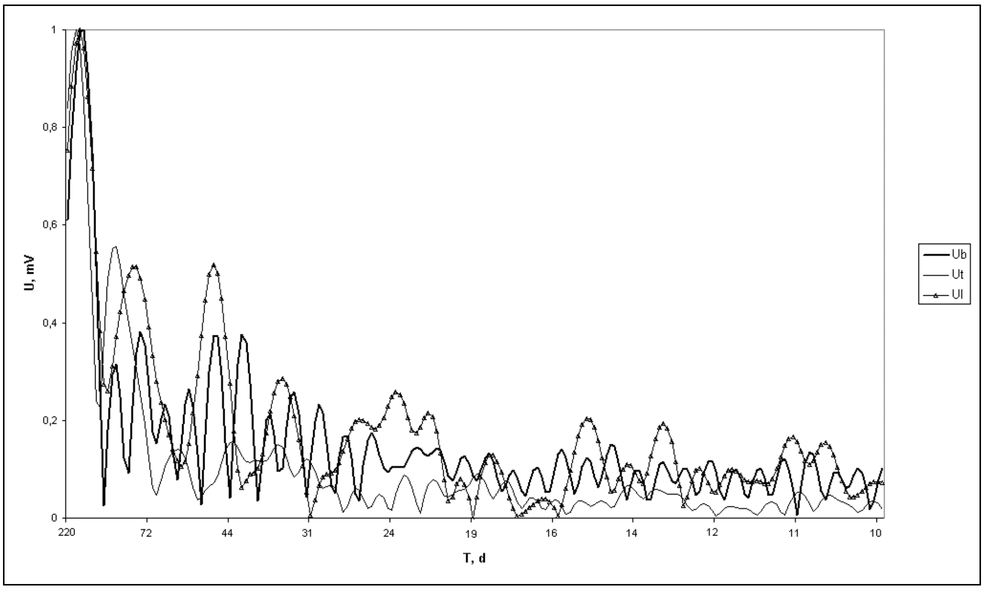

Fig. 2: Normalized amplitude spectra of the signals of bottom detector U_b, top one U_t and lab one U_l.

For causal analysis we used low-pass filtered data (at periods T>77d). Hereafter the relative error of γ and $i_{X|Y}$ estimations is less than 10%. In Fig.3 the results for the bottom and top detectors are presented. γ>1 that is U_t is the cause and U_b is the effect. At τ>0 we observe classically forbidden time reversal causality. It is just weak causality allowed only for the entangled states. Moreover, there are three causality maxima: advanced, approximately synchronous and symmetric retarded. Each maximum of causality γ corresponds to minimum of independence $i_{X|Y}$. The highest maximum of γ=1.8 and the deepest minimum of $i_{X|Y}$=0.25 are at advancement 22 d. Corresponding advanced correlation r=0.87±0.00.

The similar picture with three γ maxima was observed in our previous experiments in case of relatively close source-processes [18, 19, 21]. It is in agreement with Eq. (1) predicting detector responses with two symmetric time shifts, while the synchronous response can be a result of advanced/retarded signal interference. And in those experiments in case of distant source-processes prevailing the retarded (and hence synchronous) response was suppressed owing to absorption and only significant advanced one remained [18, 23, 29].

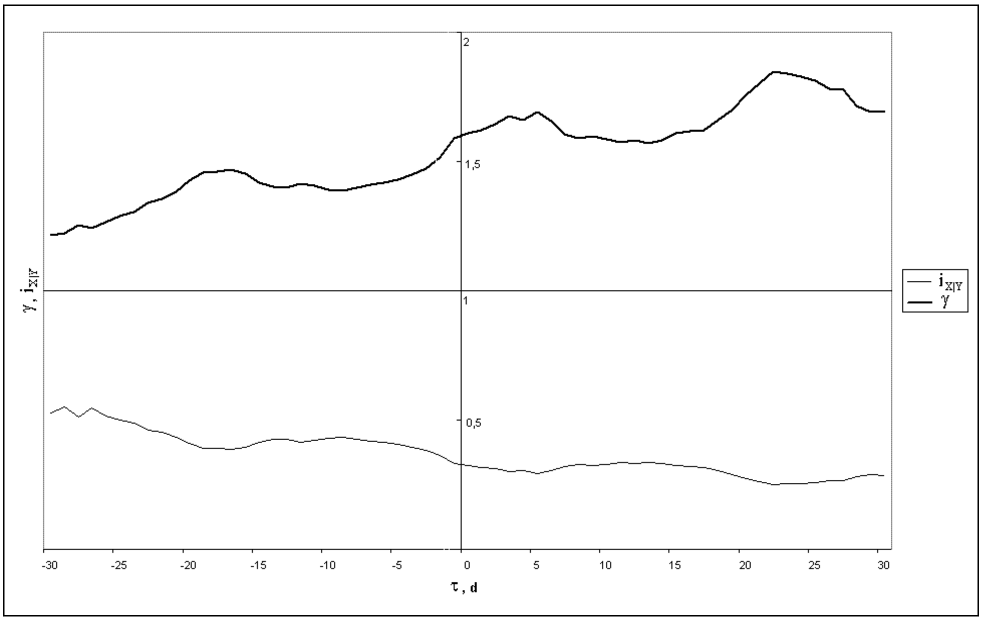

Fig. 3: Causal analysis of U_b (X) and U_t (Y). τ<0 corresponds to retardation of U_b relative U_t,, τ >0 – to advancement.

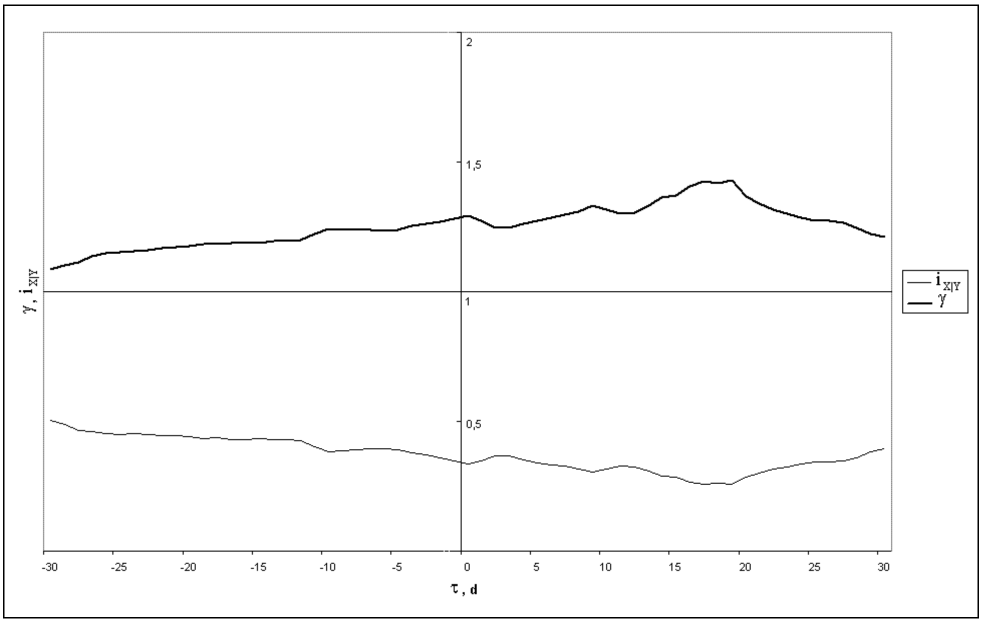

Fig. 4: Causal analysis of U_t (X) and U_l (Y). τ<0 corresponds to retardation of U_t relative U_l,, τ>0 – to advancement.

In Fig. 4 causal analysis of the top detector U_t and the distant one U_l is presented. γ>1 that is U_l is a cause with respect to U_t, and again this causal connection is time reversal. We observe the single $\max \gamma = 1.4$ at advancement 20 *d*. Corresponding $\min i_{X|Y} = 0.20$ and (not shown in this figure) $\max r = 0.97 \pm 0.00$.

And in Fig.5 causal analysis of the bottom detector U_b and distant one U_l is presented. Again U_l is a cause with respect to U_b and causality is time reversal. $\max \gamma = 1.7$ at advancement 25 *d*. Corresponding $\min i_{X|Y} = 0.28$ and $\max r = 0.66 \pm 0.01$.

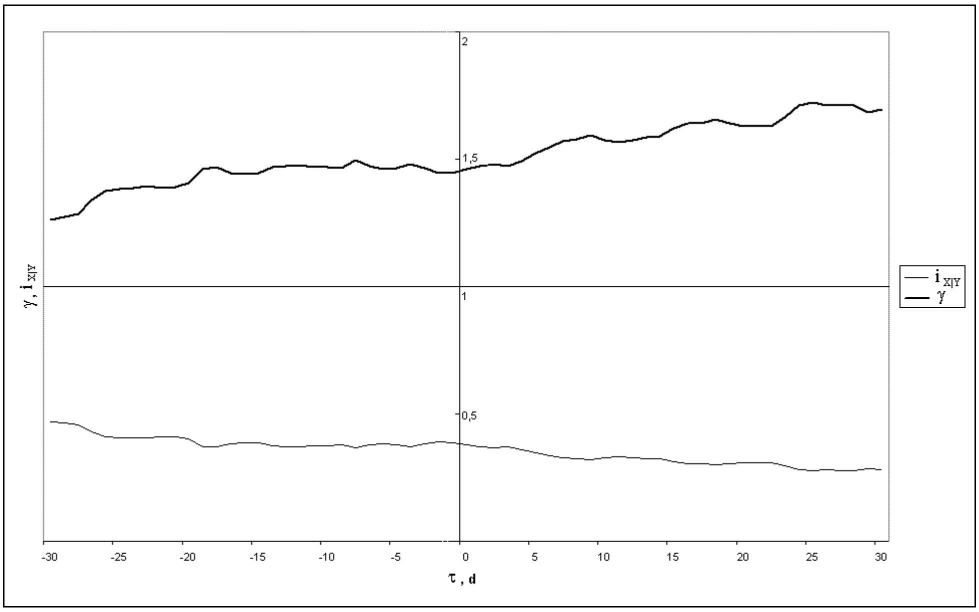

Fig. 5: Causal analysis of U_b (*X*) and U_l (*Y*). τ<0 corresponds to retardation of U_b relative U_l, τ >0 – to advancement.

Thus we may conclude that by data of three detectors the causal connection is directed downwards, from the Earth surface to the lake floor. It is quite natural for the external heliogeophysical source-processes, but this causality is time reversal: the effects appear before the causes!

Consideration of correlation between the detector signals and such global processes will be the subject of our subsequent work, by now we can present a result concerning one possible regional source-process, which is variation of the subsurface temperature measured by our setup at the depth 22 *m*. This variation absolutely can not classically influence on the bottom detector and only slightly can on the top one.

In Fig. 6 the amplitude spectra (in absolute units) of both the detectors and temperature *t* are presented. It is seen that the main maxima of the detector signals do not correspond to the temperature one. There are some small corresponding maxima of *t* and U_t (e.g. at *T*=34.4 *d*). But even this correspondence can not be explained by the classical influence. The fact is, the temperature coefficient of these electrodes equals 0.04 *mV/K*. As the temperature amplitude strongly decays with the depth (the U_t detector is set 30 *m* deeper than the *t* sensor) the spectral amplitude ratio must be less than this value. But it proves to be much greater (>1 *mV/K*). It could be explained only by some nonlocal correlation.

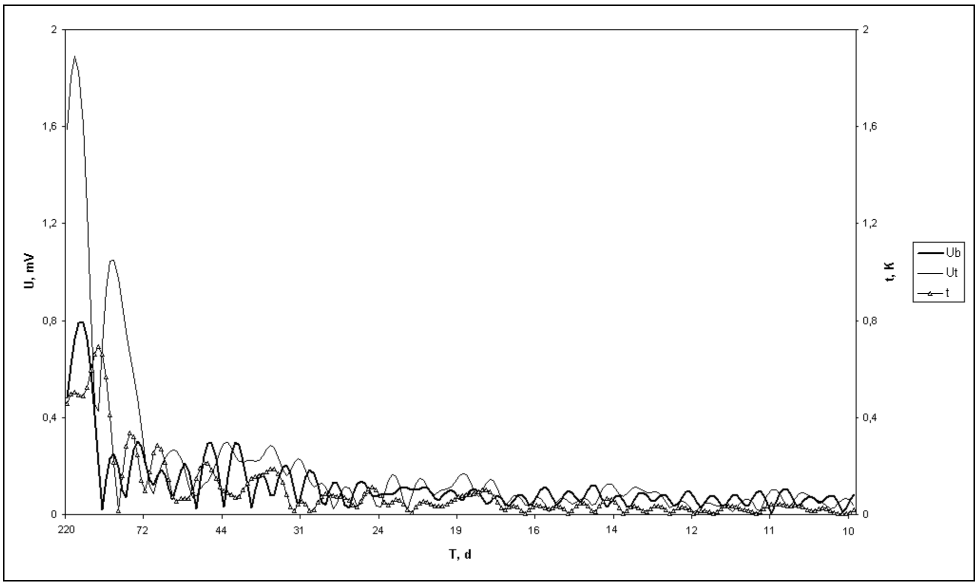

Fig. 6: Amplitude spectra of the bottom detector U_b, top one U_t and temperature t at the depth 22 m.

Consider the results of causal and correlation analysis of U_t and t (Fig. 7) with the same low-pass filtration. We observe that t is a cause with respect to U_t with two equal (1.6) γ maxima at almost symmetric retardation (-40 d) and advancement (45 d). But the advanced $\min i_{X|Y} = 0.17$ is deeper than the retarded ones (1.3 times). And the advanced correlation $\max r = 0.99 \pm 0.00$ is also much greater than the retarded extremum (1.6 times). It is just manifestation of advanced macroscopic nonlocal correlation.

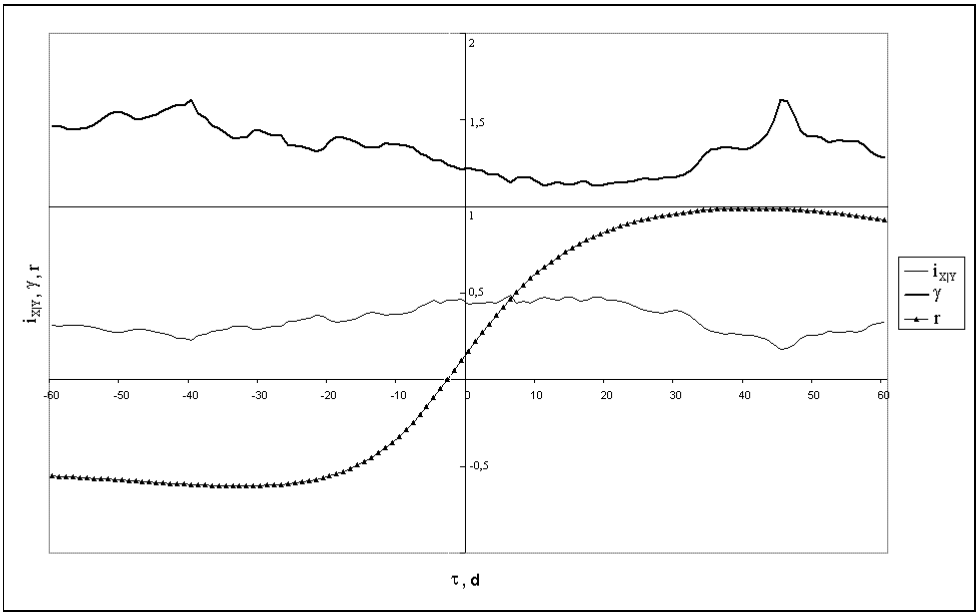

Fig. 7: Causal and correlation analysis of U_t (X) and t (Y). τ<0 corresponds to retardation of U_t relative t, τ>0 – to advancement.

We have applied to these data the forecasting algorithm based on computation of current (sliding) regression. This algorithm needs rather long training interval; hence we could test the forecast only by relatively short segment of the time series. The result is presented in Fig. 8. The forecasting curve showed in this figure is obtained by means of day by day forecasting with fixed advancement τ=45 *d*. The accuracy of the forecast is acceptable for all practical purposes.

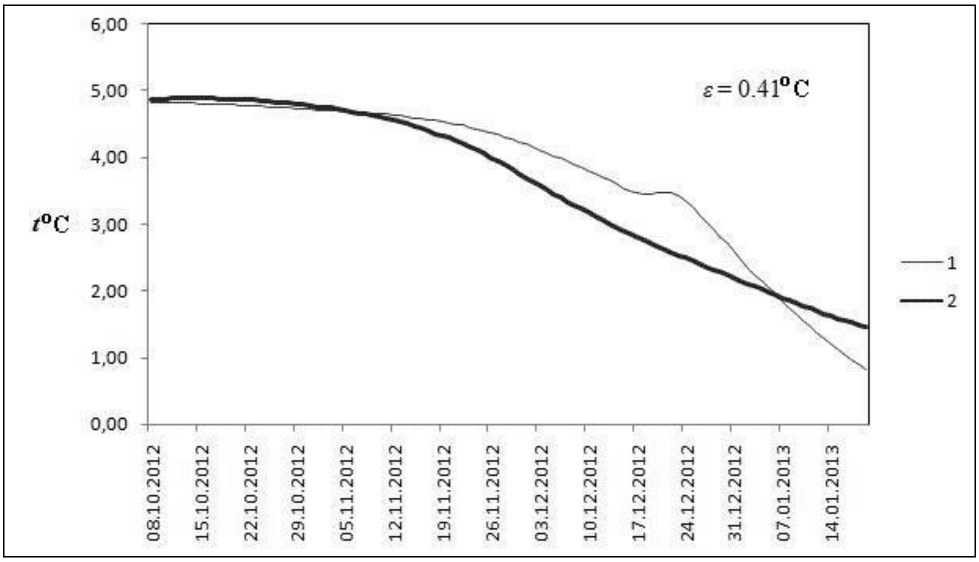

Fig. 8: The forecast of subsurface temperature with advancement 45 days (1) as compared to the factual one (2). The ε is the standard deviation of the forecasting and factual curves.

Conclusion

The long-term Baikal Deep Water Experiment, on study of macroscopic entanglement and related phenomena of advanced nonlocal correlations in reverse time, has begun. The experiment includes measurements with three nonlocal correlation detectors at the depths 52 and 1216 *m* in the Baikal Lake, and at spaced at 4200 *km* laboratory in Troitsk. Detector signals have to be correlated to each other and with large-scale random geophysical and astrophysical source-processes.

The first result is reliable establishment of detector signal time reversal causal connection. It is the most prominent property of macroscopic entanglement and manifestation of quantum principle of weak causality.

In addition, study of subsurface temperature variation in the Baikal, as a regional source-process, has revealed advanced detector signal response, which has been used for the temperature forecast.

Acknowledgements

The authors thank G.V. Domogatsky and A.I. Panfilov fo their contribution to the experiment. This work was supported by RFBR (grants 12-05-00001 and 12-05-98009).

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The Formalism of quantum particle Individual State which is compatible with Bell inequalities

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1. Introduction

The purpose of this article is to eliminate the contradiction between relative theory and quantum Einstein-Podolsky-Rozen (EPR) interaction for correlated particles [1][3]. In such process two distant quantum measurements gives the determinate connection of results without physical interaction of detectors. General research concept is hypothesis of existence for quantum particle the individual state besides ensemble state in wave function form [4][5]. Individual particle state determines the choice of eigenfunction for each measurement operator. We use following terminology. The *particle collective* is the set of particles which arise simultaneous. The *ensemble* of particles or collectives is the birth sequence of these objects. The wave function defines the measurement statistic for ensemble, and individual state give the result for each single measurement. That interpretation forms the statistic of individual states in ensemble. But each individual particle state in collective is not determined by wave function of ensemble. EPR-effect in that theory may be explained by connection between individual particle states in collective of particles in time instant of collective birth. Bell inequality has explained in that theory through ensemble statistic. We shell proof the theorem of existing for such streams of individual particle states.

The problem of agreement EPR-interaction and Relativistic Theory (RT) may see on figure 1.

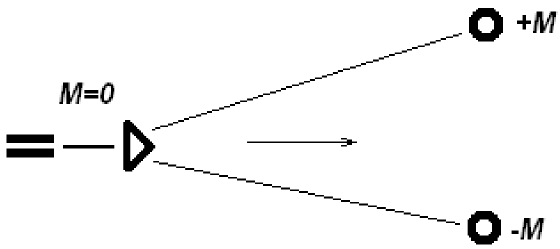

Fig. 1. The scheme of EPR-interaction.

The value of measurement parameter M for initial particle remains in summing value for two generated correlated particles. Measurement in each finish detector is probability process. But interaction between finish detectors is absent. Determined result connection on those detectors contradicts RT.

If assume that each correlated particle has individual state which interacts with detector and forming measurement result, then determine connection of parameter value on two detectors may explain by corresponding of individual states in instant of particle generation.

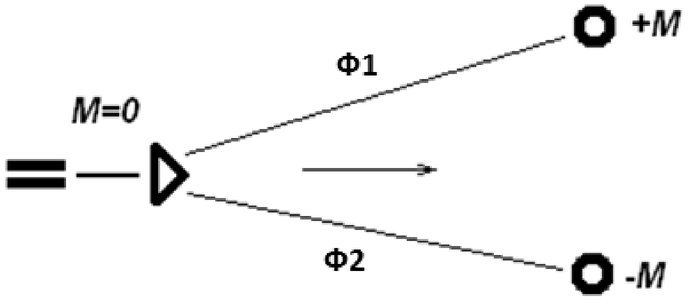

Fig. 2. The scheme of EPR-interaction with individual state of correlated particles.

On figure 2 the individual states Φ1 and Φ2 are formed in zone of birth correlated pair. In zone of parameter M detecting those individual states defines the measure value on both detectors. In that scheme contradiction with RT is absent.

The experiment testing of Bell inequality show that individual state of particle do not contain the parameter value which is common for all detectors of that parameter. Therefore individual state operator forms differ value for differ detectors. We will show that such formalism may by build.

2. Formalism of particle individual state

For introducing of formalism for individual state of quantum particle we consider the standard formalism of wave function Ψ and quantum measurement [2]. That theory has classic and relative modification. For our purpose shall important the argument space of ensemble state. In classic case

$$\Psi = \Psi(t,x),\ t \in \mathbb{R},\ x \in \mathbb{R}^3$$

$$\Psi(t,.) \in F(\mathbb{C},\mathbb{R}^3) = L_2(\mathbb{C},\mathbb{R}^3) \cap C_2(\mathbb{C},\mathbb{R}^3)$$

$$\Psi(.,.) \in C_2(\mathbb{C},\mathbb{R}^4)$$

In relative case

P — hypersurface of simultaneity in pseudo-Riemannian 4D manifold M.

$$\Psi(t,.)|_{x \in P} \in F(\mathbb{C},P)$$

$$\Psi(.,.) \in C_2(\mathbb{C},M)$$

$\Lambda_2(\mathbb{C},P)$ — The space of generalized functions (linear functional) on $F(\mathbb{C},P)$.

$E(\square, P)$ — The space and algebra of Hermitian operator on $F(\square, P)$.

In common case eigenfunctions and eigenbasis of $H \in E(\square, P)$ belong $\Lambda_2(\square, P)$.

$$\int\limits_{x \in P} \Psi(t,x)\Psi^*(t,x)dx_1dx_2dx_3 = 1$$

Orthogonal expansion of wave function:

$$\Psi(t,.) = \sum_{i=1}^{\infty} a_i \varphi_i$$

$$\Psi(t,.) = \int\limits_{s \in U} \varphi_s \alpha(ds) = \int\limits_{s \in U} \varphi_s \alpha(s) ds$$

$$\sum_{i=1}^{\infty} a_i a_i^* = 1$$

$$\int\limits_{s \in U} \alpha(s)\alpha^*(s) ds = 1$$

Eigenbasis of Hermitian operator is orthogonal basis in $F(\square, P)$. If the operator specter is discrete then that basis is countable. If the specter has continuous component then basis is continuality. Wave function expansion on eigenbasis of measurement operator formed the model of quantum measurement. Average resolution of measurement:

$$h_{\Psi,H} = \sum_{i=1}^{\infty} \lambda_i a_i a_i^*$$

$$h_{\Psi,H} = \int\limits_{s \in U} \lambda_s \left|\alpha(ds)\right|^2 = \int\limits_{s \in U} \lambda_s \alpha(s)\alpha^*(s) ds$$

Interpretation: the value λ_i (λ_s) may be obtained with probability (density) $a_i a_i^*$ ($\alpha(s)\alpha^*(s)$).

In this standard formalism we may define the class of individual state operators. Operator of individual state for quantum particle has type

$$\Phi : \mathrm{E} \to \Lambda_2(\square, M)$$

The transform of Hermitian operator by individual state operator is one its eigenfunction:

$$H \in \mathrm{E} \;\Rightarrow\; \exists \lambda | \in \square,\, H\Phi(H) = \lambda\Phi(H)$$

$$\lambda =_{def} \lambda(H \mid \Phi)$$

If measurement detector, which has operator H, measures any parameter quantity for particle, which has individual state Φ, than result is deterministic

$$\lambda = \lambda(H \mid \Phi)$$

Two differed particles θ_1 and θ_2 have differed individual state operators Φ_1, Φ_2 in common case. If that fact is sufficient then we will write $\Phi[\theta]$ with particle indicating.

The condition of concordance for individual and ensemble states in particle ensemble is following.

The probability density of measurement result in case of continuality spectrum is

$$p\{\lambda(H \mid \Phi[\theta]) = \lambda(s) \mid \Psi\} = (\Psi, \varphi_s)(\Psi, \varphi_s)^* \tag{2.1a}$$

The probability of measurement result in case of discrete spectrum is

$$P\{\lambda(H \mid \Phi[\theta]) = \lambda_i \mid \Psi\} = (\Psi, \varphi_i)(\Psi, \varphi_i)^* \tag{2.1b}$$

Consequence 2.1. If ensemble state Ψ is eigenfunction for operator H then $\Phi(H) = \Psi$ with probability 1. That fact shows dependence of individual states distribution on particles in ensemble at ensemble state.

Condition of concordance for commuting operators $H_1, \ldots, H_n$ in simultaneous measurements on one particle is following. This measurement correspond one joined operator $\lfloor H_1, \ldots, H_n \rfloor$ which has eigenbasis common for all operators $H_1, \ldots, H_n$ and

$$H_i \Phi\left(\lfloor H_1, \ldots, H_n \rfloor\right) = \lambda(i) \Phi\left(\lfloor H_1, \ldots, H_n \rfloor\right). \tag{2.2}$$

Vector $\left(\lambda(1), \ldots, \lambda(n)\right)$ defines measurement result.

That postulate may by a theorem if to require

$$act\lfloor H_1, \ldots, H_n \rfloor = actH_1 \circ \ldots \circ actH_n, \tag{2.4}$$

where $actH$ has interpretation as interaction between detector and the particle individual state in form

$$actH \circ \theta =_{def} \Phi[\theta](H) \ . \tag{2.5}$$

Than after action by H_n on θ the ensemble state is any eigenfunction $actH_1 \circ \theta = \Phi[\theta](H_1)$ for all operators $H_1, \ldots, H_{n-1}$ and that ensemble state preserves in following actions with probability 1 in accordance with consequence 2.1.

3. Ensemble of particle collectives

The wave function of collective ensemble is

$$\Psi\left(t_1, \ldots, t_N; x_1^1, x_1^2, x_1^3; \ldots; x_N^1, x_N^2, x_N^3\right) = \Psi\left(\vec{t}; \overrightarrow{x_1}; \ldots; \overrightarrow{x_N}\right) \tag{3.1}$$

The hypersurface of simultaneous for a component of collective is $P_i(t_i)$, $i = 1,\ldots,N$, with coordinate $\langle t_i, x_i^1, x_i^2, x_i^3 \rangle$, where t_i — parameter.

$$\int\limits_{P_1(t_1)\times\ldots\times P_N(t_N)} \Psi\Psi^* |_{t_1,\ldots,t_N} \, dP_1 \wedge \ldots \wedge dP_N = 1$$

Definition 3.1. Argument Extension of functional operator $H|_{F(\square, X)}$ on functional space $F(\square, X)$ is new operator $H^+|_{F(\square, X\times Y)}$ on $F(\square, X\times Y)$ which satisfy following condition.

If $H \circ f(x) = g(x)$ then

$$H^+ \circ q(x,y) =_{def} H \circ (q(x,y) \,|\, y \text{ fixed}) = Q(x)|_y = G(x,y) \tag{3.2}$$

Eigenfunction of argument extension has common definition on new argument space:

$$H^+ \circ \varphi(x,y) = \lambda\varphi(x,y) \tag{3.3}$$

Cortege of measurement operators $\left[H_1,\ldots,H_N\right]$ has interpretation as measurement of particle θ_i in collective by operator H_i, for all particles $i = 1,\ldots,N$. Cortege is defined on space $F(\square, P_1(t_1)\times\ldots\times P_N(t_N))$.

$$\Psi = \Psi(\overrightarrow{x_1},\ldots,\overrightarrow{x_N} \,|\, t_1,\ldots,t_N) =_{mark} \Psi\left(\vec{x} \,|\, \vec{t}\right)$$

Parameter $\overrightarrow{x_i} = \left(x_i^1, x_i^2, x_i^3\right)$ is space coordinate for particle θ_i, $i = \overline{1,N}$.

Parameter t_i is the time instant of measurement of particle θ_i, $i = \overline{1,N}$.

$$\int\limits_{P_1(t_1)\times\ldots\times P_N(t_N)} \left|\Psi(\overrightarrow{x_1},\ldots,\overrightarrow{x_N} \,|\, t_1,\ldots,t_N)\right|^2 dx_1^1 \wedge \ldots \wedge dx_N^3 = 1 \tag{3.4}$$

Domain of each measurement operator H_i:

$$\Xi_i = F(\square, P_i(t_i)) = L_2\left(\langle \overrightarrow{x_i} \rangle, \square\right) \cap C_2\left(\langle \overrightarrow{x_i} \rangle, \square\right) \tag{3.5}$$

Domain of operator cortege $\left[H_1,\ldots,H_N\right]$:

$$\Xi = F(\square, P_1(t_1)\times\ldots\times P_N(t_N)) = L_2\left(\langle \overrightarrow{x_1},\ldots,\overrightarrow{x_N} \rangle, \square\right) \cap C_2\left(\langle \overrightarrow{x_1},\ldots,\overrightarrow{x_N} \rangle, \square\right) \tag{3.6}$$

Lemma 3.1. Eigenfunctions of operator composition have form

$$\varphi_{s_1,\ldots,s_N}(\vec{x}) = \zeta\varphi_{s_1}(\overrightarrow{x_1})\ldots\varphi_{s_N}(\overrightarrow{x_N}) = \zeta\prod_{i=1}^{N}\varphi_{s_i}(\overrightarrow{x_i}) \tag{3.7}$$

$\zeta \in \mathbb{C}, \quad |\zeta| = 1, \ \zeta = \zeta(s_1, \ldots, s_N)$.

$\|\varphi_{s_i}\| = 1, \quad i = \overline{1, N}$;

Corresponded eigenvalue

$$H\varphi_{s_1,\ldots,s_N} = \lambda_{s_1,\ldots,s_N}\varphi_{s_1,\ldots,s_N} \tag{3.8}$$

$$\lambda_{s_1,\ldots,s_N} = \lambda_{s_1} \cdot \ldots \cdot \lambda_{s_N} \tag{3.9}$$

Proof. Let $\overrightarrow{x_{(i)}}$ is $(x_1^1, \ldots (x_i^1, x_i^2, x_i^3) \ldots, x_N^3)$ where components x_i^1, x_i^2, x_i^3 eliminate.

$$f(\vec{x} \mid \vec{t}) = \varphi_{s(\overrightarrow{x_{(1)}}, \vec{t})}\left(\vec{x_1}\right) g_1\left(\overrightarrow{x_{(1)}}\right) =$$

$$= \varphi_{s(\overrightarrow{x_{(2)}}, \vec{t})}\left(\vec{x_2}\right) g_2\left(\overrightarrow{x_{(2)}}\right) =$$

$$= \varphi_{s(\overrightarrow{x_{(2)}}, \vec{t})}\left(\vec{x_2}\right) \varphi_{s(\overrightarrow{x_{(1)}}, \vec{t})}\left(\vec{x_1}\right) g_{1,2}\left(\overrightarrow{x_{(1,2)}}\right) = \ldots =$$

$$= \varphi_{s(\overrightarrow{x_{(N)}}, \vec{t})}\left(\vec{x_N}\right) \cdot \ldots \cdot \varphi_{s(\overrightarrow{x_{(1)}}, \vec{t})}\left(\vec{x_1}\right) g_{1,\ldots,N}\left(\overrightarrow{x_{(1,\ldots N)}}\right);$$

$$g_{1,\ldots,N}\left(\overrightarrow{x_{(1,\ldots N)}}\right) = g_{1,\ldots,N}\left(\varnothing\right) = \zeta \in \mathbb{C} \ . \ \square$$

The probability or probability density (in the case of continually specter) of choice for eigenfunction $\varphi_{s_1,\ldots,s_N}$ is

$$d(s_1, \ldots, s_N) = d(\vec{s}) = \left\| proj\left(\Psi \mid \left\langle \varphi_{s_1,\ldots,s_N} \right\rangle\right) \right\|_{\vec{t}}^2 = \left|(\Psi, \varphi_{s_1,\ldots,s_N})\right|^2 \tag{3.10}$$

Let

$$W(V_1, \ldots, V_N) = \left\{\left(s_1, \ldots, s_N\right) \mid \lambda_1(s_1) \in V_1, \ldots, \lambda_N(s_N) \in V_N\right\}, \tag{3.11}$$

where $V_1, \ldots, V_N$ are measureable sets.

Let $m_1, \ldots, m_N$ are results of measurement $\left[H_1, \ldots, H_N\right]$ on particle collective.

$$p(V_1, \ldots, V_N) = \Pr\left\{m_1 \in V_1, \ldots, m_N \in V_N\right\}|_{\vec{t}} = \int\limits_{\vec{s} \in W\left(V_1, \ldots, V_N\right)} d\left(\vec{s}\right) ds_1 \wedge \ldots \wedge ds_N \tag{3.12}$$

4. Measurement on several streams of particle

Definition 4.1. The *Stationary Stream System* (SSS) is ensemble of collectives with wave function of (3.1) kind and cortege of measurement operators $\left[H_1, \ldots, H_N\right]$.

Real process, which corresponded SSS, consists of periodical emission of collective containing N particles θ_i and measurement of those particles by corresponding operators H_i of cortege in instant t_i after emission. Therefore the process realization contained the sequence of particles (k — number of emission):

$$\left\{\theta_i(k) \mid i=\overline{1,N};\, k=1,\ldots\right\} \tag{4.1}$$

The probability of event (3.12) in that process has interpretation as frequency of event observation. The frequency of subsequence $\{x_{i(k)}\}$ in sequence $\{x_i\}$ is

$$\mathrm{Fr}\{x_{i(k)}\} = \lim_{k\to\infty} \frac{k}{i(k)}.$$

Theorem 4.1. The sequence

$$\left\{\Phi[\theta_i(k)] \mid i=\overline{1,N};\, k=1,\ldots\right\} \tag{4.2}$$

of individual particle states exist for any SSS correspond (4.1) where the event frequency are equal probability (3.12) with probability 1:

$$\mathrm{Fr}\{m_1 \in V_1,\ldots,m_N \in V_N\} = p(V_1,\ldots,V_N). \tag{4.3}$$

Proof. Let $\left(m_1(k),\ldots,m_N(k)\right)$, $k=1,\ldots;$ is realization of Bernoulli sequence with probability distribution (3.12) on every step. In each measurement act the equation $m_i(k)=\lambda_i(k)$ is true for correspond eigenvalue and eigenfunction $\varphi_i(\vec{x}\mid s(k))$ of operator H_i. If $m_i(k)=m_i(k')$ then $\varphi_i(\vec{x}\mid s(k))=\varphi_i(\vec{x}\mid s(k'))$. Therefore may define $\Phi[\theta_i]\left(\left[H_1,\ldots,H_N\right]\right)=\varphi_i(\vec{x}\mid s(k))$. That individual state sequence satisfies condition (4.3) with probability 1. □

Consequence 4.1. Apparatus of individual state for quantum particles make possible realization of EPR interaction model without contradiction with relative theory and Bell inequality.

The work was fulfilled by support of Russian Fond of Basis Research, grant 13-01-00190a, , 11-06-00155a and Russian Humanitarian Science Fond, grant 11-03-00035a.

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On the Metric Structure of Space-Time

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The Hawking's hypothesis about Euclidean nature of space-time is discussed using well known correspondence between pseudo-Riemannian metrics, Riemannian metrics and unit vector field. It is shown that in the framework of the Hawking's hypothesis this correspondence may be considered as a symmetry breaking and leads naturally to polymetric space-time theories, dark matter appearance and possible existence of particles which may propagates with super light speeds and whose possible existence is widely discussed during last decade.

Gravitation field in general relativity is described by pseudo Riemannian metrics $g_{\alpha\beta}$ with Lorentzian signature $(+,-,-,-)$ on four-dimensional smooth manifold M^4. The metric $g_{\alpha\beta}$ [1] must satisfy to Einstein equations [1,2]. It is known also, that among the Lorentzian structure on M^4 some Riemannian structure which is defined by some positive-definite metric $G_{\alpha\beta}$ with signature $(+,+,+,+)$ may be introduced on the same manifold M^4 [3,4]. Moreover, there is a correspondence between arbitrary positive-definite metric $G_{\alpha\beta}$ with signature $(+,+,+,+)$ and some pseudo Riemannian metrics $g_{\alpha\beta}$ with Lorentzian signature $(+,-,-,-)$ and reverse. So the natural question is raised: whose of these two structures, Euclidean or Lorentzian, is more fundamental.

The correspondence between positive-definite metric $G_{\alpha\beta}$ and pseudo Riemannian (Lorentzian) metrics $g_{\alpha\beta}$ may be realized by two methods.

The first method is the so called Wick rotation $t \to it$ ($x^0 \to ix^0$) which is used in quantum gravity and quantum field theory. If metric $G_{\alpha\beta}$ is positive-definite with Euclidean signature $(+,+,+,+)$ then metric $g_{\alpha\beta} = -G_{\alpha\beta}\big|_{x^0 \to ix^0}$ will have Lorentzian signature $(+,-,-,-)$. Using this motivation Hawking supposed in 1978 that it may be assumed that quantum theory and even all physics is really defined in Euclidean region whereas the Lorentzian space-time structure is the result of our perception [5].

In the next method the following local representation of arbitrary Lorentzian metric $g_{\alpha\beta}$ is used

$$g_{\alpha\beta} = 2u_\alpha u_\beta - G_{\alpha\beta}\,, \tag{1}$$

where u_α is some unite vector field and $G_{\alpha\beta}$ is a positive-definite metric with signature $(+,+,+,+)$ ($g^{\alpha\beta}u_\alpha u_\beta = G^{\alpha\beta}u_\alpha u_\beta = 1$). Such representation is used for consideration of many problems of classical space-time theory. The representation of $g_{\alpha\beta}$ in the form (1) is not unique: the only restriction on the field u_α in representation (1) is $G^{\alpha\beta}u_\alpha u_\beta = 1$ for transformation $G_{\alpha\beta} \mapsto g_{\alpha\beta}$ or $g^{\alpha\beta}u_\alpha u_\beta = 1$ for transformation

[1] It is supposed that small Greek indexes take values from 0 to 3.

$g_{\alpha\beta} \mapsto G_{\alpha\beta}$ i.e. the field u_α must be time-like with unit norm. According to equation (1) any pair $(G_{\alpha\beta}, u_\alpha)$ is formed by arbitrary Riemannian metric $G_{\alpha\beta}$ and unit vector field u_α defines some Lorentzian metric $g_{\alpha\beta}$ and reverse, any pair $(g_{\alpha\beta}, u_\alpha)$ is formed by Lorentzian metric $g_{\alpha\beta}$ and time-like unit vector field u_α defines some Riemannian metric $G_{\alpha\beta}$.

If we fix some Riemannian metric $G_{\alpha\beta}$ on smooth manifold M^4 and take several unequal vector fields w^i_α ($i = 1, 2, ..., n$) on it then equation (1) introduces n nonequivalent space-time structures $(M^4, g_{\alpha\beta})$ on the same manifold. Using this motivation the author supposed in 1985 that there are several space-time structures may coexist on the same manifold M^4 [6]. This supposition leads to polymetric space-time models.

The supposition about possible coexistence of several nonequivalent causal but not necessary Lorentzian structures on the same manifold was expressed recently by Geroch [7].

Let's consider on some manifold M^4 a positive definite Riemannian metrics $G_{\alpha\beta}$, vector field u_α with unit norm ($G^{\alpha\beta}u_\alpha u_\beta = 1$) and pseudo Riemannian metrics $g_{\alpha\beta}$ of Lorentzian signature $(+,-,-,-)$ which is defined by the pair $(G_{\alpha\beta}, u_\alpha)$ by means of equation (1). The pair $(M^4, g_{\alpha\beta})$ defines usual space-time structure on the smooth manifold M^4 with maximal speed of signal propagation $c = 1$. It is easy to see that $G = \det\|G_{\alpha\beta}\| = -g = \det\|g_{\alpha\beta}\|$ [8] and $G^{\alpha\beta}u_\alpha u_\beta = 1 = g^{\alpha\beta}u_\alpha u_\beta$, i.e. the field u_α has unit norm in both metrics $G_{\alpha\beta}$ and $g_{\alpha\beta}$ [6,8].

Consider also some system of vector fields w^i_α ($i = 1, 2, ..., n$) with $G^{\alpha\beta}w^{(i)}_\alpha w^{(i)}_\beta = 1$ on the same manifold M^4. The simple modification of (1) gives the system of pseudo Riemannian metrics with Lorentzian signature $(+,-,-,-)$ on the manifold M^4:

$$g^{(i)}_{\alpha\beta} = \left(c^2_{(i)} + 1\right) w^{(i)}_\alpha w^{(i)}_\beta - G_{\alpha\beta}, \tag{2}$$

where coefficients $c_{(i)} > 0$ define the maximal speeds of signal propagation in space-times $\left(M^4, g^{(i)}_{\alpha\beta}\right)$. To exclude the coincidence of metrics $g_{\alpha\beta}$, $g^{(i)}_{\alpha\beta}$ and $g^{(j)}_{\alpha\beta}$ for $i \neq j$, the coefficients $c_{(i)}$ must satisfy to the following conditions: $c_{(i)} \neq 1$ if $w^{(i)}_\alpha = u_\alpha$ and $c_{(i)} \neq c_{(j)}$ if $w^{(i)}_\alpha = w^{(j)}_\alpha$.

For contravariant components of $g_{\alpha\beta}$ and $g^{(i)}_{\alpha\beta}$ we have

$$g^{\alpha\beta} = 2u^\alpha u^\beta - G^{\alpha\beta}, \qquad g^{\alpha\beta}_{(i)} = \frac{c^2_{(i)} + 1}{c^2_{(i)}} w^\alpha_{(i)} w^\beta_{(i)} - G^{\alpha\beta}, \tag{3}$$

where $u^\alpha = g^{\alpha\rho}u_\rho = G^{\alpha\rho}u_\rho$, $w^\alpha_{(i)} = G^{\alpha\beta}w^{(i)}_\beta = c^2 g^{\alpha\rho}_{(i)} w^{(i)}_\rho$. Correspondence between co- and contravariant components of metrics $g_{\alpha\beta}$ and $g^{(i)}_{\alpha\beta}$ may be obtained by excluding tensor $G_{\alpha\beta}$ from (1)-(3):

$$g^{(i)}_{\alpha\beta} = g_{\alpha\beta} - 2u_\alpha u_\beta + \left(c^2_1 + 1\right) w^i_\alpha w^i_\beta, \qquad g^{\alpha\beta}_{(i)} = g^{\alpha\beta} - 2u^\alpha u^\beta + \frac{1 + c^2_1}{c^2_1} k^{\alpha\beta}_{(i)}, \tag{4}$$

where

$$k_i^{\alpha\beta} = 4u^\alpha u^\beta \left(u^\rho w_\rho^i\right)^2 - 2\left(u^\alpha w_{(i)g}^\beta + w_{(i)g}^\alpha u^\beta\right)\left(u^\rho w_\rho\right) + w_{(i)g}^\alpha w_{(i)g}^\beta,$$

with $w_{(i)g}^\alpha = g^{\alpha\beta} w_\beta^i$.

In the case of empty space-times $\left(M^4, g_{\alpha\beta}\right)$and $\left(M^4, g_{\alpha\beta}^{(i)}\right)$ metrics $g_{\alpha\beta}$ and $g_{\alpha\beta}^{(i)}$ are defines by Einstein-Hilbert actions integrals

$$S_g = \int R_g \sqrt{-g}\, d^4x, \qquad S_i = \int R_i \sqrt{-g^{(i)}}\, d^4x, \tag{5}$$

where R_g and R_i are the Ricci tensors of $g_{\alpha\beta}$ and $g_{\alpha\beta}^{(i)}$ correspondingly. Using correspondence between determinants of metrics g, $g^{(i)}$ and $G_{\alpha\beta}$

$$g = g^{(i)} / c_{(i)}^2 = -G,$$

the actions (5) may be combines in the only integral

$$S_\Sigma = \int \left\{ R_g + \sum_{i=1}^{n} \sigma_i R_i \right\} \sqrt{-g}\, d^4x .$$

Using well-known equations of bimetric formalism and correspondence between metrics $g_{\alpha\beta}$ and $g_{\alpha\beta}^{(i)}$ we may represent Ricci scalars R_i in the form

$$R_i = R_g + L_i\left(u_\alpha, w_\alpha^{(i)}\right),$$

where functions $L_i\left(u_\alpha, w_\alpha^{(i)}\right)$ depend on the fields u_α, $w_\alpha^{(i)}$ and theirs derivatives. Finally we obtain for S_Σ:

$$S_\Sigma = \int \left\{ \kappa R_g + \sum_{i=0}^{n} L_i\left(u_\alpha, w_\alpha^{(i)}\right) \right\} \sqrt{-g}\, d^4x, \tag{6}$$

where κ_g is a constant which may be identified with Einstein gravitation constant.

If metrics $g_{\alpha\beta}$ and $g_{\alpha\beta}^{(i)}$ have their own sources then the action (6) takes the form

$$S_\Sigma = \int \left\{ \kappa_g R_g + \sum_{i=0}^{n} \left[L_i\left(u_\alpha, w_\alpha^{(i)}\right) + L_{migi} \right] \right\} \sqrt{-g}\, d^4x, \tag{7}$$

where L_{migi}, $i = 0,1,2,...,n$ are the Lagrangians of the sources fields of the metrics $g_{\alpha\beta}$ and $g_{\alpha\beta}^{(i)}$, and we denote $g_{\alpha\beta}^{(0)} = g_{\alpha\beta}$.

Equations (4) together with bimetric formalism equations make possible the rewrite (7) in the following equivalent form

$$S_\Sigma = \int \left\{ \kappa_{gi} R_{gi} + \sum_{j=0}^{n} \left[\tilde{L}_j\left(w_\alpha^{(i)}, w_\alpha^{(j)}\right) + \tilde{L}_{mjgj} \right] \right\} \sqrt{-g^{(i)}}\, d^4x, \tag{7a}$$

where κ_{gi} is some constant which is analogous to constant κ_g in (7), and $\tilde{L}_j\left(w_\alpha^{(i)}, w_\alpha^{(j)}\right)$ and $\tilde{L}_{mjgj}$ are analogous to $L_i\left(u_\alpha, w_\alpha^{(i)}\right)$ and L_{migi} in (7).

Analogously, equations (1)-(4) with bimetric formalism equations make possible the rewrite (7) in the form

$$S_{\Sigma}=\int\left\{\kappa_G R_G+\sum_{j=0}^{n}\left[\tilde{\tilde{L}}_j\left(w_{\alpha}^{(i)},w_{\alpha}^{(j)}\right)+\tilde{\tilde{L}}_{mjG}\right]\right\}\sqrt{G}d^4x\,, \tag{8}$$

where κ_G, $\tilde{\tilde{L}}_j\left(w_{\alpha}^{(i)},w_{\alpha}^{(j)}\right)$ and $\tilde{\tilde{L}}_{mjG}$ are analogous sense as κ_g, $L_i\left(u_{\alpha},w_{\alpha}^{(i)}\right)$ and L_{migi} in (7) or κ_{gi}, $\tilde{L}_j\left(w_{\alpha}^{(i)},w_{\alpha}^{(j)}\right)$ and $\tilde{L}_{mjgj}$ in (7a).

On our opinion, last equality confirms Hawking's supposition [5] that Euclidian (Riemannian) space-time structure is more fundamental than Lorentzian (pseudo-Riemannian) one. From this point of view Lorentzian (pseudo-Riemannian) may be considered as some form of symmetry breaking. For instance let a scalar field φ is a source of some Riemannian space-time with Riemannian metric $G_{\alpha\beta}$. The action functional may be written in the following general form

$$S=\int\left(\kappa_G R_G+\mathscr{L}\left(\varphi,\varphi_{,\alpha}\right)\right)\sqrt{G}d^4x\,, \tag{9}$$

where $\mathscr{L}\left(\varphi,\varphi_{,\alpha}\right)$ is a scalar field Lagrangian. If in the law energy limit the kinetic part of $\mathscr{L}\left(\varphi,\varphi_{,\alpha}\right)$ takes the form $g^{\alpha\beta}\varphi_{,\alpha}\,\varphi_{,\beta}$ where $g_{\alpha\beta}$ is defined by Eq. (1) with some unit vector field u_{α} then it is natural to rewrite the action (9) in the form

$$S=\int\left(\kappa_g R_g+\frac{1}{2}g^{\alpha\beta}\varphi_{,\alpha}\,\varphi_{,\beta}+V(\varphi)+F(u_{\alpha},u_{\alpha,\beta})\right)\sqrt{-g}d^4x\,, \tag{10}$$

where $V(\varphi)$ is a potential of scalar field φ and $F(u_{\alpha},u_{\alpha,\beta})=\kappa_G R_G-\kappa_g R_g$. The scalar field in (10) is considered as an example of usual matter. By force of absence of interaction between matter field φ and vector field u_{α} the last field must be considered as a part of dark matter. In the case of several Lorentzian structures coexistence which are defined by Eqs. (2)-(4) the fields $w_{\alpha}^{(i)}$ and the source fields of $g_{\alpha\beta}^{(i)}$ give additional contribution to dark matter.

Some concluding remarks must be made.

First, it is clear that above consideration does not mean that dark matter could not exist in other forms.

Second, our consideration leads naturally to polymetric space-time models with possible existence of particles and field which may propagates with superlight speed i.e. whose world lines are outside of future isotropic cone of the metric $g_{\alpha\beta}$. As it is shown in [9] because of general covariant nature of such hypothetical signals it is not unavoidable lead to causality violation.

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Two subsystems of the Universe matter and “black holes”

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We should consider so named “black holes” (super massive compact space objects) as hadrons attracting the quantum chromodynamics for their description and using $SU(3)$ symmetry ($r = 8$). As a result we can bring in question their stability relative to phase transitions the more so, that in the Universe the objects with inexplicable high energy release are observed (quasars). We offered to consider the charged pion decay as the physical process which illustrates the spontaneous breaking of the $SU(3)$ symmetry to the $SU(2)\times U(1)$ symmetry before. As a result it might be worthwhile to do not increase the count of gauge fields beyond 8 as in the grand unified theory. We evolve the Dirac hypothesis on the presence of the electrons sea with negative energies in the Universe for the explanation of the electrons stability with positive energies. The assumption on the “sea” of quarks in the ground state allows using the Landau theory of the Fermi liquid considering observable particles as quasi-particles on the background of “sterile” neutrinos and “sterile” antineutrinos. The properties of the latter’s must define the geometrical and topological properties of the space-time. Personally it must not be the simply connected space if physical systems are considered at sufficiently low energies that allow explaining the charge quantization of observable particles.

1. Two subsystems of the Universe matter.

It is known [1], what Einstein, being concerned with the development of stationary model of the Universe evolution, was very skeptical to the Friedmann model, which did not contained the answer to the question on causes of the Universe expansion. Only under the pressure of Hubble’s impressive papers, which deduced the linear low of the dependence of the velocity of the galaxies runaway on their distance in the late twenties of the twentieth century on the basis of astronomical observations [2], Einstein gave up the erroneous (as then it seemed to him) idea of the construction of the stationary model of the Universe. With this moment the Friedmann model began considering as the standard one, which demanded the modification only in parts. To this parts it was concerned the initial stage of Universe evolution, for the description of which it was offered the models with hot and cold modes. In 1965 Penzias and Wilson detected the cosmic microwave background [3]. After this in the cosmology supporters of the Universe evolution cold start was not almost remained, the same as and opponents of standard cosmological model.

But and in the Friedmann standard model there are the questions on which’s so and it can find no answers. Some questions “were easily hidden under a carpet”. So on the cause of the Universe expansion - Big Bang was the answer, which did not explain a thing. Naturally, what after this primary explosion the expansion can only be retarded in the result of action of gravitational forces, which’s anything do not counterbalanced. The latter is not verified by astronomical observation of Type *Ia* Supernovae [4]. Next awkward questions for the Friedmann model were become: the closeness problem of the Universe matter density to critical one, that is to say that one, by which the spatial curvature is bound to be equal zero, and the horizon problem – the homogeneity problem of the cosmic microwave background from causal free regions of the Universe [5]. All this caused to seek the nonstandard manners of the similar questions explanation in the form of the allotment of the vacuum by the exotic matter properties (“dark energy”, “quintessence”), if only to conserve the fundamental terms of the Friedmann model.

The given situation in the cosmology has taken to reminding that one, which preceded to the construction of the kinetic theory of gases in the XIX century. Therefore (as and in the XIX century) it is necessary to suppose the existence in the Universe the high number of no observable particles or difficulty observable particles under the laboratory conditions. Neutrinos and antineutrinos of various flavors are classified among such kind of particles, their participation only in weak interactions is the peculiarity of which's. The total density of neutrinos and antineutrinos in the Universe is not known, because the estimations can be obtained when considering the inelastic scatterings, having the enough high energy thresholds. Just in this case it can validate the spatial homogeneity of the distribution of the baryon matter (the cosmological principle for the fast subsystem), the behavior of which will defined by the behavior of the non-baryon matter (the slow subsystem).

Among various hypotheses we note those, in which's neutrinos (also and antineutrinos) are the basic form of the Universe matter. Specifically in the hypothesis of Pontekorvo and Smorodinskij [6] it was offered to explain the charge asymmetry of the baryon matter by that one, what it is only the slight fluctuation on the gigantic neutrino background of the Universe. Having the neutrino Universe and considering the Fermi-Dirac statistics, we can remember on the Sakharov's hypothesis [7], in which the vacuum elasticity and the gravitational interaction of macroscopic bodies were interconnected. As a result it might be worthwhile to use the elasticity of the neutrino Fermi liquid for that object.

Bashkin's papers [8] appearing in 80th on a propagation of the spin waves in the polarized gases initiated the supposition, that the analogous collective oscillations are possible under certain conditions as well as in the neutrinos medium [9]. Precisely it with the attraction of the Casimir effect allowed to connect the gravitational constant $G_N \sim 10^{-38}$ GeV^{-2} with parameters of the electroweak interaction ($G_N \propto \sigma_{\nu e}$, $\sigma_{\nu e}$ is the cross-section for scattering of a neutrino on an electron) [10]. Taking account of the obtained result and also the empirical formula [1] $H_\circ / G_N \approx m_\pi^3$ ($m_\pi \sim 10^{-1} GeV$ is the pion mass, $H_\circ \sim 10^{-42} GeV$ is the Hubble constant) it can offer the interpretation of the Hubble constant as a quantity characterizing kinetic process of a relaxation in the Universe (we shall use the system of units $h/(2\pi) = c = 1$, where h is the Planck constant and c is the velocity of light).

What is more, taking account of quantum lows for the explanation of the gravitational interaction, we obtain [11] the finite coherence length for the interaction of macroscopic bodies and at the same time the interpretation of the Lobachevsky – Chernikov potential

$$U(r) = \frac{G_N m}{L}\left(1 - cth\frac{r}{L}\right) = -\frac{2G_N m}{L}\frac{e^{-2r/L}}{1-e^{-2r/L}} = -\frac{2G_N m}{L}\frac{1}{e^{2r/L}-1} \qquad (1.1)$$

(m_1, m_2 are masses of macroscopic bodies; r is the distance between them; L is the Lobachevsky constant) (m is the mass of a macroscopic body; r is the distance to it; L is the Lobachevsky constant, a estimation of which must be connected with parameters, characterizing the state of the Universe dark matter). In consequence of it we obtain the explanation of the cosmological principle which is the postulate of the Universe standard model and at the same time the solution of the horizon problem, because the galaxies motion can be considered at sufficiently large distances as the motion of ideal gas particles.

Thus the partition of the Universe matter on two subsystems is a description method allowing the construction of the Universe evolution theory not resorting to fantastic forms of a matter. In the first place it is necessary to give a definition of fast subsystem particles as particles participating in strong and electroweak interactions, at the time as slow subsystem particles do not have such opportunity. Causes of this can be the very different.

According to our supposition for the most part of slow subsystem particles is what they being fermions (particles with a half-integer spin) formulate quantum liquids (the Fermi liquid, the Bose liquid from fermionic pairs). When particles go from the ground state to the excited one they acquire all properties of fast subsystem fermion – the color charge and (or) the electric charge.

In the degenerate state background fermions of Universe, generating Fermi and Bose liquids, are weakly-interacting particles, but it is not excluded by the interaction with hadrons their exhibition as color fermions – ghosts. We do not exclude also the possibility, that in the state of the Fermi liquid they must be considered as right neutrinos and left antineutrinos with the sufficiently high Fermi energy ε_F ("sterile" neutrinos and "sterile" antineutrinos) [11]. It must be exhibited in the absence of these particles by decays attributed to weak interactions of low energies (a mirror asymmetry). Thus for example, it can be interpreted a lepton production upon a charged pion decay as a freezing-out of color degrees of freedom what is expressed in the form of the spontaneous breaking of the $SU(3)$ symmetry characterizing the interaction of color quarks to the $SU(2)\times U(1)$ symmetry characterizing the electroweak interactions of leptons.

The presence of the neutrino background with the finite Fermi energy ε_F is the catalytic agent of stochastic processes, but the large value of this energy causes to the determinancy of physical processes. Specifically we connect the large value of the Fermi energy and the low temperature of the neutrino background with the stability (or if only with the metastability) of elementary particles. We note that the transition to the description of the slow subsystem by the adaptation of the space-time manifold is carried when the Fermi energy ε_F of "sterile" neutrinos tends to infinity. In this case the quotation-marks in the words "sterile" neutrinos can be discarded, because these neutrinos will not collide with the other particles even at very high energies.

2. The maximum plausible realizations

The principle of the theoretical notion adequacy to experimental data must be put in the base of the serious physical theory. It is precisely therefore we attach the fundamental importance to symmetries which in the condensed (pithy) form. For this in the elementary particle physics is used the scattering matrix which allows to guess a form of transition operators if only for linear approximation. Because we must forecast results of future experiments, the description of physical systems states will proceeds by use of smooth functions, which it is desirable to obtain as solutions of differential equations. It is precisely therefore we shall approximate the transition operators by differential operators using the variation formalism.

Note that just as in mathematics which are entered undefined concepts such as sets and elements of theoretical physics must occur undefined concepts such as matter and particle of matter. We will emphasize the contrast of the particles in the ground state of the particle in the excited state, developing the Dirac hypothesis about the existence in the Universe "sea" of electrons in the ground state. Generalizing this hypothesis on fermions of arbitrary flavors in the ground state at a temperature close to absolute zero, it will be possible to use the Landau theory of Fermi-liquid, considering the remaining particles as quasi-particles, attributing them to a condition called excited. In the experiments, the particles are observed directly in the excited state, so we can determine the properties of the particles in the ground state only indirectly.

We will assign the maximum possible velocity to the matter particles. In the excited state of the particle should emit real or virtual particles, so that they can not move linearly. They are easy to read in conjunction with the "coat" of virtual particles as a complex dynamic system, characterized by a drift velocity, which is relatively macroscopic observer can be arbitrary and in particular zero. Frequency emission virtual particles will be characterized by a charge, which depends on the type of the virtual particles will be of a specific title. For example, if the virtual particles are photons, the charge will be called electric.

We assume that the emission of elementary particles, real or virtual particles generated by their encounter with the background particles of the Universe. Since between the particles in the ground and excited states is possible only weak interaction, which is characterized by chiral asymmetry, which is why it makes sense to assume that the fermions in the ground state of the universe are "sterile" neutrinos and antineutrinos. In addition, we assume that the trajectory of the particles in the excited state can be characterized by the curvature of which is inversely proportional to the wavelength of the particle, and torsion, which is proportional to the mass of elementary particles.

In the description of physical systems for which information is statistical in nature, we will use the packet $\{\Psi(\omega)\}$ of functions given in some domain of a parametrical space M_r and let the substitutions

$$\Psi \to \Psi + \delta\Psi = \Psi + \delta T(\Psi)$$

are the most general infinitesimal ones, where δT are infinitesimal operators of a transition (we do not concretize at first which type of symmetries by them are given). We note that just the operators δT, defined by a scattering matrix, will generate the symmetries characterizing the studied interactions.

We draw smooth curves through the common point $\omega \in M_r$ with the assistance of which we define the corresponding set of vector fields $\{\delta\xi(\omega)\}$. Further we define the deviations of fields $\Psi(\omega)$ in the point $\omega \in M_r$ as $\delta_\circ\Psi = \delta X(\Psi) = \delta T(\Psi) - \delta\xi(\Psi)$ and we shall require that these deviations were minimal ones even if in "the mean". If we state the task – to find the smooth fields $\Psi(\omega)$ in the studied domain Ω_r of the parameters space M_r then it can turn out to be unrealistic one (possibly $r >> 1$ and possibly $r \to \infty$). That's precisely therefore the task of the finding of the restrictions $\Psi(x)$ on the manifold M_n ($x \in M_n \subset M_r$, $n \le r$) will present an interest.

Let the square of the semi-norm $|X(\Upsilon)|$ has the form as the following integral

$$\mathrm{A} = \int\limits_{\Omega_n} \Lambda\, d_n V = \int\limits_{\Omega_n} \kappa \overline{X}(\Psi) \rho X(\Psi) d_n V\,. \tag{2.1}$$

(we shall name A as an action and Λ as a Lagrangian also as in the field theory). Here and further κ is a constant; $\rho = \rho(x)$ is the density matrix (tr $\rho = 1$, $\overset{+}{\rho} = \rho$, the top index "+" is the symbol of the Hermitian conjugation) and the bar means the generalized Dirac conjugation which must coincide with the standard one in particular case that is to be the superposition of Hermitian conjugation and the spatial inversion of the space-time M_4. We shall name solutions $\Psi(x)$ of differential equations, which are being produced by the requirement of the minimality of the integral (2.1), as the maximum plausible realizations of Lie local loops and we shall use for the construction of the all set of functions $\{\Psi(x)\}$ (generated by the transition operators).

Of course for this purpose we can use the analog of the maximum likelihood method employing for the probability amplitude, but not for the probability as in the mathematical statistics. As is known, according to the Feynman's hypothesis the probability amplitude of the system transition from the state $\Psi(x)$ in the state $\Psi'(x')$ equal to the following integral

$$K(\Psi,\Psi') = \int\limits_{\Omega(\Psi,\Psi')} \exp(i\mathrm{A}) D\Psi = \lim_{N\to\infty} I_N \int d\Psi_1 ... \int d\Psi_k ... \int d\Psi_{N-1} \exp\left(i \sum_{k=1}^{N-1} \Lambda(\Psi(x_k)) \Delta V_k \right) \tag{2.2}$$

($i^2 = -1$; the constant I_N is chosen so that the limit is existing). So, the formula (2.2) allows describe the physical process in the quantum theory the most adequately. At the same time the functions $\Psi(x)$, being the solutions of differential equations and obtaining from the requirement of the minimum of the action A, may be the maximum likelihood ones only, but then they allow to describe the same physical system in condensed (short) form. In this approach the Lagrangian Λ plays the more fundamental role than differential equations which are generated by it. As the transition operators are constructed on the base of experimental data, then the differential equations, obtained in a result of the Lagrangian special choice in the action (2.1), can name as the differential equations of the root-mean-square regression Ψ on x.

3. The Lie local loop of realizations

Let E_{n+N} is the vector fiber space with the base M_n and the projection π_N, $\Psi(x)$ is the arbitrary section of fibre bundle E_{n+N}, ∂_i is the partial derivative symbol. Let us to consider the infinitesimal substitutions defining the vector space mapping of the neighbour points x and $x+\delta x$ ($x \in U$, $x+\delta x \in U$, $U \subset M_n$) and conserving the possible linear dependence between vectors. We write given substitutions as:

$$\Psi'(x+\delta x) = \Psi(x) + \delta\Psi(x) = \Psi(x) + \delta T(x)\Psi(x) \qquad (3.1)$$

where $\delta T(x)$ is the infinitesimal affinor fields. By this the vector field change in consequence of the transition in the neighbour point has the form: $\Psi(x+\delta x) - \Psi(x) \approx \delta x^i \partial_i \Psi(x)$ (here and further Latin indices *i, j, k, ...* will run the values of integers from 1 to n) and the change of the field Ψ in the point $x+\delta x$ will equal

$$\Psi'(x+\delta x) - \Psi(x+\delta x) = \Psi'(x+\delta x) - \Psi(x) -$$

$$[\Psi(x+\delta x) - \Psi(x)] \approx \delta T(x)\Psi(x) - \delta x^i \partial_i \Psi(x).$$

Further we shall denote

$$\delta_\circ \Psi(x) = \delta T(x)\Psi(x) - \delta x^i \partial_i \Psi(x). \qquad (3.2)$$

Let the formula (3.1) defines the infinitesimal substitution of the Lie local loop $Q_r(x)$ moreover the unit e of the Lie local loop, the co-ordinates of which equal to zero, corresponds to the identity substitution. Then the infinitesimal substitutions of the Lie loop in co-ordinates are written as

$$x^i \to x^i + \delta x^i = x^i + \delta\omega^a(x)\xi_a^i(x), \qquad (3.3)$$

$$\Psi^A(x) \to \Psi^A(x) + \delta\omega^a(x) T_{a\,B}^{\;A}(x)\Psi^B(x), \qquad (3.4)$$

where x^i are the co-ordinates of the point x, $x^i + \delta x^i$ are the co-ordinates of the point $x+\delta x$, $\Psi^A(x)$ are the components of the vector field $\Psi(x)$ and $\delta\omega^a(x)$ are the components of the infinitesimal vector field $\delta\omega^a(x)$ being the section of the vector fibre bundle E_{n+r} with the base M_n and with the projection π_r (here and further Latin indices *a, b, c, d, e* will run the values of integers from 1 to r and Latin capital indices *A, B, C, D, E* will run the values of integers from 1 to N).

As a result the formula (3.2) is rewritten in the following form:

$$\delta_\circ \Psi = \delta\omega^a X_a(\Psi), \qquad (3.5)$$

where

$$X_a(\Psi) = T_a \Psi - \xi_a^i \partial_i \Psi$$

or in the co-ordinates

$$X_a^A(\Psi) = T_{aB}^{\ A}\Psi^B - \xi_a^i \partial_i \Psi^A. \tag{3.6}$$

In the general case a type of geometrical objects can do not conserving with the similar substitutions. Therefore below we shall consider only such substitutions which conserve a type of geometrical objects.

In first in the formula (3.5) it ought to become to the covariant derivative ∇_i. Let

$$\delta_\circ \Psi^A = \delta\omega^a X_a^A(\Psi) = \delta\omega^a (L_{aB}^{\ A}\Psi^B - \xi_a^i \nabla_i \Psi^A), \tag{3.7}$$

where

$$L_{aB}^{\ A} = T_{aB}^{\ A} + \xi_a^i \Gamma_{iB}^A, \qquad \nabla_i \Psi^A = \partial_i \Psi^A + \Gamma_{iB}^A \Psi^B, \tag{3.8}$$

and we demand that $L_{aB}^{\ A}(x)$ and $\xi_a^i(x)$ should be the components of intermediate tensor fields. Hence if $\Psi(x)$ are the components of the vector field then $\Psi(x) + \delta_\circ \Psi(x)$ also are the components of the vector field.

So, let

$$[X_a X_b] = X_a X_b - X_b X_a = C_{ab}^c X_c. \tag{3.9}$$

As a result the intermediate tensor fields $L_{aB}^{\ A}(x)$ and $\xi_a^i(x)$ must satisfy to the following correlations:

$$L_{aC}^{\ B} L_{bB}^{\ A} - L_{bC}^{\ B} L_{aB}^{\ A} + \xi_a^i \nabla_i L_{bC}^{\ A} - \xi_b^i \nabla_i L_{aC}^{\ A} - \xi_a^i \xi_b^j R_{ijC}^{\ \ \ A} = -C_{ab}^c L_{cC}^{\ A},$$

$$\xi_a^i \nabla_i \xi_b^k - \xi_b^i \nabla_i \xi_a^k - 2\xi_a^i \xi_b^j S_{ij}^k = -C_{ab}^c \xi_c^k,$$

where $S_{ij}^{\ k}(x)$ are the components of the torsion tensor and $R_{ijC}^{\ \ \ A}(x)$ are the curvature tensor components of the connection $\Gamma_{iC}^A(x)$. The components $C_{ab}^{\ c}(x)$, alternating on down indices of the structural tensor, must satisfy to the generalized Jacobi identities

$$C_{[ab}^d C_{c]d}^e - \xi_{[a}^i \nabla_{|i|} C_{bc]}^e + \xi_{[a}^i \xi_b^j R_{|ij|c}^{\ \ \ \ e} = 0 \tag{3.10}$$

($R_{ijc}^{\ \ \ e}(x)$ are the curvature tensor components of the connection $\Gamma_{ia}^b(x)$).

4. "Black holes"

We shall use that smooth manifolds are locally diffeomorphic ones to the Euclidean space or to the pseudo-Euclidean space in a certain neighborhood of any point. Therefore we shall choose the connection components $\Gamma_{ia}^b(x)$ equal to zero in the region under consideration. As a result the structure equations (3.10) are rewritten in the form:

$$C_{[ab}^d C_{c]d}^e - \xi_{[a}^i \partial_{|i|} C_{bc]}^e = 0 \tag{4.1}$$

So, suffice it to assume

$$\xi_{[a}^i \partial_{|i|} C_{bc]}^e = 0 \ , \tag{4.2}$$

that in the points of space M_n, in which the correlations (4.2) are satisfied, the Lie local loop Q_r can be named the Lie local group G_r (the Jacobi identities are satisfied: $C_{[ab}^d C_{c]d}^e = 0$).

Since stable states or metastable states are characterized the specific symmetries, then giving the parameter dependence of structural tensor components C_{ab}^c, we can describe processes in the Universe matter, connected with the transition to the state of "the black hole" or to the decay of it if only approximately. Specifically, we shall consider that the process of the spontaneous symmetry breaking is characterized the quasi-group structure.

Of course we take account of the presence of the Universe neutrino background which is the catalytic agent of stochastic processes, including decays of “black holes” or theirs restorations. In consequence of this it is logically connect the stability of differential equations (4.1) solutions with the stability of “black holes”. As a result functions $C^c_{ab}(x)$ must describe the process of spontaneous breaking of symmetry at decays of not only hadrons [12], but and “black holes”.

If the Lie local loop $Q_r(x)$ operates in the space of the affine connection as transitively so and effectively ($n = r$), then it can choose the components ξ^k_a of the intermediate tensor field equaled to the Kronecker symbols δ^k_a in a neighborhood of a point ω. As a result we must test the solution stability of differential equations

$$C^d_{[ab}C^e_{c]d} - \partial_{[a}C^e_{bc]} = 0 \tag{4.3}$$

Specifically, when $n = r = 8$, it allows to do not increase the count of gauge fields beyond 8 as in the grand unified theory. Thereby we consider that gluons attend in the space domain where intermediate vector bosons are absent and on the contrary intermediate vector bosons attend in the space domain where gluons are absent. Here we can use the theory of second-kind superconductor. That’s precisely what causes to the dependence of properties both fermions and vector bosons in different space domains.

Note that in more general cases, when the connection components $\Gamma^b_{ia}(x)$ are not equal to zero and the Lie local loop $G_r(x)$ operates in the space of the affine connection as transitively so and effectively, then the correlations (3.10) become in the Ricci identity ($C^c_{ab} = 2S^c_{ab}$). Because the symmetry, characterizing the physical system, is selected in terms of experimental data, the geometrical structure tied to the symmetry is only the maximum plausible one. Hence it follows that it is desirable to use the spaces of the affine connection with the torsion for the description of hadrons and “black holes”.

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The action in a causal set approach to quantum gravity

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A causal set approach to quantum gravity is considered. A causal set is an partially ordered locally finite set. There are three fundamental problems in this approach: to get physical objects as emergent self-organized structures of a causal set, to describe physical properties as a properties of a causal set, and to construct continuous spacetime as some approximation of a causal set. In this paper, I consider the first two problems. I consider a definition of an action for a causal set with some additional conditions for its dynamics. I introduce a particular example of a causal set dynamics that satisfies these additional conditions. A numerical simulation shows that this dynamics generates self-organized structures.

Introduction

A causal set

In most physical theories, fundamental entities are objects and their properties in some instant of time. The dynamics describe the connection between these properties in different instants of time. But this picture contradict relativity theory. In relativity theory, all physical connections are causal connections. In some instant of time, the set of points has not causal connections, structures, and physical properties. Two simultaneous points are connected only by their common past. Any structure consists of causally connected points. This structure has some duration of time. This is not an object in some instant of time. This is a process. In relativity theory, processes are more fundamental than objects. An object is an approximation of a process if we can neglect the duration of this process.

We can try to design quantum gravity theory as a development of relativity theory. In this case, the causality is a most fundamental property. The second idea is discreteness. We assume that the world is discrete at the fundamental level. In Minkowski spacetime, the causal order is described as partial order of events. For two ordered events a and b ($a \prec b$), $\mathcal{A}(a,\ b) = \{c \mid a \prec c \prec b\}$ is called an Alexandrov set of the elements a and b or a causal interval. In Minkowski spacetime, an Alexandrov set of any pair of ordered events is a set of continuum. We assume that an Alexandrov set of any pair of ordered events is a finite set (local finiteness). This is a causal set approach to quantum gravity. By definition, a causal set is a partially ordered locally finite set in which the partial order represents the causal relationships between events. A causal set hypothesis has been independently introduced by G. 't Hooft (1) and J. Myrheim (2) in 1978. You can see a review of this approach (3).

The physical meaning of local finiteness is discreteness. We get the discrete model of spacetime that is a discrete analog of Minkowski spacetime. In causal set, we can define past and future light cones, a spacelike hypersurface as a set of unordered events (an antychain) and so on. By definition, a discrete process is a subset of a causal set. In causal set approach, any physical system is a discrete process. We must describe any physical properties as a properties of such subsets. In this paper, I consider an action.

A sequential growth dynamics

Any observer can only actually know a finite number of facts. Then we consider only a finite causal sets. This is a model of a part of some process. The task of a dynamics is to predict the future stages of this process or to reconstruct the past stages. We can reconstruct the process step by step. The minimal part is an event. We start from some given causal set and add new events one by one. This procedure is proposed in papers (4; 5) for some particular model. Similar procedure and the term 'a classical sequential growth dynamics' is proposed by D. P. Rideout and R. D. Sorkin (6) for other model of a causal set.

We assume the nondeterministic dynamics of a causal set. The addition of any event has some probability. In this model, particles are repetitive or quasirepetitive processes. The dynamics must describe a self-organization and stability of such processes.

An action

The principle of least action

Consider the model with events of K kinds. Each event of the kind number j can occur with a fixed probability $p(j)$. Consider N sequential events. We have

$$N(j) = Np(j), \tag{1}$$

where $N(j)$ is a mathematical expectation of the number of the events of the kind number j.

Denote by X a particular sequence of N events. Denote by $p(X,N)$ a probability of this sequence. We have

$$p(X,N) = \prod_{m=1}^{N} p_m = \prod_{r=1}^{N(0)} p_r(0) \prod_{s=1}^{N(1)} p_s(1) \cdots \prod_{t=1}^{N(K-1)} p_t(K-1). \tag{2}$$

Consider the quantity $S(X,N)$. By definition, put

$$S(X,N) = -\log_2 p(X,N) =$$

$$= -\sum_{r=1}^{N(0)} \log_2 p_r(0) - \sum_{s=1}^{N(1)} \log_2 p_s(1) - \cdots - \sum_{t=1}^{N(K-1)} \log_2 p_t(K-1) = \tag{3}$$

$$= S(0) + S(1) + \cdots + S(K-1).$$

$S(X,N)$ is additive for processes and events. Each new event adds the summand $-\log_2 p_m(j)$ to it.

The hypothesis is that $S(X,N)$ is an action in the considered model. $S(X,N)$ has a minimal value for a most probable variant of the process. In the limit of quasideterministic processes, this variant has the probability that is very close to 1. We get the principle of least action. In information theory, $S(X,N)$ is an information. The principle of least action can be called the principle of minimum information. Note that in the definition of the action, we do not use the discreteness of processes. This definition can be considered for continuous nondeterministic processes.

A frequency of discretization

Consider the process that consists of events of the kind number 0 as a clock with step τ_0. The sequence of N events has the interval of time $t_0(N) = N(0)\tau_0 = Np(0)\tau_0$. Consider the process number 1 that consists of events of the kind number 1 and its frequency of discretization $f(1)$. We have

$$f(1) = \frac{N(1)}{t_0(N)} = \frac{N(1)}{N(0)\tau_0}. \tag{4}$$

Using $S(1) = N(1)\log_2 p(1)$, we get

$$f(1) = \frac{S(1)}{\log_2 p(1) t_0(N)}. \tag{5}$$

Consider a particular case that

$$p(0) = p(1) = \cdots = p(j). \tag{6}$$

In this case, we can include the constant $p(0)$ in the definition of time: $\tau = \log_2 p(0)\tau_0$. The sequence of N events has the interval of time $t(N) = \log_2 p(0) t_0(N)$. We get

$$f(1) = S(1)/t(N). \tag{7}$$

In this equation, if we consider a continuous time instead of $t(N)$, $S(1)$ will be a phase as an action in quantum theory.

Consider any function F of events of the kind number 1. Assume that the interval of time between any two sequential events is the same. This is not an additional assumption. This is a definition of the interval of time between two sequential events. Consider a usual discrete Fourier transform of F. We get a set of modes: $F(f_k)\exp(i2\pi f_k t)$, where k is an integer. We have $k = N(1)$ for the frequency of discretization.

In quantum theory, each free particle is described by a cyclic process $\exp(iS(t)\hbar^{-1})$, where $S(t)$ is an action. In the considered model, any particle is a discrete process. We can identify a quantum cyclic process with some mode of a discrete process. For example, this can be a mode with the frequency of discretization, or a Nyquist frequency, where $k = N(1)/2$ for the even N and $k = (N(1) - 1)/2$ for the odd N. In this case, a quantum cyclic process is described by a function $\exp\left(i2\pi \frac{aS(1)t}{t(N)}\right)$, where a is some constant. We get

$$S(t) = h\frac{aS(1)t}{t(N)}. \tag{8}$$

If we assume a causal set model with some additional conditions, the quantity $S(1)$ has some properties as an action in classical and quantum physics. Consider a particular case of a causal set that satisfies this conditions.

An example

An x-graph

Each causal set can be represented by a directed acyclic graph. A directed acyclic graph is a directed graph without directed cycles. Elements of a causal set are represented by vertices. Causal connections are represented by directed paths. Vertices a and b are causally connected, a is an cause, and b is a effect if there is a directed path from a to b.

Consider a particular case of an x-graph. An x-graph is a directed acyclic graph with the degree of any vertex no more than $(2, 2)$. The degree $(2, 2)$ means that the vertex has two incident incoming and two incident outgoing edges. This model was introduced by D. Finkelstein (7) in 1988 (see e.g. (8)). All vertices can have the degree $(2, 2)$ only in an infinite x-graph. In a finite x-graph, some vertices have free valences. If we add a new vertex, this vertex can be connected by new edges only with vertices with free valences. There are two kind of free valences: incoming and outgoing valences. An incoming (outgoing) free valences can be replaced only by an incoming (outgoing) edge. We can prove that the number of incoming free valences is equal to the number of outgoing free valences in an x-graph.

The addition of a new vertex is called an elementary extension. There are four types of elementary extensions. There are two types of elementary extensions to the future of the process (Fig. 1a and 1b). In this and following figures the x-graph $\mathcal{G}$ is represented by a rectangle because it can have an arbitrary structure. The free valences are figured by arrows. The free valences that take part in the elementary extension are figured by bold arrows. First type is an elementary extension to two outgoing free valences (Fig. 1a). Second type is an elementary

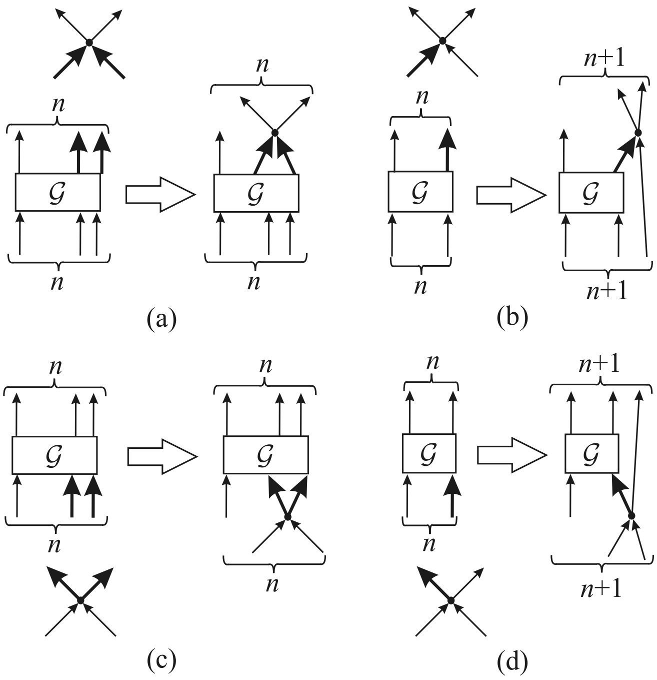

Fig. 1. The types of elementary extensions: (a) the first type, (b) the second type, (c) the third type, and (d) the fourth type.

extension to one outgoing free valence (Fig. 1b). Similarly, there are two types of elementary extensions to the past of the process (Fig. 1c and 1d). Third type is an elementary extension to two incoming free valences (Fig. 1c). Fourth type is an elementary extension to one incoming free valence (Fig. 1d). In the elementary extensions of the first or third types, the number n of incoming (outgoing) free valences is not changed. These elementary extensions describe the interior evolution of the process. In the elementary extensions of the second or fourth types, the numbers n of incoming (outgoing) free valences have increased by 1. These elementary extensions describe the interactions of the process and environment. The elementary extension of the first and second types are the addition of a maximal vertex, and the elementary extension of the third or fourth types are the addition of a minimal vertex. We can prove that we can get every connected x-graph by a sequence of elementary extensions of these four types.

An algorithm

In a sequential growth dynamics, an equation of motion is an algorithm to add new vertices. We has contradictory conditions to this algorithm. In subsection 2.2, we assume that all variants to add a new vertex have the equal probabilities (6). But in this case, a sequential growth dynamics generates random x-graphs. Any self-organized and stabile processes cannot emerge. To avoid this contradictions, suppose that only some part of elementary extensions satisfies conditions (6) and are included in the definition of an action. For example, an observer can register only the interaction of some process with environment and cannot observe internal interactions. In this case, an observer describes a process as a sequence of interactions with environment. We get the condition (6) for the elementary extensions of the second and fourth types, and we can use the elementary extensions of the first and third types to get self-organization.

Consider a particular algorithm. The first step is the choice of the elementary extension to the future or to the past. The probability is 1/2. The second step is the choice of the type of elementary extension. We choose the elementary extension of the second (fourth) type with probability p $(1 > p > 0)$, or the elementary extension of the first (third) type with probability $1 - p$. In the third step, if we choose the elementary extension of the second (fourth) type, we choose with equal probability one outgoing (incoming) free valence and connect the new vertex with the x-graph by one new edge instead of this free valence. If we choose the elementary extension of the first (third) type, the algorithm to choose the variant of the addition of the new vertex must be more complicated because it must generate self-organized processes.

Consider a particular algorithm of the elementary extension of the first type that is based on the random walks on the x-graph $\mathcal{G}$ (Fig. 2a). An algorithm of the elementary extension of the third type is the same if we change the direction of all edges in the x-graph. We choose with equal probability one outgoing free valency number i and start the walk. This free valency belongs to the vertex number A. Then we choose an opposite directed path from A. In each vertex including A, we choose the continuation of the opposite directed path or the u-turn with probability 1/2. If we choose the continuation, we choose one incoming edge or one incoming free valency with probability 1/2. If we choose an edge, we go to the next vertex and repeat this process. If we choose the u-turn, we choose the directed path. In each vertex we choose one outgoing edge or one outgoing free valency with probability 1/2. If we choose an edge, we go to the next vertex and repeat this process. If we came to the vertex, that is included in the opposite directed path, we must choose the outgoing edge that is not included in the opposite directed path or the outgoing free valency. We get the edge disjoint path. This path ends in some outgoing free valency number j that cannot coincide with i. Then we start from the free valency number j and choose outgoing free valency number k by the same algorithm. We connect a new vertex with the x-graph by two new edges instead the free valences numbers j and k.

If we choose the incoming free valency number α, the opposite directed path cannot be continued (Fig. 2b). In this case, we choose the incoming free valency number β and choose the directed path from β. This path from β cannot have common edges with opposite directed path to α. We choose β by the following algorithm. α belongs to the vertex number B. We choose an opposite directed path from B. In each vertex of the path, we choose one incoming edge or one incoming free valency with probability 1/2. This path ends in some incoming free valency number β. The directed path from β can have common edges with the opposite directed path

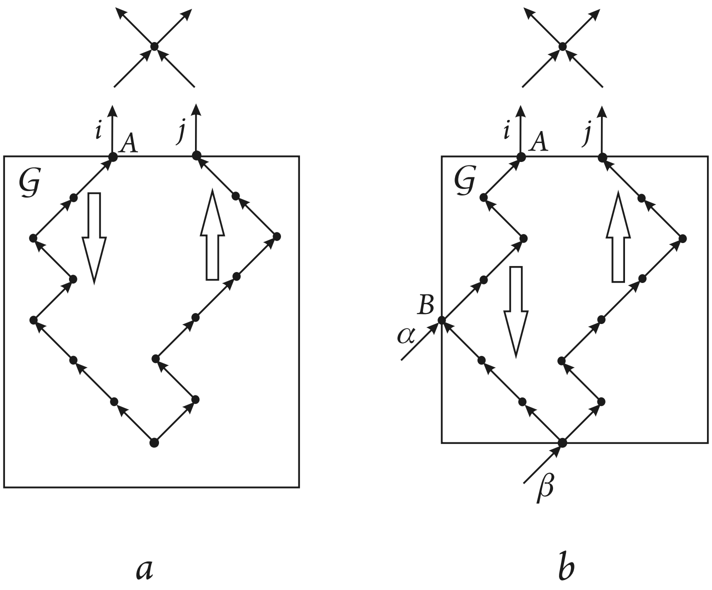

Fig. 2. The random walks on the x-graph.

to β.

If $p \ll 1$, this algorithm can generate self-organized structures. In the majority of cases, a new vertex are connected with two near free valences. This produces the stability of processes. The rare elementary extension of the second and fourth types generate new processes. The rare variant with transition between free valences during the walk (Fig. 2b) generates the rare interactions between processes.

Consider an example of a numerical simulation (Fig. 3). $p = 0.1$. We start from one vertex. This vertex is pointed by a bold arrow. We add 141 vertices. All edges directed upward. They are figured without arrows.

In this example, the sequential growth dynamics generate 7 processes. The processes number 1 and 2 are more complicated. These processes are connected by a three interactions. We get only a beginning of generation of other processes. The processes number 4 and 6 are a sequences of double edges. Such sequences are substructures of all other processes. The process number 2 emits the process number 3. The processes number 1 and 2 emit the process number 4. The process number 1 connects by single interactions with the processes number 5, 6, and 7.

Conclusion

There are three fundamental problems in a causal set approach to quantum gravity: to get physical objects as emergent self-organized structures of a causal set, to describe physical properties as a properties of a causal set, and to construct continuous spacetime as some approximation of a causal set. In this paper, the first two problems is considered. The considered

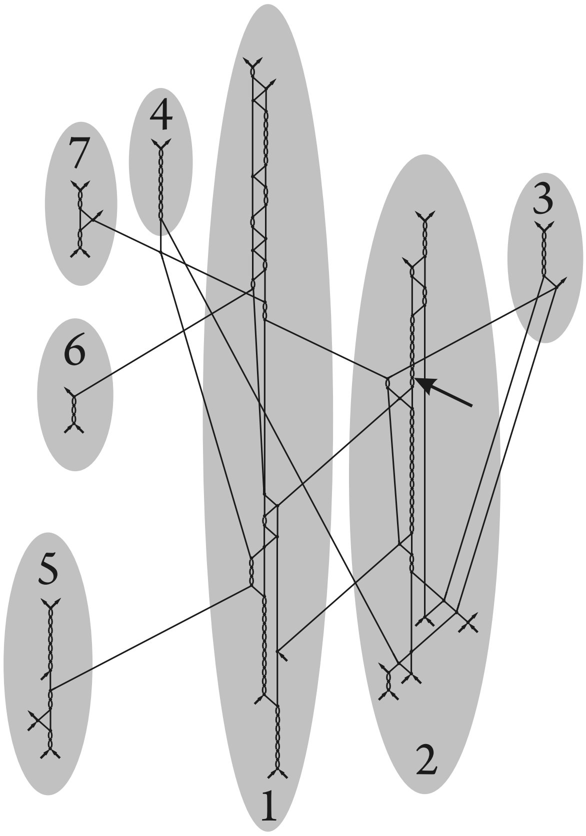

Fig. 3. An example of a numerical simulation.

example is a toy model. The task for further investigations is to get a model as a consequence of some fundamental physical principles.

I am grateful to Alexander V. Kaganov for extensive discussions on this subject.

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On correlation of the large scale structures of the quasars and galaxies

Iurii Kudriavtcev

We have checked the assumption that the quasars can be secondary images of the opposite to them galaxies. This check is done by comparison the data of 2dFGRS about the large scale spatial structure of distribution of the galaxies with the data of SDSS-DR5 about the distribution of the quasars in the opposite areas of the celestial sphere.

We have discovered the characteristics of correlation of the analyzed distributions. We have shown that the large scale spatial distributions of the galaxies and opposite to them quasars have analogical structures with a significant number of coinciding elements. We have calculated the Pearson correlation coefficient for the distributions of the galaxies and opposite to them quasars in the coordinates Z/RA and shown that it has the statistically significant positive value $R_{xy}\approx0.045$ with probability of the zero hypothesis $P<2*10^{-9}$. We have performed the analysis of their changes at deviation of the compared distributions of quasars and galaxies from the strictly opposite location with the comparative analysis of the functions of cross-correlation and autocorrelation of the distributions.

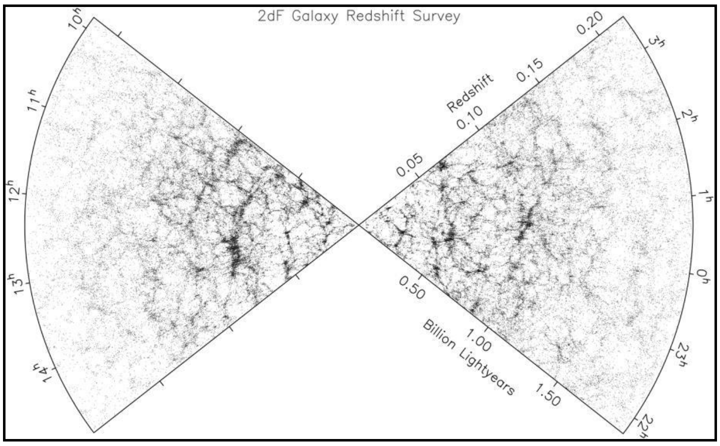
The total of the discovered characteristics give us a reason to make a statement about the correlation of the large scale spatial distributions of the galaxies and opposite to them quasars, which in its turn, confirms the appropriateness of the assumption that the quasars can be the secondary images of the galaxies observed in the opposite locations of the celestial sphere.

98.80.-k

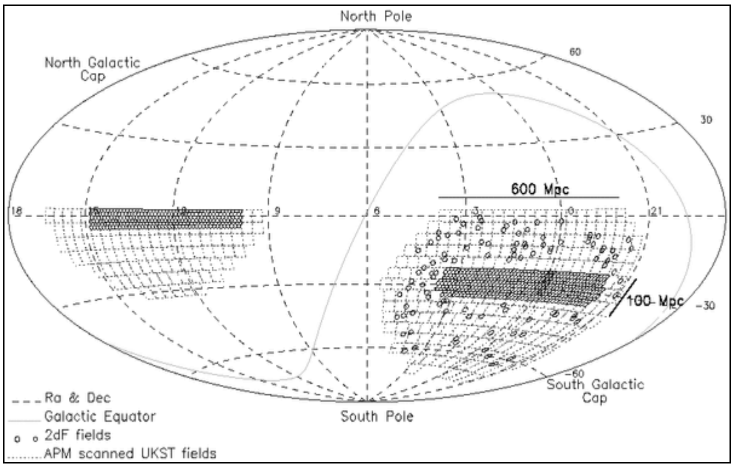
1. Introduction.

Checking the assumption of possibility of manifestation of the central symmetry [1] of the celestial sphere in existence of the pairs of opposite distant observed objects, we came to the discovery of the pairs of opposite quasars [2] with similar magnitude profiles in the ranges (u,g,r,i,z) with Pearson correlation coefficients close to 1. We discovered that the amount of the pairs with correlation coefficients $R_{xy}>0.98$ for the opposite quasars is significantly higher that for random pairs of quasars. The probability of formation of this exceeding on account of the random deviations from the average value does not exceed 10^{-9}, which excludes the possibility of its accidental nature. Comparison of the results obtained in [2] confirming the possibility of the manifestation of the central symmetry in the existence of pairs of opposite distant observed objects, with the data from [3] about the distribution by Z of the objects classified as quasars and galaxies, leads us to the assumption that many of the observed quasars can be images of the galaxies observed in the opposite to them locations of the celestial sphere.

Comparison of the individual characteristics of the objects in the pairs quasar-galaxy by the method used in [2] to analyze the opposite quasars is difficult because of the big difference of their redshifts. In this relation we consider it reasonable to check the mentioned above assumption by comparing not the individual characteristics of the objects but the data about the large scale spatial structure of the galaxies distribution presented in the materials 2dFGRS [4],[5] (Pictures 1,2) with the spatial distribution of the quasars of the opposite area of the celestial sphere according to the data of the catalog SDSS-DR5 [6],[7-11]. The presence or the absence of the correlation of the spatial structures of the galaxies and opposite to them quasars, can be the a proof or a disproof of the assumption that is being checked.



Picture 1. Large scale structure of galaxies distribution according to the data of 2dFGRS [4],[5].



Picture 2. Location of the areas of the celestial sphere for which they built the galaxies distributions shown on the Picture1 [4],[5].

2. Comparison of the spatial distributions of the galaxies and quasars.

On the Picture 3 we show the spatial distribution of the galaxies according to the data of 2dFGRS [4], for the northern part of the celestial sphere (NGP), corresponding to the left part of the Picture 1 and Picture 2, merged with the distribution of the opposite to them quasars from the catalog SDSS-DR5 [6]. We consider opposite to the galaxy a quasar whose inverted coordinates (RA_{inv}, DE_{inv}, Z_{inv}) coincide to the coordinates of the galaxy (RAJ2000, DEJ2000, Z). At that $RA_{inv} = RAJ2000_{quas}+180^0$, $DE_{inv}= -DEJ2000_{quas}$, $Z_{inv} = 2 - Z_{quas}$ [2]. This way on the coordinate axes of the Picture 4 for the galaxies RA≡ RAJ2000, Redshift≡Z, for the quasars RA≡ RA_{inv}, Redshift≡ $Z_{inv.}$ The width of the layer by the coordinate DE for the quasars is about 2.5^0 ($-1.3^0< DE_{quas}<1.3^0$), for the galaxies in the area Z<0.2 about 2^0 ($-1^0<DEJ2000<1^0$) with a smooth extension in the area 0.2<Z<0.3 до $5\text{-}6^0$ ($-3^0<DEJ2000<3^0$) to compensate the severe decrease of the density of the galaxies.

On the Picture 3 we see that the most of the chains and compact groups of distributions of the quasars are situated along the chains and near the concentrations of the distributions of the galaxies. We can note a big amount of coinciding structural elements of distributions. Big amount of coincidences is noted, as a rule, in the areas with higher density of quasars.

We can see an analogical spatial distribution for the galaxies of the southern part of the celestial sphere (SGP), corresponding to the right part of the Pictures 1 and 2, merged with the distribution of the opposite to them quasars (*available online on https://sites.google.com/site/kudrspbru/large-scale-structures-quasars-and-galaxies-2011).*

The distributions for NGP and SGP with the lines marking visible elements of the correlations of the spatial structures of the galaxies distribution, are shown on the Picture 4. In the distribution for SGP we also can note the corresponding structural elements but in lesser amounts which is possibly related to the significantly lower density of the quasars.

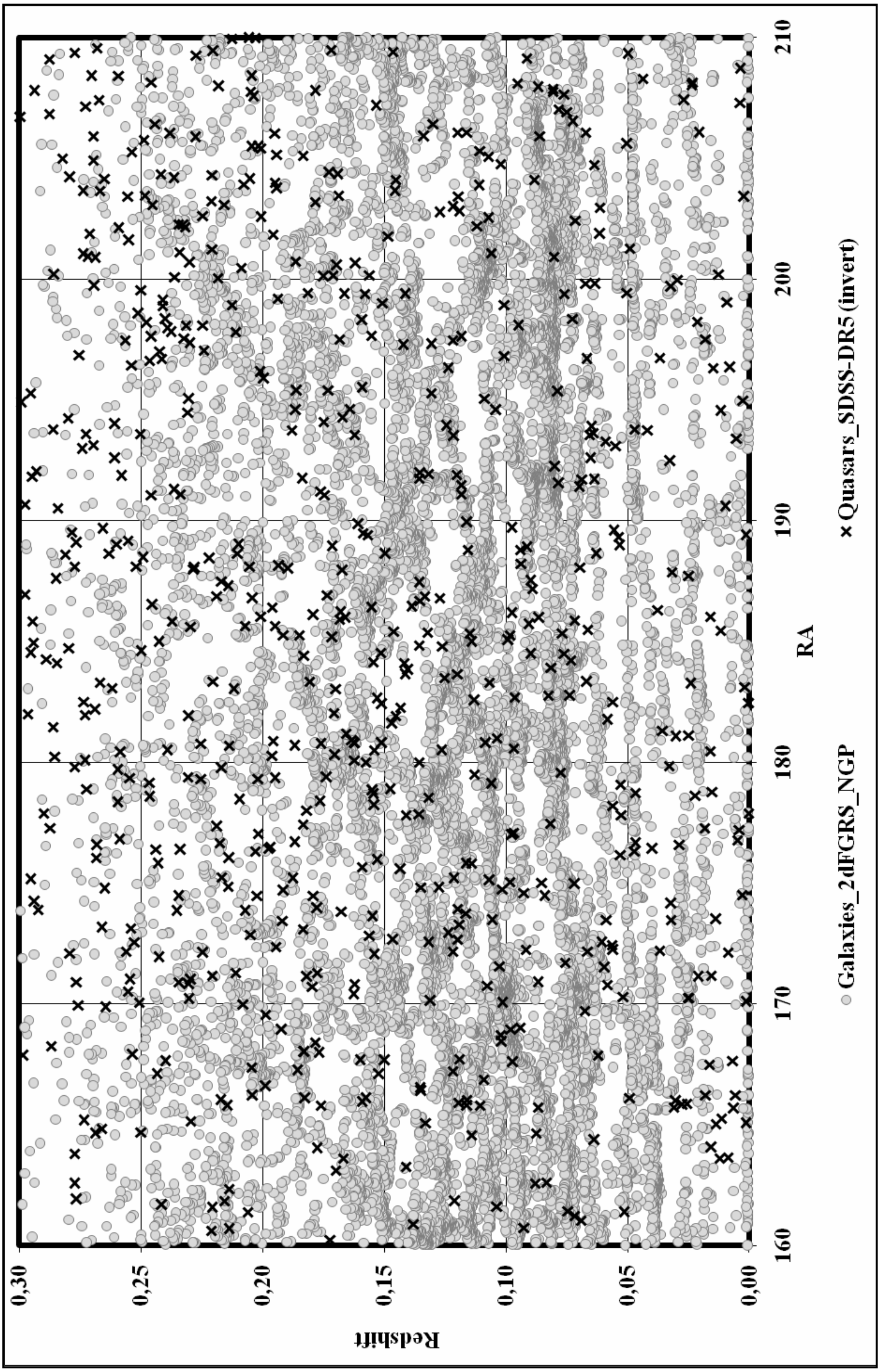
3. Calculation of the Pearson correlation coefficients for two-dimensional functions of the spatial distribution of the galaxies and quasars N(Z,RA).

For the quantitative check of the correlation of the spatial structures of quasars and galaxies, the distributions shown on the Picture 4 were transformed to the two-dimensional functions $N_{gal}(Z,RA)$ and $N_{quas}(Z,RA)$ by distributing the objects by rectangular matrix (200x150) of the spatial cells with dimensions of 0.25^0 by RA and 0.002 by Z, covering the range $160^0<RA<210^0$ and $0<Z<0.3$. To compensate small mutual deviations of the concentrations of quasars and galaxies and quantum nature of the distributions of quasars because of their low density, both obtained distributions of the amount of objects in the cells N(Z,RA) were extended, i.e. transformed into the distribution N_{5x3} for quasars and N_{3x3} for galaxies where $N_{3x3} = N_{ij}+ \sum_{i-1,j-1}^{i+1,j+1} N_{kl}$; $N_{5x3} = N_{ij}+ \sum_{i-1,j-2}^{i+1,j+2} N_{kl}$ (index $1<i<150$ corresponds to the movement by the coordinate Z, index $1<j<200$ — by the coordinate RA).

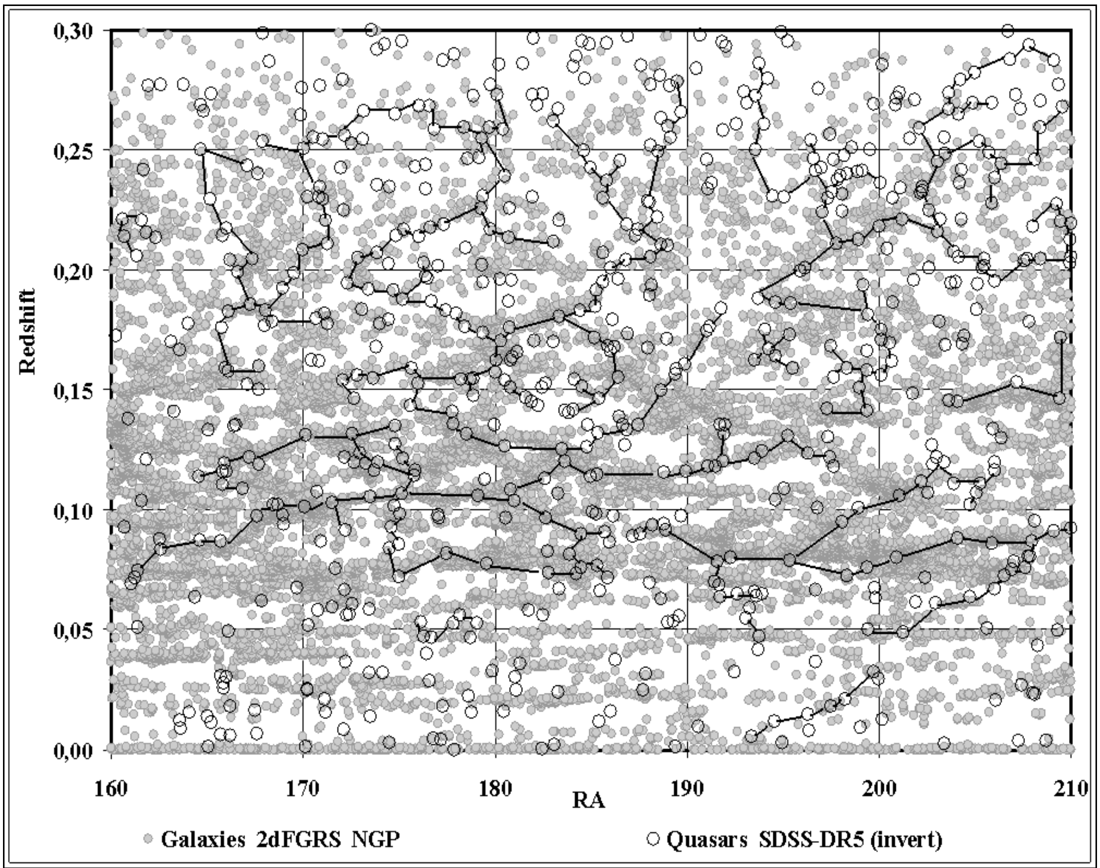
For the obtained two-dimensional functions $N_{5x3}^{quas}(Z,RA)$ and $N_{3x3}^{gal}(Z,RA)$ we calculated the Pearson correlation coefficient R_{xy} (PEARSON,Excel). It turned out positive and having a value of approximately some hundredths. To check the dependency of the correlation coefficient from the mutual location of the distributions we studied the cross-correlational functions at different values of the deviation of the distributions of the quasars in relation to the strictly opposite value by Z and by RA (ShiftZ, ShiftRA) with the pitch, equal to the size of the cell, i.e. 0.25^0 by RA and 0.002 by Z. At that the inverted values of the coordinates of the quasars were calculated by the formulas:

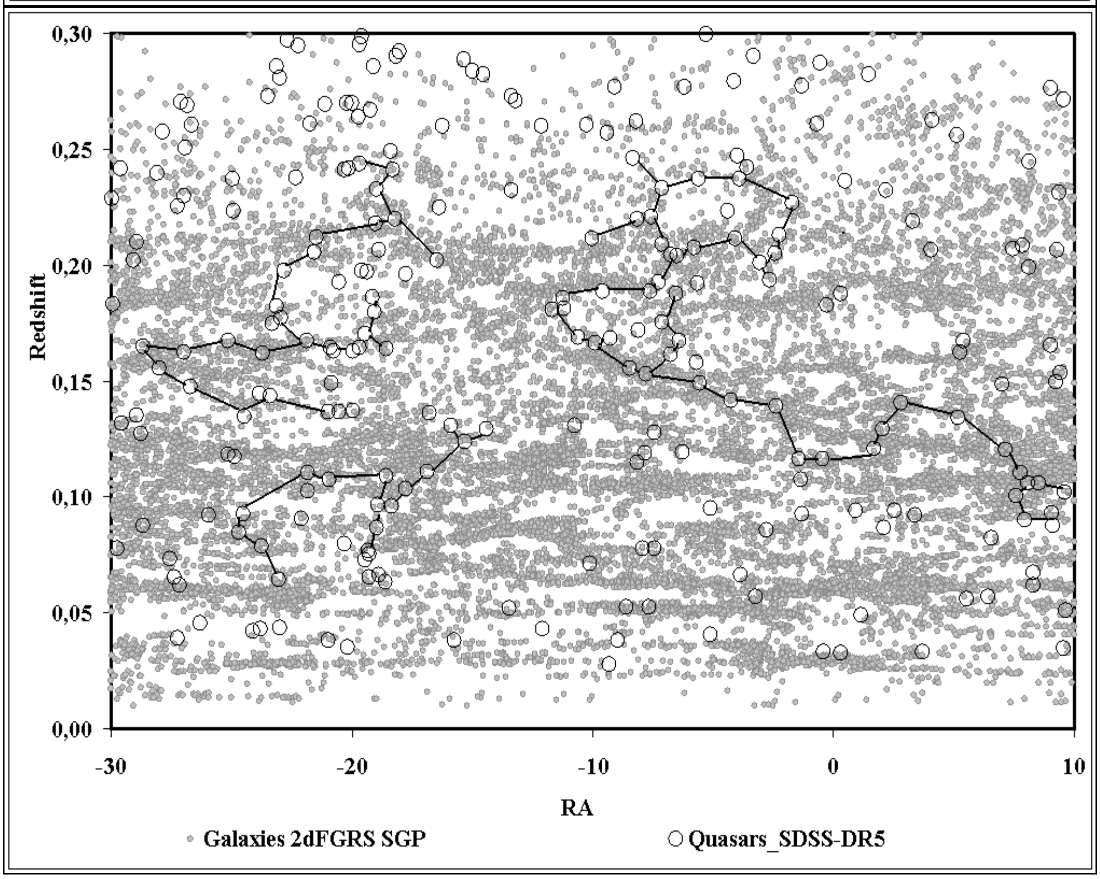
$RA_{inv} = RA_{quas}+180^0+ShiftRA$, $DE_{inv}= -DE_{quas}$, $Z_{inv} = 2 - Z_{quas}+ShiftZ$.

Obtained curves $R_{xy}(ShiftZ)$ и $R_{xy}(ShiftRA)$ are shown on the Picture 7.



Picture 3. Spatial distribution of the galaxies of the northern part of the celestial sphere (NGP) 2dFGRS [4], merged with the distribution of the opposite quasars SDSS-DR5 [6].

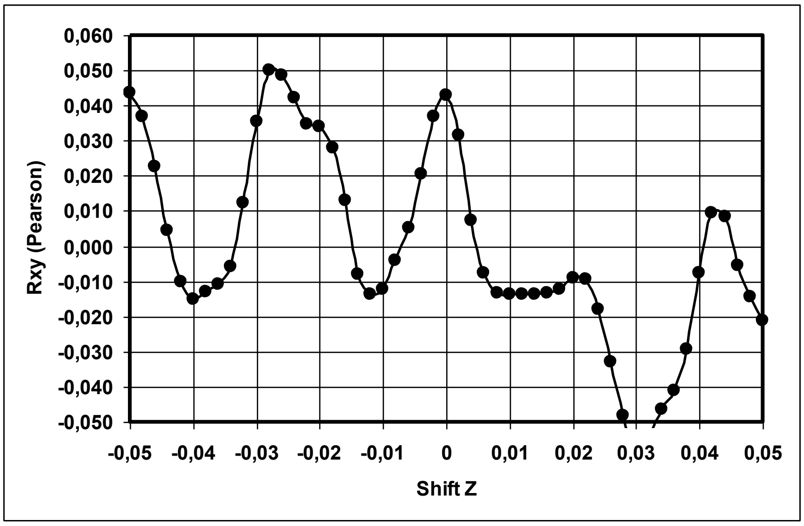


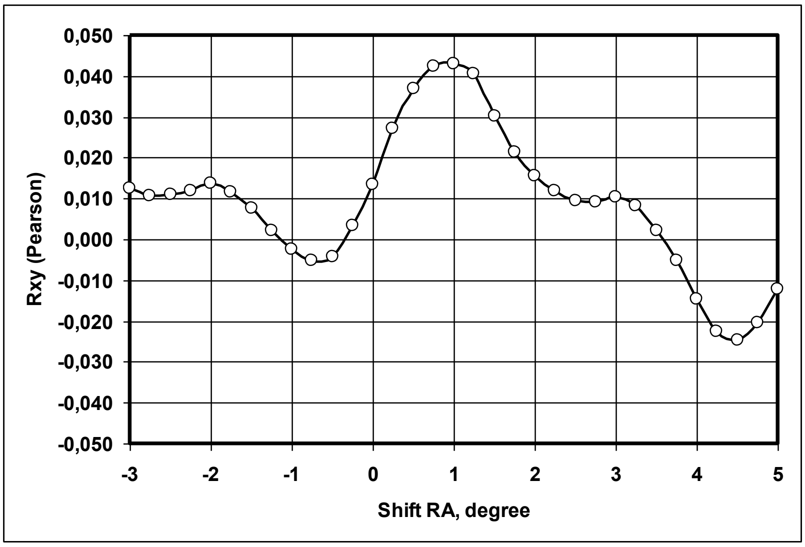


Picture 4. Spatial distributions of the galaxies and quasars on the areas NGP and SGP with the lines marking visually correlating structural elements.

On both curves there are clearly expressed local maxima of the correlation coefficient in the area of zero shift with the value $R_{xy} \approx 0{,}045$. However on the curve R_{xy}(ShiftRA) the top of the maximum is somehow shifted from the strictly opposite value and is located at ShiftRA $\approx 1^0$. The reason of the shift can be related to, for example, the peculiar movement of the observer in relation to the distant sources.

It is necessary to point out that the small value of the calculated coefficient of the positive correlation does not mean the absence of its statistical significance, as it is defined not only by the absolute value Rxy, but also by the amount of selection where it was calculated. In this case the amount of the selection is defined by the size of the spatial cells matrix, in each of which the correlated between themselves quasars and galaxies are defined. To calculate the obtained value of Rxy we used a part of this matrix with dimensions of 180x100 cells, covering the range (162^0<RA<207^0; 0.1<Z<0.3). This way in this case the amount of selection is N = 18000.





Picture 5. Dependencies of the Pearson correlation coefficient from the deviation of the coordinates of quasars from the strictly opposite location R_{xy}(ShiftZ) and R_{xy}(ShiftRA).

Calculation of the significance of the correlation coefficient Rxy with use of Student t-distribution for this value of N gives

$t = R_{xy}((N-2)/(1- R_{xy}^2))^{1/2} \approx 0.045 \sqrt{18000} \approx 6.04;$ (1)

This value of t corresponds to the almost zero probability of zero hypothesis. For the two sided Student distribution (TDIST, Excel) we obtain the probability of zero hypothesis $P \approx 1.6*10^{-9}$.

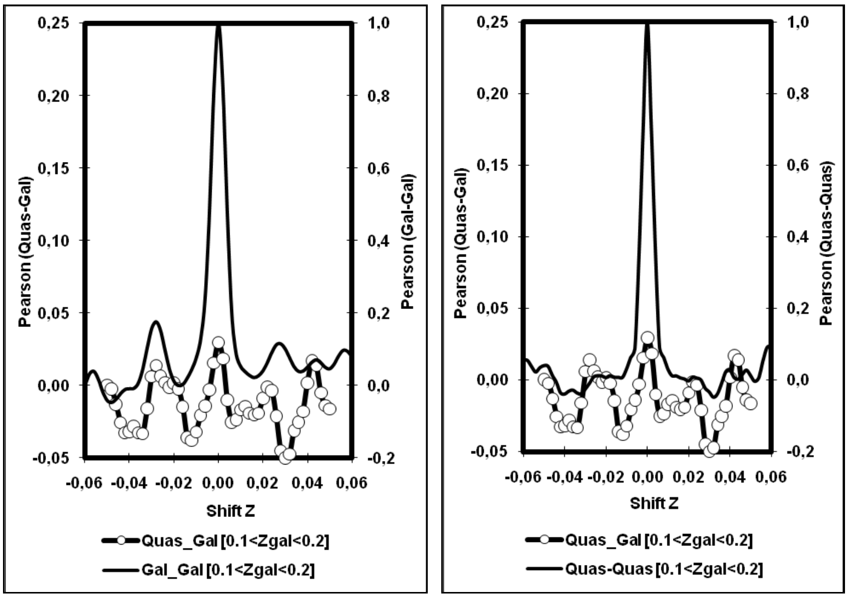
At the amount of the selection N = 18 000 we can, on basis of the central limit theorem, make a statement that the obtained value of the correlation coefficient $R_{xy}\approx 0.045$ is statistically significant and indicates the real correlation between the large scale spatial distribution of the galaxies and opposite to them quasars.

4. Comparative analysis of cross-correlational and autocorrelational functions.

A significant characteristic of the curves on the Picture 5 seems to be a presence, apart from the central one< of other chaotically located maxima and minima, equal by amplitude to the central one and even exceeding it. As the complex nature of the dependencies R_{xy}(ShiftZ) and R_{xy}(ShiftRA) on the Picture 5 can be related to the proper periodical or quasi-periodical inhomogeneities of the compared spatial distributions of galaxies and quasars, we, using the identical method, studied these dependencies not only for the pair "Distribution of the galaxies in relation to the shifted distribution of the quasars" but also for the autocorrelational dependencies "Distribution of the galaxies in relation to the shifted distribution of the galaxies" and "Distribution of the quasars in relation to the shifted distribution of the quasars".

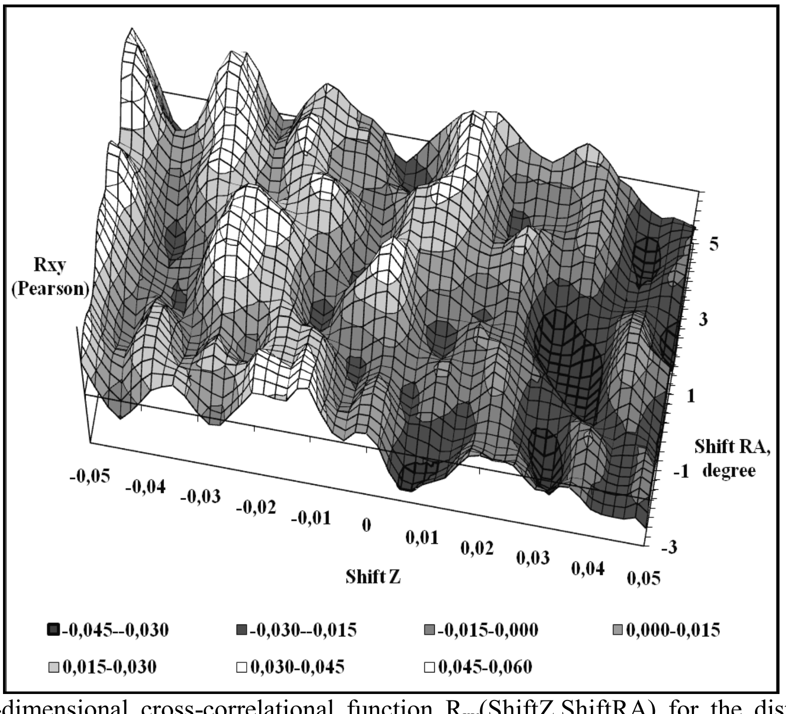
On the Picture 6 we show the curves R_{xy}(ShiftZ) for the pairs of the distributions "Galaxies-Galaxies" and "Quasars-Quasars" in comparison with the curve "Quasars-Galaxies" shown on the Picture 5. From the comparison of the pairs of curves we see that nearly all the particularities of the cross-correlational curve R_{xy}(ShiftZ), located on the left and on the right from the central peak coincide to the corresponding particularities of the autocorrelational curves for the pairs "Galaxies-Galaxies" and "Quasars-Quasars" and that confirms the assumption that they are caused by the concrete spatial inhomogeneities common for these distributions.

From the other point, the form of the central peak on the curve on the Picture 5 is analogical to the form of the central peaks of the curves for the pairs "Galaxies-Galaxies" and "Quasars-Quasars" (where the central peaks reflect 100% correlation of distributions with themselves and have amplitudes R_{xy}=1), which can also be interpreted as a confirmation of the fact that the central maximum on the curve on the Picture 5 reflect the correlation of the distribution of the galaxies and opposite to them quasars.



Picture 6. Dependencies R_{xy}(ShiftZ) for the pairs of distributions "Galaxies-Galaxies" and "Quasars-Quasars" in comparison with the curve "Galaxies-Quasars" shown on the Picture 5.

On the Picture 7 we show the two-dimensional dependency R_{xy}(ShiftZ,ShiftRA) for the distributions "Galaxies-quasars" in the area of the central peak. We draw attention to the characteristic form of the central peak with an almost circular groove around (analogical to the local minima on the left and on the right from the central peak on the curve R_{xy}(ShiftZ) for the pair of distributions "Galaxies-Galaxies" (Picture 8) which confirms the special character of the peak in relation to the other elements of the relief and reflected correlation of the spatial distributions of the galaxies and opposite to them quasars.



Picture 7. Two-dimensional cross-correlational function R_{xy}(ShiftZ,ShiftRA) for the distributions of the galaxies and opposite quasars in the area of the central peak ($-0.05<\text{ShiftZ}<0.05$; $-3^0<\text{ShifrRA}<5^0$).

5. Conclusion

We performed a comparison of a large scale spatial structure of galaxies distribution according to the data of 2dFGRS in two areas of the space: (NGP) with coordinates (160^0<RAJ2000<210^0;-3^0<DEJ2000<3^0; 0.1<Z<0.3) and (SGP) with coordinates (-30^0<RAJ2000<10^0;-33^0<DEJ2000<-29^0;0.1<Z<0.3) with spatial distributions of opposite to them quasars according to the data of SDSS-DR5.

We have shown that in both areas the most of the chains and compact groups of distributions of the quasars are situated along the chains and near the concentrations of the distributions of the galaxies. There is a big amount of coinciding structural elements of the compared distributions. A big amount of the coinciding elements is noted in the areas with higher density of quasars as in the areas with lower density the big distances between the separate quasars do not allow to surely interpret them as the ones belonging to the same structural element of the distribution.

For the distributions in the area (NGP) we calculated Pearson correlation coefficient R_{xy}. For the opposite location of the distributions of the quasars and galaxies $R_{xy} \approx 0.045$. The amount of the selection is 18,000, the probability of zero hypothesis $P \approx 1.6*10^{-9}$. At shift in relation of this location the correlation coefficient R_{xy} decreases down to the negative values, forming on the function of the cross-correlation a peak located in the area of the opposite location of the distributions and coinciding by form and location to the analogical peaks of the autocorrelation at shift of the distributions of the galaxies and quasars in relation to themselves.

This way we have discovered the following characteristics of the correlation of the large scale spatial distributions of the galaxies and opposite to them quasars:

1). In both studied areas (NGP) and (SGP) the most of the chains and compact groups of quasars are located along the chains and near the concentrations of the galaxies.

2). In both areas there is a big amount of the coinciding structural elements of the compared distributions.

3). For the distributions in the area (NGP) Pearson correlation coefficient, at opposite location of the distributions of the quasars and galaxies, has a statistically significant positive value $R_{xy} \approx 0.045$ with probability of zero hypothesis $P \approx 1.57*10^{-9}$.

4). At shift in relation to this location the correlation coefficient decreases down to negative values, forming a peak located in the area of the opposite location of the distributions of the galaxies and quasars and coinciding by form and location with analogical peaks for the correlation coefficients at shift of the distributions of the galaxies and quasars in relation to themselves.

The total of these characteristics gives us a reason to make a statement about the discovery of the correlation of large scale spatial distributions of the galaxies and opposite to them quasars, which in its turn, confirms the legitimacy of the assumption made earlier [2], that many quasars can be secondary images of the galaxies observed in the opposite to them locations of the celestial sphere.

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Some elementary questions on consistency or inconsistency of the standard cosmological model

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It is shown that the metric in the basis of the standard cosmological model belongs to the stationary Universe and corresponds to the absence of the energy transfer between the matter and the gravitational field which is equivalent to the impossibility of the Universe development in time. This way in the basis of the standard cosmological model describing the developing Universe there is a deep inner contradiction that is being eliminated by using the metric that considers the non-zero value of the scale factor differential. The necessity of the model revision is confirmed by the results of the observation data analysis about the microwave background symmetry and galaxies and quasars distribution that contradict to the Big Bang model but are naturally interpreted in the modified model on basis of the changed metric. We suggest a number of elementary questions to check the consistency or inconsistency of the Big Bang model.

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Introduction

Cosmological model describing the development of the Universe as the result of a certain catastrophic event (“Big Bang”), is now for more than half a century being considered the main model of the Universe and is called the standard cosmological model (also Lambda-CDM model, concordance model, etc.), despite an enormous amount of the revealed contradictions and corresponding doubts about its correctness [1]-[6]. The Nobel Prize Winner Steven Weinberg pointed out [1]: “Of course it is absolutely possible that the standard model is totally or partly incorrect. But its value is not in its unflinching truth but in its being fundamental for discussion of a great variety of the observation data”. (translation from a Russian edition).

However the value of the matter in dispute is quite high. More and more information about the Universe structure obtained by the physicists and astronomers is interpreted from the point of view of the standard model. More and more resources are spent to obtain it. The results of the interpretation define the direction of the further research that also requires resources; so as a result, the matter has not only an academic importance but also an increasing economic importance.

The fact that the period in the matter of the consistency or inconsistency of the standard cosmological model is not put yet, can be explained by the maximal complexity of the Einstein’s General Theory of Relativity in its basis and the transition of the matter on new study levels which even more restricts the community of the specialists working on it.

As the topicality of the matter increases but its study by means of deepening and complication still did not lead to any solution, we suggest studying it from an opposite point of view – from the one of some elementary question. Answers to them, as it seems to us, can be a contribution to the formation of the beliefs on consistency or inconsistency of the cosmological model of the Big Bang.

Metric and the energy conservation law

Metric fundamental for the standard cosmological model [7] is expressed by the correlation:

$$ds^2 = c^2dt^2 - a^2[d\chi^2 + \sin^2\chi\,(\sin^2\theta\, d\varphi^2 + d\theta^2)]; \qquad (1)$$

where a – space curvature radius (scale factor), χ – range coordinate, θ, φ – angular coordinate, c – light speed. Corresponding values of the components of the metric tensor: $g_{00} = 1$, $g_{11} = - a^2$, $g_{22} = - a^2 \sin^2\chi$, $g_{33} = - a^2 \sin^2\chi \sin^2\theta$.

The equations of the General Theory of Relativity represent the mathematical expression of the energy conservation law which is repeatedly pointed out by Einstein in "Formal Foundations of the General Theory of Relativity" [8]. The energy conservation law including the energy of the matter and the gravitational field, in the General Theory of Relativity is expressed by the tensor equation [7], [8]:

$$(1/\sqrt{-g})\, \partial[T_i^k\sqrt{-g}]/\partial x^k - (1/2)\, \partial g_{kl}/\partial x^i\, T^{kl} = 0; \qquad (2)$$

The second member in the left side of this equation represents an expression for the impulse and, correspondingly, for the energy, which in a unit of time and in a unit of volume are being transferred to the matter from the gravitational field [8]. Its zero component corresponds to the spatial density of the energy transfer. Inserting the components of the metric tensor and the energy-impulse tensor of the standard cosmological model (1) we obtain that it equals to zero (because all the components T^{kl} except T^{00} are equal to zero and g_{00}=1=const, from where $\partial g_{00}/\partial x^i \equiv 0$), which corresponds to the absence of the energy transfer between the matter and the gravitational field and, consequently, to the impossibility of the development of the Universe in time. This way in a foundation of the standard cosmological model that describes the expanding Universe, there is a deep inner contradiction that cannot be eliminated without changing the beliefs about the metric.

Metric and Universe expansion

Expression (1) corresponds to the evenly curved space obtained by Einstein by inserting an imaginary 4th spatial coordinate and its further exclusion through the space curvature radius [9]. This mathematical formalism that allows describing the curvature of the 3-dimensional space by the gravitational fields, was as is known inserted by Einstein when studying the stationary Universe. At insertion of the expression (1) the differential of the excluded 4th spatial coordinate that enters into the expression for the element of the spatial distance dl, is expressed through the differentials of other three spatial coordinates [1],[8],[9], but not through the differential of the curvature radius da, which equals zero in the stationary Universe:

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + (x_1\, dx_1 + x_2\, dx_2 + x_3\, dx_3)^2 / (a^2 - x_1^2 - x_2^2 - x_3^2). \qquad (3)$$

Expressing the fourth spatial coordinate through differentials of other spatial coordinates and the differential of the curvature radius of the space da, which for $a(t) \neq const$ is not equal to zero [10], we will obtain another expression for dl:

$$dl^2 = dx_1^2 + dx_2^2 + dx_3^2 + (a\,da - x_1\,dx_1 - x_2\,dx_2 - x_3\,dx_3)^2 / (a^2 - x_1^2 - x_2^2 - x_3^2); \qquad (4)$$

and corresponding to it expression for the interval:

$$ds^2 = c^2dt^2(1-a'^2) - a^2[d\chi^2 + \sin^2\chi\,(\sin^2\theta\,d\varphi^2 + d\theta^2)]. \qquad (5)$$

It is not difficult to check [10], that the Law of energy conservation in the formulation (2) in this case at $g_{00} = (1-a'^2)$, is being realized.

We point out that there was an argument against the study of the Universe on basis of the metric (5) that it has no sense as from the point of view of the equations of the General Theory of Relativity, suitable for any coordinates, the metric tensors by the expressions (1) and (5), differing only by redefinition of the time coordinate, should be considered equivalent. But this argument contains a contradiction and appears to be false. Weinberg [1] points out that the symbol of "t" in Robertson-Walker metric is time "or its function". But this circumstance is of no importance only until the moment when we want to show the dynamics of the development of the Universe and define the stages of its development in years, which is what cosmology does. What is t = 13.5 billions of years? Is it the age of the Universe or some function of its age, and which one? Existence of metric (1) along with metric (5) puts a question of where the symbol of "t" is time and where it is the function of time?

The main particularities and the development dynamics of the modified model

The change of the time component of the metric tensor g_{00} significantly changes the dynamics of the development of the Universe and lead us to the modified model [10] (*available online on https://sites.google.com/site/kudrspbru/on-inner-contradiction-2011)*, which describes the Universe, closed at any matter density, not requiring an introduction of additional non-observed substances (cosmological constant, dark energy, vacuum energy), infinite in time and at this development stage expanding in an accelerated manner, which corresponds to the existing observation data about its accelerated expansion [11].

When studying the model, we determine two time scales – the time on the surface of the expanding hypersphere, i.e. in the observer's coordinates, and the time in the geometrical center of the hypersphere, lying beyond our 3-dimensional space. We are, of course, interested in the time scale of the observer's coordinates.

The main parameters of the model [10] are given by the expressions:

$$a'_{obs} = da / cdt_{obs} = \alpha\,(1-\alpha^2)^{1/2}\,; \qquad (6)$$

where a – scale factor (hypersphere radius), t_{obs} – time in the immobile observer's coordinates, c – light speed, α – relative value of the scale factor $\alpha = a/2a_0$, $a_0 = const$. From where

$$dt_{obs} = (2a_0/c)\,d\alpha / [\alpha\,(1-\alpha^2)^{1/2}]; \qquad (7)$$

$$\alpha(\tau_{obs}) = 1/\mathrm{ch}(\tau_{obs}); \quad (8)$$

where

$$\tau_{obs} \equiv (c/2a_0)t_{obs}. \quad (9)$$

Counting of τ_{obs} is performed from the moment of the maximal expansion of the hypersphere (α =1).

Geometrical parameters of the Universe in the modified model are related to the relative matter density Ω and Hubble constant H by the expressions:

$$\alpha(\Omega) = \Omega^{1/2}(\Omega+1)^{-1/2}; \quad (10)$$

$$a_0 = (c/2H)(1-\alpha^2)^{1/2} = (c/2H)(\Omega+1)^{-1/2}; \quad (11)$$

$$a = 2a_0\alpha = (c/H)\Omega^{1/2}(\Omega+1)^{-1}. \quad (12)$$

The expansion, unlike the standard model, happens slowly which allows eliminating the problems in the foundation of the scenario of the inflating [12] ("horizon problem", "singularity problem", etc.), and also the problems of the astronomy and astrophysics related to the lack of time (see for example [13]). The dynamics of the expansion are graphically shown on the Fig. 1.

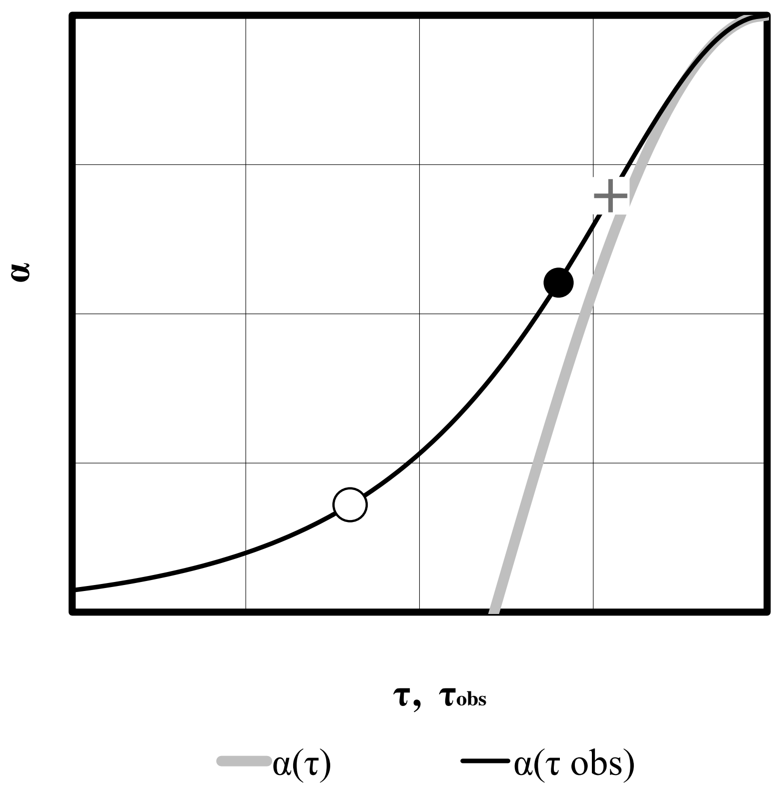

Fig. 1. Dynamics of the expansion of the closed Universe with consideration of the dependency of the scale factor from time ($g_{00}= \gamma^{-2}$) in different coordinates. Time is expressed in relative units $\tau \equiv (c/2a_0)t$ ans is counted from the moment of the maximal expansion (α =1). On the curvature $\alpha(\tau_{obs})$ we mark the positions of the current condition of the Universe at Ω=0,03 (light circle) and at Ω=0,4 (dark circle). The point of bend is marked with the cross.

“Axis of Evil” and the central symmetry of the microwave background

The recently discovered symmetry of the microwave background inhomogeneities is interpreted in the standard model as a possible evidence of the violation of the fundamental space isotropy requirement (“Axis of Evil”[14], but in the modified model it is interpreted as a natural phenomenon resulting from slower development dynamics and not related to the violation of the relativity theory principles. The time passed from the moment of the signal radiation by the distant source, for instance, an inhomogeneity of the matter density at the period of hydrogen recombination, appears to be sufficient for the source radiation, spreading in all the directions, to reach the observer not only by the small but also by the big arc of the big circle of the closed Universe, i.e. from the opposite side, which is perceived by the observer as the phenomenon of the central symmetry – a possibility to see the same source in two opposite (centrally symmetrical) points of the celestial sphere.

Let us study the light radiated by the inhomogeneity located in some point of the hypersphere at the period of recombination. If in some point of the celestial sphere we see an inhomogeneity corresponding to the increase of the microwave background temperature (velocity of the matter that radiated it is directed to us – it approaches by this arc), we simultaneously see the same inhomogeneity in the opposite point of the celestial sphere (the signal that came by another arc of the big circle, i.e. “from the back”), but this time corresponding to the decrease of the microwave background temperature (as it moves away by this arc). This is a central symmetry with the negative sign – “antisymmetry”. But at the same time we can assume the existence of the central symmetry with the positive sign that can have place for the inhomogeneities whose reason was not the movement of the matter but the inhomogeneity of its density.

In the work [15] it is shown that both types of the central symmetry take place, positive and negative, and the contribution of the negative component is higher which is confirmed by the results of the visual and numeric analysis of the microwave background distribution. The calculated value of the central symmetry coefficient averaged by all the celestial sphere is $4\pm1\%$.

Manifestation of the central symmetry in the distribution of the quasars

The phenomenon of the central symmetry of the celestial sphere is also proved by the existence of the centrally symmetrical pairs of quasars with the identical luminosity magnitude profiles in the ranges (u,g,r,i,z) [16], that can be interpreted as pairs of opposite images of the same distant object.

The check of the dependency of the quantity of pairs of opposite quasars with high degree of correlation of the luminosity profiles on the central symmetry breaking has shown that the percentage of the pairs with Pearson correlation coefficients $R_{xy}>0.98$ for the opposite quasars is significantly higher than for the randomly composed pairs of quasars. The obtained data was compared to the results of the statistical analysis. It was shown that the probability of the formation of the discovered peak due to the random deviation of the quantity of pairs from the average values does not exceed 10^{-9}, which excludes the probability of their random nature and confirms the existence of the pairs of opposite quasars with identical characteristics of luminosity which can be the pairs of images of one object (see Fig. 2.) [16].

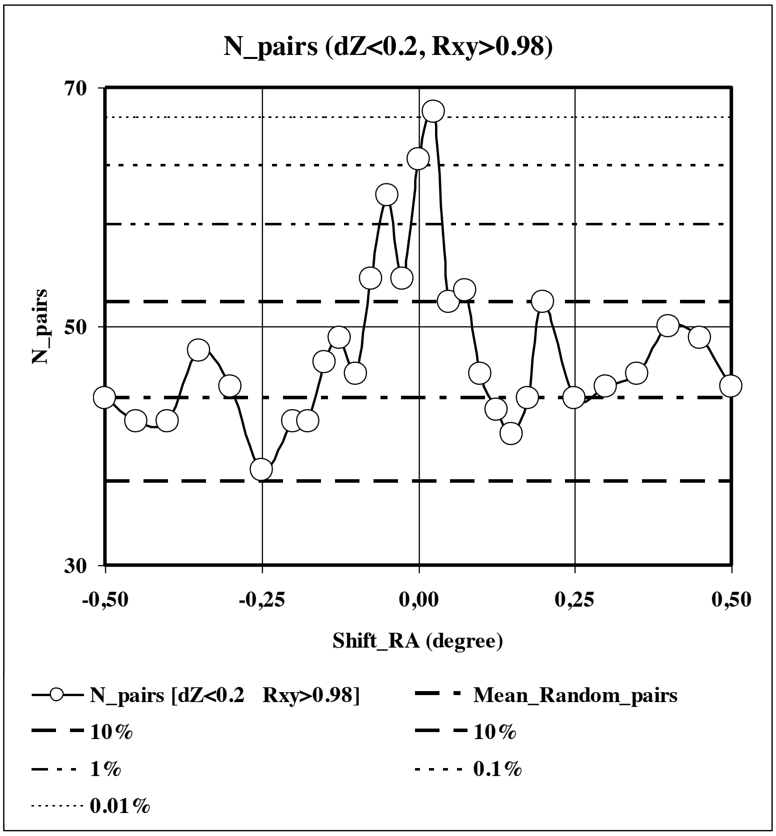

Fig. 2. Dependency on Shift_RA of the quantity of pairs of opposite quasars, corresponding to the criterion (dZ<0,2;Rxy>0,98). The quantities of pairs corresponding to the average volume of the selection (N≈210) and probabilities of the correspondence to the given criterion (p = 0,211) for the body of randomly generated pairs are shown with the horizontal lines. Dot-dash line – average quantity for the selection of given volume, dotted line – probabilities of the random deviation from the average.

Dependency of the magnitudes and luminosities of the objects on the redshift

In the work [17] we have studied the dependencies on the redshift of the averaged by all observation angles stellar magnitudes of the galaxies according to the 2DFGRS (2003) [18] , Millenium (2003) [19], Gyulzadian (2011) [20] and the quasars according to the catalogs SDSS DR-5 (2007) [21] and SDSS DR-7 (2010) [22] (105 thousand quasars and 175 thousand galaxies).

The results of the analysis of the average magnitude values show that for all the distant objects, both quasars and galaxies, the dependencies of the average magnitude values on Z are close by absolute values and form, and have a converging oscillations nature with negative slope areas near $Z \approx 1$ and $Z \approx 2$, corresponding to the increase of the apparent luminosity of the objects with the increase of the distance.

The results of the calculation of the average luminosities of the objects for the flat space and different variants of the ΛCDM model show that the average luminosity powers of the galaxies and quasars, averaged by all the observed areas of the celestial sphere, at calculation both for the flat (by method [1]), and for the ΛCDM model (by method [23]) drastically increase with the increase of the redshift. Increase of the average galaxies power at increase of Z from 0.02 till 2.0 reaches 7 thousand times, which appears to be impossible and leads us to the assumption that these results are related not to the real change of the luminosity power of the galaxies and quasars but to the specifics of the used calculation methods and fundamental to them specifics of the standard cosmological model metric (1).

In the modified model built on the metric (5), considering the non-zero value of the scale factor differential in the expanding Universe, it is possible that the regularities absent in the standard model would appear, including the effect of the increase of the apparent luminosity of the sources with the increase of the distance. The performed calculation of the dependencies of the apparent magnitudes of the distant radiating objects on the redshift allows obtaining the theoretical dependency mag(Z), practically coinciding with the dependencies on the redshift of the average magnitudes of the quasars and galaxies, obtained by the results of the observation data analysis for the object of constant luminosity L = const(Z), see Fig. 3.

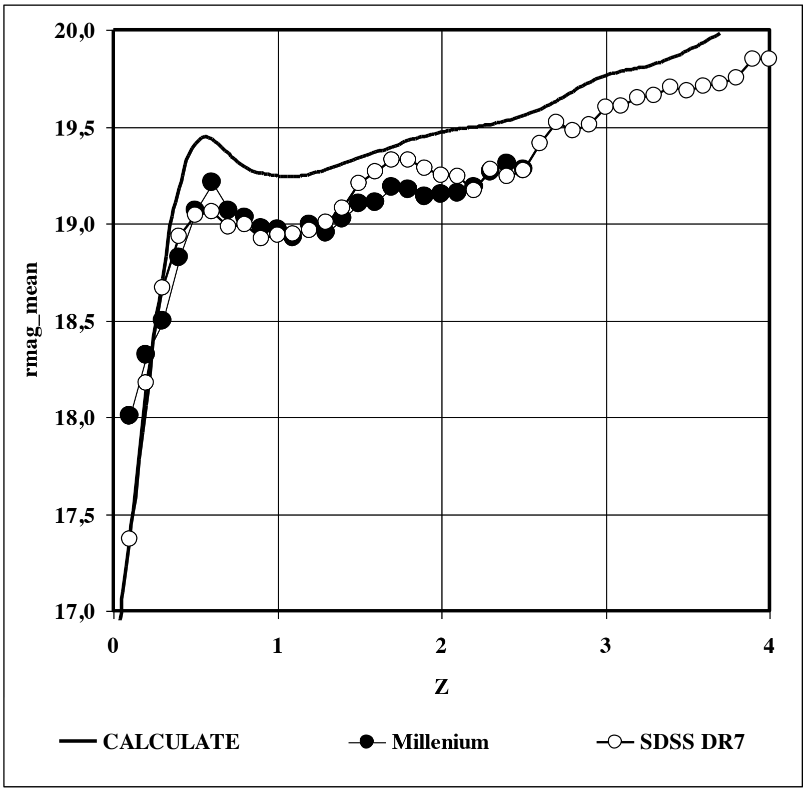

Fig. 3. Comparison of the calculated curve mag (Z) with the dependencies Rmag_mean(Z) for the galaxies from the catalog Millenium (2003) and quasars from the catalog SDSS DR-7 (2010). Shown curve calculated for L = const(Z) ≈ $3*10^{10}$Lsun.

Conclusion

The truth of the results of the elementary calculation and the simple analysis of the publically available observation data brought in this work is equivalent, as we suppose, to the proof of absolute inconsistency of the Big Bang model and the necessity of its revision beginning from the source of contradiction in its basis. This primary source, in our opinion, is the metric tensor of the stationary Universe which is for some unknown reason put in the basis of the non-stationary Universe.

That is why to resolve the matter of consistency or inconsistency of the standard cosmological model which has the increasing scientific and economical value, we suggest answering the following simple questions:

- Is it logical to build a model of the non-stationary Universe on the metric revealed by Einstein for the stationary Universe with the non-zero value of the scale factor differential?

- Is this metric obtained without consideration of the non-zero value of the scale factor differential corresponds to the absence of the energy transfer between the matter and the gravitational field and the impossibility of the Universe development in time?

- Is the energy conservation law of the General Theory of Relativity realized with the transition to the metric obtained with the consideration of the non-zero value of the scale factor differential?

- Does the phenomenon of the central symmetry of the celestial sphere manifesting in the central symmetry of the microwave background and the central symmetry of the quasars exist?

- Are there intervals Z, in which the distance to the object corresponds to an increase of the visible luminosity?

- Whether in the standard cosmological model, an increase in the redshifts of galaxies and quasars corresponds to an increase in average output power of hundreds and thousands of times?

- Is the transition to the model with the slower development dynamics can lead to the appearance of the regularities that are absent in the standard model, including the effect of the increase of the apparent luminosity with the increase of the distance?

- Is it possible in the model with the slower development dynamics to obtain the theoretical dependency mag(Z), close to the dependencies on the redshift of the galaxies and quasars average magnitudes obtained by the results of the observation data analysis?

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The Everett's Axiom of Parallelism

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Abstract: In this work we consider the meaningfulness of the concept "parallel worlds". To that extent we propose the model of the infinite-dimensionaly multievent space, generating everettics altervers in each point of Minkowski's space time. Our research reveals fractal character of such alterverse. It was also found that in Minkowski's space $\{x, ict\}$ the past actively influences the present, whereas the future is a conservative factor – it slows down already occuring processes and interferes with actualization of the latent ones. Fast fusions formation is predicted based on modeling of fractal dynamics of. It was also found that the alterverse branches grow in non-Markov fashion; some of this feature are discussed. The concept "fractal parallelism according to Everett" is proposed. Inevitable inaccuracy of the model is also discussed.

Context of interpretations of quantum mechanics is pluralistic, as a result, notoriously abundant, but unsuccessful attempts to find the one and only "true" interpretation seem to have led by now to realization that this effort is as utopian as perpetuum mobile.
Plurality of interpretations of quantum mechanics is as inevitable as the strangeness of the world that quantum mechanics discovered (or created).

V.I. Arschinov [1]

Among the dozens of interpretations of quantum mechanics seriously discussed by physicists and philosophers in recent years, two are the most significant and drawing most attention: Copenhagen interpretation and the many-worlds one. In philosophy the many-worlds interpretation is presented the form of everettic: axiomatic ideological construction, whose axioms include the most important point of the many-worlds interpretation, specifically, branching of the wave function during the interaction process [2, 3, 4].

Concepts of the many-world (everettic, as we call it hereafter) branching and fusions are basic axiomatic concepts of everettics [2]. However, Hugh Everett's paper [5] does not detail the mechanism of branching, which certainly strengthened the concept of "parallel worlds" along with the respective term. That is particularity true for popular presentations of many-worlds interpretation of quantum mechanics.

"Geometric" understanding of the "parallel worlds" concept has in its core a statement about "disjointness" of alterverse[1] branches. The concept has in its basis the passage from Everett's work: " This total lack of effect of one branch on another also implies that no observer

[1] Alterverse is a set of classical realities of the physical world (CRPW), reflecting the state of the single quantum reality (SQR). The alterverse is structured in the branches as specific CRPW that are relative states of Mensky's crystal faces and consciousness of the observer. The term reflects the fact that different "Everett worlds" are different alternative "projections" of the quantum world (SQR) on the memory of the observer. The term was proposed by Mensky [7].

will ever be aware of any "splitting" process " [5]. As a result of interpretation of the concept of the branch in terms of epistemological optimism, everettics put forward the idea of branch fusions [6, pp. 106-107] and the postulate of "disjointness" was replaced by another one: "Axiom of everettical fusions", which proclaims the inevitable interaction between the alterverse's branches [2, p.56].

Additionally, everettics postulates Fifth axiom about metasystem of the universes. This axiom reflects the current most common conception of the structure of being: "Being as a whole, is a Godel's fractal metasystem of universes and their inhabitants" [2, p. 56].

The present work is an attempt to specify the manifestations of everettic axioms based on fractal model of the mechanism of the everettic branches formation.

Let us consider a structure of alterverse of an object A in the Minkowski event space. The question of the general physical interpretation of the event for object "A" is separate everettic issue that requires special attention.

For the purpose of this work it is important to consider an event which has universal character and clear physical meaning. In that regard, the event should be generated by the environment that is present at any point in Minkowski space-time. Physical vacuum is a logical choice in this case. From a philosophical point of view, we can consider any other model of "the aether " in its Einstein's interpretation as a filling of the void [8]. However, the model of the physical vacuum is preferred because inevitable quantum fluctuations of the physical fields in this environment play an important role in explaining some of the fundamental phenomena, not only the "exotic" ones(chaotic inflation by Linde, Hawking radiation, the Lamb shift, van der Waals forces, etc.), but "every day life" ones as well(spontaneous emission of excited atoms).

Thus, we assume that the object A is a light bulb, which is located in the cabin of a spaceship, and the event is a "flash of light" produced by this bulb. We will leave aside the technical details of the observation of this event, as well as feasibility of this observation.

Let us also assume that the spaceship can move at any sub-light speed. This means that the light in the cabin is to be located in any point of future light cone of Minkowski's event space of the ship.

It is known that the each event of the photon emission by the filament of an incandescent bulb is due to fluctuation of the electromagnetic vacuum. (In the absence of such fluctuations, the excited state of the atom would be stable, and the bulb would not emit light.)

The substantiation of this event at the given point of Minkowski space is determined by the presence of a set of excited atoms (filament) and a random value of the energy of vacuum fluctuations of the electromagnetic field at that point. Point $\{x_1, y_1, z_1, ict_1\}$ where the event 1 occurred in alterverse of the object A is a branching point: the object A goes into a state that can produce flashes of light in some other points k $\{x_k, y_k, z_k, ic(t_1 + \Delta t_k)\}$. Coordinates x_k, y_k, z_k depend on the specific route chosen by the crew, or other reasons influencing the speed and direction of the lamp location point, and the coordinate $t_1 + \Delta t_k$ depends on an arbitrarily chosen interval Δt_k and random vacuum fluctuations at $\{x_k, y_k, z_k, ic(t_1 + \Delta t_k)\}$. If the intensity of the fluctuations at this point is below a certain threshold, the flash of light does not occur. Therefore, the event k only occurs at certain points of Minkowski space. The points $\{x_k, y_k, z_k, ic(t_1 + \Delta t_k)\}$ at which event k may occur we will call active branching poins.

The axiom of everettic branchings dictates that cross section of the space-time structure of the object A alterverse by isotemporal surface $ic(t_1 + \Delta t_{1k})$ should contain the active branching points.

Not reducing generality of the model, we extend the analysis to the case of two dimensional Minkowski space $\{x, ict\}$ (Fig. 1).

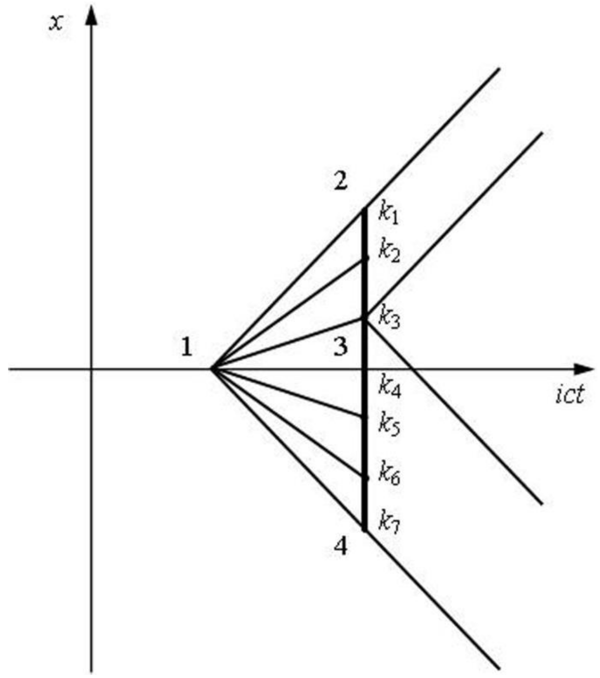

Fig.1. The object A alterverse in two dimensional Minkovsky space.

Fig. 1 showes *l* and *k* events in two dimensional Minkowski space. The bulb located inside A is on at point 1. After that, the object can move along different trajectories $1 \to k_1$, $1 \to k_2$, ..., $1 \to k_7$ during time interval Δt_k, which corresponds to change of coordinate *ict* by segment (1-3). Fig. 1 presents the case where each specific alterverse branch (direction and speed of object A movement from point 1) is chosen by the ship crew or is a result of deterministic laws of mechanics. Thus, the shown structure of alterverse branches is a macroscopic deterministic part of its overall structure, and does not reflect the branches arising from the quantum fluctuations of the electromagnetic vacuum.

Rays (1-2) and (1-4) limit the light cone of event 1. Isotemporal surface of the section of alterverse represented by the segment (2-4). The points k_1,..., k_7 can potentially contain "event of flash". For clarity, it is assumed that this happened at point k_3, which in this case is the active branching point. This is reflected by the construction of the light cone of the event k_3.

Let us consider a region of space-time to the right of the surface (2-4), i.e. future of the elements of the surface. In the viscinity of k_3 we select a thin layer with thickness Δict, adjacent to the isotemporal surface with t_0 coordinate. Obviously, on the segment of isotemporal secant (2-4) in the viscinity of k_3 there will be other points in which the fluctuations of the electromagnetic vacuum are intense enough to cause a flash of light. We denote them as $k_{3i}(i = a, в, c...)$. It is also obvious that these points are randomly distributed on the the segment (2-4).

We split the layer (Δict) in squares with a side of the axis of time $(\Delta ict)_j$ equivalent to the threshold fluctuation energy causing the flash (calculated from the uncertainty relation for energy and time), and the side along the spatial axis *X* equal to the linear size of the fluctuation (Fig. 2).

We now need to answer the question whether the structure of fluctuations (distribution of fluctuation energies in networks of cell built on the segment (Δict) in the chosen field of the future of A object) is static or dynamic?

If considered a Minkowski space $\{x, ict\}$ was purely geometric, like the Euclidean (or any other metric space, which metric does not have time), the answer would be unambiguous: the parameters of fluctuations must be static.

However, the event spaces feature some properties fundamentally different from those of geometrical spaces.

Note that using the mathematical methods of the event spaces one usually does not discus or acknoledge presence of the External Observer associated with these spaces. This metaphysical object arises in everettics when analyzing the very statement of the problem of describing the universe as an isolated system. "The need for such a *special* External Observer logically inevitable, and results from the text of the Everett's article, the author and the audience who consider Everett 'isolated system' from outside are such observers" [9, p 64].

Presence of External Observer is even more evident in the event space - supratemporal analysis of mathematical and physical properties and phenomena of event spaces with *temporal* coordinate is performed from his perspective.

However, External Observer, always present in the description of the realities of event space, is not introduced into this model from outside, although it is in line with the Amakko principle: "For the sake of completeness one must multiply as much as possible the substances logically compatible with the fact considered " [10]. Here authors just highlight the presence of an External Observer in *all* models of event space, including the Minkowski space-time. The only Amakko property, which we assign to the External Observer in our model, is its ability to capture the locations of flashes of light and to store in memory the their time sequence.

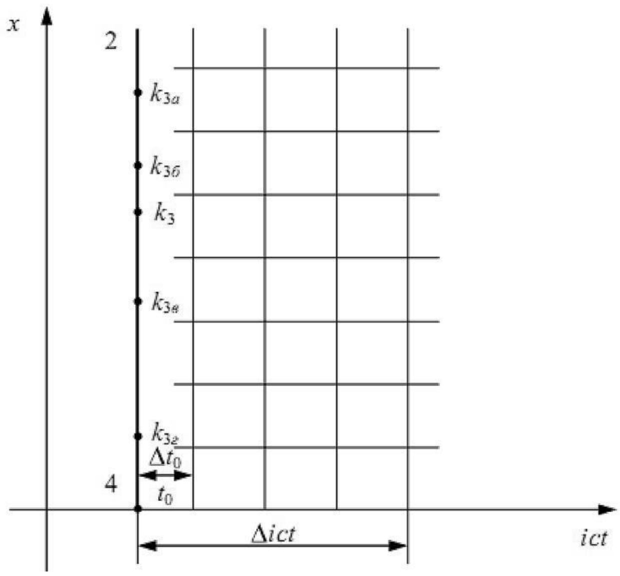

Fig 2. The area near future of A object.

One also needs to take into account the properties of the coordinates, specifically time coordinate: in the Minkowski event space it is physically impossible to capture a point t_0. The concept of "a moment of zero duration" (i.e. "time point") does not exist. Temporal point is defined with a precision Δt_0, and the its value depends on the accuracy of measurement of the energy of the event, in accordance with the Heisenberg uncertainty principle . Wallace defined the essence of temporal coordinate in event spaces as follows: "We may speak of 'moments of time' and the number of moments of time ('the next moment', etc.) but this is just a metaphor for temporal duration, and cannot be interpreted literally. " [11]. Therefore the discretization performed earlier uses a lattice, where the parameters of vacuum fluctuations in each cell are random variables, determined by the physical properties of the vacuum in the area of the partition.

The specific value of this parameter is determined by the "here-and-now-for-me" principle. In other words, the values of fluctuation parameters at each $\{x, ict\}$ of event space will be different for different observers, or for the different calls to this point made by the same

External Observer. This property of event space can be described phenomenologically by the notion of "intrinsic time τ" at each point of event space. Mathematically, this is equivalent to introduction of one more dimension at each point $\{x, ict\}$, orthogonal to both x and ict.

This dimension should have characteristic of the time (in this case, the most important characteristic is fluidity) and have dimensionality of $ic_\tau\tau_\tau$. We leave aside the issue of the the value of the constant. Thus formed space $\{x, ict, ic_\tau\tau_\tau\}$ is infinite multi-event space, and its corresponding section $\{x, ic_\tau\tau_\tau\}$ at $ict = const$ corresponds to everettic alterverse of events at k_3. This approach, as opposed to approach of External Observer, is a direct consequence of the Amacco principle applied to this system. Moreover, in this case the Amacco principle is used in its strictest form - the model considered has an infinite number of new entities.

The space is essentially a universal state of object A space. According to Wallace: " We are undoubtedly more at home with Minkowski spacetime than with the universal state. Partly this may be because we have worked with the concept in physics for rather longer, but more importantly we have long been used to the idea that multiple times exist (in some sense) — the innovation in relativity theory is the unification of these instants into a whole, and the identification of the instants as secondary concepts. Everett asks us to take both steps at once: to accept that there exist many worlds, and then to fuse them together into a whole and accept that the worlds are only secondary. ". [11]

An important feature of the space $\{x, ict, ic_\tau\tau_\tau\}$ is the fact that there is no single point of "origin" - each event has its alterverse, i.e. $ic_\tau\tau_\tau$ axis occurs at each point of axis ict.

Introduction of the alterversal space $\{x, ic_\tau\tau_\tau\}$ allows us to move on with alterverse of the flash of light on the object A in the viscinity of k_3 in the Minkowski event space.

To proceed further, It is important to understand a certain feature of uncertainty relation for energy and time:

$\Delta E\Delta t \geq \hbar$

Applying this relationship to the point t_0 (Fig. 2), one can see two potential outcomes of the energy fluctuation :

First: $\Delta t > 0$, and $\Delta E > 0$. That means that at $\Delta ic(t_0 + \Delta t_{10})$ (i.e. in the future of the point t_0), the energy of the lattice element to the right of t_0 is greater that the energy of t_0. Based on the principle of local energy conservation this fluctuation means less energy in present time and increase it in the future.

Second: $\Delta t < 0$, and $\Delta E < 0$. This means that at $\Delta ic(t_0 - \Delta t_{10})$ (i.e. in the past of the point t_0), the energy of the lattice element to the left of point t_0 is less than the enrgy of t_0. Based on the principle of local energy conservation this fluctuation means more energy in the present time and decrease in the past.

We now see that in the space $\{x, ict\}$ only the past actievely influences the present (adding energy stimulates actualization of latent processes), while the future conservatively influences the present (energy decrease slows down already occuring processes and hinders actualization of the latent ones).

However, External Observer in the space $\{x, ict, ic_\tau\tau_\tau\}$ will see it differently. In a supratemporal plane $\{x, ict\}$ selected by the External Observer in the absence of object A, fluctuation of energy in every cell of the lattice will randomly vary over time τ_j in alterverse spaces $\{x, ic_\tau(\tau_j)\tau_j\}$. External Observer will therefore capture a picture of the cells that contain the energy necessary for flash of light at the point k_3, which will correspond to the equilibrium Brownian motion of points (cells with threshold energy sufficient for the flash) on the part of the plane $\{x, ict\}$ within the light cone of point k_3. Fig. 3 shows possible displacements of one of the observed elements of the "apparent perturbation" along the mesh of elements of alterverse spaces.

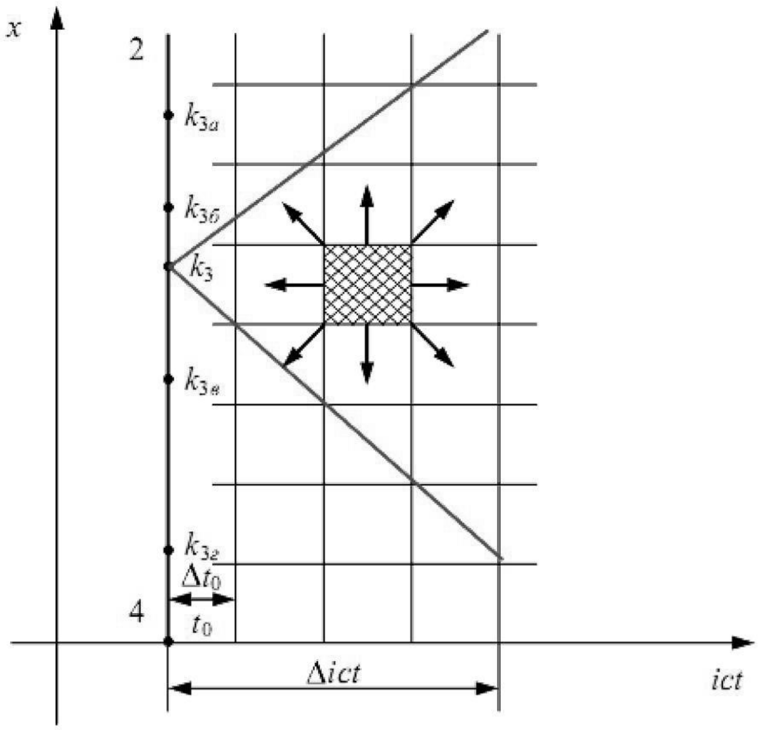

Fig. 3. Displacements of "apparent perturbation" in network of alterverse cells.

When the A object appears on line (2-4), the physical conditions of that line change: a "scavenger fluctuations" arises at the point k_3 – an excited atom in the filament of lamp. A similar pattern can be observed for all points k_{3i}.

In this case Brownian motion of the points of the "effective disturbance" will transform, according to Le Chatelier-Brown principle, into the diffusive motion towards excited atoms.

Considering that a real flash in physical space (which in this case is represented by a plane $\{x, ict\}$) occurs during a finite period of time Δt_0, "flowing" along the axis *ict*, and can occur in any cell adjacent to the cell containing the point k_3, one can see that the sequence of flashes (alterverse event at point k_3) will look to External Observer as a dendrite growing from point k_3. As shown in [12], the type of fractal of branching in this case will depend on the conditions of the structure formation.

To describe this process, we applied the model of a random fractal developed by A. Dulfan in his work "The random fractal with a given preferential direction of growth" [13]. The model is based on the Witten-Sander method.

The method is based on a concept of a fluctuation randomly occuring in the lattice And then stochastically moving up until it "encounters" the element, which the External Observer captures. The algorithm of the simulation is detailed in [12].

Numerical simulation performed in [12], which we interpret in terms of our alterverse model, shows the pattern of growth of fractals of alterverse branching of events at k_3 , occuring at various local conditions for the origin and motion of fluctuations.

Fig. 4 showes a graphical representation of the simulation results for a single "active spot".

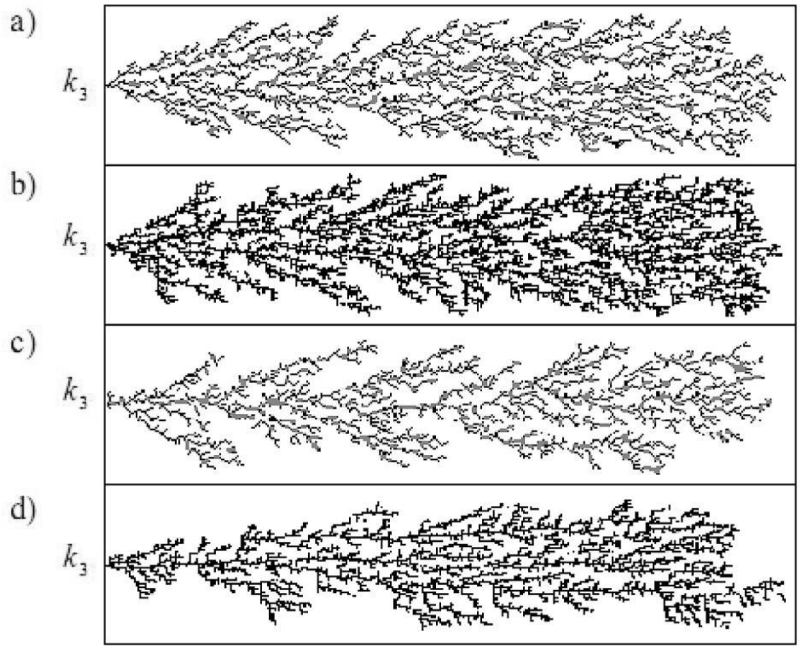

Fig. 4. Fractal growth of the various points in the diffusion mode.

The monostructures of alterverse resulting during the generation of fluctuations at various degree of isotropy are shown in Fig. 4:

a) isotropic situation: movement of fluctuations is only possible in the horizontal and vertical directions, "fluctuations absorbtion" occurs on the same lines,

b) partially isotropic situation 1: fluctuations only move in the horizontal and vertical directions, and absorption is possible in both of these and the diagonal direction as well,

c) partially isotropic situation 2: fluctuations can move not only in the horizontal and vertical directions, but also along the diagonals; absorption is possible only in the horizontal and vertical directions,

d) anisotropic situation: fluctuations move horizontally, vertically and diagonally; absorption of fluctuations takes place in all these directions as well.

Fig. 4 demonstrates that the alterverse fractal depends only weakly on the diffusion and steric factors (direction of interaction between the excited atom and fluctuation), which reveals the stability of the model in the presence of heterogeneity of local conditions.

This us gives reason to consider growth of alterverse from several points k_{3i}.

Fig. 5 shows four of the 9000 sequential steps of the modeling (Witten-Sander method, at the increment Δt_0) of the location of the "apparent fluctuation". Note that the term "dynamics" in our alterverse model has a specific meaning. The pattern of events represented by dots in Fig. 5, is not directly related to the dynamics of flashes in the event space $\{x, ict\}$. Rather, it is the "road map" of one of the layers of space $\{x, ict, ic_\tau \tau_j\}$, captured by External Observer.

Its physical meaning is that it predicts flashes of light at certain points of the segment (2-4) at $n\Delta t_0$ time intervals in event space $\{x, ict\}$ under the condition of frozen times τ_j (Isochronous section of space $ic\tau_\tau$). (Fig. 6)

An obvious feature of this fractal structure is the large number of alterverse branches intersections, considered to be realities fusions s in everettics.

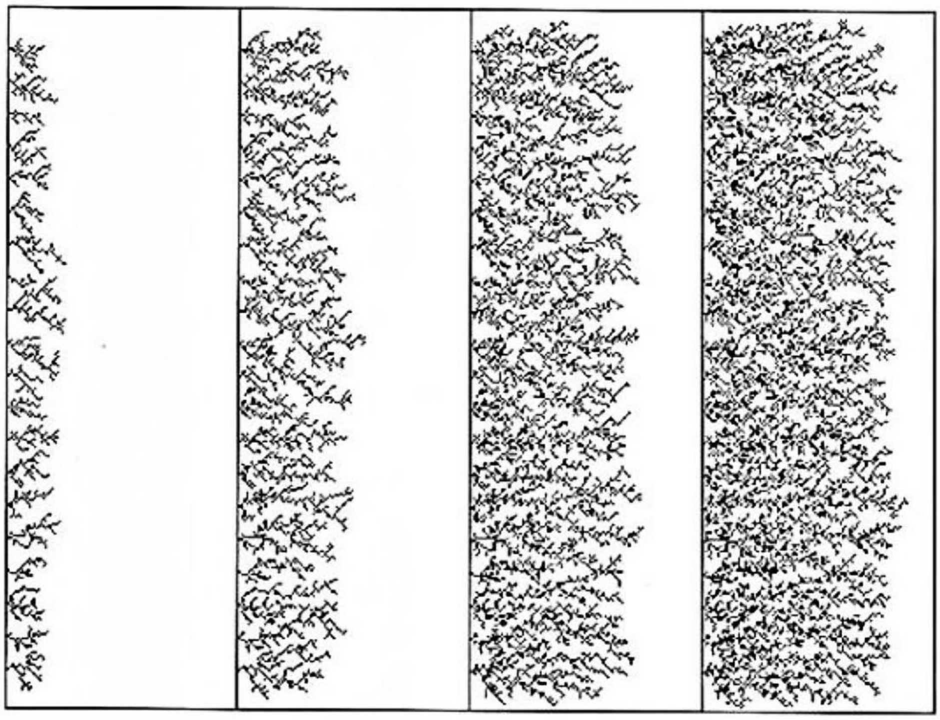

Fig. 5. Dynamics of the alterverse branching.

An important detail emerged from consideration of a detailed modeling of the alterverse evolution is the fact that branch fusions occure already at a relatively small number of steps. Thus, in Fig. 6 alterverse branches of k_{3b} and k_3 intersect at step 19, and the alterverse branches of k_{3c} and k_3 intersect at step 13.

This same pattern was found for branches of any *n's* section of alterverse if n is sufficinly large.

One can guess that this feature is characteristic of most other fractal models of everettic branches.

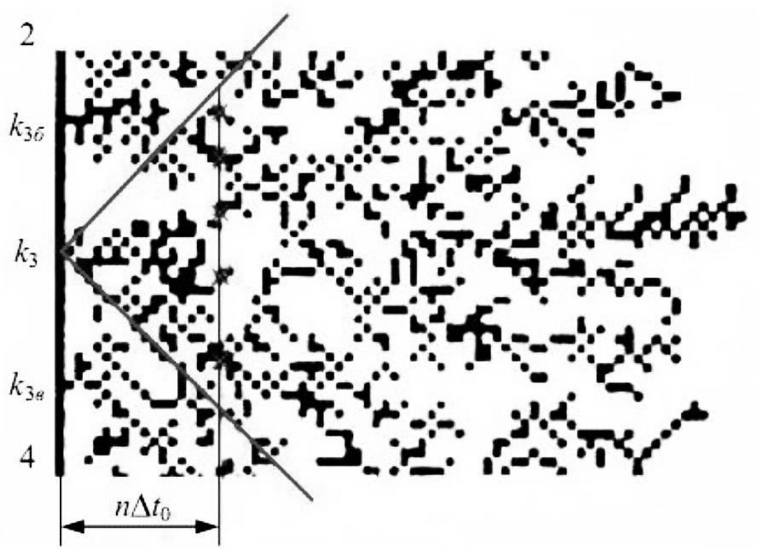

Fig. 6. Spots of light flashes (red crosses) on the surface (2-4) within the light cone of the point k_3 at intervals $n\Delta t_0$ in space $\{x, ict\}$.

Complete the "road map" for a future of k_3 should be in the n-dimentional space and be a dynamic object in each of n times τ_j of alterversal spaces $\{x, ic_\tau(\tau_j)\tau_j\}$. Moreover, analysis of Heisenberg-Bohr's uncertainty principle $\Delta E \Delta t \geq \hbar$ revealed that k_3 and the object A should have a similar structure of their "road map" of the past.

Since in alterverses of the times τ_j both "road maps" are dynamic objects that have a common point, there is no reason to dismiss their interaction and mutual influence. Moreover, for the half cone of the future, the fractal considered "is essentially a non-Markovian and therefore it is very difficult to study analytically " [12]. This means that not only deterministic, but also random events in space $\{x, ict\}$ depend on the evolution of the system as a whole. (In our case, the former are the ones of object A appearing at points k_i(Figure 1), which are due to the decisions of the crew, and the later are the random events of the flashes of light at k_{3i} in Figures 2-5).

Non-Markovian character of the evolution of alterverse branches allows us to resolve persistent questions regarding the description of the features of certain quantum paradoxes. For instance, the following problem is posed by the famous paradox of Schrödinger cat.

In a closed box Schrodinger cat exists in a superposition of its possible states. Let us assume that , after the is opened box, we find a live cat. This would mean that a dead cat was in the other multiverse branch. Close the box again, and wait for a while, then open the box. Suppose that we again see the cat alive. So, there arises another alterverse branch with a dead cat. Let us now repeat this procedure until we finally find a dead cat. Now we are in a branch of a dead cat, and the number of such branches is N. With reagrd to the cat all these branches are the same - cat is dead in all of them.

What is different in each branch is the external event: in one branch the technician caught a flue, in another he had dinner and so on. However, it is never mentioned in the procedure description, and the fate of the obsever during the experiment is normally omitted from consideration. Non-Markovian nature of everettics branches predicts that the presence the information about the death of a cat in the memory of the observer limits his subsequent behavior and, therefore, structures his future. For instance, in those alterverse branches where a cat died, technician will never come to the experimental box with a bowl of milk. It is however very likely in the branches, where cat was alive in the preceding opening of the box.

This means that the entropy of the future of non-Markov processes in the alterverse (processes, depending on the history and memory of the observer) is always less than the entropy of the future of Markov processes that are independent of history. For a more detailed discussion of the entropy in the alterverse evolution an improved algorithm of fractal simulation is needed, one accounting for the memory of External Observer.

Due to its symmetry, fractal of alterverse past for the object A and for the point k_3 is non-Markov, and the entire space $\{x, ict\}$ is "historically conditioned" regardless of the origin and the direction of the axis ict.

Obviously, the scales of the axes X and ict on Figs. 1 and 2-6 differ by many tens of orders. Moreover, the volumes of the event spaces of the ship (object A) and nano-sized element of the lamp filament in its cabin, containing the point k_3 (hundreds of orders for the four dimensional Minkowski spacetime) differ as well.

Once we realize that is practically impossible to build a "road map" of alterverse of past and future for the point k_3 at the current computational level, calculations of these cards for both micro-and macro-objects becomes seemingly hopeless.

However, the many-worlds interpretation of quantum mechanics, being a part of the ideological foundation for quantum computers, may obtain a tool for quantification of alterversal spaces as a result of the development of such computers.

The proposed model offers a new perspective of the "parallel worlds". The key property of fractal is scale invariance or, in other words, a complete $\{x, ict\}$ self-similarity of the geometrical descriptions of the fractal process. Fractal in event spaces in our model adequately describes the physical processes of galactic to the atomic scale, and it can be perceived as a kind of "parallelism". However, this parallelism is not linear, as in Euclid geometry, but fractal. Note that there is not the term "parallel" in fifth Euclid postulate: “5. That, if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the two straight lines, if produced indefinitely, meet on that side on which are the angles less than the two right angles.” [14]. The property of the lines described by Euclid is only geometric meaning of the term in its current understanding. Currently, the term "parallel" is defined as "the same, a comparable" [15, page 516], which is very close to the meaning of the term "fractal".

From this perspective, fifth everettic axiom, as well as Euclid's fifth postulate, may be regarded as "an axiom of parallelism." As such, it deserves the name of the Everett’s axiom of parallelism.

The meaning of the Everett’s remark, cited at the beginning of this article, does not imply the absence of the splitting process. Knowledge about branching is a characteristic of the observer, not the process.

Everett’s remark was sagacious in the sencese that observer taking part in the process of branching (such as those associated with the point k_3 in Fig. 6), loses its primary identity in a few "steps of branching" (in our example, 13 and 19 steps) and becomes the new "mixed observer" $k_{3c} - k_3$, or $k_{3b} - k_3$ mixing and loosing his initial identity progressievely. Therefore, "the initial observer" in fact ceases to exist after the first fusions and "does not know about any process of "splitting "".

The fractal nature of the Everettic "parallelism" reconciles us with the common term "parallel worlds", assuming generilized interpretation of parallelism.

To describe the everettic branching of alterverse in event space, one can use its dimension α. In this case the "branching factor" is one temporal coordinate, therefore events with only one outcome will be characterized by an integer dimension equal to unity. The presence of branching increases the value of α proportionally to the density of the branches in the event space. This density limit (if there is branching at every point) would be the value $\alpha = 2$. Thus, everettic branching in the two dimensional event-space should be in the range $1 \leq \alpha \leq 2$.

Clearly, in the n-dimensional space the relation is $1 \leq \alpha \leq n$. In the case of $\alpha = n$ branching occurs at every point of event space and alterverse takes up all cells in Fig. 2 (in the model case in Fig. 5, 6, $\alpha = 1{,}3$).

For a continuous space the equality $\alpha = n$ means infinite number of branches and the density of the number of branches. Thus, the considered fractal model confirms utility of the modeling of Minkowski space-time by discrete networks such as Fig. 2.

It can be assumed that the fractal dimension of time keeps the information, determining the hierarchical structure of event space.

In conclusion we would like to note , that the of spontaneous radiation in space $\{x, ict, ic_\tau\tau_\tau\}$ is not quite correct example, as "point of the flash" are not captured in the cross-section $\{x, ict\}$. This capturing is only possible in the supratemporal consciousness of External Observer. The authors are aware that the granting of an External Observer the ability of such capturing may be a mistake.

Moreover, the magnitude of the interval Δt_0 in the model should be of the order of Planck time (~ 10^{-43} s) in order to ensure the applicability of the assumed model of diffusive motion. For larger intervals relativistic limitations on the vertical movement of fluctuation will take effect.

Therefore, our model is only a first approximation of fractal description of alterverse with its inevitable coarsening and inaccuracies. We hope that its further development will identify the " diffusion modes" in which conditions of Conway-Cohen theorem are satisfied[16].

However, the authors firmly believe that the "trial and error, the usual method of investigations in science, requires the consideration of all kinds of ideas, of which only one will be correct and will remain for the future" [17].

Trial and error method can be likened to a collapse of the wave function. Any scientific research (and not only scientific) is similar to quantum event, which can develop in many ways, but at the time when we observe it we see only one option, and all the other solutions of the wave equation collapse (in the Copenhagen interpretation).

However, many-worlds interpretation of scientific research is also possible: all of our research leads to the goal, but each goal is achieved in its own universe, where the laws of physics correspond precisely to such a solution. In this case the trial and error method is akin to a particular solution of "the wave equation of knowledge", randomly selected from the whole set of solutions, because the trial and error method does not investigate all possible options.

Zwicky morphological method [18, 19] allows us to consider all possible options for research, that is, by analogy, comparable to a full solution of the wave equation. Then "Zwicky morphological box" is similar to everettic multiverse: all cells of this "box" correspond to solutions of a certain problem, but every decision is executed in its own universe. Our universe corresponds to one of the cells of the morphological box.

High dimensionality of the morphological box precludes practical applications of the Zwicky method to real everettical problems. "A box composed by Zwicky to predict only one type of rocket engines, had - with 11 axes - 36,864 combinations!.." [20, p. 53]. But, as noted earlier with respect to the calculation of the "road maps of alterverse", everettics itself can evolve into a tool for the quantitative description of highly complex tasks.

The axiom of parallelism for Everett is one of the steps of this development.

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PHOTOMETRIC SCALE OF COSMOLOGICAL DISTANCES: ANISOTROPY AND NONLINEARITY, ISOTROPY AND ZERO-POINT

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1. Introduction.

Distance measurements in observational cosmology are important in establishing the cosmologic distance scale of the universe. Methods of measuring distances to remote objects are based on calibration of the techniques developed for larger distances by the reliable data obtained from other methods that work at close distances, like parallax and Standard Candles. In 1929, Edwin Hubble was able to combine his own distance measurements to relatively close galaxies with Vesto Slipher's measurements and discovered a correlation between the red shifts for these objects and well established distances to them: $D_L(z) = c{\cdot}z/H_0$, where z is the red shift, c – the speed of light in vacuum, H_0 – the Hubble constant, $c{\cdot}z$ is the redshift speed which is combination of c and the speed due to the cosmological expansion of space.

In 1958 Wolfgang Mattig derived one of the most important equations in observational cosmology which provides the relationship between the luminosity distance D_L and the redshift z:

$$D_L(z) = [c/(H_0 \cdot q_0^2)] \cdot [q_0 \cdot z + (q_0 - 1) \cdot (\sqrt{2q_0 \cdot z + 1} - 1)], \qquad (1.1)$$

where q_0 is deceleration parameter. For $q_0 = 1$ it reduces to Hubble's law; the two most interesting special cases are for $q_0 = 0$ in a zero density universe, and $q_0 = 1/2$ for a flat space-time universe.

In 1960 Fred Hoyle proposed to add the 2nd degree component to the Hubble's equation [1]:

$$c \cdot z = H_0 \cdot r + K \cdot r^2, \qquad (1.2)$$

where r is the distance to the galaxy, K – Hoyle's coefficient, H_0 – Hubble's constant, at that time estimated to be 170 km·s^{-1}·Mps^{-1} [2]. In 1962, Allan Sandage suggested the value of H_0 = (98 ± 15) km·s^{-1}·Mps^{-1} [3]. He also discovered that at a distance of 10^9 light years the galaxies are moving 10^4 km·s^{-1} faster compared to the Hubble's law prediction [4, p. 288]. In 1966, based on these data, the Hoyle's coefficient was estimated to be $K \approx c{\cdot}(H_0/c)^2$, and it was suggested [5, p. 22] that the formula (1.2) represents the first two terms of an expansion in series of the nonlinear model with a discontinuity point of 2nd kind – interpolation model:

$$z = (H_0/c) \cdot r \cdot [1 - (H_0/c) \cdot r]^{-1} \underset{r << c/H_0}{\approx} (H_0/c) \cdot r + (H_0/c)^2 \cdot r^2 = (H_0/c) \cdot r + (K/c) \cdot r^2, \qquad (1.3)$$

at that, the results to a considerable degree depend on statistical methods used to evaluate these results. After the cosmologic distance scale was first established, the collection of data continued, and processing of these data produced some "unexpected" results.

1. The value of the Hubble constant dropped from 530 km·s^{-1}·Mps^{-1} in 1929 [6] to 72 km·s^{-1}·Mps^{-1} in 2001 with the range (64,7 ± 2,4 … 124,4 ± 19,0) km·s^{-1}·Mps^{-1} [7-8]. Because of that, in the 1970th the Hubble constant was called the "Hubble variable". In the XXIst century, the suggested value of the Hubble's constant keeps decreasing: in 2012, as measured by NASA's Wilkinson Microwave Anisotropy Probe (WMAP) [9], it was 69,32 ± 0,80 km·s^{-1}·Mps^{-1}. As of March 2013, the value of Hubble's constant measured by the Planck Mission [10] was 67,80 ± 0,77 km·s^{-1}·Mps^{-1}. Also the value of the deceleration parameter q_0 changed from (2,6 ± 0,8) in 1956 [2] to (–1,0 ± 0,4) in 1998 [11]. In the Measurement Theory [12-14], such variations can be explained by incorrect parameterization of the model, incorrect use of the regression analysis [15], and by stochastic multicol linearity [16] which, while the data obtained by WMAP was being processed, was called the "ΛCDM-model degeneration".

2. The first evidence that this "degeneration" is actually true appeared when the spectral index n_s and the optical thickness τ were evaluated for the sphere of last scattering [17, 18]. The new data reduced the $n_s{\times}\tau$ correlation, increased the correlation between the amplitude of fluctuations of the density of galaxies and the density $\sigma_8{\times}\Omega_c{\cdot}h^2$ of the "cold dark matter" within the 8 Mps radius, and increased the correlation between the spectral index and the density $n_s{\times}\Omega_b{\cdot}h^2$ of the "baryon matter." Also the new data reduced the correlation between the density of "dark energy" $\Omega_\Lambda{\times}\Omega_c{\cdot}h^2$,

the density of "cold dark matter" and the amplitude of fluctuations of density of galaxies $\Omega_\Lambda \times \sigma_8$. As a result, all 6 parameters of the ΛCDM-model were affected by the stochastic multicollinearity.
3. The WMAP data led to the conclusion that on a large scale the universe is homogeneous and flat (with only a 0,4 % margin of error), and can be described by the Euclidean geometry [19].
4. If we compare the microwave background fluctuation spectrum in the ΛCDM-model to the one obtained from the WMAP data, the probability of the agreement between the two will be only 0,02…0,172 when evaluated with the χ^2-criterion [20], if the maps are non-Gaussian [21]. So far the Planck Mission data did not confirm that fluctuations are non-Gaussian, but for the angles 90˚…6˚ the probability of the agreement between the model spectrum and experimental data is even less than the probability mentioned above [10].
5. While the amount of data received by the WMAP in 7 years increased by the factor of 3, the accuracy of the 6-parameterl ΛCDM-model improved only by 50 %. At the same time introduction of just one additional parameter improved the accuracy of the model by 90 %, and introduction of two parameters – by 200 % [22]. If, of course, we not to consider as additional the parameters Ω_c and Ω_Λ which were introduced to simply adjust the model for better description of experimental data. The improvement of the parameters of the ΛCDM-model was expected to be directly proportional to the square root of the amount of collected data under assumption that the fluctuations are Gaussian, but the data collected by the WMAP in 9 years did not produce results that were expected.
6. Iye Masanori, while analyzing extreme redshifts of remote galaxies, predicted that we may not be able to discover the galaxies with redshifts z = 15…20 because at the time when the light signal could have been radiated toward us these galaxies were not formed yet, and this would affect many aspects of the ΛCDM-model and would be a "full-scale crisis of this model" [23]. The Plank mission confirmed the WMAP data that the average temperatures measured for the opposite sky hemispheres are different, and also confirmed existence of a billion-light-year-wide Big Cold Spot, the size of which appeared to be greater than thought before. The Coma Supercluster located in the Coma Berenices constellation and containing two major clusters, the Coma Cluster and the Leo Cluster, was discovered [10]. In Eridanus and Aquarius there is a system of big voids of 11…150 Mps. Voids presumably were formed in the Big Bang by baryon acoustic oscillations and started from initially small anisotropies due to quantum fluctuations in the early Universe. The largest confirmed supervoids are about 100 Mpc across and include the Bootes Supervoid, and the Northern and Southern Local Supervoids. The disputed Eridanus Supervoid, or Great Void, is located at a distance of 2…3 Gpc and was suggested as an explanation to the "cold spot" in the cosmic microwave background radiation. This can be an extremely large almost empty region of the universe, more than 150 Mpc across, possibly up to 300 Mpc. The void of this size can be hardly explained by the idea of baryon fluctuations.
The large scale heterogeneities correlate with the dipole anisotropy [24] of the cosmic microwave background radiation [25-27], with the redshift of galaxies [28], radio galaxies and quasars [29-33], and also with the blue shift of the Local Group [30].
Formation of large-scale heterogeneities like voids and superclusters requires a lot of time and occurred in the early universe. At the same time, according to the ΛCDM-model, the Universe should be homogenous. Slightly reduced value of the Hubble constant reported by Plank researchers led to a slightly greater accepted value of the age of the universe, which is now (13,796 ± 0,058) b. y.
7. On the Hubble's semi-log diagram, if we apply linear interpolation with a standard slope of 0,2 to the sets of points for different groups of extragalactic objects (galaxies, radio galaxies, quasars, etc.), the lines will have different zero-points and different statistical dispersion, which is a violation of the condition of statistical homogeneity [24]. Nevertheless the redshift is being used for distance calculations as if it has purely cosmologic nature. Other violations of the applicability of the statistics used for data processing in cosmology are reviewed in [33].
These "unusual" results usually are not discussed.
The problem is that the maximum credibility for the ΛCDM-model yields very little probability for the model to match the experimental data even if we introduce additional parameters. The cosmologic distance scale is an important tool when measuring distances to the remote objects, and the Hubble's Diagram in which the redshift or the speed of an object is plotted with respect to its dis-

tance from the observer is the core of it. The objective of this study is to review and investigate the cosmologic distance scale.

2. Gravitational redshift

In 1967, Halton Arp who used to be Edwin Hubble's assistant put together a catalog of galaxies and quasars which were located in the vicinity of each other but had significantly different red shifts [34]. He suggested that some exceptionally large redshifts for some objects cannot be used to determine the distances to them. For example, the quasar with $z = 2{,}11$ is located near the galaxy with NGC 7319 at $z = 0{,}0225$ [35]. Margaret and Jeffrey Burbidge, quasar researchers, supported the Arp's hypothesis. They discovered that for the quasars the independent variable in the Hubble's diagram must be quasar luminosity and suggested the following expression for the quasar redshift:

$$z = (1 + z_g)(1 + z_{\kappa}) - 1 \qquad (2.1)$$

The function has two components: cosmological z_{κ} and gravitational z_g:

$$z_g = [1 - 2GM/(rc^2)]^{-1/2} - 1 \text{ or } z_g \xrightarrow[GM/(rc^2) \ll 1]{} GM/(rc^2), \qquad (2.2)$$

where G is the gravitational constant, M and r – mass and radius of the object [36]. Later Jesse Greenstein and Maarten Schmidt noted that the cosmological component is due to the expanding universe but gravitational red shift also must be considered [36]. Before this suggestion was made, the contribution of the gravity believed to be negligible, and the approach (2.1) was never considered. In 1992 Halton Arp showed [37] that the red shift of a quasar can be a function of its absolute luminosity μ (K-effect), and in [38] this function is written as

$$z_0 = K \cdot 10^{-0{,}2\mu}, \qquad (2.3)$$

where $K = 2{,}6 \cdot 10^{-6}$ is a slope of a line of linear regression obtained from the radial velocities data for stars and interacting galaxies. Taking this into account and replacing the gravitational component $(1+z_g)$ with cosmological component $(1+z_0)$ we can obtain a rigorous solution [39] of Mattig's equation (1.1) [40] for $q_0 < 1$:

$$z_{\kappa} = q_0 \cdot (H_0/c) \cdot D_L - (q_0 - 1) \cdot [\sqrt{1 + 2(H_0/c) \cdot D_L} - 1], \qquad (2.4)$$

however the value of red shift for Sun $z_{0C} = 2{,}6 \cdot 10^{-6-0{,}2 \cdot 4{,}79} = 2{,}864 \cdot 10^{-7}$ calculated under formula (2.3) appeared to be different from the value of $z_{gC} = 2{,}17 \cdot 10^{-6}$ for Sun's absolute magnitude of $\mu_C = 4{,}79$ [6].

At the same time, the quasar's luminosity can be determined from the two characteristics: the radius r_0 and the effective temperature T_e [6]: $L = 4\pi\sigma \cdot r_0^2 T_e^4$, were σ is the Stefan-Boltzmann constant. The mass-luminosity relationship can be written as $L = L_C \cdot (M/M_C)^{\alpha}$, where L_C and M_C are the luminosity and the mass of the sun. In situations when the opacity of plasma depends only on scattering on free electrons, and the pressure of radiation is dominant, the exponent $\alpha \to 1$. If we use the relationship between the luminosity and the absolute magnitude μ for galaxies $L = L_C \cdot 10^{0{,}4 \cdot (\mu_C - \mu)}$ [6], then $M = M_C \cdot 10^{0{,}4 \cdot (\mu_C - \mu)/\alpha}$ and $GM/(r_0 c^2) = (G/c^2) \cdot M_C \cdot 10^{0{,}2 \cdot (\mu_C - \mu)} \sqrt{4\pi\sigma T_e^4 / L_C}$, and finally:

$$z_g = [1 - 2K_g T_e^2 \cdot 10^{-0{,}2\mu}]^{-1/2} - 1 \text{ or } z_g \cong K_g T_e^2 \cdot 10^{-0{,}2\mu} \text{ at } K_g T_e^2 \cdot 10^{-0{,}2\mu} \ll 1,$$

where $K_g = (G/c^2) \cdot M_C \sqrt{4\pi\sigma / L_C} \cdot 10^{0{,}2\mu_C} = 5{,}76 \cdot 10^{-13}$ K^{-2}. Since effective temperatures for the stars of spectral types M8…O8 vary from 2660…38000 K [6], the value of $T_{eC} = 5784$ K for the sun yields acceptable value of $z_{gC} = 2{,}12 \cdot 10^{-6}$. In other words, the linear approximation (2.2) of the K-effect [37] appears to be the same as the gravitational red shift at the effective temperature of $T_{eK} = 2\,125$ K, and now we can find a rigorous solution for the gravitational red shift as a function of the effective temperature, photometric distance $D_L = 10^{-5+0{,}2 \cdot (m-\mu)}$ [6] and the apparent magnitude m:

$$z_g = [1 - 2K_g T_e^2 D_L 10^{5-0{,}2m}]^{-1/2} - 1 \text{ or } z_g \cong K_g T_e^2 D_L 10^{5-0{,}2m}, \qquad (2.6)$$

and the formula (2.1) will be as follows:

$$z = [1 - 2K_g T_e^2 D_L 10^{5-0{,}2m}]^{-1/2}(1 + z_{\kappa}) - 1 \text{ or } z \cong (1 + K_g T_e^2 D_L 10^{5-0{,}2m}) \cdot (1 + z_{\kappa}) - 1. \qquad (2.7)$$

Some other approaches for evaluating the cosmological component of the redshift are discussed in [41], in particular the solutions of Mattig's equation (2.4) for $q_0 = \{-1/2;\ 0;\ 1/2;\ 1\}$ and the interpolation model [5] $z_\kappa = (H_0/c)\cdot D_L \cdot [1 + k\cdot(H_0/c)\cdot D_L]^k$ with values of $k = -1$ (1.3) evaluated for angular coordinates, magnitudes and redshifts of different objects (Table); the value of the Hubble's constant used in this work was $H_0 = (74{,}2 \pm 3{,}6)$ km·s^{-1}·Mps^{-1} [42].

Data used for developing the cosmological distance scale

Objects	z	m	Sample size	Source
Quasars (Fig. 1)	0,036…2,877	12,8…20,07	201	[6]
Radio galaxies	0,00086…0,4614	6,98…20,11	172	[6]
Brightest galaxies of clusters	0,004…0,140	12,5…21,0	10	[43]
Galaxies of Local Volume	0,0017…0,0026	7,63…18,13	67	[44]

The interval of values of z is limited: the limit $z < 0{,}0017$ ($cz < 500$ km·s^{-1}) is due to the distance measurement techniques in the Local Volume (Cepheid variables, red giants, supernovas, Tully-Fisher and Faber-Jackson relation); another limit is due to the Gunn-Peterson effect. The criterion for the structural-parametric identification was the minimum of the average structural-parametric modulus of the inadequacy error [12] of the model of the cosmological component of the redshift d_z.

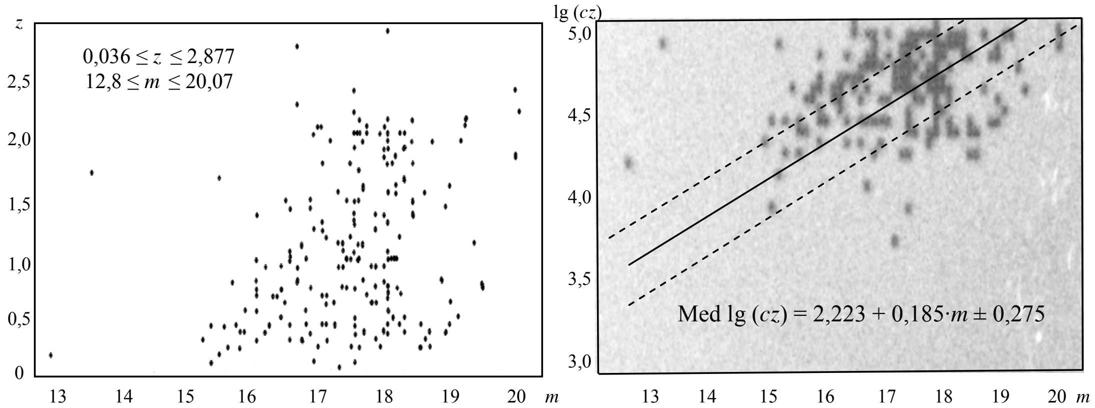


Fig. 1: Hubble's diagram for a sample set of quasars ($N = 201$) for corrected magnitudes m [6].

Because of the gravitational redshift (2.6), the cosmological component of the redshift tends to zero, which means that the scale is isotropic (fig. 2); also it means that the following two models appeared to be most accurate: the model (1.1) for q_0=1 and the interpolation model (1.3).

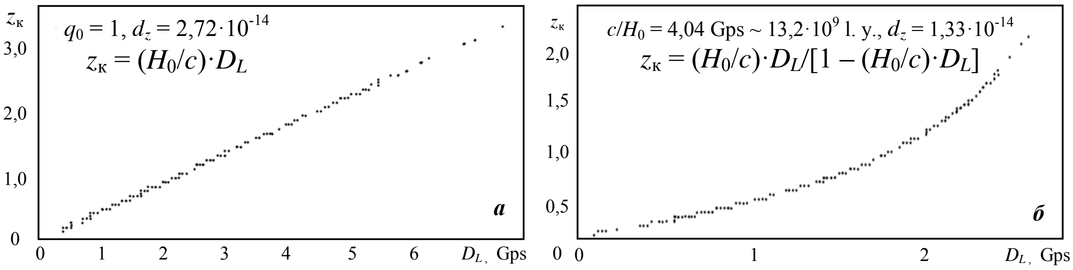


Fig. 2: The cosmological component of the redshift for quasars: *a* – for (1.1) and *b* – for (1.3).

The average structural-parametric modulus of inadequacy errors was compatible to the accuracy of the computer calculations. This required the change of the criterion for structural-parametric identification of the set of model properties and was done without the increase of the number of parameters. The use of computers with increased precision led to the increase of the number of competing models and decreased the average modulus of the inadequacy error up to $d_z \sim 10^{-16}$ [45].

Comparing the models (1.1) and (1.3) we can show that:

1. In the model (1.3) the discontinuity point is located at distance of $c/H_0 = (13{,}16^{+0.67}/_{-0.61})\cdot 10^9$ l. y. for $c = 299\ 792{,}458$ km·s^{-1} [46]. This is less that the "age of the universe" determined from WMAP data $T = (13{,}74 \pm 0{,}11)\cdot10^9$ [9] and the Plank data $T = (13{,}796 \pm 0{,}058)\cdot10^9$ years [10]. For the estimation of Hubble's constant $H_{0\ \mathrm{WMAP}} = 70{,}0 \pm 2{,}2$ and $H_{0\ \mathrm{Plank}} = 67{,}9 \pm 1{,}5$ km·s^{-1}·Mps^{-1}, the discontinuity points must be $R_{0\ \mathrm{WMAP}} = (13{,}953^{+0.452}/_{-0.425})\cdot10^9$ and $R_{0\ \mathrm{Plank}} = (14{,}385^{+0.325}/_{-0.311})\cdot10^9$ l. y.

2. In the model (1.1) for $q_0 = 1$ the parameter H_0/c is simply a slope, and there are no restrictions for the radial speed cz. For $q_0 < 1$, the solutions were greater than the value of the Hubble's radius R_0=

$= c/H_0 = 4040^{+206}/_{-187}$ Mps. For $q_0 < 0$, the equation (2.7) for quasars 5C 02.56 and QS 1108 +285 did not have rational solutions. In the model (1.3), the Hubble radius is a restriction parameter.
3. The cosmological distance scale based on model (1.3) changes the values of absolute magnitudes and distances to the objects. This reduces restrictions for ΛCDM-model redshifts because in this case the "cosmological age" does not correspond to the time of the formation of heterogeneities like galaxy superclusters, quasars and large voids.
4. In Doppler interpretation of cosmological redshift, the speed $v_э$ and acceleration $w_э$ of the "universe expansion" in model (1.3) are as follows:

$$v_э = c\cdot\frac{D_L/R_0 - 0{,}5\cdot D_L^2/R_0^2}{1-D_L/R_0+0{,}5\cdot D_L^2/R_0^2} \text{ and } w_э = \frac{d\,v_э}{dD_L} = c\cdot\frac{(c/R_0)\cdot(1-D_L/R_0)}{[1-D_L/R_0+0{,}5\cdot D_L^2/R_0^2]^2},$$

where $w_э$ increases from $w_e(0) = c\cdot H_0 = 7{,}21\cdot10^{-10}$ m·s^{-2} to its maximum of $w_{e\,max} = 9{,}59\cdot10^{-10}$ m·s^{-2} at $z_к = 0{,}732$ and $D_L = 5{,}43\cdot10^9$ l. y., then reduces to $w(R_0) = 0$ while not changing the sign (fig. 3).

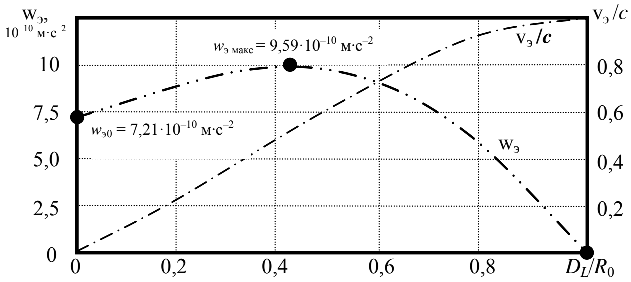


Fig. 3: Acceleration w_e and speed v_e/c of the «universe expansions» vs. distance [45].

This correlates with the point in time when deceleration switched to acceleration [47] – at $z \approx 0{,}73$ (fig. 3), or 5,4 billion years ago. At that, the location of the maximum of the acceleration which is equivalent to the Doppler effect does not depend on the gravitational component of the red shift.
5. The acceleration of the universe expansion $w(0)$ is approximately the same as the "abnormal acceleration" of the spacecrafts Pioneer: $(8{,}74 \pm 1{,}33)\cdot10^{-10}$ m·s^{-2} assuming this acceleration was not changing. The Doppler effect data accumulated over a longer (twice as long) period of time provide evidence that the hypothesis that the acceleration is not constant (linear or exponential function of time) provides 10 % better accuracy [48]. Because of that the abnormal component of the acceleration of Pioneer-10 after 23 years decreased up to the value indicated in the model (1.3) $w(0) = cH_0 = 7{,}21\cdot10^{-10}$ m·s^{-2} [41] and demonstrated inclination to further growth. The fact that the Pioneer-10 and Pioneer-11 abnormal accelerations are close can be just a coincidence provided the discrepancy between the two values is not due to the anisotropy of q_0 [49] since the probes are moving in opposite directions.
6. Interpolation model (1.3) and Hubble's Law have one parameter – H_0/c, whereas the model (1.1) also includes the deceleration parameter q_0. If two or more models provide the same accuracy, the preference is always given to a less complicated model [12-14].
If we combine the diagram and Hubble's Law, then in case of linear approximation the photometric distance scale will be isotropic [41]:

$$D_L = z\cdot\{(1+z)\cdot(H_0/c) + \mathrm{K}\cdot10^{5-0{,}2m}\}^{-1}. \quad (2.8)$$

The rigorous equation (2.7) for a cosmological scale in model (1.3) will be as follows:

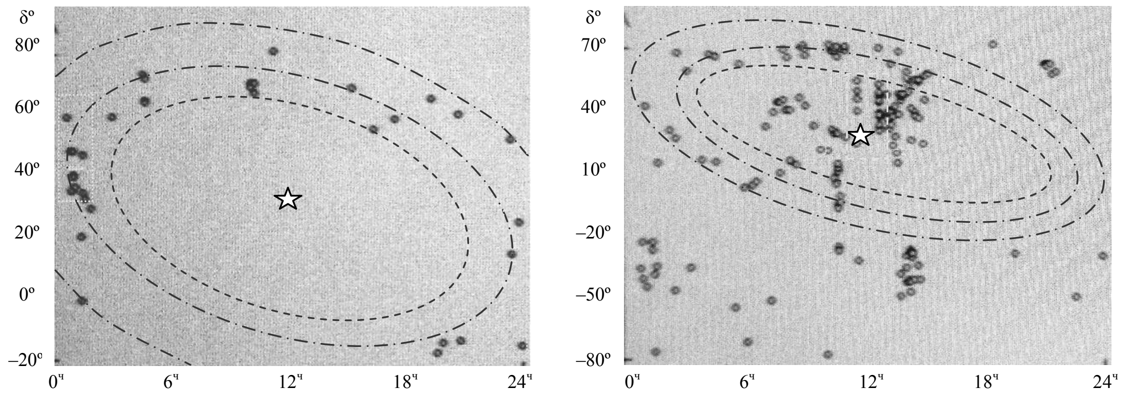
$$(1-D_L/R_0)\sqrt{1-2\mathrm{K}_g T_e^2 D_L 10^{5-0{,}2m}} = 1/(1+z). \quad (2.9)$$

This equation can be reduced to the cubic equation with the variable D_L, and its solution can be analytically tractable although in some cases it is preferable to use the numerical method.
The solution of the system of equations (2.9) for the quasar 201 [6] at the "standard" effective quasar temperature T_{eQ} = 30 000 K [36] yields the scale under which the distance for the most remote quasar from in set is $D_{L\,max}$ = 90 Mps. It means that either the «standard» effective quasar temperatures [36] do not conform to their apparent magnitudes or Arp's hypothesis about the nature of their redshift is true [34, 35]. It also means that the most plausible reason for the redshift anisotropy is the component (2.6), the dipole of which concurs with the galactic polar axis. The «Distance scale»

program [50] was developed for the purpose of using equation (2.7) to find "optimal" values of H_0 and K_g. This program allows modeling based on different databases, but so far it did not prove to be successful. Nevertheless there is hope that improved data on effective temperatures of extragalactic objects will make it applicable and useful.

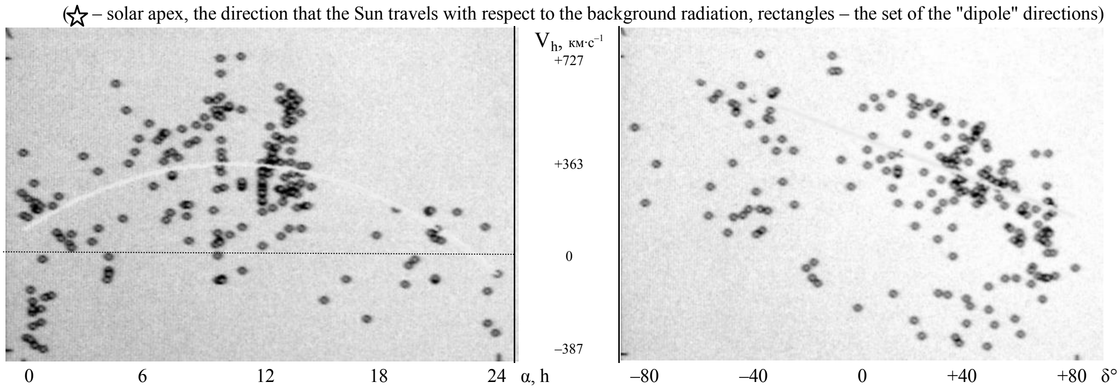
3. Zero-point of the cosmological distance scale

In the 1970s, Allan Sandage had noted that Hubble's constant is the same for different directions and also for different distances, even for distances less than 300 Mps [51] over which the universe is considered to be uniform: the "uniform Hubble's Law" is observed starting from distances of 1,5…2,0 Mps [30]. At the same time 37 galaxies in the Local Volume have blueshifts [44]; they are located in Andromeda, Camelopardalis, Ursa Minor, Draco and Pegasus in Northern hemisphere. This area is shaped as an extended "horseshoe" between the ecliptic nodes (0^h and 12^h), its center is located on the borderline between Virgo and Leo (fig. 4a); this is the area with the maximum temperature of the background radiation, it is in the direction of the solar apex and in the direction of the redshift dipole [39] (fig. 4cd). 167 Galaxies with redshift are located predominantly in Canes Venatici, Coma, Virgo and Centaurus and are distributed more uniformly [44] (fig. 4b).



a) blueshift b) redshift

(☆ – solar apex, the direction that the Sun travels with respect to the background radiation, rectangles – the set of the "dipole" directions)



a) on an axis of blue-red «dipole» b) at zero-point of cosmological distances scale

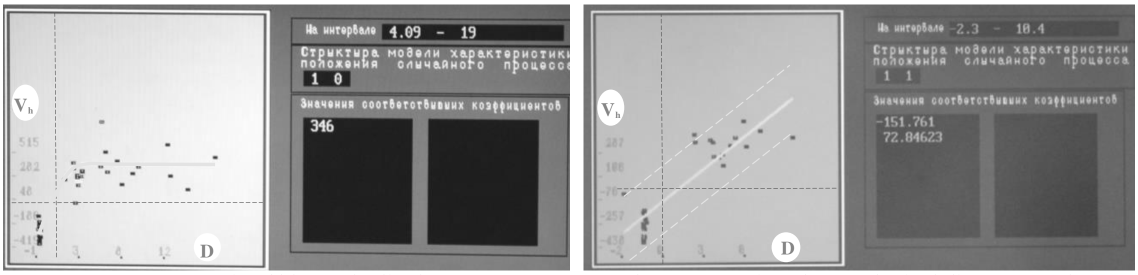
Fig. 4: Distribution of heliocentric speeds (program MMK-STAT [12])

The analysis of these data with employment of the "MMK–STAT M"-program [12] demonstrated that the radial speed V_h, km·s^{-1}, of the galaxies in Local Volume can be represented as a function of several parameters – declination δ, right ascension α, apparent magnitude m and photometric distance D_L; the radial speed can be described using the regression model presented in the form of the MMKMNK-estimation [12]:

$$V_h = \begin{cases} 129{,}28175 - 2{,}9535704 \cdot \delta + 5{,}8127337 \cdot m + 30{,}177181 \cdot D_L;\ V_h > 0 \\ -181{,}39638 + 2{,}1469548 \cdot \alpha - 7{,}1032076 \cdot m + 45{,}310307 \cdot D_L;\ V_h < 0 \end{cases}.$$

It means that in Local Volume there is an implicitly defined "dipole" formed by two groups of galaxies with blueshifts and redshifts. A composite MMKMNK-estimation [12] for the position characteristic of the Hubble's law evaluated for galaxies in the Local Volume located along the "dipole" axis on knots ecliptic within the interval D_L = [–0,62…3,92] Mps yields $\overline{V}_h$ = –264,5 km·s$^-$

[1] for galaxies within D_L = [–0,91…–0,62] Mps and $\overline{V}_h$ = 346 km·s^{-1} for galaxies outside this interval (fig. 5a). If we combine galaxies with blueshits and redshifts within the interval of [–2,3…10,4] Mps, then for the 90% tolerance [52] and for $\widetilde{V}_h = -151{,}761 + 72{,}846 \cdot D \pm 193{,}876$ [km·s^{-1}] and for peculiar velocity of V_h = –151,76 km·s^{-1} we will obtain the slope of 72,85 km·s^{-1}·Mps^{-1} (fig. 5b), which is close to recently measured estimations of Hubble's constant (72 ± 3 ± 7) km·s^{-1}·Mps^{-1} [42] and (74,2 ± 3,6) km·s^{-1}·Mps^{-1} [7].

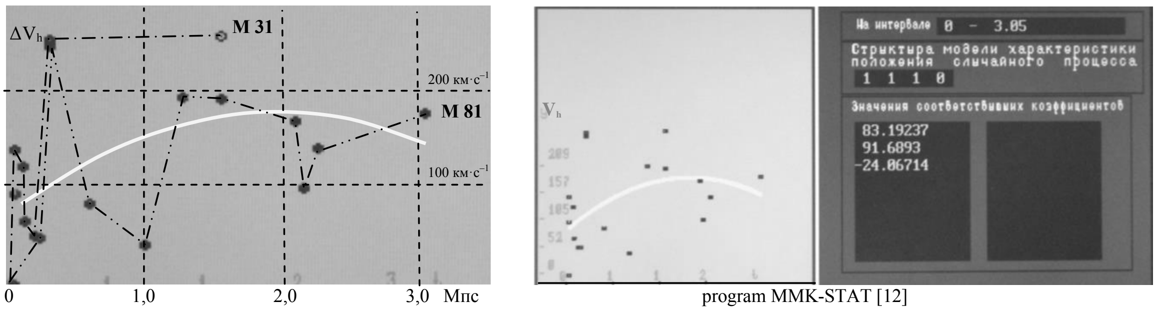


a) on an axis of blue-red «dipole» b) at zero-point of cosmological distances scale

Fig. 5: Distribution of heliocentric speeds (program MMK-STAT [12])

The value of the heliocentric speed at zero-point is not zero because most of the galaxies in the Local Volume are dwarf galaxies or irregular galaxies for which the gravitational shift in their spectra can be considered negligible. This conclusion can be true for other objects because, if we extend the model (2.1) by considering the Doppler Effect, then we will have only 9 objects left in Local Volume out of 167.

In 1973 Michael Roberts and Arnold Rots showed [53] that the functions of rotating for the neighboring spiral galaxies are dropping not as fast as it was predicted by Newtonian mechanics because of the large amounts of matter which can be found at large distances from their centers. In 1983, Halton Arp discovered [54] that the redshift for the spiral galaxies located near the center of the groups of galaxies M31 and M81 in the Local Volume is less that for their dwarf companions (fig. 5). Similar results are obtained for the system Milky Way – Magellan Clouds. In 1986 Allan Sandage indicated [55] that the local value of the Hubble's constant within distances of (1…2) Mps is increasing due to gravitational attraction of the Local Group.



program MMK-STAT [12]

Fig. 5: The excess heliocentric speed (cz) of the companions of galaxies in M 31 and M 81.

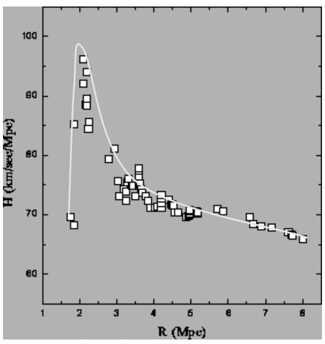


Fig. 6: local Hubble's constant in Local Volume [44].

In 1988, Brent Tully discovered a maximum of 90 km·s^{-1}·Mps^{-1} for the local Hubble's constant in the range of (7…30) Mps [56]. In 1997, Igor Karachentsev and Dmitriy Makarov discovered in Local Volume a maximum for Hubble's constant of H_0(2 Mps) ~ 90 km·s^{-1}·Mps^{-1} (fig. 6) [44], and this maximum was mostly due to the galaxies in the group IC 342 Maffei and Sculptor. These "maxima" can be explained by «Toroid of dark matter» surrounding the spiral galaxies. Same authors showed [44] that the root-mean-square deviation for the peculiar velocities of galaxies is practically the same for gigantic and dwarf galaxies and is equal to 72 ± 2 km·s^{-1} for relative variations which are not more than 3%. Within the radius of 1 Mps, this value is close to the Hubble's constant.

So, the blueshift in the Local Volume is the extension of the redshift, the excess redshifts for the groups of galaxies are due to the gravitational anomalies of the "dark

matter" and the "uniform Hubble's law" is true at the distances which are significantly less that predicted by Allan Sandage.
The problem of the zero-point of the cosmologic distance scale yet is not solved.

4. Conclusion

By taking into consideration the apparent magnitudes of the extragalactic objects and by correcting the redshifts of these objects, it was possible to decrease the deviation of the evaluations and integrate the objects of different morphologic types into one isotropic function.
Purely metrological interpretation of the data makes it possible to determine the red shift model without artificial parameterization and artificial adjustments to the experimental results.
The list of "unexpected" results includes some "unexpected coincidences" most "unexpected" of which are:
1) the apex of the sun, the cosmic microwave background radiation dipole anisotropy, the delay parameter, the redshift of the galaxies, radio galaxies and quasars as well as the blueshift of the galaxies in the Local Volume correlate with the polar galactic axis, with the biggest structural element of the observable universe in the northern direction and with the set of gigantic voids in the southern direction of the polar galactic axis,
2) the maximum in the interpolation model correlates with the beginning of the accelerating expansion of the universe, and the minimum correlates with the "anomalous acceleration" of Pioneer 10 and Pioneer 11,
3) own redshift of quasars correlates with own gravitational redshift,
4) the corrected experimental redshifts correlate with the hypothesis that the cosmologic component is isotropic,
5) the interpolation model correlates with the simple expansion model at constant speed allowing for the delay and imposed restriction on the range of the gravitational interaction.
Paul Dirac's hypothesis about the cause of the redshift was that the photons loose energy while traveling in the space and overcoming the resistance of the space. Jeffrey and Margaret Burbidge believed that the redshift is due to both the Doppler Effect and the gravity. Other suggested causes of the redshift were the viscosity of the ether, spontaneous radio luminescence of the hydrogen atoms, vacuum fluctuations, etc.
But the real nature of the redshift is yet to be explained.

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Measuring the gravitational redshift effect with space-borne atomic clocks

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The problem of measuring the gravitational redshift effect with space-borne atomic clocks is discussed. The two common approaches to the experiment are presented with relevant examples of past, current and future missions. Special attention is given to a less common satellite communication system design of a 1-way 2-frequency downlink, relevant, for example, to RadioAstron space radio telescope. Various effects influencing the precision of the test are discussed. Some results of simulated calculations are given.

1. Introduction

According to the Einstein's principle of equivalence an electromagnetic wave which travels in a region of space with varying gravitational potential experiences a frequency shift

$$\frac{\Delta f}{f} = \frac{\Delta U}{c^2}, \tag{1}$$

where $\Delta f/f$ is the fractional frequency shift, $\Delta U/c^2$ is the gravitational potential difference and c is the speed of light. The best test of this relation to-date was performed as a part of the GP-A project in 1976 and reached the precision of $1.4 \cdot 10^{-4}$ [1]. In that experiment frequencies of two H-masers were compared, one on-board a sounding rocket with nearly up-down trajectory and another resting on the surface of the Earth. The interest in an even more precise test of (1) is motivated by searches for the limits of applicability of the equivalence principle which are predicted by the majority of grand unification theories.

2. Approaches to gravitational red shift measurement with space-borne atomic clocks

The two main approaches to gravitational redshift experiment are absolute measurement and modulation of the effect. The *absolute measurement* approach is based on the following principal formula (postponing consideration of the clock motion and propagation media contribution to section 3):

$$\frac{f_s - f_e}{f_e} = (1+\varepsilon)\frac{U_s - U_e}{c^2}, \tag{2}$$

where f_s is the frequency of the "space" clock as received on earth, f_e — the frequency of the "earth" clock, U_s and U_e — gravitational potential at the location of the space and earth clock, respectively. Here we also included a parameterization in terms of ε of the hypothetical deviation from general relativity which is, in fact, the value to be tested.

Now we can estimate limitations to measurement precision of ε due to different clock types. For example, for H-masers which now demonstrate accuracies of $\frac{\Delta f}{f} \sim 3 \cdot 10^{-13}$, the earth clock residing at mean sea level and the space one — at a distance of 350 000 km ($\sim$ apogee of the RadioAstron satellite), which corresponds to gravitational potential difference of $6.8 \cdot 10^{-10}$, equation (2) implies

$$\delta\varepsilon \sim 4 \cdot 10^{-4}. \tag{3}$$

Consider now a more accurate clock, namely, caesium atomic clocks and, in particular, caesium atomic fountain clocks. The best such clock qualified for space operation is PHARAO by SYRTE, LKB, and CNES [2] with accuracy $\frac{\Delta f}{f} = 10^{-16}$. This clock type would make it possible to perform the absolute measurement of the frequency shift due to gravitational potential difference of $6.8 \cdot 10^{-10}$ with precision

$$\delta\varepsilon \sim 1.4 \cdot 10^{-7}. \tag{4}$$

It should be noted, however, that PHARAO is not going to travel as far as 350 000 km away from Earth. Rather, it's going to be installed at ISS's Columbus module as a part of the mission ACES [3], with the launch now being postponed to 2016 [4]. The height of the ISS orbit being $\sim$ 400 km, this means that the mission is going to achieve a more modest value of

$$\delta\varepsilon \sim 1 \cdot 10^{-6}. \tag{5}$$

Still this is 2 orders of magnitude better than the now-record-holder GP-A and 1 order of magnitude better than envisaged for RadioAstron mission.

Now consider the experimental approach based on *modulation of the gravitational potential.* Here the principal formula (again temporarily ignoring clock motion and media contribution) is effectively a difference of (2) taken at two locations of the spacecraft:

$$\frac{f_{s1} - f_{s2}}{f_e} = (1+\varepsilon)\frac{U_{s1} - U_{s2}}{c^2} \tag{6}$$

with f_{s1} and f_{s2} being the frequencies of the space clock measured on earth and radiated when the space clock resided at points with gravitational potentials U_{s1} and U_{s2} respectively.

The greatest precision in this mode of experiment is achieved when gravitational potential difference between the points 1 and 2 is maximum. That is, in case of an Earth-orbiting satellite the most favorable configuration is a highly elliptic orbit. The modulation method has two advantages over the absolute measurement one. First, the clocks used need not be highly accurate, the requirement, however, is high *frequency stability* and low drift at times of order of satellite passage from perigee to apogee. This is exactly the situation for H-maser clocks, which today have relevant frequency stability of $\sim 3 \cdot 10^{-15}$ at times ~ 1000 secs. Another advantage is the possibility of repeating the experiment and thus improving the precision roughly as a square root of the number of measurements because of the statistical nature of the clock error.

Consider, for example, a satellite with perigee 1000 km and apogee 350 000 km, having an H-maser with the characteristics stated above. Then a single measurement of ε is to be expected to have precision of

$$\delta\varepsilon \sim 1 \cdot 10^{-5}, \tag{7}$$

and repeating the experiment 100 times yields

$$\delta\varepsilon \sim 1 \cdot 10^{-6}. \tag{8}$$

Although these estimations are quite promising it should be noted that they take into account only the limitations of the precision coming from the clocks themselves. Let's now consider how the situation is changed when the effects of clock motion and propagation media are taken into account.

3. Clock motion and propagation media contributions

The clock motion and propagation media effects depend crucially on frequency transfer scheme realized by the satellite. The two principal designs are the 3-way configuration pioneered by R. F. C. Vessot [5] and a conventional 1-way/2-way scheme. Both schemes possess 2-frequency downlinks for ionospheric correction (see below). The scheme by Vessot is a combination of simultaneously operating 2-way up-down link (phase-locked loop), synchronized from the ground clock and a 1-way downlink, synchronized from the space clock. In this design the total frequency shift of the space clock relative to the ground clock is expressed by the equation:

$$\frac{\Delta f}{f} = -\frac{1}{c^2}\left(\frac{|\vec{v}_s - \vec{v}_e|^2}{2} + \vec{r}_{se} \cdot \vec{a}_e\right) + \frac{\Delta f_{grav}}{f} + \frac{\Delta f_{ion}}{f} + \frac{\Delta f_{trop}}{f} + O\left(\frac{v}{c}\right)^3, \tag{9}$$

$\vec{v}_s$ is the spacecraft velocity at the moment of radiation of the signal, $\vec{v}_e$ — earth station velocity at the moment of reception of the signal, $\vec{r}_{se}$ — vector originating at the "s" point and pointing to the "e" point, $\Delta f_{grav}/f$ — gravitational frequency shift, equal to $\Delta U/c^2$ according to general relativity, $\Delta f_{ion}/f$ and $\Delta f_{trop}/f$ — ionospheric and tropospheric contributions.

The principal point to note from (9) is that it doesn't contain a first-order (non-relativistic) doppler shift which considerably lowers the requirements to orbit determination accuracy. Since this equation is rather well known it will not be considered here.

In one-way frequency transfer the total frequency shift of the space clock relative to the ground clock can be obtained by simple application of general relativity:

$$\frac{\Delta f}{f} = -\frac{\dot{D}}{c} - \frac{v_s^2}{2c^2} + \frac{v_e^2}{2c^2} + \frac{\Delta f_{grav}}{f} + \frac{\Delta f_{ion}}{f} + \frac{\Delta f_{trop}}{f} + O\left(\frac{v}{c}\right)^3, \tag{10}$$

where D is the distance from the spacecraft to the ground station, $\dot{D}$ is the radial velocity and all other symbols are as in (9). The task is therefore to obtain a constraint on the value of ε using the equation

$$\frac{\Delta f}{f} - \left(-\frac{\dot{D}}{c} - \frac{v_s^2}{2c^2} + \frac{v_e^2}{2c^2} + \frac{\Delta f_{ion}}{f} + \frac{\Delta f_{trop}}{f}\right) = (1+\varepsilon)\frac{\Delta U}{c^2}, \tag{11}$$

where the LHS is the experimentally determined gravitational redshift while the RHS is predicted by general relativity.

Relative magnitude of some of the terms in (10) can be judged from Figs. 1 and 2, which correspond to a model of a spacecraft orbiting the Earth in a highly elliptic orbit with a period of 8.5 days. The base frequency is supposed to be equal to 8.4 GHz.

Let's estimate the potential measurement precision of determination of ε implied by (11) assuming the modulation approach to experiment and the following values for relevant parameters of the problem

radial velocity error $\delta\dot{D}$	0.1 mm/s	
perigee	1000 km	(12)
apogee	350 000 km	

Then the error in estimating the gravitational frequency shift is

$$\delta\left(\frac{\Delta f_{grav}}{f}\right) = 3 \cdot 10^{-13}, \tag{13}$$

and the value of gravitational potential modulation

$$\frac{\Delta U}{c^2} = 6.0 \cdot 10^{-10}. \tag{14}$$

Putting (13), (14) into (11) we obtain for single-measurement precision

$$\delta\varepsilon = 5.0 \cdot 10^{-4}, \tag{15}$$

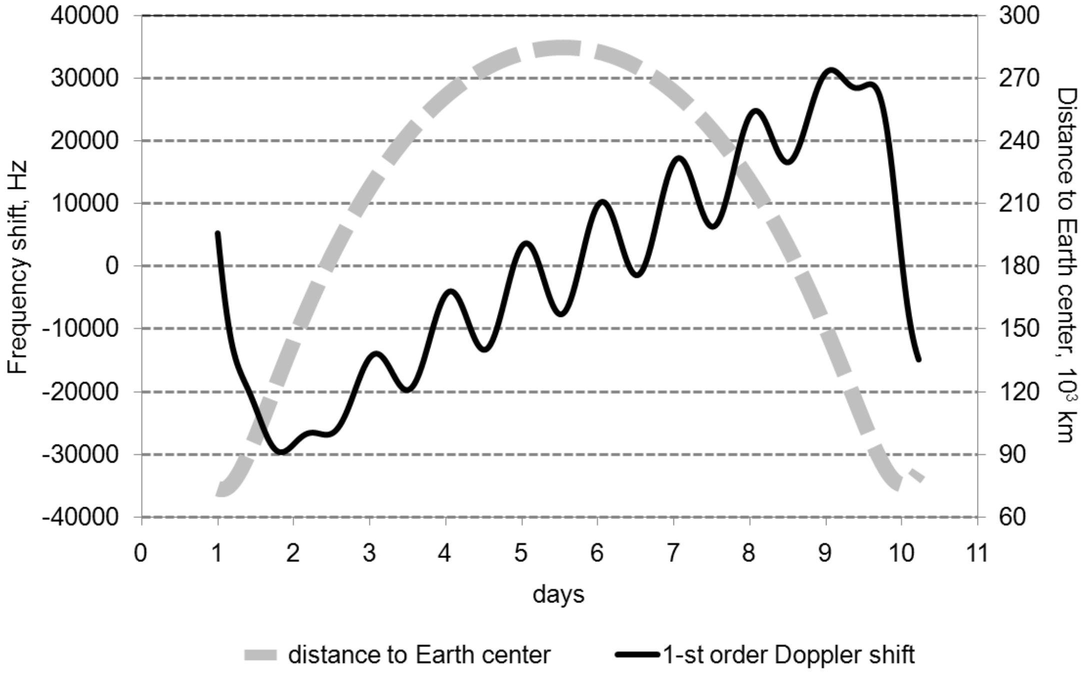

Fig. 1. First order Doppler shift modelling for an Earth-orbiting satellite with a period of 8.5 days, $70 \cdot 10^3$ km perigee and $280 \cdot 10^3$ km apogee. Base frequency is chosen to be 8.4 GHz.

and after accumulating 100 orbits

$$\delta\varepsilon = 5.0 \cdot 10^{-5}. \tag{16}$$

Consider now briefly propagation media contribution [5], limiting our discussion to ionosphere (for the complex problem of troposphere influence on radio signal propagation see [6]). Ionospheric contribution is determined by the refractive index

$$n_{ion} = \left(1 - \frac{f_n^2}{f^2(1 \pm (f_m/f))}\right). \tag{17}$$

Here f is the propagation frequency, f_n — the plasma frequency

$$f_n = \sqrt{\frac{\rho e^2}{\pi m}}, \tag{18}$$

ρ — the number density of electrons, e — electronic charge, m — electron mass. The term in (17) with gyromagnetic frequency f_m is less then 10^{-3} when $f > 2$ GHz and can be neglected. Ionospheric doppler shift is equal to the change of ionospheric density n_{ion} along the satellite-ground

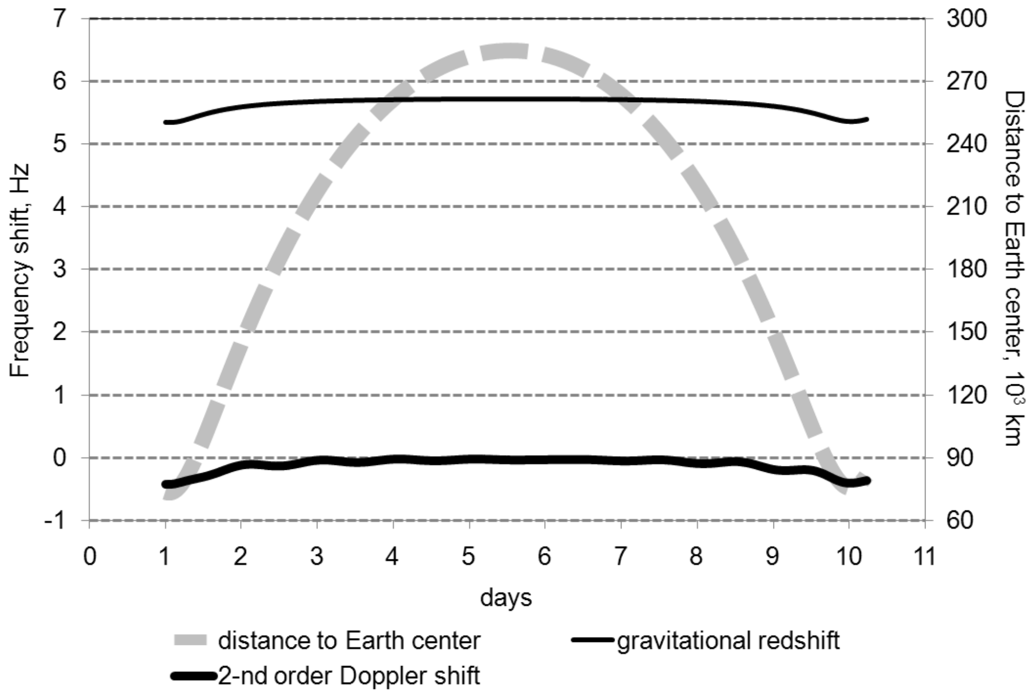

Fig. 2. Second order Doppler and gravitational red shift modelling for an Earth-orbiting satellite with a period of 8.5 days, $70 \cdot 10^3$ km perigee and $280 \cdot 10^3$ km apogee. Base frequency is chosen to be 8.4 GHz.

station path

$$\Delta f_{ion} = \frac{f}{c}\frac{d}{dt}\int_P n_{ion} ds = \frac{f}{c}\frac{d}{dt}\int_P \left(1 - \frac{40.4\rho}{f^2}\right) ds = \frac{40.4}{cf}\frac{d}{dt}\int_P \rho ds, \tag{19}$$

where numerical values for the parameters have been inserted. Since we have a 2-frequency downlink, say with frequencies A and B, we can write the last equation for each of the frequencies A and B. Making a difference of these equation we then obtain

$$\frac{\Delta f_{A,ion}}{f_A} - \frac{\Delta f_{B,ion}}{f_B} = \frac{40.4}{c}\frac{d}{dt}\int_P \rho ds \left(\frac{1}{f_A^2} - \frac{1}{f_B^2}\right). \tag{20}$$

Now performing the same procedure with (10) and noting that only ionospheric shift exhibits f^{-1} behaviour while all the others are independent of f in the 1–15 GHz frequency range and thus cancel, we obtain

$$\frac{\Delta f_A}{f_A} - \frac{\Delta f_B}{f_B} = \frac{40.4}{c}\left(\frac{d}{dt}\int_P \rho ds\right)\left(\frac{1}{f_A^2} - \frac{1}{f_B^2}\right) \tag{21}$$

where the LHS is determined from the experiment. The factor $\left(\frac{d}{dt}\int_P \rho ds\right)$ can thus be determined experimentally and then, using (19), the individual ionospheric frequency shifts for channels A and B can be found, with error low enough for experiments aimed at testing the gravitational redshift with 10^{-6} precision.

4. Conclusions

We discussed various aspects of the problem of measuring the gravitational redshift effect with space-borne atomic clocks. Measurement precision limitations arising from finite clock accuracy and stability and orbit reconstruction errors were discussed. The results given can be used as a starting point for any real experimental mission aimed at measuring the gravitational redshift effect with precision of up to 10^{-6}, especially in the case of a standard 1-way 2-frequency satellite communication mode. In case of a higher measurement precision goal more precise formulas should be used, taking into account terms of order $(v/c)^3$ in the frequency shift expressions.

5. Acknowledgements

The authors wish to thank A. V. Biriukov for invaluable discussions on H-masers and satellite communication systems as well as for constant support for our work.

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New Family of Conserved Quantities for Perturbations in Metric Theories of Gravity

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A construction of conservation laws and conserved quantities for perturbations in arbitrary metric theories of gravity is developed. A Lagrangian description of perturbations in a background spacetime is based on incorporating a background (fixed) metric related to this spacetime into the initial background independent Lagrangian. It was stated that the incorporation can be carried out by various ways. Each of the ways leads to its own *covariantized* Nœther identities. With using the identities, identically conserved currents with corresponding superpotentials have been constructed and united into a family. Just this is a natural basis for constructing conserved quantities for perturbations. In the result, a *new family* of conserved currents and correspondent superpotentials for perturbations on arbitrary curved backgrounds in metric theories is suggested. The family is described by the continuous parameter p: $0 \leq p \leq 1$. In general relativity, there is no a real family: all its members (say, currents) are united into an unique quantity for all p from $0 \leq p \leq 1$. However, in more complicated theories, for example, of the Lovelock type, the family is real. To test the new family each of the superpotentials of the family is applied to calculate the mass of the D-dimensional Schwarzschild-anti-de Sitter black hole in the Einstein-Gauss-Bonnet gravity. Using all the superpotentials of the family gives the standard accepted mass.

1 Introduction

Last time, numerous metric theories, which are various modifications of general relativity (GR), become more and more popular. They are quadratic in curvature theories, or theories of the Lovelock type, or $f(R)$ theories, *etc.* In all such theories, there is a necessity to study perturbations and construct conservation laws for them. Many results have been obtained already (see,e.g., [1] - [7]). Lagrangians in these theories have very various forms, therefore it is very desirable to elaborate united standards for constructing the conservation laws. The present work is just devoted to this problem.

One of the earlier attempts to suggest covariant Nœther identities in GR for perturbations is the Ray work [8]. Here, in a definite sense, we develop Ray's ideas. Examining perturbed models with a fixed background (a known solution), we operate with a background metric. Namely its presence permits us to construct covariant expressions. The presentation follows, in general, the text of our paper [9]. However, here, it is more simple and more explanations are given, whereas in [9] one can find a formalism in detail.

2 Nœther identities in an arbitrary field theory

In this section, we repeat the main results of the book [10]. We derive the Nœther identities and conserved quantities for arbitrary theories in D dimensions, an action of which is

$$S = \int d^D x \hat{L}(Q^A; Q^A{}_{,\alpha}; Q^A{}_{,\alpha\beta})\,. \tag{2.1}$$

As usual, the Lagrangian is a scalar density of the weight +1. Here and below: 'hat' means that a quantity is a density of the wight +1, for example, $\hat{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu}$, $\hat{L} = \sqrt{-g}L$; dynamical fields of the system (2.1) are presented by a set of tensor densities Q^A, where the generalized index 'A' is a collective tensor index; Greek indexes numerate coordinates in D-dimensional spacetime; $(,\alpha) \equiv \partial_\alpha$, $\partial_{\alpha\beta} \equiv \partial_\alpha\partial_\beta$ are partial (ordinary) derivatives; derivatives of Q^A are included in the Lagrangian up to a second order.

Considering Lie displacements as perturbations of the system, we define variations of fields as their Lie derivatives (here, with the opposite sign 'minus' [10]):

$$\delta_\xi Q^A = \pounds_\xi Q^A = -\xi^\alpha \partial_\alpha Q^A + Q^A\big|^\alpha_\beta \partial_\alpha \xi^\beta\,. \tag{2.2}$$

The notation $Q^A\big|^\alpha_\beta$ is defined by the transformation properties of Q^A. Variating the Lagrangian one obtains the identity:

$$\delta_\xi \hat{L} = \pounds_\xi \hat{L} \equiv -(\xi^\alpha \hat{L})_{,\alpha}\,. \tag{2.3}$$

Keeping in mind (2.1) and providing identical transformations in (2.3), one obtains

$$-\left[\frac{\delta\hat{L}}{\delta Q_B} Q_{B,\alpha} + \partial_\beta\left(\frac{\delta\hat{L}}{\delta Q_B}\, Q_B\big|^\beta_\alpha\right)\right]\xi^\alpha + \partial_\alpha\left[\hat{U}_\sigma{}^\alpha \xi^\sigma + \hat{M}_\sigma{}^{\alpha\tau}\partial_\tau\xi^\sigma + \hat{N}_\sigma{}^{\alpha\tau\beta}\partial_{\beta\tau}\xi^\sigma\right] \equiv 0\,. \tag{2.4}$$

In (2.4), the coefficients are defined by the Lagrangian without ambiguities in unique way:

$$\hat{U}_\sigma{}^\alpha \equiv \hat{L}\delta^\alpha_\sigma + \frac{\delta\hat{L}}{\delta Q_B}\, Q_B\big|^\alpha_\sigma - \frac{\delta\hat{L}}{\delta Q_{B,\alpha}}\partial_\sigma Q_B - \frac{\partial\hat{L}}{\partial Q_{B,\beta\alpha}}\partial_{\beta\sigma} Q_B\,, \tag{2.5}$$

$$\hat{M}_\sigma{}^{\alpha\tau} \equiv \frac{\delta\hat{L}}{\delta Q_{B,\alpha}}\, Q_B\big|^\tau_\sigma - \frac{\partial\hat{L}}{\partial Q_{B,\tau\alpha}}\partial_\sigma Q_B + \frac{\partial\hat{L}}{\partial Q_{B,\beta\alpha}}\partial_\beta(Q_B\big|^\tau_\sigma)\,, \tag{2.6}$$

$$\hat{N}_\sigma{}^{\alpha\tau\beta} \equiv \frac{1}{2}\left[\frac{\partial \hat{L}}{\partial Q_{B,\beta\alpha}}\, Q_B|^\tau_\sigma + \frac{\partial \hat{L}}{\partial Q_{B,\tau\alpha}}\, Q_B|^\beta_\sigma\right]. \tag{2.7}$$

The last coefficient satisfies $\hat{N}_\sigma{}^{\alpha\tau\beta} = \hat{N}_\sigma{}^{\alpha\beta\tau}$ that follows from (2.4); $\delta\hat{L}/\delta Q_B$ and $\delta\hat{L}/\delta Q_{B,\alpha}$ mean the usual Lagrangian derivatives with respect to Q_B and $Q_{B,\alpha}$.

Opening the identity (2.4), and, since ξ^σ, $\partial_\alpha\xi^\sigma$, $\partial_{\beta\alpha}\xi^\sigma$ and $\partial_{\gamma\beta\alpha}\xi^\sigma$ are arbitrary at every world point, we equalize to zero the coefficients at them and obtain the system of identities:

$$\partial_\alpha \hat{U}_\sigma{}^\alpha \equiv \frac{\delta\hat{L}}{\delta Q_B} Q_{B,\alpha} + \partial_\beta\left(\frac{\delta\hat{L}}{\delta Q_B}\, Q_B|^\beta_\alpha\right), \tag{2.8}$$

$$\hat{U}_\sigma{}^\alpha + \partial_\lambda \hat{M}_\sigma{}^{\lambda\alpha} \equiv 0, \tag{2.9}$$

$$\hat{M}_\sigma{}^{(\alpha\beta)} + \partial_\lambda \hat{N}_\sigma{}^{\lambda(\alpha\beta)} \equiv 0, \tag{2.10}$$

$$\hat{N}_\sigma^{(\alpha\beta\gamma)} \equiv 0. \tag{2.11}$$

As we know, the system (2.8) - (2.11) was pioneered by Klein [11]. Therefore, we shall refer to this as *the Klein identities*. After differentiating (2.9) and using (2.10) and (2.11) one obtains the identity $\partial_\alpha \hat{U}_\sigma{}^\alpha \equiv 0$. This means that the right hand side of (2.8) is equal identically to zero also

$$\frac{\delta\hat{L}}{\delta Q_B} Q_{B,\alpha} + \partial_\beta\left(\frac{\delta\hat{L}}{\delta Q_B}\, Q_B|^\beta_\alpha\right) \equiv 0. \tag{2.12}$$

These are just so-called Nœther identities. Thus instead of the identity (2.4) one can use independently (2.12) and

$$-\partial_\alpha\left[\hat{U}_\sigma{}^\alpha\xi^\sigma + \hat{M}_\sigma{}^{\alpha\tau}\partial_\tau\xi^\sigma + \hat{N}_\sigma{}^{\alpha\tau\beta}\partial_{\beta\tau}\xi^\sigma\right] \equiv \partial_\alpha \hat{I}^\alpha(\xi) \equiv 0\,. \tag{2.13}$$

The expression under the divergence we call as a current. Because the divergence (2.13) is equal to zero identically, the current has to be expressed through a superpotential $\hat{I}^{\alpha\beta}$, a double divergence of which has to be equal to zero identically $\partial_{\alpha\beta}\hat{I}^{\alpha\beta}(\xi) \equiv 0$. Indeed, with the use of the Klein identities it is rewritten in the form of the identity

$$\hat{I}^\alpha(\xi) \equiv \partial_\beta \hat{I}^{\alpha\beta}. \tag{2.14}$$

Thus (2.14) can be considered as the identity equivalent to the conservation law (2.13) for the current $\hat{I}^\alpha(\xi)$. All the above expressions and identities are not covariant. The next sections are devoted to a construction of covariant identities and conserved quantities.

3 Covariantization of Nœther identities

The Lagrangian (2.1) of a covariant theory is not covariant evidently. One of the ways to present it in an explicitly covariant form is to incorporate an external (auxiliary, background) metric by exchanging $Q_{B,\alpha} \equiv Q_{B;\alpha} - \overline{\Gamma}{}^{\tau}_{\alpha\rho}\, Q_B|^{\rho}_{\tau}$, and more complicated exchanging $Q_{B,\alpha\beta}$ by $Q_{B;\alpha\beta}$. Because the Lagrangian in (2.1) is a scalar density after such identical substitutions it is transformed into an explicitly covariant form:

$$\hat{L}(Q_B,\, Q_{B,\alpha},\, Q_{B,\alpha\beta}) \equiv \hat{\mathcal{L}}(Q_B,\, Q_{B;\alpha},\, Q_{B;\alpha\beta},\, \overline{g}_{\mu\nu},\, \overline{R}{}^{\alpha}{}_{\mu\beta\nu})\,. \tag{3.1}$$

Here, $\overline{g}_{\mu\nu}$, $\overline{\Gamma}{}^{\mu}_{\alpha\nu}$ and $\overline{R}{}^{\alpha}{}_{\mu\beta\nu}$ are metric, Cristoffel symbols and the curvature tensor of the auxiliary spacetime; $({}_{;\alpha}) = \overline{D}_\alpha$, $\overline{D}_{\alpha\beta} = \overline{D}_\alpha \overline{D}_\beta$ are covariant derivatives with respect to $\overline{g}_{\mu\nu}$; here and below 'bar' means that a quantity is a background one.

To conserve the explicit covariance under variation of $\hat{\mathcal{L}}$ the direct way is to variate the external metric $\overline{g}_{\mu\nu}$ together with fields Q_B. However, this way is very cumbersome, and we are going by a more economical one. It is evidently that the identical substitutions of $Q_{B,\alpha}$ and $Q_{B,\alpha\beta}$ does not incorporate an additional external metric. Therefore the new presentation of the initial Lagrangian, namely $\hat{\mathcal{L}}$ in (3.1), does not contain $\overline{g}_{\mu\nu}$ and its derivatives in whole. This means that finally variation of $\hat{\mathcal{L}}$ has to be transformed into the identity (2.4) anyway. Thus, we follow the inverse way. Using (3.1), we represent (2.4) into an explicitly covariant form, and then obtain covariant identities.

So, the identity (2.4) is covariant in whole since it has been obtained from the covariant identity (2.3) directly and conserving all the terms. Now, turn to the identity (2.12). It is known that the Lagrangian derivative of the scalar density is covariant. Then it is not difficult to show that (2.12) is covariant and has also a covariant form. After that one concludes that the identity (2.13), the same as (2.4), is covariant in whole. Now, substitute $\partial_\rho \xi^\sigma = \overline{D}_\rho \xi^\sigma - \xi^\sigma|^{\alpha}_{\beta}\, \overline{\Gamma}{}^{\beta}_{\rho\alpha}$ into (2.13) and rewrite it as

$$-\partial_\alpha \left[\hat{u}_\sigma{}^{\alpha} \xi^\sigma + \hat{m}_\sigma{}^{\alpha\tau} \overline{D}_\tau \xi^\sigma + \hat{n}_\sigma{}^{\alpha\tau\beta} \overline{D}_{\beta\tau} \xi^\sigma\right] \equiv \partial_\alpha \hat{\imath}^\alpha \equiv \overline{D}_\alpha \hat{\imath}^\alpha \equiv 0 \tag{3.2}$$

where the expression under divergence $\hat{\imath}^\alpha$ we call as a current. It is a vector density because the coefficients in the terms of the covariantized Lagrangian (3.1) are covariant evidently

$$\begin{aligned}
\hat{u}_\sigma{}^{\alpha} \;&\equiv\; \hat{\mathcal{L}}\delta^\alpha_\sigma + \frac{\delta \hat{\mathcal{L}}}{\delta Q_B}\, Q_B|^{\alpha}_{\sigma} - \left[\frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\alpha}} - \overline{D}_\beta \left(\frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\alpha\beta}}\right)\right] \overline{D}_\sigma Q_B \\
&- \;\frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\beta\alpha}} \overline{D}_{\beta\sigma} Q_B + \frac{1}{2} \frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\tau\alpha}}\, Q_B|^{\beta}_{\lambda}\, \overline{R}{}^{\lambda}{}_{\sigma\tau\beta}\,,
\end{aligned} \tag{3.3}$$

$$\hat{m}_\sigma{}^{\alpha\tau} \;\equiv\; \left[\frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\alpha}} - \overline{D}_\beta \left(\frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\alpha\beta}}\right)\right] Q_B|^{\tau}_{\sigma} - \frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\tau\alpha}} \overline{D}_\sigma Q_B + \frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\beta\alpha}} \overline{D}_\beta (Q_B|^{\tau}_{\sigma})\,, \tag{3.4}$$

$$\hat{n}_\sigma{}^{\alpha\tau\beta} \;\equiv\; \frac{1}{2}\left[\frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\beta\alpha}}\, Q_B|^{\tau}_{\sigma} + \frac{\partial \hat{\mathcal{L}}}{\partial Q_{B;\tau\alpha}}\, Q_B|^{\beta}_{\sigma}\right]. \tag{3.5}$$

Opening (3.2) and equating independently to zero the coefficients at ξ^σ, $\overline{D}_\alpha \xi^\sigma$, $\overline{D}_{(\beta\alpha)} \xi^\sigma$ and $\overline{D}_{(\gamma\beta\alpha)} \xi^\sigma$, we get a set of identities:

$$\overline{D}_\alpha \hat{u}_\sigma{}^{\alpha} + \tfrac{1}{2} \hat{m}_\lambda{}^{\alpha\rho} \overline{R}{}_{\sigma}{}^{\lambda}{}_{\rho\alpha} + \tfrac{1}{3} \hat{n}_\lambda{}^{\alpha\rho\gamma} \overline{D}_\gamma \overline{R}{}_{\sigma}{}^{\lambda}{}_{\rho\alpha} \equiv 0, \tag{3.6}$$

$$\hat{u}_\sigma{}^\alpha + \overline{D}_\lambda \hat{m}_\sigma{}^{\lambda\alpha} + \hat{n}_\lambda{}^{\tau\alpha\rho} \overline{R}_\sigma{}^\lambda{}_{\rho\tau} + \tfrac{2}{3} \hat{n}_\sigma{}^{\lambda\tau\rho} \overline{R}^\alpha{}_{\tau\rho\lambda} \equiv 0, \tag{3.7}$$

$$\hat{m}_\sigma{}^{(\alpha\beta)} + \overline{D}_\lambda \hat{n}_\sigma{}^{\lambda(\alpha\beta)} \equiv 0, \tag{3.8}$$

$$\hat{n}_\sigma^{(\alpha\beta\gamma)} \equiv 0. \tag{3.9}$$

One can be convinced that the system (3.7) - (3.9) consists of linear combinations of the Klein identities (2.9) - (2.11). The identity (3.6) is not independent - it is a consequence of (3.7) - (3.9), the same as $\partial_\alpha \hat{U}^\alpha_\sigma \equiv 0$ is a consequence of (2.9) - (2.11). Since the equality (3.2) is identically satisfied, the current $\hat{\imath}^\alpha$ must be a divergence of a superpotential:

$$\hat{\imath}^\alpha \equiv \partial_\beta \hat{\imath}^{\alpha\beta}, \tag{3.10}$$

for which $\partial_{\beta\alpha} \hat{\imath}^{\alpha\beta} \equiv 0$. Indeed, substituting $\hat{u}_\sigma{}^\alpha$ from (3.7) into the current $\hat{\imath}^\alpha$, using (3.8) and algebraic properties of $\hat{n}_\sigma{}^{\alpha\beta\gamma}$ and $\overline{R}^\alpha{}_{\beta\rho\sigma}$, and conserving the covariance, we reconstruct $\hat{\imath}^\alpha$ into the form (3.10), where the superpotential is

$$\hat{\imath}^{\alpha\beta} = \left(\tfrac{2}{3} \overline{D}_\lambda \hat{n}_\sigma{}^{[\alpha\beta]\lambda} - \hat{m}_\sigma{}^{[\alpha\beta]}\right) \xi^\sigma - \tfrac{4}{3} \hat{n}_\sigma{}^{[\alpha\beta]\lambda} \overline{D}_\lambda \xi^\sigma. \tag{3.11}$$

It is explicitly antisymmetric in α and β (antisymmetrical tensor density).

4 Another variant of covariantization.

In Sect. 2, all the expressions are derived through partial derivatives. The order of partial derivatives is not important because they are symmetrical with respect to replacements. For example, expressions, like $\partial \hat{L} / \partial Q_{B,\alpha\beta}$, are symmetrical in α and β. Nevertheless, in previous subsection, we *did not used* the symmetry of partial derivatives, *conserving an original order* of derivatives in the identities. This has permitted us to present the covariant versions of identities and conserved quantities. However, there are another possibilities. Let us consider an arbitrary Lagrangian $\hat{L}^{test} = \hat{P}^{B\alpha\beta} Q_{B,\alpha\beta} + \ldots$. After direct covariantization it acquires the form $\hat{\mathcal{L}}^{test} = \hat{P}^{B\alpha\beta} Q_{B;\alpha\beta} + \ldots$. The variation with respect to $Q_{B,\alpha\beta}$ gives $P^{B(\alpha\beta)}$, whereas, the variation with respect to $Q_{B;\alpha\beta}$ gives simply $P^{B\alpha\beta}$ that is not necessarily symmetrical in α and β. Thus, another order of covariant derivatives can lead to a different result.

Let us change the order of second covariant derivatives in (3.1):

$$\begin{aligned} \hat{L}(Q_B,\, Q_{B,\alpha},\, Q_{B,\alpha\beta}) &\equiv \hat{\mathcal{L}}(Q_B,\, Q_{B;\alpha},\, Q_{B;\alpha\beta},\, \overline{g}_{\mu\nu},\, \overline{R}^\alpha{}_{\mu\beta\nu}) \\ &\equiv \hat{\mathcal{L}}(Q_B,\, Q_{B;\alpha},\, Q_{B;\beta\alpha} + Q_B|^\rho_\sigma\, \overline{R}_\rho{}^\sigma{}_{\alpha\beta},\, \overline{g}_{\mu\nu},\, \overline{R}^\alpha{}_{\mu\beta\nu}) \\ &\equiv \hat{\mathcal{L}}^*(Q_B,\, Q_{B;\alpha},\, Q_{B;\alpha\beta},\, \overline{g}_{\mu\nu},\, \overline{R}^\alpha{}_{\mu\beta\nu})\,. \end{aligned} \tag{4.1}$$

It is clear that the Lagrangian acquires the *other form* that is accented by the star. Then derivatives with respect to second covariant derivatives of Q_B change their order, also one obtains an additional derivative with respect to Q_B, proportional to the Riemannian tensor. Although the exchange in (4.1) does not change the equations of motion, this leads to different conserved quantities. Below we describe this.

Because the Lagrangian $\hat{\mathcal{L}}^*$ is a scalar density, like $\hat{\mathcal{L}}$, one can construct coefficients, like (3.5), (3.4) and (3.3), which are connected with the last by

$$\hat{n}^{*\alpha\tau\beta}_{\sigma} \equiv \hat{n}_{\sigma}{}^{\alpha\tau\beta} + \frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\beta]}} \, Q_B|^{\tau}_{\sigma} + \frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\tau]}} \, Q_B|^{\beta}_{\sigma}\,. \tag{4.2}$$

$$\hat{m}^{*\alpha\tau}_{\sigma} \equiv \hat{m}_{\sigma}{}^{\alpha\tau} - 2\frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\tau]}}\overline{D}_{\sigma}Q_B + 2\overline{D}_{\beta}\left(\frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\beta]}} \, Q_B|^{\tau}_{\sigma}\right). \tag{4.3}$$

$$\hat{u}^{*\alpha}_{\sigma} \equiv \hat{u}_{\sigma}{}^{\alpha} - 2\overline{D}_{\beta}\left(\frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\beta]}}\overline{D}_{\sigma}Q_B\right) + \frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\tau]}} \, Q_B|^{\beta}_{\lambda}\,\overline{R}^{\lambda}{}_{\sigma\tau\beta}\,. \tag{4.4}$$

A direct substitution of (4.2) - (4.4) into the identities (3.6) - (3.9) shows that n^*, m^* and u^* satisfy them also. In the result, a starred current is

$$\begin{aligned} \hat{\imath}^{*\alpha} &= -\left[\hat{u}^{*\alpha}_{\sigma}\xi^{\sigma} + \hat{m}^{*\alpha\tau}_{\sigma}\overline{D}_{\tau}\xi^{\sigma} + \hat{n}^{*\alpha\tau\beta}_{\sigma}\overline{D}_{\beta\tau}\xi^{\sigma}\right] \\ &= \hat{\imath}^{\alpha} - 2\overline{D}_{\beta}\left(\frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\beta]}}\pounds_{\xi}Q_B\right). \end{aligned} \tag{4.5}$$

Then $\partial_\alpha\hat{\imath}^{*\alpha} \equiv \partial_\alpha\hat{\imath}^{\alpha}$, and consequently $\partial_\alpha\hat{\imath}^{*\alpha} \equiv 0$. Analogously to (3.10), the identity

$$\hat{\imath}^{*\alpha} \equiv \partial_{\beta}\hat{\imath}^{*\alpha\beta}; \tag{4.6}$$

$$\hat{\imath}^{*\alpha\beta} = \left(\tfrac{2}{3}\overline{D}_{\lambda}\hat{n}^{*[\alpha\beta]\lambda}_{\sigma} - \hat{m}^{*[\alpha\beta]}_{\sigma}\right)\xi^{\sigma} - \tfrac{4}{3}\hat{n}^{*[\alpha\beta]\lambda}_{\sigma}\overline{D}_{\lambda}\xi^{\sigma}\,. \tag{4.7}$$

exists. Superpotential (4.7), corresponding to (4.5) is

$$\hat{\imath}^{*\alpha\beta} = \hat{\imath}^{\alpha\beta} - 2\frac{\partial\hat{\mathcal{L}}}{\partial Q_{B;[\alpha\beta]}}\pounds_{\xi}Q_B. \tag{4.8}$$

Because both of possibilities of covariantization in (4.1) have equal rights we suggest a covariantized Lagrangian of the uniform form:

$$\hat{L}(Q_B,\, Q_{B,\alpha},\, Q_{B,\alpha\beta}) \equiv \hat{\mathcal{L}}^{\dagger}(Q_B,\, Q_{B;\alpha},\, Q_{B;\alpha\beta},\, \overline{g}_{\mu\nu},\, \overline{R}^{\alpha}{}_{\mu\beta\nu}) \equiv p\,\hat{\mathcal{L}} + q\,\hat{\mathcal{L}}^* \tag{4.9}$$

where $p+q = 1$ with real p and q. The Lagrangian (4.9) leads to the original field equations, whereas the conservation law for (4.9):

$$\hat{\imath}^{\dagger\alpha} \equiv \partial_{\beta}\hat{\imath}^{\dagger\alpha\beta} \tag{4.10}$$

presents a family of identically conserved quantities:

$$\hat{\imath}^{\dagger\alpha} \equiv p\,\hat{\imath}^{\alpha} + q\,\hat{\imath}^{*\alpha}, \tag{4.11}$$

$$\hat{\imath}^{\dagger\alpha\beta} \equiv p\,\hat{\imath}^{\alpha\beta} + q\,\hat{\imath}^{*\alpha\beta}. \tag{4.12}$$

5 Arbitrary D-dimensional metric theories

To present D-dimensional metric theory we consider the Lagrangian:

$$\hat{L}_D = -\frac{1}{2\kappa_D}\hat{L}_g(g_{\mu\nu}) + \hat{L}_m(g_{\mu\nu}, \Phi)\,, \tag{5.1}$$

which depends on the metric $g_{\mu\nu}$ and Φ and their derivatives up to a second order, where Φ defines matter sources without concretization. Thus $\hat{L}_g$ can be thought as an algebraic function of the metric and Riemannian tensors, $\hat{L}_g(g_{\mu\nu}) = \hat{L}_g(g_{\mu\nu}, R^\alpha{}_{\rho\beta\sigma})$, that can be arbitrary. Variation of (5.1) with respect to $g^{\mu\nu}$ leads to the gravitational equations:

$$\hat{\mathcal{G}}_{\mu\nu} = \kappa_D \hat{T}_{\mu\nu}\,. \tag{5.2}$$

Variation of (5.1) with respect to Φ gives corresponding matter equations. Below we will use also the background Lagrangian defined as $\overline{\hat{L}}_D = \hat{L}_D(\bar{g}_{\mu\nu}, \overline{\Phi})$ and corresponding background gravitational equations (barred (5.2) ones). The background fields $\bar{g}_{\mu\nu}$ and $\overline{\Phi}$ satisfy the background equations and, thus, are known (fixed).

Now, the subject of our attention is the gravitational part of the Lagrangian (5.1). Basing on the results of previous section, we set $Q^A = \{g_{\mu\nu}\}$ and incorporate an external metric $\overline{g}_{\mu\nu}$ into $\hat{L}_g$ in (5.1). We change partial derivatives by covariant derivatives defined with respect to $\overline{g}_{\mu\nu}$. Thus, we transform the pure metric Lagrangian $\hat{L}_g$ into an explicitly covariant form:

$$\hat{L}_g = \hat{\mathcal{L}}_g = \hat{\mathcal{L}}_g(g_{\mu\nu},\, g_{\mu\nu;\alpha},\, g_{\mu\nu;\alpha\beta},\, \overline{g}_{\mu\nu},\, \overline{R}^\lambda{}_{\tau\rho\sigma}). \tag{5.3}$$

Next, we derive the coefficients (3.3) - (3.5) for the Lagrangian $\hat{\mathcal{L}} = -\hat{\mathcal{L}}_g/2\kappa_D$, setting there $Q^A = \{g_{\mu\nu}\}$. We calculate directly n_g and m_g coefficients, and present u_g coefficient in a structured form:

$$\hat{u}_{g\sigma}{}^\alpha = -\frac{1}{\kappa_D}\left[\hat{\mathcal{G}}^\alpha_\sigma + \kappa_D \hat{\mathcal{U}}_\sigma{}^\alpha + \kappa_D \hat{n}_\lambda{}^{\alpha\tau\beta}\overline{R}^\lambda{}_{\tau\beta\sigma}\right]\,; \tag{5.4}$$

$$\hat{\mathcal{G}}^\alpha_\sigma \;\equiv\; \frac{1}{2}\frac{\delta\hat{\mathcal{L}}_g}{\delta g_{\mu\nu}}\, g_{\mu\nu}|^\alpha_\sigma \equiv -\frac{\delta\hat{\mathcal{L}}_g}{\delta g_{\mu\alpha}} g_{\mu\sigma} \equiv \frac{\delta\hat{\mathcal{L}}_g}{\delta g^{\mu\sigma}} g^{\mu\alpha}\,, \tag{5.5}$$

$$\hat{\mathcal{U}}_\sigma{}^\alpha \;\equiv\; -\frac{1}{2\kappa_D}\left(\frac{\partial\hat{\mathcal{L}}_g}{\partial g_{\mu\nu;\beta\alpha}}\overline{D}_{\sigma\beta}g_{\mu\nu} + \frac{\delta\hat{\mathcal{L}}_g}{\delta g_{\mu\nu;\alpha}}\overline{D}_\sigma g_{\mu\nu} - \delta^\alpha_\sigma\hat{\mathcal{L}}_g\right)\,. \tag{5.6}$$

As usual, $\delta\hat{\mathcal{L}}_g/\delta g_{\mu\nu}$ means Lagrangian derivatives, $\hat{\mathcal{G}}^\alpha{}_\sigma$ is exactly the left hand side of (5.2), and $\hat{\mathcal{U}}^\alpha{}_\sigma$ is the canonical energy-momentum related to the gravitational Lagrangian (5.3).

Incorporation of the background metric is a key point, basing on which one has a possibility to describe perturbations. Perturbations are determined by the way when a one solution (dynamical) of the theory is considered as a perturbed system with respect to another solution (background) of the same theory. Then the background spacetime acquires a *real* sense, not auxiliary. Perturbations in such a derivation are exact, not infinitesimal or approximate ones. We denote δ as an exact difference between dynamical and background quantities: $\delta Q^A = Q^A - \overline{Q^A}$.

Following to the Katz-Bičák-Lynden-Bell ideology [12] we construct the metric Lagrangian for perturbations:

$$\hat{\mathcal{L}}_G = -\frac{1}{2\kappa_D}\left(\hat{\mathcal{L}}_g - \overline{\hat{\mathcal{L}}}_g + \partial_\alpha \hat{d}^\alpha\right). \tag{5.7}$$

By the definition, the Lagrangian has to vanish for vanishing perturbations, therefore usually $\hat{d}^\alpha$ disappears for vanishing perturbations. Substituting n_g, m_g and u_g into $\imath^\alpha$, applying the barred procedure, subtracting one from another and taking into account the divergence, one obtains the current corresponding to (5.7): $\delta\hat{\imath}^\alpha = \hat{\imath}^\alpha - \overline{\hat{\imath}^\alpha} + \hat{\imath}^\alpha_d$. Then we use the dynamical equations (5.2) in $\hat{u}_{g\sigma}{}^\alpha$. We change $\hat{\mathcal{G}}_{\mu\nu}$ (as a part of $\hat{u}_{g\sigma}{}^\alpha$, see (5.4)) by the matter energy-momentum $\hat{T}_{\mu\nu}$ at the right hand side of (5.2). Next, we do the same combining $\overline{\hat{u}}_{g\sigma}{}^\alpha$ and the barred equations. In the result one obtains that the identically conserved current $\delta\hat{\imath}^\alpha$ related to (5.7) transforms into the current for perturbations:

$$\hat{\mathcal{I}}^\alpha(\xi) = \hat{\Theta}_\sigma{}^\alpha\xi^\sigma + \hat{\mathcal{M}}^{\sigma\alpha\beta}\partial_{[\sigma}\xi_{\beta]} + \hat{\mathcal{Z}}^\alpha(\xi) \tag{5.8}$$

where Z-term disappears, if vector ξ^α is a background Killing vector. Now, the conservation law: $\partial_\alpha\hat{\mathcal{I}}^\alpha(\xi) = 0$ takes a place due to the field equations, *not identically*. The generalized canonical energy-momentum tensor density and spin-term for perturbations are

$$\hat{\Theta}_\sigma{}^\alpha \equiv \delta\hat{T}^\alpha_\sigma + \delta\hat{\mathcal{U}}_\sigma{}^\alpha + \kappa_D^{-1}\overline{D}_\beta(\delta^{[\alpha}_\sigma \hat{d}^{\beta]}), \tag{5.9}$$

$$\hat{\mathcal{M}}^{\sigma\alpha\beta} \equiv \delta\hat{m}_{g\rho}{}^{\alpha\beta}\bar{g}^{\sigma\rho} - \kappa_D^{-1}\bar{g}^{\sigma[\alpha}\hat{d}^{\beta]}, \tag{5.10}$$

To present the concrete value of the current (5.8), one has to use solutions to both the dynamic and background equations. Next, starting from (3.11), by the same way we construct a superpotential corresponding to the current (5.8):

$$\begin{aligned}
\delta\hat{\imath}^{\alpha\beta} &= \hat{\imath}^{\alpha\beta} - \overline{\hat{\imath}^{\alpha\beta}} + \hat{\imath}'^{\alpha\beta} \rightarrow \\
\hat{\mathcal{I}}^{\alpha\beta} &= \left(\tfrac{2}{3}\overline{D}_\lambda\delta\hat{n}_{g\sigma}{}^{[\alpha\beta]\lambda} - \delta\hat{m}_{g\sigma}{}^{[\alpha\beta]} - \frac{2\delta^{[\alpha}_\sigma\hat{d}^{\beta]}}{\kappa_D}\right)\xi^\sigma - \tfrac{4}{3}\delta\hat{n}_{g\sigma}{}^{[\alpha\beta]\lambda}\overline{D}_\lambda\xi^\sigma.
\end{aligned} \tag{5.11}$$

Then, the conservation law for perturbations acquires the form: $\hat{\mathcal{I}}^\alpha(\xi) = \partial_\alpha\hat{\mathcal{I}}^{\alpha\beta}(\xi)$. It is not identity, but the conservation law determined by the solutions to the field equations. Analogously the starred conservation law can be constructed. Thus, more generally, the family of the Nœther canonical conservation laws for perturbations corresponding to the presentation (4.9) has a form:

$$\hat{\mathcal{I}}^{\dagger\alpha}(\xi) = \partial_\alpha\hat{\mathcal{I}}^{\dagger\alpha\beta}(\xi). \tag{5.12}$$

6 Applications

It turns out that in GR generally there is no a difference between starred and non-starred quantities. Therefore for GR the conservation law (5.12) is a single one, not a family. To examine a real family (5.12) it is necessary to consider a more complicated theory, like the Einstein-Gauss-Bonnet (EGB) gravity.

Earlier, in [4] -[6], just the *starred* variant of conservation laws for perturbations based on the identities (4.6) has been suggested. Basing on the correspondent (starred) superpotentials, the mass of the Schwarzschild-anti-de Sitter (S-AdS) black hole [13] has ben obtained, it coincides exactly with the standard results. In [9], we have calculated the mass for the S-AdS black hole in EGB gravity with the use of the generalized superpotentials in (5.12). Now, we shortly repeat these results.

The conservation law (5.12) allows us to construct the conserved charges in generalized form in D-dimensions:

$$\mathcal{P}(\xi) = \int_{\Sigma} d^{D-1}x\, \hat{\mathcal{I}}^{\dagger 0}(\xi) = \oint_{\partial\Sigma} dS_i\, \hat{\mathcal{I}}^{\dagger 0i}(\xi)\,. \tag{6.1}$$

Represent the superpotentials in (5.12) in the form:

$$\hat{\mathcal{I}}^{\dagger\alpha\beta} = \hat{\mathcal{I}}^{*\alpha\beta} + p\left[(\hat{\imath}^{\alpha\beta} - \hat{\imath}^{*\alpha\beta}) - (\overline{\hat{\imath}^{\alpha\beta}} - \overline{\hat{\imath}^{*\alpha\beta}})\right] = \hat{\mathcal{I}}^{*\alpha\beta} + p\Delta\hat{\mathcal{I}}^{\alpha\beta}\,. \tag{6.2}$$

The expressions in square brackets are defined by the formula (4.8). Because for the S-AdS black hole the starred superpotentials already have been checked we need only to calculate the contribution defied by $\Delta\mathcal{P}(\xi) = p\oint_{\partial\Sigma} dS_i\, \Delta\hat{\mathcal{I}}^{0i}(\xi)$. Although generally in EGB gravity

$$\frac{\partial\hat{\mathcal{L}}_{EGB}}{\partial g_{\mu\nu;[\alpha\beta]}} = \frac{2\alpha\sqrt{-g}}{\kappa_D}\left(R^{\alpha(\mu}g^{\nu)\beta} - R^{\beta(\mu}g^{\nu)\alpha}\right) \tag{6.3}$$

that is important (see (4.8)), in the case of the S-AdS black hole one has $\hat{\imath}^{\alpha\beta} - \hat{\imath}^{*\alpha\beta} \equiv 0$. This means that, calculating mass of the S-AdS black hole in the EGB gravity, all the superpotentials of the new family give the same standard result. It is not surprisingly. Indeed, in the Deruelle, Katz and Ogushi paper [1] in the EGB gravity it was constructed the *non-starred* superpotential of the type (3.11). Calculating the mass of the S-AdS black hole on its basis gives also the standard result.

Thus, all the superpotentials of the family give the same standard accepted mass for the S-AdS black hole. However, numerus solutions of popular gravitational theories frequently have very exotic properties. Therefore, wider possibilities to study such solutions are desirable, and, in this relation, the suggested family presents a more universal instrument. We do not exclude the situation when any solution any modified theory of gravity could be a crucial test solution for a choice between members (conserved quantities) of the family. We plan such applications in future.

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The properties and specific features of the field-theory equations. The structure of physical fields

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The existing field theories are based on the properties of closed exterior skew-symmetric differential forms, which correspond to the conservation laws for physical fields. Such conservation laws are those that claim an existence of conservative quantities or objects. It is shown that closed exterior forms of field theories are obtained from the equations of conservation laws for energy, linear momentum, angular momentum, and mass, which are conservation laws for material systems.This demonstrate the connection between the field-theory equations and the equations for material systems and discloses the general foundations of field theories and the properties of physical fields.

Introduction

The results of present paper were obtained using the properties of conservation laws for physical fields and material systems. It turns out that physical fields and material systems are governed by the different conservation laws**;** a certain connection between these laws can be realized.

The physical fields obey conservation laws that are described by closed exterior forms. Such conservation laws are those that claim an existence of conservative quantities or objects. Whereas the conservation laws for energy, linear momentum and angular momentum, which, unlike the conservation laws for physical fields, are described by differential equations, are conservation laws for material systems.

From the equations of conservation laws for material systems one gets the nonidentical evolutionary relation, which includes the unclosed evolutionary skew-symmetric form [1]. Under degenerate transformation the closed exterior forms can be realized from the evolutionary form. Such closed exterior forms can be the solutions to the field-theory equations, which describe the physical fields. This follows from the fact that there is a correspondence between the field-theory equations and the nonidentical evolutionary relation obtained from the equations for material systems. It has been shown that the field-theory equations are just such evolutionary relations or its analogs. Such a correspondence discloses the connection between the field-theory equations and the equations for material systems, and this points to the fact that physical fields and material systems are connected.

Closed exterior forms and relevant dual forms obtained from the equations for material systems made up differential-geometric structures, which describe physical structures, namely, the pseudostructures (dual forms) with conservative quantities (closed exterior forms). It is evident that physical fields are built by such physical structures.

1. Closed inexact exterior forms as the basis of field theories

The equations of existing field theories are those obtained using the properties of closed exterior skew-symmetric differential forms. Such a role of closed exterior forms is explained by the fact that they correspond to the conservation laws.

From the closure conditions for exterior differential form

$$d\theta^k = 0 \qquad (1)$$

one can see that the closed exterior differential form is a conservative quantity (θ^k is the exterior differential form of degree k (k -form)). This means that it can correspond to conservation law, namely, to existence of a certain conservative physical quantity.

If the exterior form is a closed inexact form, i.e. is closed only on pseudostructure, the closure condition is written as

$$d_\pi \theta^k = 0 \qquad (2)$$

And the pseudostructure π obeys the condition

And the pseudostructure π obeys the condition

$$d_\pi {}^*\theta^k = 0$$

where ${}^*\theta^k$ is the dual form. From conditions (2) and (3) one can see that the closed exterior form and the dual form constitute a conservative object, namely, the pseudostructure with conservative quantity. Hence, such an object can correspond to some conservation law. Such conservation law is that for physical fields. (Below they will be referred to as ``exact" ones.)

The closed inexact exterior or dual forms are solutions to the field-theory equations. And there is the following correspondence.

-Closed exterior forms of zero degree correspond to quantum mechanics.

-The Hamilton formalism bases on the properties of closed exterior and dual forms of first degree.

-The properties of closed exterior and dual forms of second degree are at the basis of the equations of electromagnetic field.

-The closure conditions of exterior and dual forms of third degree made up the basis of equations for gravitational field. (It should be noted that the field-theory equations have nonunique solution.)

One can see that the field theory equations are connected with closed exterior forms of a certain degree. This allows to introduce a classification of physical fields and interactions in degrees of closed exterior forms. If to denote the degree of closed exterior forms by k , then $k = 0$ corresponds to strong interaction, $k = 1$ does to weak one, $k=2$ does to electromagnetic one, and $k = 3$ corresponds to gravitational interaction. Such a classification shows that there exists an internal connection between field theories that describe physical fields of various types. It is evident that the degree of closed exterior form is a parameter that integrates field theories into unified field theory.

A significance of exterior differential forms for field theories consists in the fact that they disclose the properties that are common for all field theories and physical fields irrespective to their specific type.

Below it will be shown that closed exterior forms corresponding to field theories are obtained from the equations of conservation laws for energy, linear momentum, angular momentum, and mass, which are conservation laws for material systems. (This allows to disclose the connection between physical fields and material systems.)

2. The equations of conservation laws for material systems

The conservation laws for material systems (as opposed to the conservation law for physical fields) are conservation laws for energy, linear momentum, angular momentum, and mass, which establish the balance between the variation of a physical quantity and the corresponding external action. (Below they will be referred to as the balance conservation laws.) These are conservation laws for material systems such as thermodynamic and gas dynamical systems, systems of charged particles, cosmic systems, systems of elementary particles and others. [The material system is a variety of elements which have internal structure and interact to one another. Examples of elements that made up a material system are fluid particles, cosmic objects, electrons, protons, atoms and others.]

It turns out that, even without the knowledge of concrete form of these equations, with the help of skew-symmetric differential forms one can see the specific features of these equations that elucidate the properties of balance conservation laws and their role in evolutionary processes.

The equations of balance conservation laws are differential (or integral) equations that describe the variation of functions corresponding to physical quantities [2, 3]. The solutions (functions) to the equations for material media sought are usually functions which are related to such physical quantities like the particle velocity (of elements), temperature or energy, pressure and density. Since these functions relate to one material system, it has to exist a connection between them. This connection is described by state functional that specifies the material system state. The action functional, entropy, the Pointing vector, Einstein tensor, wave function and others can be regarded as examples of such functionals [4].

From the equations of balance conservation law it follows the evolutionary relation for functionals, which enables one to disclose the mechanism of generation of closed inexact exterior form corresponding to field theories.

2.1. Evolutionary relation

The functional properties and specific features of differential equations or sets of equations depend on whether or not the derivatives of differential equations or the equations in the sets of differential equations are consistent.

Let us now analyze the consistency of the equations that describe the conservation laws for energy and linear momentum.

In the accompanying frame of reference, the equation for energy is written in the form

$$\frac{\partial \psi}{\partial \xi^1} = A_1 \qquad (3)$$

Here ξ^1 are the coordinates along the trajectory, ψ is the functional of the state, A_1 is the quantity that depends on specific features of the material system and external (with respect to the local domain) energetic actions onto the system [5, 6]. [Thus, accounting for the fact that the total derivative with respect to time is that along the trajectory, the energy equation expressed in terms of the action functional S has the following form: $DS/Dt = L$, where $\psi = S$ and $A_1 = L$ is the Lagrange function. The energy equation for ideal gas can be presented in the form: $Ds/Dt = 0$, where s is the entropy [5].]

Similarly, in the accompanying frame of reference the equation for linear momentum appears to be reduced to the equation of the form

$$\frac{\partial\psi}{\partial\xi^{\nu}} = A_{\nu}, \quad \nu = 2,... \tag{4}$$

where ξ^{ν} are the coordinates along the direction normal to the trajectory, A_{ν} are the quantities that depend on the specific features of material system and external force actions.

Equations (3) and (4) can be convoluted into the relation

$$d\psi = A_{\mu} d\xi^{\mu}, \quad \mu = 1,\nu \tag{5}$$

Relation (5) can be written as

$$d\psi = \omega \tag{6}$$

Here $\omega = A_{\mu} d\xi^{\mu}$ is the skew-symmetric differential form of the first degree.

Since the balance conservation laws are evolutionary ones, the skew-symmetric differential form ω and the relation obtained are also evolutionary ones.

Relation (6) was obtained from the balance conservation law equations for energy and linear momentum. In this relation the form ω is that of the first degree. If the balance conservation law equation for angular momentum be added to the equations for energy and linear momentum, this form in the evolutionary relation will be a form of the second degree. And in combination with the equation of the balance conservation law for mass this form will be a form of degree 3.

Thus, in the general case, the evolutionary relation can be written as

$$d\psi = \omega^{p} \tag{7}$$

where the form degree p takes the values $p = 0,1,2,3$. (The evolutionary relation for $p = 0$ was obtained from the equations for energy and time.) (A concrete form of relation (6) and its properties were considered in papers [6, 7]).

2.1.1. Nonidentity of evolutionary relation. Noncommutativity of the balance conservation laws

Evolutionary relation obtained from the equation proves to be nonidentical since the skew-symmetric differential form in the right-hand side of this relation is not a closed form, and, hence, this form can not be a differential like the left-hand side.

Let us analyze the relation (6).

The form $\omega = A_\mu d\xi^\mu$ isn't a close form since its differential is nonzero. The differential $d\omega$ can be written as $d\omega = K_{\alpha\beta} d\xi^\alpha d\xi^\beta$, where $K_{\alpha\beta} = \partial A_\beta / \partial\xi^\alpha - \partial A_\alpha / \partial\xi^\beta$ are the components of the differential form commutator built of the mixed derivatives (here the term related to the nonintegrability of the manifold has not yet been taken into account). The coefficients A_μ of the form ω can be obtained from the equation of the balance conservation law for energy or from that for linear momentum. This means that in the first case the coefficients depend on the energetic action and in the second case they depend on the force action. In actual processes energetic and force actions have different nature and appear to be inconsistent. The commutator of the form ω constructed from the derivatives of such coefficients is nonzero. The differential of the form ω is nonzero as well. Thus, the form ω proves to be unclosed and cannot be a differential. *This means that the evolutionary relation cannot be an identical one*. In the left-hand side of this relation it stands a differential, whereas in the right-hand side it stands an unclosed form that is not a differential. The nonidentity of the evolutionary relation is the result of the fact that the balance conservation law equations turn out to be inconsistent. [The skew-symmetric form in evolutionary relation is defined on the manifold made up by trajectories of the material system elements. Such a manifold is a deforming manifold. The commutator of skew-symmetric form defined on such manifold includes an additional term connected with the differential of basis. This term specifies the manifold deformation and hence is nonzero. Both terms of the commutator (obtained by differentiating the basis and the form coefficients) have a different nature and, therefore, cannot compensate one another. This fact once more emphasize that the evolutionary form commutator, and, hence, its differential, are nonzero. That is, the evolutionary form remains to be unclosed.]

The nonidentity of evolutionary relation means that the balance conservation law equations are noncommutative. Noncommutativity of the balance conservation laws reflects the state of material system. Since the evolutionary relation is nonidentical, from this relation one cannot get the differential of the state functional $d\psi$. This means that the functional ψ is not a state function. And this points to the fact that the material system is in nonequilibrium state. It is evident that the internal force producing such nonequilibrium state is described by the evolutionary form commutator.

2.2 Generation of closed inexact exterior form

The evolutionary nonidentical relation is a selfvarying one (this relation includes two objects one of which appears to be nonmeasurable). During selfvariation of evolutionary relation the conditions when an inexact (closed on *pseudostructure*) exterior form is obtained from evolutionary form can be realized. However, the transition from unclosed evolutionary form (with nonzero differential) to closed exterior form (with vanishing differential) is possible only as degenerate transformation, namely, a transformation that does not conserve the differential. The degenerate transformation can take place under additional conditions. [The conditions of degenerate transformation are reduced to vanishing of such functional expressions as determinants, Jacobians, Poisson's brackets, residues and others. They are connected with the symmetries, which can be due to the degrees of freedom (for example, the translational, rotational and oscillatory degrees of freedom of material system).]

The conditions of degenerate transformation are closure conditions of dual form that define the integral structure (the pseudostructure), on which the closed exterior form is realized.

If the conditions of degenerate transformation are realized, it will take place the transition

$$d\omega^p \neq 0 \rightarrow d_\pi \omega^p = 0,\ d_\pi {}^*\omega^p = 0$$

The relations obtained

$$d_\pi \omega^p = 0\ ,\ \ d_\pi {}^*\omega^p = 0 \qquad (8)$$

are closure conditions for exterior inexact form and for dual form. The dual form is a metric form of manifold. The closed dual form describes the pseudostructure π, on which closed inexact (only on pseudostructure) exterior form is defined. (Integral structures and manifolds, such as the characteristics, potential surfaces, eikonal surfaces, singular points, are examples of pseudostructures and relevant manifolds.)

The realization of the closure conditions (8) points to the fact that the exterior form closed on pseudostructure is realized.

Closed inexact exterior forms of the field theories are just such closed exterior forms that were obtained from the equations of the conservation laws for material systems. This follows from the fact that there exists a correspondence between the field-theory equations and the evolutionary relation.

3. The correspondence between the field-theory equations and the evolutionary relation

As it has been shown in section 1, the solutions to the field-theory equations are closed inexact exterior forms, i.e. they are differentials. Only the equations that have the form of relations (nonidentical) may have the solutions which are differentials rather then functions.

One can verify that all equations of existing field theories have the form of nonidentical relations in differential forms or in the forms of their tensor or differential (i.e. expressed in terms of derivatives) analogs.

The Einstein equation is a relation in differential forms.

The Dirac equation relates Dirac's *bra-* and *cket*- vectors, which made up the differential forms of zero degree.

The Maxwell equations have the form of tensor relations.

The field equation and Schrodinger's one have the form of relations expressed in terms of derivatives or their analogs.

Another specific feature of the field-theory equations consists in the fact that all field-theory equations are nonidentical relations for functionals such as the wave function, action functional, the Pointing vector, Einstein's tensor and so on [4]. (Entropy is such a functional for the fields generated by thermodynamic and gas-dynamical systems.)

The evolutionary relation obtained from the equations for material systems is a nonidentical relation for all these functionals. That is, all field-theory equations are an analog to the evolutionary relation.

From this it follows that closed inexact exterior forms, which are realized from the evolutionary forms in the evolutionary relation, corresponds to closed inexact exterior forms of field theories.

It appears that the closed inexact exterior forms of field theories are generated by the equations of conservation laws for material systems.

Below it will be shown that the generation of such closed exterior forms describes the generation of physical structures that made up physical fields.

3.1. Generation of physical structures. The process of origin of observable formations in material systems

Closed inexact exterior form and relevant closed dual form describe the differential-geometrical structure: the pseudostructure (dual form) and the conservative quantity (closed exterior form). It is evident that the pseudostructures with conservative quantity are structures, on which exact conservation laws are satisfied.

Such structures have a physical meaning, and therefore, they can be named as physical structures.

In section 2 it has been shown that, from evolutionary form obtained from the relations for material systems, closed inexact exterior forms are realized.The realization of the closed dual form and the exterior inexact skew-symmetric form points out to an occurrence of pseudostructure (dual form) with conservative quantity (closed exterior form), i.e. an occurrence of physical structures on which the exact conservation law is fulfilled. The correspondence between the field-theory equations, which describe physical fields, and the equations for material systems, which generate physical structures, points to the fact that such structures are structures that made up physical fields.

From the evolutionary relation it follows that the generation of physical structures is accompanied by origin of some observable formations in material systems.

If the closed (on pseudostructure) exterior form is realized, on the pseudostructure π evolutionary relation (7) converts into the relation

$$d_\pi \psi = \omega_\pi^p \tag{9}$$

which proves to be an identical relation, since the form ω_π^p is a closed one on the pseudostructure and this form turns out to be a differential.

From identical relation one can obtain the differential of the state functional $d_\pi \psi$ and find the state function. This points to that the material system state is an equilibrium state. (But this state is realized only locally since the state differential is interior one defined exclusively on pseudostructure. The total state of material system turns out to be nonequilibrium because the evolutionary relation itself remains to be nonidentical one.)

The transition of material system from nonequilibrium state into the locally equilibrium one means that the nonmeasurable quantity described by the nonzero commutator of the unclosed evolutionary differential form ω^p , which acted as an internal force, converts into the measurable quantity. In material system this reveals as an occurrence of certain observable formation that develops spontaneously. Such formations and their manifestations are fluctuations, turbulent pulsations, waves, vortices, and others. The intensity of such formations is controlled by a quantity accumulated by the evolutionary form commutator. (In paper [6] the process of production of vorticity and turbulence is described.)

One can see that the occurrence of physical structure is connected with the process of origination of observable formation. This fact is also fixed by identical relation (9), which possesses the duality. The left-hand side of identical relation (9) includes the differential, which specifies material system and whose presence points to the locally-equilibrium state of material system. And the right-hand side includes the closed inexact form, which describes physical structure. The existence of the state differential (left-hand side of relation (9)) points to the transition of material system from nonequilibrium state to the locally-equilibrium state (and origination of observable formations). And the emergency of the closed (on pseudostructure) inexact exterior form (right-hand side of relation (9)) points to the origination of the physical structure. [Physical structures and the formations are the manifestation of the same phenomena. The light is an example of such a duality. The light manifests itself in the form of a massless particle (photon) and as a wave. On the other hand, the formation and the physical structure are not identical objects. When the wave be such a formation, the element of wave front made up the physical structure in the process of its motion.]

3.2. Characteristics of physical structures

Since closed inexact exterior forms corresponding to physical structure are obtained from the evolutionary relation for the material system, it follows that the characteristics of physical structure and physical fields are determined by the characteristics of the material system, which generates the physical structures, and the characteristics of evolutionary and closed inexact exterior forms obtained from the equations of balance conservation laws for material system.

The physical structure is an object obtained by conjugating the conservative physical quantity, which is described by inexact closed exterior form, and the pseudostructure, which is described by relevant dual form.

Conservative physical quantity describe a certain charge. Under transition from some structure to another, the conservative on pseudostructure quantity, which corresponds to the closed exterior form, changes discretely, and the pseudostructure changes discretely as well.

Discrete changes of the conservative quantity and pseudostructure are determined by the value of the evolutionary form commutator, which is the commutator at the time instant when the physical structure originates. The first term of the evolutionary form commutator made up by the derivatives of the evolutionary form coefficients controls the discrete change of the conservative quantity. The second one obtained from the derivatives of the metric form coefficients of the initial manifold controls the pseudostructure change. Spin is the example of the second characteristic. Spin is a characteristic that determines the character of the manifold deformation before the origination of physical structure. (The spin value depends on the form degree.)

A discrete change of the conservative quantity and that of the pseudostructure produce the quantum that is obtained when going from one structure to another. The evolutionary form commutator formed at the time instant of the structure origination determine the characteristics of this quantum.

Since closed inexact exterior forms corresponding to physical structure are obtained from the evolutionary relation for material system, it follows that the characteristics of physical structure are determined by the characteristics of the material system that generates these physical structures. The equation of pseudostructure is obtained from the conditions of degenerate transformation, which are due to the degrees of freedom of material system. Vanishing certain functional expressions (like Jacobians, determinants, the Poisson brackets and so on) must correspond to the degenerate transformation.

The characteristics of inexact closed exterior forms are defined by the characteristics of evolutionary forms obtained from the balance conservation laws for material media and, hence, depend on the characteristics of material media.

3.3. Classification of physical structures and physical fields

The connection of the physical structures with the skew-symmetric differential forms allows to introduce a classification of these structures and corresponding physical fields in dependence on parameters that specify skew-symmetric differential forms.

The closed forms that correspond to physical structures are generated by the evolutionary relation having the parameter p that defines the number of interacting balance conservation laws. Therefore, the physical structures can be classified by the parameter p.

The other parameter is a degree of closed forms generated by the evolutionary relation. To determine this parameter, one has to consider the problem of integrability of the nonidentical evolutionary relation.

The identical relation can be directly integrated because it involves closed forms that are differentials. In the integration process the transition to the relation with differential forms of less degrees takes place.

An integration and transitions with lowering the form degree are allowed in nonidentical relations (those with evolutionary forms) as well but only in the case of degenerate transformations. Under the degenerate transformation on the pseudostructure it can be obtained the identical relation that can be integrated, and this enables one to get a relation with the forms of the degree less by one. The relation obtained turns out to be nonidentical as well. By integrating (under realization of relevant degenerate transformation) the nonidentical evolutionary relation with the forms of degree p, one can successively obtain nonidentical relations with the forms of degree $k=0$, where k takes the values from p to 0. At each transition the closed forms on the pseudostructure of sequential degrees $k=p$, $k=p-1$, … , $k=0$ arise, which indicates the creation of physical structures of a relevant type.

One more parameter is the space dimension n. When generating closed forms of sequential degrees $k=p$, $k=p-1$, … , $k=0$ the pseudostructures of dimensions $(n+1-k)$ are obtained.

The parameters of evolutionary and exterior forms p, k, n enables one to introduce the classification of physical structures that defines the type of physical structures and, accordingly, the type of physical fields and interactions [8].

Conclusions

The methods of exterior and evolutionary skew-symmetric differential forms, which reflect the properties of conservation laws for physical fields and material media, allow to disclose the peculiarities of field-theory equations and the properties of physical fields.

The fundamental result that clarifies the problems of field theories is the fact that the field theories, which are based on the conservation laws for physical fields, are connected with the equations of noncommutative conservation laws for material systems (the conservation laws of energy, linear momentum, angular momentum, and mass). This connection has to be taken into account while building the general field theory.

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Gravitational Wave Radiation by Dark Matter Bodies in the Sun

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It is shown that an analysis of the data for the solar surface oscillations observed at the frequency $\nu_{CrAO} = 104.1891$ μHz by Crimean Astrophysical Observatory (CrAO), and at the frequency $\nu_{SoHO} = 220.7$ μHz by Solar Helioseismic Observatory (SoHO) leads to a prediction of detectable Gravitational Waves (GW) radiated by the Sun at the frequencies $2\ \nu_{CrAO}$ and $2\ \nu_{SoHO}$. Main idea of this contribution is based on the fact that the main details of these solar oscillations can be explained as tidal excitations of the solar eigenmodes forced by two compact Dark Matter Bodies (DMB) – DMB_{CrAO} and DMB_{SoHO} with masses $\approx 4\ 10^{-9}\ M_{\Theta}$, and radii $\sim 10^3 km$ circulated under the solar surface at radii $0.97\ R_{\Theta}$ and $0.58\ R_{\Theta}$ correspondingly. These DMB radiate GW with metric tensor amplitudes $h_{CrAO} \sim 2\ 10^{-20}$, and $h_{SoHO} \sim 7\ 10^{-21}$ at the Earth (near-zone) which are strong enough to be detected by the Laser Interferometer Space Antenna (LISA) for time series longer than *40 days* at frequency $2\ \nu_{CrAO}$ and *20 days* at frequency $2\ \nu_{SoHO}$. Thus the suggested experiment allows to solve two fundamental physical problems simultaneously – the direct detection of GW, and the direct detection of two compact DMB (DMB_{CrAO} and DMB_{SoHO}) in the Sun, and, in the case of success, open a new GW epoch in the solar physics.

1. Introduction

Experiments for the direct detection of Gravitational Waves (GW), and the direct detection of Dark Matter (DM) are the most challenging tasks in modern physics. In this contribution a possibility to solve these two fundamental physical problems simultaneously by the detection of GW radiated by compact Dark Matter Bodies (DMB) circulated in the Sun is discussed.
Gravitational Waves (GW) predicted by General Theory of Relativity (GR), are expected to detect within the nearest years. GW in GR are ripples in the curvature of space-time and manifest themselves as fluctuating tidal forces on masses in the path of the wave. GW sources such as compact binary systems, stellar collapses, and pulsars are possible candidates for detection. New unexpected sources are expected to be found as well. The problem for the experimental physicist is that the predicted magnitudes of the amplitudes or strains in space in the vicinity of the Earth caused by gravitational waves even from the most violent astrophysical events are extremely small, of the order of 10^{-21} or lower, because of their distances from the solar system are too large. On the other hand all known near-zone GW radiated in the solar system are too weak or have too low frequencies to be detected. In particular, the measurement bandwidth of the most low frequency GW detector, the Laser Interferometer Space Antenna (LISA), extends from 30 μHz to 100 mHz whereas the Moon-Earth binary system radiates GW at 0.8 μHz. Nevertheless DM could allow to solve this problem in the solar system if Dark Matter Bodies (DMB) can be formed in Universe and captured by the Sun on the orbits under the solar surface where frequencies of GW are limited by the solar mass, radius and central density 200 μHz < ν_{GW} < 2000 μHz in desirable for LISA bandwidth. According to the recent SNe Ia [1] and WMAP [2] data interpretation within the standard model of cosmology, the Universe is thought to contain about 5% of Ordinary Matter (OM), 25% of DM, and 70% of dark energy. All known candidates for the role of DM particles (the mirror particles [3-6], or particles of other extensions of the standard particle model, in particular, gravitino in the partial split supersymmetry with bilinear *R*-parity violation [7]) interact with OM at forces almost as weak as the gravitational force. Therefore it is extremely hard to detect DM particles directly via their interactions with OM. Nevertheless, as a result of the cosmological evolution, interactions of DM particles may lead to a formation of compact astronomical DM Bodies (DMB – DM-planets or DM-stars). Then these DMB may be discovered by their reinforced gravitational interaction with OM. The question is which possible astrophysical objects are the most effective for this task?
The first simple estimations of the Sun surface oscillations as an example of manifestation of DMB orbiting under the Sun surface has been done [8] in applications to the *104 μHz* oscillation first observed in Crimean Astrophysical Observatory (CrAO) [9], and independently in Pic-du-

Midi observatory in France by Birmingham group [10]. Finally, after 37 years of observations (1974-2011) the frequency of these oscillations is measured as $\nu_{CrAO} = 104.1891 \pm 0.0003\ \mu Hz$ [11].

Another solar surface oscillation with frequency $220.7\ \mu Hz$ is seen in VIRGO/SPM setup at Solar Helioseismic Observatory (SoHO) . A long time series of 4098 days of VIRGO/SPM data are available since April 11, 1996 up to June 2007.

It should be noted that the both solar oscillations correspond to the frequencies of solar g-modes and these amplitudes can be resonantly amplified.

Nevertheless, there is no more attempts to consider DMB in the Sun structure possibly because the simplest estimations of amplitudes of solar surface oscillations forced by the DMB are too small to be observed, or because the real existence of the $104\ \mu Hz$ oscillations is not generally accepted mostly because their characteristics are considered as too contradictive for a clear interpretation within helioseismology and solar models.

The purposes of this contribution are i) to show that for a normal solar model (for example, the Christensen-Dalsgaard solar model (S) [19]) amplitudes of the surface oscillations can be calculated as tidal excitations of the solar eigenmodes by the DMB orbiting under the Sun surface, ii) to compare frequencies (ν) and amplitudes of surface velocity oscillations (V) of the solar surface with CrAO [11] data ($\nu_{CrAO} = 104.1891 \pm 0.0003\ \mu Hz$, $V_{CrAO} = 270 \pm 50\ mm/s$), and the SoHO [12] data ($\nu_{SoHO} = 220.7\ \mu Hz$, $V_{SoHO} = 4.5 \pm 1.5\ mm/s$) in order iii) to estimate DMB masses and radii, iv) calculate main characteristics of GW radiated by these DMB in the near-zone in order to propose a new GW experiment for LISA to confirm finally existence of DMB_{CrAO} and DMB_{SoHO} in the solar structure.

2. Tidal excitation of solar surface oscillations forced by compact DMB in the Sun

For simplicity, let us assume that the DMB orbit im the Sun is circular. When the rotation of the star is synchronized with the orbital motion, the tidal bulges are perfectly aligned with the companion star; their elongation δR1 and mass δM1 are easily estimated, neglecting numerical factors of order unity:
where d is the distance between the two components, and f1 and f2 the forces
that are exerted on the tidal bulges,

$R_☼ = 695700\ km$ is the solar radius. The observed solar oscillations have frequencies in excess of the fundamental dynamical frequency

$$\nu_{dyn} = (G\, M_☼/R_☼^{3})^{1/2}/(2\pi) = 99.9178\ \mu Hz \qquad (2)$$

(here G is the gravitational constant, and $M_☼$ is the Sun mass).

ν_{dyn} is close enough to $\nu_{CrAO} = 104.1891\ \ \mu Hz$.

This is a reason to consider solar surface oscillations observed in CrAO as a result of tidal excitation of the eigenmodes by the gravitational field of a small mass ($m_{DMB} << M_☼$) DMB which orbits the Sun at the depth $d_{DMB} \approx 19150\ km$ under the surface where the DMB orbital frequency $\nu_{DMB} = \nu_{CrAO}$

$$\nu_{DMB} = (G\, M_☼/(\, R_☼ - d_{DMB})^{3})^{1/2}/(2\pi) = \nu_{CrAO} = 104.1891\ \ \mu Hz, \qquad (3)$$

and its velocity

$$V_{DMB} = (G\, M_☼/(\, R_☼ - d_{DMB}))^{1/2} \approx 443\ km/s \qquad (4)$$

is supersonic (the respective sound speed in the solar model S [19] $c_s \approx 55\ km/s$). In this case the forced oscillations of the solar matter are described by the following equations:

$$\partial_t \rho + \partial_{\mathbf{r}} (\rho\, \mathbf{V}) = 0\ , \qquad (5)$$

$$\partial_{\mathbf{r}} \mathbf{V} = 0\ , \qquad (6)$$

$$\rho[\partial_t \mathbf{V} + (\mathbf{V}\,\partial_\mathbf{r})\,\mathbf{V}] = -\,\partial_\mathbf{r}\, p + \rho\,(\mathbf{g} - G_N\, m_{DMB}\, |\mathbf{r} - \boldsymbol{r}_{DMB}(t)|^{-3}\,(\mathbf{r} - \mathbf{r}_{DMB}(t)), \qquad (7)$$

where ρ is the solar mass density, $\boldsymbol{V}$ is the velocity, p is the pressure, $\boldsymbol{g}$ is the gravitational acceleration, and $\boldsymbol{r}_{DMB}(t)$ is the DMB trajectory. Further simplifications are possible if radius of DMB (a_{DMB}) is large enough ($a_{DMB} >> R_A$, where R_A is the accretion radius) to consider only potential flows in the solar matter without a significant accretion or strong shock waves. In the supersonic case

$$R_A = 2\, G\, m_{DMB}/V_{DMB}^2 = 2\, R_☼\, (m_{DMB}/\, M_☼), \qquad (8)$$

and if $m_{DMB} = 10^{-9}\, M_☼$ then $R_A \approx 1.4\ m$.

Value of a_{DMB} can be estimated only under different assumptions on the DM structure. For the case of mirror DM matter with mass density close to the mass density of OM ($\rho_{OM} \sim 1\text{-}10\ g/cm^3$) a_{DMB} can be estimated as

$$a_{DMB} \approx (3\, m_{DMB}/(4\,\pi\,\rho_{OM}))^{1/3}, \qquad (9)$$

and if $m_{DMB} \sim 10^{-9}\, M_☼$ then $a_{DMB}\ \sim 400\text{-}800\ k\,m >> R_A$.

Alternative estimation can be done for DM from gravitino the mass of which is estimated in the partial split supersymmetry with bilinear R-parity violation [7] as $m_g = 2.3\text{-}5.3\ GeV/c^2$, and radius of the ground state of the nonrelativistic system from gravitino can be calculated [24] as

$$a_{DMB} \approx 2.84\, l_P\, (m_g/\, m_{DMB})^{1/3} (m_P/m_g)^3, \qquad (10)$$

where l_P is the Plank length, and m_P is the Plank mass, and if $m_{DMB} \sim 10^{-9}\, M_☼$ then

$a_{DMB}\ \sim 10^3\ k\,m >> R_A$.

Then, as usual, equations (5-7) must be combined with respective boundary conditions, should be linearized, decomposed on spherical harmonics $Y_l^m(\theta,\varphi)$, and Fourier transformed. Then, following [18], the radial amplitude of the surface velocity for the forced oscillations can be estimated as

$$V_r \approx 2\ V_{DMB}\ (R_☼/d_{DMB})(m_{DMB}/M_☼)\ S_{10}^{res}, \qquad (11)$$

where

$$S^{res}\ \approx v_{DMB}^2/(\,v_{DMB}^2 - v_0^2) \approx 13 \qquad (12)$$

for $v_{DMB} = v_{CrAO}$, and $v_0 = v_{dyn}$. A small shift of v_0 induced by the DMB periodic perturbations is neglected here because there is a much more uncontrolled uncertainty in v_{10} concerned with some special problems of near-surface layers of the Sun which are not yet resolved. In particular, i) modeling of the structure of these layers is complicated by the presence of convective motions with Mach numbers approaching 0.5 in the uppermost part ($\sim 100\ km$) of the convection zone, and ii) the adiabatic approximation used in most computations of solar oscillation frequencies is not valid near the surface. These potential problems with the models must be taken into account when the observed and computed frequencies are compared.

As well, in order to estimate m_{DMB} from (10) via comparison of the velocity amplitudes (11) with the observed ones such experimental conditions as the way of spatial and time averaging has to be considered, and the correct angular dependence of the solar surface velocity for the f_{10} mode ($\sim Y_1^{\pm 1}(\theta,\varphi)\ e^{-i\omega t}$ in coordinates with polar axis being orthogonal to the DMB orbital plane) should be taken into account for calculations of the line-of-sight component of the solar surface velocity.

In particular, for the differential measurements of solar-photosphere oscillations carried out in CrAO during 12235 h in total (2006 days in the period of 1974–2007) [11] these are i) the acquisition time of the radial velocity signal was 1 min, and further processing of these data involved averaging over consecutive 5-min intervals, ii) the "Sun versus Sun" measurement technique which automatically eliminates all major instrumental and atmospheric distortions of the radial velocity, iii) residual slow daily trends of mainly terrestrial origin eliminated using

quadratic polynomials, iv) in daylight hours, the CrAO solar magnetograph measures the radial velocity difference between the central part of the disk with diameter of $0.66\ D_☼$ and the outer annular zone (here, $D_☼ = 2\ R_☼$), and v) the Sun is observed in a "parallel" beam, when the spectrograph input slit is illuminated by the radiation of the entire disk.
These experimental conditions lead to the corresponding spatial response function $S_1^{(V)}$ [20] for the CrAO data $S_1^{(V)} \approx 0.7$ which is not as extremely small as $S_1^{(V)} \sim 0.003$ [8] based on approximation of hydrostatic adiabatic tide [21]. As a result the observed amplitude for the radial velocity can be estimated as

$$V_{r(10)}^{obs} \approx 2\ V_{DMB}\ (R_☼/d_{DMB})(m_{DMB}/M_☼)\ S_{10}^{res}\ S_1^{(V)}. \qquad (13)$$

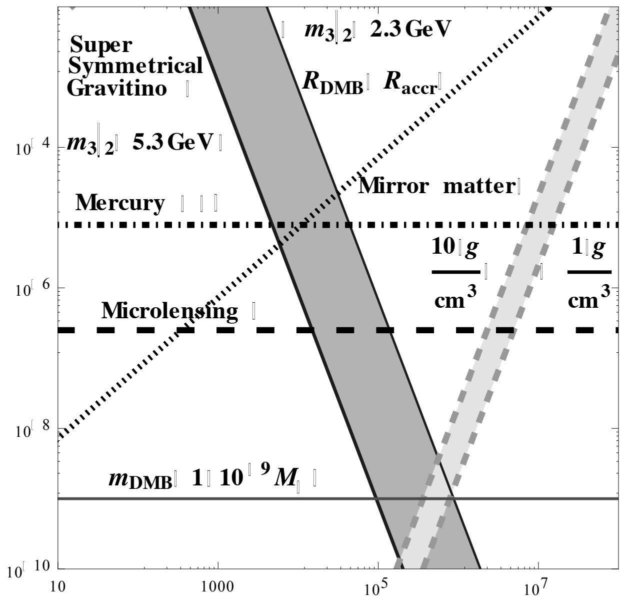

Fig.1: Plot of m_{DMB} (vertical axis in $M_☼$) – a_{DMB} (horizontal axis in m) for different limitations concerned with i) the MACHO project limits on planetary mass dark matter in Milky Way halo from gravitational microlensing [22], ii) perihelion precession of the Mercury, iii) negligibly small accretion of solar matter on DMB, and iv) possible nature of DM – mirror particles (dashed line stripe with slope on right) or neutralino (solid line stripe with slope on left). In particular, for $m_{DMB} \approx 1\ 10^{-9}\ M_☼$ possible value of a_{DMB} can be estimated from this picture as $a_{DMB} \sim 10^3\ km$ for the both kinds of DM - mirror or gravitino DM.

Then an estimation of the DMB mass (m_{DMB}) can be done from (13) for the measured value $V_{r(CrAO)} = 0.27 \pm 0.05\ m/s$ [11]

$$m_{DMB} \approx M_☼/(2\ S_{10}^{res}\ S_1^{(V)})(\ d_{DMB}/R_☼)(\ V_{r(CrAO)}\ /V_{DMB}) \approx 4\ 10^{-9}\ M_☼\ , \qquad (14)$$

what is about 10% of the Moon mass. As well, this value of m_{DMB} well satisfies all limitations (Fig.1.) concerned with theoretical assumptions about possible nature of DMB (mirror [3-6] or gravitino [7] matter), permissible perturbations of the Sun, and Solar System, and some other astrophysical data (the MACHO project limits on planetary mass dark matter in the galactic halo from gravitational microlensing [22]), and perihelion precession of the Mercury. In particular, these limitations (Fig.1.) lead to a rough estimation for radius of DMB:

$$a_{DMB} \sim 10^3\ km \approx 0.6\ R_{Moon}, \qquad (15)$$

and mean mass density is estimated to be

$$\rho \sim 500\ kg/m^3 \qquad (16)$$

for the both kinds of DMB nature considered in this paper. Lets denote this DMB as DMB_{CrAO}.

It should be noted that for some other possible indications for DMB with estimated masses of two orders of magnitude greater or less than $10^{-9} M_☼$ it seems possible to distinguish between the mirror or gravitino matter (Fig.1.). The value $a_{DMB} \sim 10^{-9} M_☼$ can be used for estimation of additional observable effects of perturbation of the solar matter by DMB orbiting at the depth $\sim d_{DMB}$ under the solar surface where the mass density ρ and temperature T are predicted by the solar model S [19] to be $\rho \approx 2.7\ kg/m^3$, and $T \approx 140000\ ^oK$. If $a_{DMB} \sim 10^3\ km$ then $a_{DMB} /R_A >> V_{DMB}/c_s$, and the velocity of perturbations of the infalling gas ΔV turn to be everywhere less than the speed of sound c_s: $\Delta V < V_{DMB} (R_A/ a_{DMB})$. This means that even if the accretion shock wave is formed, then the shock is weak and maximum heating of the matter is low being of order $\partial_t m\ (\Delta V)^3/c_s$, where $\partial_t m \sim \pi a_{DMB}^2 \rho V_{DMB}$ is the mass perturbed (but not captured) by the DMB in a unit time. So the heating rate is less than $10^{13}\ W$ which is much less than the solar luminosity $L_☼ \approx 3.8\ 10^{26}\ W$. Thus the DMB induces inside the Sun only gravitational and weak acoustic effects. The same is true even if DMB mass is at the limit concerned with perihelion precession of the Mercury $\Delta\varphi_♀^{exp} = 574.10\pm0.65\ arcsec/century$ [25]. For mass m distributed along the ring of radius $R_☼ - d_{DMB}$ (which is reasonable because $P_{DMB} \approx 160\, min << P_♀ = 88\ days$ - the period of Mercurian revolution) the perturbation of the perihelion precession of the Mercury [26] per one revolution is

$$\Delta\varphi = (3\ \pi/2)(\ m/\ M_☼)(\ R_☼/\ (a(1- e^2)))^2, \tag{17}$$

where a - is the large semi-axis of the orbit ($a_♀ \approx 5.8\ 10^7\ km$), and e - is its excentricity ($e_♀ \approx 0.0002$). And if $m_{DMB} < 10^{-5} M_☼$ the perturbation does not exceed the experimental error $0.65\ arcsec/century$ (Fig.1.). Leaving aside the question of the origin of the DMB inside the Sun, it should be noted that the value $m = 4\ 10^{-9} M_☼ \approx 8\ 10^{21}\ kg$ corresponds to 0.04-0.4% of the amount of DM which might be accreted by the Sun during its lifetime if the parameters of the interstellar DM in the Galaxy are the same as for OM [8]. The presence of a body inside the Sun may be in principle discovered as well by its gravitational effect on a probe body approaching the Sun (comet or spacecraft) at a distance of several $R_☼$ [8]. For a test of this opportunity the detailed analysis of possible noncircular (and non-elliptic) orbits inside the Sun is needed. In connection with this the numerically exact orbits of $\mathrm{DMB_{CrAO}}$ near the Sun surface are calculated within the solar model S [19] in order to estimate time variations of the f_{10} velocity amplitude, and to compare them with corresponding characteristics of the solar oscillations displaying in the CrAO data [27] small variations of the radial velocity amplitude with period about 27.04 days. Suitable trajectories of DMB are found to be not circular and almost elliptic with maximal depth 57700 km under the solar surface, maximal height over the surface 22600km (large semi-axis $a_{\mathrm{DMBCrAO}} \approx R_☼ - 17550\ km$, excentricity $e_{\mathrm{DMBCrAO}} \approx 0.059$), and the perihelion precession $\Delta\varphi_{\mathrm{DMBCrAO}} \approx -1.5^o$ per one revolution. So that these data definitely indicate existence of $\mathrm{DMB_{CrAO}}$ with mass about $4\ 10^{-9} M_☼$ and radius $\sim 10^3\ km$ which is almost freely traveling near the Sun surface. At last, for additional quantitative verification of the solar surface excitation by $\mathrm{DMB_{CrAO}}$. orbiting near the solar surface with frequency $\nu_{CrAO} = 104.1891\pm0.0003\ \mu Hz$, lets estimate velocity amplitude of another solar surface oscillation mode observed in the VIRGO-GOLF/SOHO, and GONG data $V_{SoHO} = 4.5\pm1.5\ mm/s$, $\nu_{SoHO} = 220.7\ \mu Hz$ [12]. Because of the small excentricity of the orbit the time dependent perturbation by the $\mathrm{DMB_{CrAO}}$ is periodic but not harmonic. The second term in Fourier series with twice more frequency ($2\ \nu_{\mathrm{DMBCrAO}} = 208.2782\pm0.0006\ \mu Hz$ very close to $\nu_{SoHO} = 220.7\ \mu Hz$) most effectively excites this mode (the most closely approaching f_0 in the space of eigenfunctions). This leads to estimation of the velocity amplitude like (11) but with additional small factor $e_{\mathrm{DMBCrAO}}/2$

$$V_{r)} \approx e_{\mathrm{DMBCrAO}} V_{DMB} (R_☼/d_{DMB})(m_{DMB}/M_☼)\ S^{res}, \tag{18}$$

where

$$S^{res} \approx (2\nu_{DMB})^2/(\ (2\nu_{DMB})^2 - \nu_{11}^2) \approx -8.2 \text{ at } \nu_{11} = \nu_{SoHO}. \tag{19}$$

As a result the corresponding amplitude of the radial velocity is estimated to be $\approx 3.4\ mm/s$ which is in the very good agreement with the SoHO data $V_{SoHO} = 4.5\pm1.5\ mm/s$. This agreement additionally support existence of the DMB_{CrAO}. It should be noted that the *220.7 μHz* peak is usually discussed as a *g*-mode oscillation [23] which could be or a component of the $l = 2$, $n = -3$ *g*-mode, or a component of the $l = 3$, $n = -5$ or a bitting between this latter and the $l = 5$, $n = -8$. Regardless of the solution, an important question remains to be answered: why is this particular peak so excited when there are no other visible *g*-mode components? The same analysis of data being applied for the major solar flares of high X-ray classes (M3.0) leads to the conclusion about possible existence of another DMB (DMB_{Flare}) orbited the Sun more eccentrically than DMB_{CrAO} or DMB_{SoHO} and periodically ejected high temperature jets of solar matter in the form of the major flares with the measured frequencies 103.72 μHz (solar cycle 19), and 103.959 μHz (cycle 21) [26]. This small growth of the frequency from cycle 19 to cycle 21 is explained in the suggested mechanism with DMB by a drag force arising due to pure gravitational interaction of the supersonic DMB with solar matter. This force most strongly drags DMB_{Flare} when it passes through the more central regions of the Sun with the more dense solar matter. As well some phenomena concerned with DMB traveling in planet structures are possible to be observed, for example, a specific behavior of the Moon dust disturbed by a DMB crossing the Moon, a formation of short-time vertical up or down flows in deserts, oceans and atmosphere of the Earth, and other planets.

3. Conclusions

Summarizing lets conclude that i) there is tidal excitation of the solar eigenmodes by a dark matter body orbiting close under the solar surface, which enables to consider the Sun surface as a very sensitive detector for the dark matter bodies almost freely traveling in the solar matter, ii) there are quantitative arguments for presence inside the Sun of at least two dark matter bodies (DMB_{CrAO} and DMB_{SoHO}) with well estimated mass and radius for the both of them ($4\ 10^{-9}\ M_☼$, and $10^3\ km$), iii) GW radiated by these DMB can be detected by LISA, and iv) there is no alternative theoretical models to explain these oscillations and flares on the Sun.

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Conservation Laws in the Einstein-Cartan Theory

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Abstract. *Local and non-local integral conservation laws in the Riemann-Cartan spaces are given. Hypothesis of the friedmons as dark matter particles is proposed. The mass of friedmon is near* $10^9 GeV$ *and the symmetry group is dual for SU(2) one.*

Key words: *non-local integral conservation laws, tetrad currents, Planckian density of the original de Sitter world, spin-mass, hypothesis of friedmons as dark matter particles.*

Gravitational field as some gauge field is a connection. The metric of the expanding Universe is near to de Sitter one. The Casimir operators in this case contain the combination of the spin and mass or spin-mass as total parameter with physical significance due to the mixing of the spin-mass components by action of the de Sitter group SO(1,4) transformations [1]. For example in the theory of supergravity with two complex Grassmanian values physical significance have eight real coordinates only (or four complex ones – with imaginary time and two space null coordinates) due to its mixing by the expanded Poincare group transformations. The Einstein equations connect masses with Riemannian curvature of space-time (events world for Minkowski). In frame of the Einstein-Cartan theory the spin-mass must be connected with the Riemann-Cartan space with the curvature and the torsion of the manifold of events [2-4]. There are 16 equations with the non-symmetric Einstein tensor $G_{\mu\nu}$ *and the spin tensor* $S_{\mu\nu\lambda}$ *as a result of the variation energy-momentum tensor with respect to the spinor connection 1-form. The Einstein equivalency principle (gravity and inertia are locally equivalent) is violated: the Cartan torsion is nontrivial for any observers. The Bianchi contracted identities give differential conservation laws for energy-momentum tensor. There is the non-local integral equivalent of its in General Relativity [5] (for example the integral on space contains also the integral on time). In the Riemann-Cartan space-time we deduce the follow formula (* $e_{a\mu}dx^{\mu}$ *- eigen 1-forms for* $T_{(\mu\nu)}$ *with the eigen values* p_a*,* $K_{a\mu\nu}$ *- focusing tensors for 3-forms* $*e_a$*,* $*$ *is the Hodge operator):*

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

$$\nabla_{\mu} G^{\mu\nu} = 0$$

$$G_{[\mu\nu]} = 4\pi G(\nabla^{\rho} S_{\mu\nu\rho} + S_{\lambda\rho}{}^{\rho} S_{\mu\nu}{}^{\lambda})$$

$$T_{(a\mu)} = e_a^{\nu} T_{(\nu\mu)} = p_a e_{a\mu}$$

$$\int_{\Sigma_a(s_a)} [*e_a p_a \exp \int_0^{s_a} (p_a^{-1} p^b K_{abb} + (4\pi G p_a^{-1}) \sqrt{h_a} S_a{}^{\mu\nu} (\nabla^{\rho} S_{\mu\nu\rho} + S_{\lambda\rho}{}^{\rho} S_{\mu\nu}{}^{\lambda})) ds] = const$$

Here $\sqrt{h_a}$ *is 3-volume, the signature of space-time is (-+++), the mass density* $p^0 \geq 0$ *. But due to the spin repulsing potential the effective density may be negative. The matter collapse will be succeeded due to the limit Planckian matter density and before it on the Cartan radius* l_{car} *[6] (below J is the spin of the particle with Compton length* l_{com}*,* l_{pl} *is the Planckian length):*

$$l_{car}^3 = 8\pi J^2 l_{pl}^2 l_{com}$$

For example for proton $l_{car} \approx 10^{-26}$*cm. There is possible give also the local integral conservation low with aid the part* δde_a *(the codifferential of the differential of the tetrad) in the Einstein tensor, contrsacted with the tetrad (* $G_{\nu\mu}e_a^\nu = G_{a\mu}$ *) giving the tetrad current* $S_a[7]$ *. There are the corresponding formulas with the Gibbons-Hawking Lagrangian [8]:*

$$L = R + 2\delta K = -\nabla_\mu e_{a\nu} \cdot \nabla^\nu e^{a\mu} + K_\mu K^\mu$$
$$K_\mu = e_\mu^a K_a, K_a = \delta e_a = -\nabla^\mu e_{a\mu}$$
$$\Delta = \delta d + d\delta, \delta = *^{-1} d *$$
$$\Delta e_a = -\nabla^2 e_a + R_a, R_a = R_{\mu\nu} e_a^\mu dx^\nu$$
$$G_{\mu\nu} = -\Lambda g_{\mu\nu} + 8\pi G T_{\mu\nu} \Leftrightarrow$$
$$G_a = -\Lambda e_a + 8\pi G T_a \Leftrightarrow$$
$$\delta de_a = 8\pi G S_a, \delta S_a = -\nabla^\mu S_{a\mu} = 0$$
$$8\pi G S_a = (\Lambda - \nabla^2) e_a + 8\pi G (T_a - \frac{1}{2} T e_a) - dK_a$$

$$L = R + 2\nabla^\mu K_\mu = -\nabla_\mu e_{a\nu} \cdot \nabla^\nu e^{a\mu} + K_\mu K^\mu$$
$$K_\mu = e_\mu^a K_a, K_a = -\nabla^\mu e_{a\mu}$$

Now let us discuss the gravitational energy problem. The search of the gravitational energy-momentum pseudo-tensor [9, 10] was the mistake: its dependence on a choice of coordinates deprives it of the physical significance since - this significance has a frame of reference only as an additional invariant structure on the space-time. It is impossible introduce the Dirac equations in the Riemann-Cartan spaces and to solve the gravitational energy problem without the lowering from the metric level to the tetrad one. Even the accelerating frame of reference in the flat space-time permits us due to the Einstein equivalency principle to speak about nontrivial gravitation field (without the acceleration) with the gravitational energy $-g^2/8\pi G$ *(where* g *is the acceleration of the free full) and with the nontrivial Gibbons-Hawking Lagrangian. Einstein received the correct total energy of the island physical system due to using the asymptotically Cartesian frame of referent in the neighborhood of the space infinity [11]. Here the Cartesian coordinates are primary as equal to real translations.*

My teacher Abraham Leonidovich Zelmanov (1913-1887) have been searching unsuccessfully the chronometric invariant gravitational pseudo-tensor by the undetermined coefficients method. He introduced the monad vector field and the set of the integral time-like world lines as a frame of reference. Tetrad field includes the monad field, and the difference $t_{0\mu} = S_{0\mu} - T_{0\mu}$ *gives desired quantity. There is interesting to note the Steven Weinberg pseudo-tensor [12] as the difference between energy-momentum tensor and the nonlinear parts of the Einstein tensor. But both the indices remained coordinate. Tetrad current is the analogical difference but our contracting of the energy-momentum tensor with the tetrad potential is an analog of the member* $A_\mu J^\mu$ *in frame of the electrodynamics. This approach we named as semitetrad one: double contracted Einstein tensor with the tetrad gives 16 scalar equations instead one tensor equation in Einstein-Cartan theory.*

There are many different vacuums even in the flat space-time connecting by the Bogolyubov transformations. Vacuum plus matter is the whole physical system and conservation laws must be realized for vacuum plus matter only [13]. We propose introduce the gauge constraints for the vacuum background: that Γ *(connection module* “gamma”) be the sum of the *product of the 12 complex Penrose spin coefficients [14] and complex conjugated ones. Condition* $\Gamma = \min$ *gives primary frame of reference and corresponding physical vacuum. Vacuum is not invariant under the Lorentz transformations. Due to this non-invariance on tell about non-localization of the gravitational energy. We introduce the requirement* $\delta G_a = -\nabla^\mu G_{a\mu} = 0$ *as the conservation gauge for gravitational theory. In this case we have the normal local integral conservation laws in gravity. For the de Sitter world we have:*

$$\begin{aligned} &\delta G_a = -\nabla^\mu G_{a\mu} = 0 \\ &G_a = -\Lambda e_a, \\ &\delta G_a = -\Lambda K_a = 0 \Leftrightarrow K_a = 0, L = R = 4\Lambda \end{aligned}$$

Here the conservation gauge gives the Lorentz tetrad gauge (tetrad without the dilatation).

The time-form $e_0 = dt$ *implies* $S_{00} = 0$, *but for the Lorentz gauge we receive ordinary density for the weak gravitational radiation [10] and ordinary negative density of the gravitational energy in the static field. For the Hodge decomposition of tetrad into the sum of gradient, coclosed and harmonic parts*

$$\begin{aligned} &e_a = d\alpha_a + \delta\beta_a + \gamma_a \\ &\Delta\gamma_a = 0 \end{aligned}$$

the second term gives nontrivial gravity and the last term – the gravitational radiation.

The Gibbons-Hawking Lagrangian implies the density of the Hamiltonian as a partial divergence [11]. An integration in the de Sitter world (the set of the 3-spheres with empty cluster set) implies the trivial total mass-energy. There is important that Einstein tensor and energy-momentum tensor (directly proportional to the cosmological term) are nontrivial and conserving.

The Einstein-Cartan equations admit the next interpretation: the geometry of the space-time compensates the matter by the Riemannian curvature and the Cartan torsion. The string theory implies the string additions (that must be carried over right part of equations) that are important in origin of the Big Bang [15] (lower l – the string length, α' *- the inclination of the Regge trajectory):*

$$\begin{aligned} &G_{\mu\nu} + \Lambda g_{\mu\nu} + \frac{\alpha'}{2}(R_{\mu\alpha\beta\gamma}R_\nu{}^{\alpha\beta\gamma} - \frac{1}{2}g_{\mu\nu}R_{\alpha\beta\gamma\delta}R^{\alpha\beta\gamma\delta}) = 8\pi G T_{\mu\nu} \\ &\alpha' = l^2/2 \\ &G_{\mu\nu} + 3(\frac{1}{a^2} + \frac{\alpha'}{a^4})g_{\mu\nu} = 0 \end{aligned}$$

Here is string term ($\Lambda = 3/a^2$ *) is absent then it will be arise. The curvature radius must be changed with vacuum transformation when it gives its mass to the mass-energy of the matter. The geometric empty is forbidden by the quantum mechanics, and any real physical vacuum implies the nontrivial geometry. The de Sitter world is the physical system with the conserving nontrivial scalar matter field.*

In conclusion we present the friedmons hypothesis as dark matter particles [16]. There is easy find *the connection between the gravitational radius of the Sun* r_{gr} *and the nucleon size* l_n*:* $r_{gr}/l_n = l_n/l_{pl}$, $10^5 cm \cdot 10^{-33} cm = (10^{-14} cm)^2$. *We propose that before the Big Bang the Univers was the 3-sphere (as the de Sitter world) with the limit Planckian density and respectively with the gravitational radius* $10^{-13} cm$ *(the initial Lemaitre atom). Let us propose that this size is connected with the hypothetical particle friedmon (* $h/2\pi = c = 1$ *):*

$$10^{-13} cm \cdot 10^{-33} cm = (10^{-23} cm)^2$$
$$G \cdot (2 \cdot 10^{56} g) = 10^{28} cm$$
$$m_f = 0{,}677 \cdot 10^9 GeV$$

We propose that the symmetry group for the friedmon is the dual SU(2). In Standard model the particles with symmetry groups SU(3), U(1) are stable, but for the dual symmetry groups the particle with dual SU(2) group are stable only [17, 18]. The interaction between the particles relating to the basic and the dual symmetry groups may be gravitational one only. The friedmons are heavy be-lepton formations with the quark type confinement. We propose that the Lemaitre atom after the transformation of the topological energetic modes into oscillation ones transmitted its mass to the ordinary substance. Even if the particle of dust gives its mass m to the de Sitter vacuum then we will receive the de Sitter world with the curvature radius a and the density $\rho : 2\pi^2 a^3 \rho = m$. *For the Universe with its isotropy on the grand sizes its transformation into the scalar field (the de Sitter world) will be occur after the decay of the all elementary particles. May be there was the approximate equality of the total masses of the particles and dual particles with the hypothetical Grand Unification group as a product of the E8 on the dual one, containing the Standard model groups and dual groups as subgroups. The following evolution reduced to the predominance of the dark matter before the ordinary one. The future quantum gravity theory will give here the final answer.*

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A critical value for dark energy

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Experimental evidence over a number of recent years has shown the density parameter of the universe Ω converging to the critical value of 1, which defines a flat, Euclidean universe. No such calculations have defined a critical value for the most significant component of Ω, that for the dark energy, Ω_Λ, but the new data provided by the Planck probe open up the previously unconsidered possibility that a particular value with special physical significance occurs at $\Omega_\Lambda = 2/3$. If future observations should converge on exactly this value, then we may have the first indication that the explanation for this phenomenon lies in necessary constraints provided by fundamental laws of physics on possible cosmologies for the universe.

Introduction

The dark energy content of the universe, discovered in 1998 [1,2], and generally described as completely unexpected and without obvious explanation, is a phenomenon with significant implications for physics as well as cosmology. In fact, the new results available from the Planck probe allow the previously unexpected possibility that the dark energy may show fundamental physics driving possible cosmologies, rather than cosmology determining possible physics, and this can be done directly from the data without requiring any conjectural or speculative physics input. In addition, we can immediately see how such a possibility can be put to rigorous testing using data from future probes.

Dark energy and acceleration

In relation to the scale factor R, in an expanding universe, the Hubble parameter H is defined as the normalised rate of expansion $H = \dot{R}/R$, and is measured as the ratio v / r of the recessional velocity v and the comoving distance, r, of distant galaxies. The Hubble constant, H_0, the Hubble parameter at the present time, is defined in terms of the Hubble radius, r_H, as c / r_H. So, at the present time,

$$v = H_0 r = \frac{cr}{r_H} \tag{1}$$

Friedmann's solutions of the Einstein field equations [3] suggest that there is a particular density at which the universe must be flat or Euclidean, with curvature parameter k equal to zero. According to the first Friedmann equation, in the absence of a cosmological constant Λ,

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2}. \tag{2}$$

At the present time, the critical density for zero curvature is given by

$$\rho_{crit} = \frac{3H_0^2}{8\pi G}. \tag{3}$$

and the ratio of the actual density ρ to this critical value, the density parameter, $\Omega = \rho / \rho_{\text{crit}}$, will determine the universe's evolution and ultimate fate. The value $\Omega = 1$ clearly has a special significance, creating a clear separation of a flat or Euclidean universe from a closed universe with $\Omega > 1$ and spherical geometry, and an open universe with $\Omega < -1$ and hyperbolic geometry. The fact that, since 2000 [4], the experimental value of this parameter has been apparently converging towards the precise value of 1, has been taken as indicating that the universe is flat and possibly infinite, apparently in line with an inflationary view of cosmology, but in contradiction with the consensus that became accepted during the main part of the twentieth century for the geometry of the universe. It also means that general relativistic calculations are taken at the Newtonian limit, if we incorporate energy into the mass term. Then equation (3) becomes equivalent to $\rho_{\text{crit}} = 3\, v^2 / 8\, \pi \text{G} r^2$, which implies the Newtonian relation $Gm / r = ½\, v^2$, where m is the mass enclosed within radius r.

The particular form of the Friedmann equation in equation (2) allows the possibility of ρ being replaced by $\rho + 3P / c^2$, and the inclusion of a vacuum energy term with positive (inward) pressure P from, say, radiation, without any significant change to the meaning of Ω. However, the inclusion of a 'cosmological constant' Λ or a vacuum energy term with negative (outward) pressure, as in equation (4), while, not changing the definition of Ω, would have major physical consequences.

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\rho - \frac{kc^2}{R^2} - \frac{\Lambda c^2}{3} \tag{4}$$

It was a term of this kind that was discovered in 1998, through an outward acceleration in the red-shift velocity of distant galaxies [1.2]. Named 'dark energy', and incorporated into Ω as the component Ω_Λ, it has remained an unexplained phenomenon, and the early values of its magnitude, ranging from 71.4 to 74 per cent of the total energy of the universe, gave no obvious clue as to its origin. However, it may be that, just as with Ω itself, there is a particular critical value of Ω_Λ with a precise physical significance, and it may be that the new data from the Planck probe indicate that it could converge towards this particular value.

The main result quoted for Ω_Λ, from Planck Collaboration XVI, gives 0.6825 as the best fit and 0.686 ± 0.020 for the 68 % confidence limits [5]. Including lensing as well as Planck gives 0.6964 as best fit and 0.693 ± 0.019 for the 68 % confidence limits. Including WMap as well as Planck gives 0.6817 as best fit and 0.685 + 0.018 and – 0.016 for the 68 % confidence limits. The overview reported by Planck Collaboration I states that determining the dark energy contribution from temperature anisotropies data alone gives 0.67 + 0.027 and – 0.023 for the 68 % confidence limits [6].

The new value for Ω_Λ suggests an intriguing possibility. The value for this vacuum energy is close to two-thirds of the total energy of the universe, and, if this fraction should turn out to be the preferred value, then a simple calculation suggests some interesting consequences. If we suppose that

$$\frac{\rho_{vac}}{\rho_{crit}} = \frac{2}{3}, \tag{5}$$

then, using (3), the vacuum density becomes

$$\rho_{vac} = \frac{H_0^2}{4\pi G}. \tag{6}$$

This is equivalent to a 'dark' energy density or negative pressure

$$-P = \frac{H_0^2 c^2}{4\pi G}, \tag{7}$$

and cosmological constant

$$\Lambda = 8\pi G \rho_{vac} = 2H_0^2. \tag{8}$$

We can incorporate P (or Λ) into equation (4), with $k = 0$, to give

$$H^2 = \left(\frac{\dot{R}}{R}\right)^2 = \frac{8\pi G}{3}\left(\rho + \frac{3P}{c^2}\right) \tag{9}$$

or into Friedmann's second, acceleration, equation, to give

$$\frac{\ddot{R}}{R} = \frac{4\pi G}{3}\left(\rho + \frac{3P}{c^2}\right). \tag{10}$$

In line with our observation that this is at the Newtonian limit, we can see the connection between these equations and an equivalent Poisson equation:

$$\nabla^2 \phi = 4\pi G\left(\rho + \frac{3P}{c^2}\right) = 4\pi G\left(\rho - \frac{3H_0^2}{4\pi G}\right) = 4\pi G(\rho - 3\rho_{vac}). \tag{11}$$

If ρ is the mass density of a uniform and isotropic Hubble universe with mass $m = 4\pi G\rho r^3 / 3$, within radius r, then we can express equation (10) in terms of a force on a unit mass, combining the effect of gravity and dark energy:

$$F = \frac{Gm}{r^2} - H_0^2 r = \left(\frac{4}{3}\pi G\rho - H_0^2 \right) r \,. \tag{12}$$

This means that the acceleration responsible for dark energy can be expressed as

$$a = \frac{v^2}{r_H} = H_0^2 r \,. \tag{13}$$

This is a remarkable result, suggesting that the acceleration observed in the red-shift, like the velocity, depends only on Hubble's constant H_0 and the distance, and is thus perhaps an integral component of the same process that produces the red-shift velocity v. Integrating

$$a = v\frac{dv}{dr} = H_0^2 r \tag{14}$$

with respect to v and r between the limits 0 and v and 0 and r gives the exact Hubble red-shift law:

$$v = H_0 r = \frac{cr}{r_H} \,. \tag{15}$$

Here, we should point out that the significant equation (14) has been obtained solely from the data, with no conjectural or speculative element whatsoever, and no additional theoretical content or modelling. If the data stands as it is today, the equation is valid to within the experimental confidence limits, and certainly to within a factor 1.02 ± 0.02. It is so close to being exactly true, and of such exceptional physical significance if it is, that we are justified in saying that the test of its exact validity should be one of the aims of future probes. We may, for comparison, recall that the Boomerang collaboration [4], finding the 95% confidence interval for Ω to be between 0.88 and 1.12, were immediately able to claim, with full justification, that this provided 'evidence for a euclidean geometry of the Universe'. Even now, $\Omega = 1$ is hardly established to better than about 1 %, erring on the side of an increased Ω value, and inferentially that of Ω_Λ. Planck Collaboration I, including data from lensing, constrains 'departures from spatial flatness at the percent level', that is $\Omega_k = -\ 0.0096$ with 68 % confidence limits of + 0.010 – 0.0082, that is, a total Ω of 1.0096 + 0.0082 – 0.010 [6].

Inertia and Mach's principle

In the present case, the physical significance of equation (14) stems from the fact that the velocity term can be derived directly from the acceleration, implying that the acceleration, whatever its origin, is actually *responsible* for the velocity. Clearly, if this is true, there must be a significant impact on possible models of cosmological evolution. There is, however, another equation which we can derive directly from (5) and (14), which, again without any hypothetical or model-dependent input, suggests even further significance. We begin by writing the acceleration in the form

$$a = \frac{dv}{dt} = \frac{c^2 r}{r_H^2}, \tag{16}$$

following which we recall that Sciama considered the possibility of explaining inertia along the lines of Mach's principle using a gravitomagnetic inductive force between two masses with relative acceleration, which could be derived from general relativity [7,8]. In this case, there is an inductive force between masses m_1 and m_2,

$$F = \frac{G}{c^2 r} m_1 m_2 \sin\theta \frac{dv}{dt} \tag{17}$$

of the same kind as the one between charges e_1 and e_2,

$$F = \frac{G}{c^2 r} e_1 e_2 \sin\theta \frac{dv}{dt}, \tag{18}$$

which can be derived from Faraday's law of induction. Sciama considered that, using the gravitomagnetic inductive force, and assuming that isotropy removes the angular dependence θ, the inertia of a body of mass $m = m_1$ could be attributed to the action of the total mass $m_H = m_2$ within the observable universe, specified by radius r_H, so making the inductive force equation equivalent to the Newtonian inertial equation $F = Kma$, with K a constant and $a = dv / dt$. The inertial force on a unit mass due to the entire mass in the Hubble universe m_H would then be:

$$F = \frac{G}{c^2 r} m_H \frac{dv}{dt}. \tag{19}$$

Inertia provides one standard for defining a unit mass, gravitation provides another, and the connection can be made via the equivalence principle. If we now suppose that mass m_H defines a radial inertial field of constant magnitude from the centre of a local coordinate system, and, at the same time, use the principle of equivalence to equate the magnitude of this to the gravitational field (Gm_H / r_H^2), which, independently of the local coordinate system, defines a unit of gravitational mass within the same event horizon, we obtain

$$\frac{Gm_H}{c^2 r}\frac{dv}{dt} = \frac{Gm_H}{r_H^2}, \tag{20}$$

which gives us the exact expression for the acceleration which would result if the dark energy constitutes exactly two-thirds of the total energy of the universe:

$$a = \frac{c^2 r}{r_H^2} = H_0^2 r. \tag{21}$$

The calculation suggests that an exact value of two thirds for the dark energy contribution would not only link the dark energy and the Hubble red-shift as aspects of the same phenomenon, but could be of additional interest in connection with Mach's principle and the origin of inertia. Again, there is an indication that possible cosmologies are necessarily constrained by fundamental laws of physics. A Machian origin of inertia, for example, would allow a universe to evolve by creating inertial mass at the same time as its space-time structure, with the creation process also generating the force which drives its evolution. It is significant, of course, that equation (20) is simply another version of equation (16), and does not *require* the development through equations (17)-(19) for its derivation. These serve to provide a *possible* context, but none is needed to generate the equation, and the form of the equation is in itself significant. However, prior prediction leading to experimental confirmation remains one of the strongest arguments available for any theoretical construction, and, in the present case, there is also a prior prediction.

Conclusion

A version of the calculation presented here was done in reverse as part of a larger study when values of Ω_Λ looked less favourable to its conclusions [9], and this was preceded by a series of calculations deriving the red-shift acceleration as H_0^2r on the basis of a flat universe, some of which predated the experimental discovery of the dark energy. The most accessible, though not the earliest version of $a = H_0^2r$, from a series of publications beginning in 1979, was incorporated into a book with a largely historical slant [10]. In this predictive context, the dark energy would seem to have a possible explanation in both physical and cosmological contexts. The calculations seem to imply that there may be a critical value for Ω_Λ just as there is for Ω. This potentially critical value now falls within the limits of the data provided by the Planck probe, and future experimental findings may converge on this value, just as they have converged on the physically significant value of unity for Ω. Even a value which came very close would require explanation in the same way as values of Ω close to 1 were thought to be too close for coincidence even before observations were able to establish an exact value. It would be interesting to see how the constraints on other cosmological parameters would be affected by applying an exact value of two thirds for the dark energy density to the Planck data, and how any possible deviations in the assumed universal isotropy and uniformity might be manifested.

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Emergence of symmetries. Multidimensional approach.

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The observed symmetries are considered in the framework of multidimensional gravity. It is shown that a symmetry formation is connected to the entropy decrease of compact space. The conditions for charges non conservation during the inflationary stage are discussed.

Introduction

The existence of (gauge) symmetries may be related to isometries of extra space. To discuss the subject we consider compact d-dimensional universal extra spaces. The size of extra dimensions varies in the wide range from the Planck scale up to 10^{-18}cm, the upper limit known from the particle physics.

Maximally symmetric metrics of extra space as a starting point are among the most popular nowadays. At the same time we must take into account the quantum origin of space itself due to fluctuations in the space-time foam. There is no reason to assume that the geometry or/and topology of extra space is simple just after its nucleation from space time foam. Moreover it seems obvious that a measure $\mathcal{M}$ of all symmetrical spaces equals zero so that the probability of their nucleation $\mathcal{P} = 0$. Hence some period of extra space symmetrization ought to exist [1].

Here we investigate the entropic mechanism of space symmetrization after its formation. It is shown that the stabilization of the extra space and its symmetrization are proceeding simultaneously. This process is accompanied by a decrease in entropy for the extra space and an increase in entropy for main one.

Evolution of extra space

Consider a Riemannian manifold

$$T \times M \times M' \tag{1}$$

with the metric

$$ds^2 = G_{AB}dX^A dX^B = dt^2 - g_{mn}(t,x)dx^m dx^n - \gamma_{ab}(t,x,y)dy^a dy^b, \tag{2}$$
$$\gamma_{ab}(y,t) = \eta_{ab} + h_{ab}(t,y).$$

Here M, M' are the manifolds with spacelike metrics $g_{mn}(t,x)$ and $\gamma_{ab}(t,x,y)$ respectively, T denotes the timelike direction. The set of coordinates of the subspaces M is denoted by x; y is the same for M'. We will refer to M and M' as a main space and a compact extra space respectively. The curvature of the manifold is assumed to be arbitrary.

Consider the action in the form

$$S = \int d^{D+1}z\sqrt{G}f(R), \tag{3}$$

where $z = (t,x,y)$ and $G = |\det g \cdot \det \gamma|$. The metric of extra space (3) is chosen in the form $\gamma_{ab} = \gamma_{ab}(t,y)$. We also use inequality

$$R_M \ll R_{M'} \tag{4}$$

for the Ricci scalar of the main space R_M and the Ricci scalar of the extra space $R_{M'}$. It is known that in the framework of D-dimensional gravity linear in the Ricci scalar the stabilization of an extra space is impossible without involving additional fields. On the other side a D-dimensional gravity with higher derivatives gives such an opportunity. The process of stabilization of the extra space depends on its dimensionality and initial parameters of the pure gravitational lagrangian.

Recall that action (3) is equivalent to linear action with an additional scalar field [4]

$$S = \int d^{D+1}z\sqrt{\tilde{G}}\left[\tilde{R}(\tilde{G}) + \tilde{G}^{ab}\partial_a\phi\partial_b\phi - 2U(\phi)\right], \tag{5}$$

where

$$\phi = \frac{1}{A}\ln f'(R); \quad A = \sqrt{\frac{D-1}{D}} \tag{6}$$

$$U(\phi) = \frac{1}{2}e^{-B\phi}\left[R(\phi)e^{A\phi} - f(R(\phi))\right], \quad B = \frac{D+1}{\sqrt{(D-1)D}} \tag{7}$$

The details can be found in the papers cited above. The classical equation of motion of action (5) has the form

$$\ddot{\phi} + 3H\dot{\phi} + \Box_{d+1}\phi + U'(\phi) = 0, \quad H = \frac{da/dt}{a}. \tag{8}$$

The term containing the Hubble parameter H is responsible for friction in the system.

Let the potential (7) has a minimum at ϕ_m. Due to the presence of the friction, the additional scalar field ϕ tends to a constant. According to (6) the Ricci scalar of the extra space is connected to the scalar field and also tends to a stationary value,

$$R \to R(\phi_m) \tag{9}$$

The observed main space is described by the FRW metric and its Ricci scalar tends to zero, $R_M \sim 1/a(t \to \infty)^2 \to 0$ so that we may neglect its contribution at large times. Thus the extra space M' acquires a maximally symmetrical form.

The dynamics of the main space that is responsible for the friction in the extra space and its stabilization. This indicates the presence of entropy flow from the extra space M' to the main one M [2].

Entropy outflow and symmetry formation

It was shown above that stabilization of extra space, an extension of its symmetry group and an entropy increase in a whole space proceed simultaneously. Below we show that the entropy of a compact extra space is decreasing with time. To this end we prove the following

Statement

Let M be a smooth manifold, G_1 and G_2 are two given metrics on it. If the number of Killing vectors of metric G_1 is less then the number of Killing vectors of metric G_2 then the entropy of G_1 is greater than the entropy of G_2.

We will use the well known connection Ω, $S = k_B \ln \Omega$ of the Boltzmann entropy S and a number of microstates Ω

Let us consider a compact smooth manifold M. We suppose that two metrics G_1 and G_2 on M define the same microstate if and only if they are equal in each of the points $P \in M$.

The definition of a macrostate is as follows. Let v be an arbitrary smooth vector field defined globally on the smooth manifold M. Any shift along the integral path of vector field v corresponds to a diffeomorphism M on itself. We define a macrostate as a set of metrics G that are connected by shifts. As an example, a 2-dim torus with a bulge, being shifted, still represents the same macrostate. Another macrostate is determined by the addition of another bulge. So this definition seems reasonable.

The statistical weight of a given macrostate is the number of microstates. The latter is a continuum set for any classical system. The concept of microstates is correctly defined at a quantum level where the set of energy levels is known. However, the quantization of geometry is a yet unsolved problem. That is why any discussion of a metric on scale less than the Planck scale L_P is pointless. Thus shifts less than Planck scale should not be taken into account when counting statistical weight. Therefore a number of shifts along various integral paths is assumed to be finite.

Let us compare statistical weights of two metrics G_1 and G_2 with the same number of shifts at manifold M. Let G_1 have no Killing vectors and G_2 possesses a global Killing field. Shifts along Killing vector of G_2 lead to the same microstate by definition. So the statistical weight of G_1 is greater than the statistical weight of G_2. A similar argument is correct in the general case as well when the number of Killing vectors of metrics G_1 is less then the number of Killing vectors of metrics G_2. This statement is also valid for the entropy which is the nondecreasing function of the statistical weight. Therefore

$$S_1 > S_2. \tag{10}$$

The statement is proved.

So the entropy decreases in the presence of Killing vectors. Namely if M is a smooth manifold with some metric g and f is a shift along its Killing vector then the topological entropy $h_g(f) = 0$.

Combining the results discussed above we can conclude that the entropy of the whole manifold is increasing with time, while the extra space metric tends to maximally symmetric one. The latter means an entropy decreasing of the extra space. So an isometry group of extra space becomes larger due to an influx of entropy into a main space.

The entropic flux can be realized by various mechanisms. In this section we show that the space expansion is responsible for the friction what leads to the attenuation of motion in a compact extra space. The friction term is proportional to the Hubble parameter so that most intensive flux takes place during the inflationary stage.

Another way to support the entropic flux is the decay of excitations of the compact extra space, the Kaluza-Klein modes. The dynamical evolution of compact hyperbolic extra space leads to an intensive injection of entropy into the observable Universe [5].

Observable consequences. Inflation

According to condition (9) a Ricci scalar of internal metric of extra space is a constant with good accuracy whenever condition (4) holds. Meantime the latter can be violated at the inflationary stage due to metric fluctuations of the main space.

Let us estimate the characteristic size L_{extra} of extra space, insensitive to this fluctuations. During the inflationary stage we have

$$R_3 = 12H^2 = \frac{32\pi}{M_{\text{Pl}}^2} V(\phi) = 16\pi \left(\frac{m}{M_{\text{Pl}}}\right)^2 \phi^2.$$

The last equality holds for a quadratic inflaton potential $V(\phi) = \frac{1}{2}m^2\phi^2$. Metric fluctuations of the main space have the form

$$\delta R_3 = 32\pi \left(\frac{m}{M_{\text{Pl}}}\right)^2 \phi\, \delta\phi \sim \frac{m}{M_{\text{Pl}}} m^2.$$

Here we take into account the approximate equality $\phi \sim M_{\text{Pl}}$ during the inflationary stage. Field fluctuations $|\delta\phi| = H/(2\pi)$ are connected to the scale factor

$$H = \sqrt{\frac{8\pi}{3}\frac{V(\phi)}{M_{\rm Pl}^2}} = \sqrt{\frac{4\pi}{3}}\frac{m}{M_{\rm Pl}}\phi$$

in the usual manner. For the size of extra space not to be disturbed, it should satisfy the inequality

$$L_{\rm extra} \sim \frac{1}{\sqrt{R_{M'}}} < \frac{1}{\sqrt{\delta R_3}} \sim \frac{1}{m}\sqrt{\frac{M_{\rm Pl}}{m}} \sim 10^{-24} \text{ cm.} \tag{11}$$

Otherwise metric fluctuations of the main space would influence the geometry of the extra space and any symmetries would be absent during inflation.

LHC collider could find extra space provided its size is larger than $\sim 10^{-18}$ cm. Suppose that the LHC succeeded in finding an extra space. In the framework of our approach it would mean the absence of symmetries at the inflationary stage. The same can be said about the gauge symmetries provided the gauge fields are connected to off diagonal components of metric tensor in the spirit of the Kaluza-Klein model. These symmetries arise during the stage of reheating or later.

Partial symmetrization. The electroweak standard model and the Higgs field.

According to the previous discussion, it seems natural to suppose that an extra space must be maximally symmetrical what corresponds to a state with minimal entropy. Nevertheless the real situation is different. Indeed, consider as an example the electroweak standard model that is based on $SU(2) \times U(1)$ group. More accurately we have to consider some extra space with an isometry group T being isomorphic to the electroweak group.

Following [3] consider a $D = 10$-dimensional Riemannian manifold $V_{10} = M_4 \times V_4 \times V_2$ with the metric tensor of the form:

$$G_{AB} = \left(\begin{array}{c|c|c|c} g_{\mu\nu}(x) & & & \\ \hline & G^{(4)}_{ab} & e_a(x) & \\ \hline & e_b(x) & -r_d^2 & \\ \hline & & & -r_d^2\sin^2\theta. \end{array}\right) \tag{12}$$

Here $\mu,\nu = 1,2,3,4$. The subspace V_4 with metric $G^{(4)}_{ab} = -r_c^2 diag(1,1,1,1)$ is described by coordinates $y_a, a = 5,6,7,8$. Our 4-dim space is described by the coordinates x and a metric tensor $g_{\mu\nu}$. The second subspace V_2 has the geometry of sphere with radius r_d. It will be shown below that the components $e_a \equiv g_{a9}$ of the metric are connected to the Higgs field. The metric components which are important for the following consideration are written explicitly in expression

(12).

The group of linear coordinate transformations of extra space V_4 is.

$$y'^a = T^a_b y^b, \qquad a, b = 5, ..., 8. \tag{13}$$

It is supposed that the matrices $T = T_1 T_2$ form a Lie group of isometries of the extra space. The set of 4×4 matrices $T_1,\ T_2$ is defined as follows

$$T_1 = \begin{pmatrix} I\cos\phi & -I\sin\phi \\ I\sin\phi & I\cos\phi \end{pmatrix}, T_2 = \begin{pmatrix} A & -B \\ B & A \end{pmatrix}, \tag{14}$$

where I is the unit 2×2 matrix and matrices A, B satisfy the conditions

$$A^T A + B^T B = 1, \qquad A^T B - B^T A = 0, \qquad \det(A + iB) = 1. \tag{15}$$

Under these conditions a real 4-parametric group T is isomorphic to the group $SU(2) \times U(1)$ [3]. Transformation properties of the proto-higgs field e_a are the same as in (13).

As the result the Higgs sector of the Stadard Model could be reproduced on the basis of extra space paradigm. The symmetry group T of the extra space V_2 is not maximally symmetrical. On the other hand one could naively propose that the entropy grows must lead to restoration of maximal symmetry of the extra space. The way to solve this contradiction is not found yet.

Discussion

It is known that the idea of extra space leads to a set of observational effects. Most promising are those extra spaces which possessing some symmetries, but they are hardly produced from space-time foam. We elaborated the mechanism for the symmetries formation related to the entropy flow from the extra space to the main one. Due to the entropy decreasing in the compact subspace, its metric undergoes the process of symmetrization during some time after its quantum nucleation. Relaxation time of symmetry restoration depends on many aspects and could overcome the period of inflation.

One can reasonably suppose that the entropy of the extra space decreases until a widest symmetry is restored. On the other side we need specific isometries to explain observable symmetries of low energy physics, $SU(2) \times U(1)$ for example. Could they be represented by a widest symmetry mentioned above? The answer is not evident though the result of the paper [6] is rather promising: every compact Lie group can be realized as the full isometry group of a compact, connected, smooth Riemannian manifold. Nevertheless this problem needs further investigation.

This work was supported by the grants 14.132.21.1446 and 14.A18.21.0789 of Ministry of Ed-

ucation and Science of the Russian Federation.

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Topological solitons in nonlinear spinor model

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Nonlinear spinor model based on 16-spinor formalism is suggested, the unification of Skyrme and Faddeev models for the description of baryons and leptons respectively being achieved. The particles being considered as topological solitons, the main topological argument concerns using the special 8-spinor identity discovered by the Italian geometer F. Brioschi. Constructing the Lagrangian of the model is motivated by the mirror symmetry specifying baryon or lepton sectors and by the analogy with the Skyrme – Faddeev model. Some arguments are discussed for solving the problem of the electric charge quantization and the principle of correspondence with quantum mechanics.

1. Introduction

The Skyrme's fruitful idea [1] to describe baryons as topological solitons was based on the identification of the baryon number $\mathbb{B}$ with the topological charge of the degree type $\mathbb{B} = \deg(S^3 \to S^3)$, which serves as the generator of the homotopy group $\pi_3(S^3) = \mathbb{Z}$. The similar idea to describe leptons as topological solitons was announced by Faddeev [2], who identified the lepton number $\mathbb{L}$ with the Hopf invariant Q_H, which serves as the generator of the homotopy group $\pi_3(S^2) = \mathbb{Z}$. The unification of these two approaches was suggested in [3], baryons and leptons being considered as two possible phases of the effective 8-spinor field model, for which the special 8-spinors Brioschi identity [4] holds:

$$j_\mu j^\mu - \tilde{j}_\mu \tilde{j}^\mu = s^2 + p^2 + \vec{v}^2 + \vec{a}^2, \tag{1.1}$$

where the quadratic spinor quantities are introduced:

$$s = \overline{\Psi}\Psi, \quad p = i\overline{\Psi}\gamma_5\Psi, \quad \vec{v} = \overline{\Psi}\vec{\lambda}\Psi, \quad \vec{a} = i\overline{\Psi}\gamma_5\vec{\lambda}\Psi, \quad j_\mu = \overline{\Psi}\gamma_\mu\Psi, \quad \tilde{j}_\mu = \overline{\Psi}\gamma_\mu\gamma_5\Psi,$$

with $\overline{\Psi} = \Psi^+\gamma_0$ and $\vec{\lambda}$ standing for Pauli matrices in the isotopic spinor space. Here $\gamma_{\mu,}\ \mu = 0,1,2,3,$ designate the unitary Dirac matrices acting on Minkowski spinor indices. One can verify the time-like character of the Dirac 4-current j_μ and, in view of the identity (1.1), use the special structure of the Higgs potential V implying the spontaneous symmetry breaking in our model:

$$V = \frac{\sigma^2}{8}(j^\mu j_\mu - æ_0^2)^2, \tag{1.2}$$

where σ and $æ_0$ stand for some constant parameters. If one searches for localized soliton-like configurations, one should use the natural boundary condition at space infinity:

$$\lim_{|\vec{x}|\to\infty} j_\mu j^\mu = æ_0^2, \tag{1.3}$$

which determines the fixed (vacuum) point in the phase space of our model. In particular, if for the vacuum state Ψ_0 one gets $s(\Psi_0) \neq 0$, then the chiral invariant $s^2 + \vec{a}^2$ determines sphere S^3 as the field manifold, that corresponds to the Skyrme model and the baryon phase. If $v_3(\Psi_0) \neq 0$, then the $SO(3)$ invariant $\vec{v}^2$ determines the sphere S^2 as the field manifold, that corresponds to the Faddeev model and the lepton phase.

2. Effective nonlinear 16-spinor field model

However, the vacuum state Ψ_0 should be universal, that is common both for baryon and lepton phases. It means that conditions $v_3(\Psi_0) \neq 0$ and $s(\Psi_0) \neq 0$ should be compatible. Taking into account that the Brioschi identity (1.1) holds only for 8-spinor fields, one concludes that the realistic model should include the two 8-spinor fields Ψ_1 and Ψ_2, which can be naturally unified forming 16-spinor field

$$\Psi = \Psi_1 \oplus \Psi_2. \tag{2.1}$$

Finally, each topological sector should be described by the effective 8-spinor field.

In view of these arguments, using the analogy with Skyrme – Faddeev model, we consider the following Lagrangian density for the 16-spinor field model, which resembles the one used in [3]:

$$L_{spin} = \frac{1}{2\lambda^2}\overline{\partial_\mu \Psi}\gamma^\nu j_\nu \partial^\mu \Psi + \frac{\varepsilon^2}{4} f_{\mu\nu} f^{\mu\nu} - V, \tag{2.2}$$

where $f_{\mu\nu}$ stands for the anti-symmetric tensor of Skyrme – Faddeev type:

$$f_{\mu\nu} = (\overline{\Psi}\gamma^\alpha \partial_{[\mu}\Psi)(\overline{\partial_{\nu]}\Psi}\gamma_\alpha \Psi), \tag{2.3}$$

with λ and ε being constant parameters of the model. It should be underlined that the first term in (2.2) is similar to the sigma-model one and includes the projector $P = \gamma^0\gamma^\nu j_\nu$ on the positive energy states. We also remark that the Dirac 16-spinor current conserves its time-like character due to additive property $j_\mu = j_\mu^{(1)} + j_\mu^{(2)}$.

In 16-spinor space we introduce the two kinds of internal Pauli matrices:

$$\vec{\lambda} = I_4 \otimes \vec{\sigma} \otimes I_2, \quad \vec{\Lambda} = I_8 \otimes \vec{\sigma}, \tag{2.4}$$

where I_n designates the n-dimensional unity and $\vec{\sigma}$ - usual Pauli matrices. We suppose also that the vacuum state has the following structure:

$$\Psi_0 = \Psi_{10} \oplus 0, \tag{2.5}$$

with the projective property:

$$\Lambda\Psi_0 = 0, \quad \Lambda = \frac{1}{2}(1-\Lambda_3). \tag{2.6}$$

Now the main question arises: how to choose the lepton sector or the baryon one? To find the answer, we consider the topological structure of our model implying the distinction between these sectors. First, we pay attention to the fact that in the lepton sector $\vec{a} = i\overline{\Psi}\gamma_5\vec{\lambda}\Psi = 0$ that implies $\mathbb{B} = 0$. This condition can be satisfied if we suppose the mirror symmetry of the lepton sector:

$$\Psi \to \gamma_0 \Psi. \tag{2.7}$$

Therefore, if we introduce the following structure of the 16-spinor:

$$\Psi = \bigoplus_{j=1}^{2}(\varphi_j \oplus \chi_j \oplus \xi_j \oplus \zeta_j), \tag{2.8}$$

where $\varphi_j, \chi_j, \xi_j, \zeta_j$ stand for 2-spinors, then (2.7) implies the invariance condition $\Psi = \gamma_0\Psi$, that is $\varphi_j = \chi_j$, $\xi_j = \zeta_j$ for the Weyl representation of γ-matrices. Thus, we conclude that the lepton sector is described by the effective 8-spinor:

$$\Psi_L = \bigoplus_{j=1}^{2}(\varphi_j \oplus \varphi_j \oplus \xi_j \oplus \xi_j), \tag{2.9}$$

which is evidently consistent with the Brioschi identity, since the new vector $\vec{V} = \overline{\Psi}\vec{\Lambda}\Psi \neq 0$. In particular, one gets

$$j_0 = 2\sum_{j=1}^{2}(|\varphi_j|^2 + |\xi_j|^2); \quad \vec{j} = 0; \quad V_1 = 4\,\mathrm{Re}\,(\varphi_1^+\varphi_2 + \xi_1^+\xi_2);$$
$$V_2 = 4\,\mathrm{Im}(\varphi_1^+\varphi_2 + \xi_1^+\xi_2); \quad V_3 = 2(|\varphi_1|^2 + |\xi_1|^2 - |\varphi_2|^2 - |\xi_2|^2),$$

with the evident property:

$$\frac{1}{4}(j_0^2 - \vec{V}^2) = 4(|\varphi_2|^2 + |\xi_2|^2)\,(|\varphi_1|^2 + |\xi_1|^2) - 4\,|\varphi_1^+\varphi_2 + \xi_1^+\xi_2|^2 \geq 0$$

due to the Schwartz inequality.

In view of the boundary condition (1.3) we conclude that the correspondent localized configuration can be endowed with the nontrivial Hopf index $Q_H = \mathbb{L}$, the S^2-manifold being determined by the invariant structure $\vec{V}^2$.

Let us now consider the baryon sector. In this case we take into account the charge independence of strong interactions, which is equivalent to the mirror symmetry in the isotopic space:

$$\Psi \to \gamma_0 \gamma_5 \lambda_2 \Psi^* . \tag{2.10}$$

Inserting (2.10) into the invariance condition for the group (2.10), one finds the following structure for the baryon sector 16-spinor field:

$$\Psi_B = \bigoplus_{j=1}^{2} (\varphi_j \oplus \chi_j \oplus i\sigma_2 \varphi_j^* \oplus i\sigma_2 \chi_j^*) , \tag{2.11}$$

which is again equivalent to the effective 8-spinor. In this case we find the following bilinear spinor quantities determining the topology of the baryon sector:

$$j_0 = 2\sum_{j=1}^{2} (|\varphi_j|^2 + |\chi_j|^2); \quad \vec{j} = 0; \quad s = 4\sum_{j=1}^{2} \mathrm{Re}\,(\varphi_j^+ \chi_j);$$
$$a_1 = -4\sum_{j=1}^{2} \mathrm{Re}\,(\varphi_j^T \sigma_2 \chi_j); \quad a_2 = -4\sum_{j=1}^{2} \mathrm{Im}\,(\varphi_j^T \sigma_2 \chi_j); \quad a_3 = 4\sum_{j=1}^{2} \mathrm{Im}\,(\varphi_j^+ \chi_j).$$

Therefore, we can calculate the chiral invariant structure determining S^3-manifold:

$$s^2 + \vec{a}^2 = 16\,(|\varphi_1^+ \chi_1 + \varphi_2^+ \chi_2|^2 + |\varphi_1^T \sigma_2 \chi_1 + \varphi_2^T \sigma_2 \chi_2|^2) .$$

Using the Schwartz inequality, one can prove that $j_0^2 \ge s^2 + \vec{a}^2$. This structure is again consistent with the Higgs potential (1.2).

3. Interaction with physical vector fields

The Lagrangian (2.2) admits very large group of transformations of the 16-spinor field Ψ, including phase transformations with some charge generator Γ_e and left, right isotopic rotations with generators $P_{L,R}\lambda^a \Lambda/2;\ a = 1,2,3,$, where $P_{L,R} = (1 \pm \gamma_5)/2$ stand for the corresponding projectors. These symmetries give rise to the interactions with the electromagnetic field A_μ and left, right Yang – Mills fields $A_\mu^{aL,R}$ respectively, the latter ones being responsible for the strong interactions. Due to the general gauge invariance principle these interactions can be included in the Lagrangian through the extension of the derivative $\partial_\mu \to D_\mu$, where

$$D_\mu \Psi = \partial_\mu \Psi - i e_0 \Gamma_e A_\mu \Psi + (A_\mu^L + A_\mu^R)\Psi . \tag{3.1}$$

We adopt the following structure of Γ_e and $A_\mu^{aL,R}$:

$$\Gamma_e = \lambda_3 \Lambda; \quad A_\mu^{L,R} = P_{L,R} \frac{e_{1L,R}}{2i} A_\mu^{aL,R} \lambda^a \Lambda , \tag{3.2}$$

which is in accordance with the breaking of isotopic symmetry by the electromagnetic interaction, e_0, e_{1L}, e_{1R} being the corresponding coupling constants. However, the fields $A_\mu^{aL,R}$ seem to be not independent due to the natural condition that Yang – Mills fields should not give any contribution to the lepton sector. This condition can be satisfied, in view of (2.7) and (3.1), only if the sum $A_\mu^L + A_\mu^R$ is proportional to γ_5. Taking into account (3.2), one concludes that this is possible, only if the following constraint holds:

$$e_{1L} A_\mu^{aL} + e_{1L} A_\mu^{aR} = 0\,. \tag{3.3}$$

In view of (3.3) one can easily find that

$$A_\mu^L + A_\mu^R = \frac{e_{1L}}{2i} A_\mu^{aL} \gamma_5 \lambda^a \Lambda \equiv g_0 \gamma_5 \mathbb{A}_\mu\,, \tag{3.4}$$

where the denotation is used:

$$e_{1L} \equiv g_0; \qquad \mathbb{A}_\mu \equiv \frac{\lambda^a}{2i} \Lambda A_\mu^a; \quad A_\mu^{aL} \equiv A_\mu^a\,.$$

If one introduces the intensity of the Yang – Mills fields:

$$F_{\mu\nu}^{L,R} = \partial_\mu A_\nu^{L,R} - \partial_\nu A_\mu^{L,R} + [A_\mu^{L,R}, A_\nu^{L,R}]$$

and use the natural supposition that $e_{1L} = e_{1R}$, then the standard Yang – Mills Lagrangian can be rewritten, in view of the constraint (3.3), as follows:

$$L_{YM} = \frac{1}{32\pi g_0^2} Sp\,(F_{\mu\nu}^L F^{\mu\nu L} + F_{\mu\nu}^R F^{\mu\nu R}) = \frac{1}{32\pi} Sp\{(\partial_\mu \mathbb{A}_\nu - \partial_\nu \mathbb{A}_\mu)(\partial^\mu \mathbb{A}^\nu - \partial^\nu \mathbb{A}^\mu) +$$

$$+\; g_0^2 [\mathbb{A}_\mu, \mathbb{A}_\nu][\mathbb{A}^\mu, \mathbb{A}^\nu]\},$$

where the symbol *Sp* signifies the corresponding matrix trace.

Now the extended derivative (3.1) takes the form:

$$D_\mu \Psi = \partial_\mu \Psi - i e_0 \Gamma_e A_\mu \Psi + g_0 \gamma_5 \mathbb{A}_\mu \Psi\,, \tag{3.5}$$

the effective Yang – Mills interaction being pseudo-vector one.

4. Quantization of electric charge and quantum mechanics

Finally we intend to discuss a new possibility given by the nonlinear spinor model in question to derive usually adopted hypothesis of the electric charge Q_e quantization, i. e. $Q_e = n\, e_0$, $n \in \mathbb{Z}$. To this end we take into account that in the Maxwell electrodynamics the electric charge

$$Q_e = \frac{1}{4\pi} \int d^3x\, div \vec{E} = \frac{1}{4\pi} \oint_{S\to\infty} (\vec{n}\vec{E})\, dS \tag{4.1}$$

is considered as invariant under Lorentz and dilatation transformations:

$$x^\mu \to \alpha\, x^\mu, \quad A^\mu \to \alpha\, A^\mu(\alpha\, x), \quad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \to \alpha^2 F_{\mu\nu}(\alpha\, x)\ .$$

Within the framework of 16-spinor electrodynamics there also exists an interesting possibility to construct for localized configurations (solitons) a special invariant u under the above transformations, the latter one coinciding with the electric charge (4.1) of the system at large distances:

$$u = (n_\mu A^\mu)^2 (E_\nu E^\nu)^{-1/2}, \tag{4.2}$$

where $E_\nu = n^\mu F_{\mu\nu}$ and the unit time-like vector n^μ can be identified with the normalized Dirac current: $n^\mu = j^\mu / j, \quad j \equiv (j_\nu j^\nu)^{1/2}$. In the proper reference frame of the static soliton with the charge q and the dipole moment $\vec{p}$ one finds

$$u = \Phi^2 / E, \quad \Phi = A^0, \quad E = |\nabla\Phi|.$$

Therefore, at large distances $r \to \infty$ one can use the multi-pole expansion in spherical coordinates:

$$\Phi = \frac{q}{r} + \frac{p}{r^2}\cos\vartheta + O(r^{-3}); \quad E^2 = \frac{q^2}{r^4}\left[1 + \frac{4p}{qr}\cos\vartheta + O(r^{-2})\right],$$

whence one gets the remarkable behavior of the invariant u:

$$u = q\left[1 + O(r^{-2})\right]. \tag{4.3}$$

Let us now consider some generalization of the Maxwell Lagrangian density:

$$L_{em} = \frac{E^2}{8\pi} f(u), \tag{4.4}$$

with $f(u)$ being some unknown function. The equation of motion for the scalar potential Φ reads:

$$\Phi\, div[\vec{E}(2f - u\, f'(u)] = 2u\, f'(u). \tag{4.5}$$

At large distances, in view of (4.3), the left hand side of (4.5) behaves as $O(r^{-5})$. Therefore, if $q \neq 0$, then $f'(u) \to 0$ as $r \to \infty$, or

$$f'(q) = 0. \tag{4.6}$$

The equation (4.6) could be considered as the quantization condition for the electric charge, if it had the solution $q = Q_e = n\, e_0, \quad n \in \mathbb{Z}$. The simplest choice of the function $f(u)$ satisfying the correspondence with the Maxwell theory reads:

$$f(u) = 1 + \mu_0 \sin^2[\pi u/(2e_0)],$$

with μ_0 being some small constant. Thus, the electromagnetic part of the Lagrangian density has the form:

$$L_{em} = -\frac{1}{16\pi} F_{\mu\nu} F^{\mu\nu} \left[1 + \mu_0 \sin^2 \left(\frac{\pi u}{2e_0} \right) \right]. \tag{4.7}$$

Finally, let us consider small excitations of our soliton near the vacuum: $\Psi = \Psi_0 + \xi$, $\xi \to 0$ as $|\vec{x}| \to \infty$. Then the linearized equation for ξ after the substitution $\xi = k\,\Psi_0$ takes the form:

$$\partial_\mu \partial^\mu k + \frac{1}{2} M_0^2 (k^* + k) = 0, \quad M_0 \equiv 2\sigma\lambda\, æ_0. \tag{4.8}$$

Thus, from (4.8) one derives the following equations for real and imaginary parts of $k = k_1 + i\,k_2$:

$$\partial_\mu \partial^\mu k_1 + M_0^2 k_1 = 0, \quad \partial_\mu \partial^\mu k_2 = 0. \tag{4.9}$$

According to (4.9), our model admits two types of vacuum excitations: massive and massless ones. For massive soliton the first equation in (4.9) coinciding with the well-known Klein – Gordon one, the spinor excitation ξ could be interpreted as the wave function of the point-like particle representing the motion of the center of the soliton, if the mass parameter M_0 corresponded to the real mass M of the particle-soliton (or inverse Compton length in natural units $\hbar = c = 1$). To satisfy this condition, we first introduce the interaction with the gravitational field via the new extended derivative generalizing (3.5):

$$\nabla_\mu \Psi = (D_\mu - \Gamma_\mu)\Psi, \tag{4.10}$$

where Γ_μ stands for spinor connection with gravity. Then we intend to generalize the Higgs potential (1.2), in which the constant multiplier σ^2 will be replaced with the special invariant:

$$\sigma^2 = -\frac{2(8/7)^3 D^3}{\lambda^2 G^2 K^2}\, æ_0^{-2}, \tag{4.11}$$

where G stands for the Newton gravitational constant, D is the sigma-model part of the L_{spin}:

$$D = \overline{\nabla_\mu \Psi}\, \gamma^\alpha j_\alpha \nabla^\mu \Psi \tag{4.12}$$

and K is the so-called Kraichnan invariant constructed with the help of the Riemannian curvature tensor:

$$K = \frac{1}{48} R_{\mu\nu\sigma\lambda} R^{\mu\nu\sigma\lambda}. \tag{4.13}$$

The validity of the choice (4.11) can be verified by calculating invariants (4.12) and (4.13) for Schwarzschild metric at large distances $r = |\vec{x}| \to \infty$:

$$D = -\frac{7r_g^2}{16r^4}\, æ_0^2; \quad K = \frac{r_g^2}{r^6}; \quad r_g = GM. \tag{4.14}$$

The resulting Lagrangian of our 16-spinor model reads:

$$L = L_{spin} + L_{em} + L_{YM} + L_g\,, \tag{4.15}$$

where the gravitational Lagrangian L_g coincides with the Einstein one: $L_g = R/(16\pi G)$, R being the scalar curvature.

It is worth-while to stress that the gravitational field plays an important role in our model, since the wave-particle duality principle of quantum mechanics has the gravitational origin. As for the vacuum excitation ξ, one can prove [5, 6] that it plays the role of the wave function in the special stochastic representation of quantum mechanics.

5. Conclusion

Some arguments in favor of 16-spinor nonlinear model were discussed to obtain the unification of Skyrme and Faddeev models via Brioschi identity and special choice of the Higgs-like potential. The resulting Lagrangian density includes spinor part and the special form of interactions with the electromagnetic, Yang – Mills and gravitational fields. The main result of the modifications proposed concerns the quantization of the electric charge and the correspondence with quantum mechanics in the linear limit of small vacuum excitations.

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Termination of the physics relativization and logical reloading in the space-time geometry

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In the beginning of the twentieth century the relativity theory had not been completed in the sense that dynamic equations were relativistic, but the particle state remained to be nonrelativistic. Consecutive relativistic approach admits one to construct unified formalism of the particle dynamics which can be applied for deterministic and stochastic motion of particles. This formalism admits one to found the quantum mechanics and to explain quantum phenomena without a use of quantum principles. Refusing from the constraint on continuity of the space-time geometry and using the metric approach to geometry, one explains stochastic motion of elementary particles and constructs the skeleton conception of particle dynamics. The skeleton conception admits one to investigate the elementary particle structure (but not only to systematize the elementary particles, ascribing quantum numbers to them)

Introduction

In the beginning of the twentieth century the relativity theory had not been completed in the sense that dynamic equations were relativistic, but the description of the particle state remained to be nonrelativistic. Nonrelativistic concept of the particle state is a point in the 3-dimensional space or in the phase space. The real relativistic definition of the particle state looks otherwise.

Conventionally the special relativity principle is formulated as the Lorentz-invariance of dynamical equations. On the other hand, a general physical principles can be hardly formulated as a statement connected with such details of description as a coordinate transformation. We formulated the relativity principle as follows. *The space-time is described by one space-time structure* ST. It means that the space-time geometry is described by the only quantity: space-time distance ρ, or only by the world function $\sigma = \frac{1}{2}\rho^2$. In the non-relativistic physics the space-time is described by means of two independent quantities (structures): spatial distance S and and temporal interval T. Among three structures: T, S, and ST only two of them are independent. Such a formulation of the relativity principle is more general, because it is valid not only for the space-time geometry of Minkowski. It is valid for any space-time geometry, including a discrete space-time geometry. Besides, this formulation is coordinateless.

One cannot be sure that the space-time geometry is continuous in microcosm. Restricting our consideration by the continuous space-time geometries, we are mistaken. This mistake is justified by the fact that the formalism of a discrete geometry has not been developed, and one believes that the space-time geometry cannot be discrete. In reality, a discrete geometry, as any geometry, is a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. But the Euclidean geometry is to be described in terms of distance ρ and only in terms of distance, because other concepts of $\mathcal{G}_{\mathrm{E}}$ contain a reference to continuity of $\mathcal{G}_{\mathrm{E}}$, and they cannot be used for a construction of a discrete geometry.

The simplest discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is described by the world function

$$\sigma_{\mathrm{d}}(P,Q) = \sigma_{\mathrm{M}}(P,Q) + \frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}(P,Q)\right), \quad \forall P,Q \in \Omega \tag{1}$$

where Ω is a set of all points of the space-time, σ_{M} is the world function of the Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$, and λ_0 is the elementary length. In the inertial coordinate system the world function σ_{M} has the form

$$\sigma_{\mathrm{M}}(x,x') = \frac{1}{2}g_{ik}\left(x^i - x'^i\right)\left(x^k - x'^k\right), \quad g_{ik} = \mathrm{diag}\left(c^2, -1, -1, -1\right) \tag{2}$$

In the discrete space-time geometry a pointlike particle cannot be described by a world line, because any world line is a limit of the broken line, when lengths of its links tend to zero. But in the discrete geometry $\mathcal{G}_{\mathrm{d}}$ there are no infinitesimal lengths, because all lengths are longer, than λ_0. In $\mathcal{G}_{\mathrm{d}}$ a pointlike particle is described by a world chain (broken line) instead of a smooth world line. Description of a pointlike particle state by means of the particle position and its momentum becomes inadequate. The reason lies in the fact that in the continuous (differential) space-time geometry the particle 4-momentum p_k is described by the relation

$$p_k = g_{kl}\frac{dx^l}{d\tau} = g_{kl}\lim_{d\tau\to 0}\frac{x^l(\tau + d\tau) - x^l(\tau)}{d\tau} \tag{3}$$

where $x^l = x^l(\tau)$, $l = 0,1,2,3$ is an equation of the world line. The limit in the formula (3) does not exist in $\mathcal{G}_{\mathrm{d}}$, and the 4-momentum p_k is not defined (at any rate in such a form). In general, the mathematical formalism of a differential geometry, based on the infinitesimal calculus (differential dynamic equations), is inadequate in the discrete space-time geometry, where infinitesimal distances are absent.

In the case of arbitrary space-time geometry the particle state is described by two space-time points. The two points P, Q determine the vector $\mathbf{PQ} = \{P, Q\}$, which can be interpreted as the particle momentum. In the case of a discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ the vector $\mathbf{PQ}$ can be also interpreted as a momentum, but its presentation in the form (3) is impossible.

In the arbitrary space-time geometry the pointlike particle is described by a world chain $\mathcal{C}$

$$\mathcal{C} = \bigcup_s P_s, \quad |\mathbf{P}_s\mathbf{P}_{s+1}| = \mu = \text{const}, \quad s = ...0, 1, ... \tag{4}$$

Here μ is a geometric mass of the particle (length of the world chain link), which is connected with the particle mass m by the relation

$$m = b\mu \tag{5}$$

where b is an universal constant.

In $\mathcal{G}_\mathrm{d}$ only coordinateless description is possible [1], which is produced in terms and only in terms of the world function σ_d, or in terms of the space-time distance ρ_d, because a use of all geometric concepts of the Riemannian geometry (except for distance) contains a reference to continuity of the geometry. In the coordinateless description the scalar product $(\mathbf{PQ}.\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ has the form

$$(\mathbf{PQ}.\mathbf{RS}) = \sigma(P,S) + \sigma(Q,R) - \sigma(P,R) - \sigma(Q,S) \tag{6}$$

and

$$|\mathbf{PQ}|^2 = (\mathbf{PQ}.\mathbf{PQ}) = 2\sigma(P,Q) \tag{7}$$

The coordinateless definitions of the scalar product $(\mathbf{PQ}.\mathbf{RS})$ and of the vector length $|\mathbf{PQ}|$ coincide with their conventional definitions in the proper Euclidean geometry. They can be used in any space-time geometry $\mathcal{G}$, which is completely described by its world function σ. Such a space-time geometry will be referred to as a physical geometry.

Equivalency $(\mathbf{PQ}\mathrm{eqv}\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ is defined by two coordinateless relations

$$(\mathbf{PQ}\mathrm{eqv}\mathbf{RS}): \quad (\mathbf{PQ}.\mathbf{RS}) = |\mathbf{PQ}|\cdot|\mathbf{RS}| \wedge |\mathbf{PQ}| = |\mathbf{RS}| \tag{8}$$

The discrete space-time geometry is multivariant in the sense, that there are many vectors $\mathbf{PQ}, \mathbf{PQ}', \mathbf{PQ}''$,... at the point P which are equivalent to the vector $\mathbf{RS}$ at the point R, but vectors $\mathbf{PQ}, \mathbf{PQ}', \mathbf{PQ}''$,... are not equivalent between themselves.

In the proper Euclidean geometry $\mathcal{G}_\mathrm{d}$ the equivalence relation (8) is single-variant, and there is only one vector $\mathbf{PQ}$ at the point P which is equivalent to the vector $\mathbf{RS}$ at the point R.

If the world chain (4) describes a free particle, its links satisfy the relations

$$(\mathbf{P}_s\mathbf{P}_{s+1}\mathrm{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}), \quad s = ...0, 1, ... \tag{9}$$

These relations are multivariant in $\mathcal{G}_\mathrm{d}$. It leads to a wobbling of the world chain. This wobbling means that the particle motion is stochastic (random). Amplitude of wobbling is restricted by the elementary length λ_0 in $\mathcal{G}_{\rm d}$ for timelike vectors, But this amplitude is infinite for spacelike vectors. In the geometry of Minkowski $\mathcal{G}_{\rm M}$ the wobbling is absent for timelike vectors ($\lambda_0 = 0$), and amplitude of this wobbling is infinite for spacelike vectors.

In the nonrelativistic approximation a statistical description of timelike world lines in $\mathcal{G}_{\rm d}$ leads to the Schrödinger equation [2], if the elementary length

$$\lambda_0^2 = \frac{\hbar}{bc} \tag{10}$$

where b is the universal constant defined by (5). A single particle with the spacelike world chain (tachyon) cannot be detected, because of the infinite amplitude of its wobbling. However, gravitational field of the tachyon gas can be detected (dark matter) [3, 4].

It is very important that the statistical description of wobbling world chains is produced relativistically, when the pointlike particle state is described by two points (but not by a point in the phase space). In this case the statistical ensemble is a dynamic system of the type of a continuous medium, and one may introduce the wave function as a method of the continuous medium description [5]. In the nonrelativistic description the particle state is a point in the phase space. In this case the statistical description is a probabilistic construction describing evolution of the particle state probability [6, 7, 8].

Let us stress that the statistical ensemble as a dynamic system (but not as a probabilistic construction) is a result of a correct (relativistic) definition of the particle state (but not a result of some new hypothesis). Description of the particle motion by means of the world chain (4) is a corollary of the consecutive application of the relativity principle.

Unified formalism of particle dynamics

The main optical element of the experimental setup is a rotating optical disc. Since the optical disc is a wedge-shaped, then when disc rotates the fringes move in the plane of the photodetector by a harmonic law. Specifications and a more detailed description of the SADE experimental setup can be found in [**?**]. When using spatial interference pattern shift measurement methods there is a loss of interference pattern contrast due to rotation of the disc caused by disc wedge shape. Wedge shape is hard to remove production defect. Instead of trying to create the ideal discs without the wedge shape, which is an expensive process, the discs are made slightly wedge-shaped. Owing to the high resolution of temporary measurements with respect to spatial, this approach allows us to get the best value for S/N ratio.

Foundation of the quantum mechanics on the basis of the stochastic particle dynamics is obtained as corollary of unified formalism of the particle dynamics [9]. Stochastic particle $\mathcal{S}_{\rm st}$ is not a dynamic system, and there are no dynamic equations for $\mathcal{S}_{\mathrm{st}}$. However, statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, i.e. a set of many independent stochastic particles $\mathcal{S}_{\mathrm{st}}$, is a dynamic system of the type of the continuous medium. Deterministic particle $\mathcal{S}_{\mathrm{det}}$ as well as statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{det}}]$ are dynamic systems, and there are dynamic equations for them. At the conventional approach to the particle dynamics, when the basic element of dynamics is a single particle, one cannot construct a unified dynamic conception for stochastic and deterministic particles, because there are no dynamic equations for a single stochastic particle. However, after the logical reloading, when the statistical ensemble becomes a basic object of the particle dynamics, one obtains dynamic equations for statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ and for statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{det}}]$ of deterministic particles $\mathcal{S}_{\mathrm{det}}$ [9].

For instance, the action for the statistical ensemble of stochastic particles $\mathcal{S}_{\mathrm{st}}$ has the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_\xi} \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\}\rho_1(\boldsymbol{\xi})\,dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{11}$$

The variable $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$ describes the regular component of the particle motion. The independent variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ label elements (particles) of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$. The variable $\mathbf{u} = \mathbf{u}(t,\mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant, $\rho_1(\boldsymbol{\xi})$ is a weight function. One may set $\rho_1 = 1$. The second term in (11) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t,\mathbf{x})$ and the regular component $\dot{\mathbf{x}}(t,\boldsymbol{\xi})$ of the particle velocity. The operator

$$\boldsymbol{\nabla} = \left\{\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right\} \tag{12}$$

is defined in the space of coordinates $\mathbf{x}$.

Formally the action (11) describes a set of deterministic particles $\mathcal{S}_{\mathrm{d}}$, interacting via the force field $\mathbf{u}$. The particles $\mathcal{S}_{\mathrm{d}}$ form a gas (or a fluid), described by the variables $\dot{\mathbf{x}}(t,\boldsymbol{\xi}) = \mathbf{v}(t,\boldsymbol{\xi})$. Here this description is produced in the Lagrange representation. Hydrodynamic description is produced in terms of density ρ and velocity $\mathbf{v}$, where

$$\rho = \rho_1 J, \quad J \equiv \frac{\partial(\xi_1,\xi_2,\xi_3)}{\partial(x^1,x^2,x^3)}, \quad v^\alpha = \frac{\partial(x^\alpha,\xi_1,\xi_2,\xi_3)}{\partial(t,\xi_1,\xi_2,\xi_3)}, \quad \alpha = 1,2,3 \tag{13}$$

Nonrotational flow of this gas is described by the Schrödinger equation [9].

The dynamic equation for the force field $\mathbf{u}$ is obtained as a result of variation of (11) with respect to $\mathbf{u}$. It has the form

$$\mathbf{u} = \mathbf{u}(t,\mathbf{x}) = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho \tag{14}$$

The vector $\mathbf{u}$ describes the mean value of the stochastic velocity component of the stochastic particle $\mathcal{S}_{\mathrm{st}}$. In the nonrelativistic case the force field $\mathbf{u}$ is determined by its source: the fluid density ρ.

In terms of the wave function the action (11) takes the form [9]

$$\mathcal{A}[\psi,\psi^*]=\int\left\{\frac{i\hbar}{2}(\psi^*\partial_0\psi-\partial_0\psi^*\cdot\psi)-\frac{\hbar^2}{2m}\boldsymbol{\nabla}\psi^*\cdot\boldsymbol{\nabla}\psi+\frac{\hbar^2}{8m}\rho\boldsymbol{\nabla}s_\alpha\boldsymbol{\nabla}s_\alpha\right\}\mathrm{d}^4x \tag{15}$$

where the wave function $\psi=\left\{\begin{smallmatrix}\psi_1\\ \psi_2\end{smallmatrix}\right\}$ has two complex components.

$$\rho=\psi^*\psi,\qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho},\qquad \alpha=1,2,3 \tag{16}$$

σ_α are 2×2 Pauli matrices

$$\sigma_1=\left(\begin{array}{cc}0&1\\1&0\end{array}\right),\qquad \sigma_2=\left(\begin{array}{cc}0&-i\\i&0\end{array}\right),\qquad \sigma_3=\left(\begin{array}{cc}1&0\\0&-1\end{array}\right), \tag{17}$$

Dynamic equation, generated by the action (15), has the form

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi+\frac{\hbar^2}{8m}\boldsymbol{\nabla}^2s_\alpha\cdot(s_\alpha-2\sigma_\alpha)\,\psi-\frac{\hbar^2}{4m}\frac{\boldsymbol{\nabla}\rho}{\rho}\boldsymbol{\nabla}s_\alpha\sigma_\alpha\psi=0 \tag{18}$$

In the case of one-component wave function ψ, when the flow is nonrotational and $\boldsymbol{\nabla}s_\alpha=0$, the dynamic equation has the form of the Schrödinger equation

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\boldsymbol{\nabla}^2\psi=0 \tag{19}$$

Thus, the Schrödinger equation is a special case of the dynamic equation, generated by the action (11) or (15). Thus, linearity of dynamic equation in terms of the wave function is a special case of dynamic equation for the statistical ensemble of stochastic particles, although it is considered usually as a principle of quantum mechanics.

Reason of the elementary particles stochasticity

Stochasticity of elementary particles and wobbling of their world chains are conditioned by discreteness of the space-time geometry, more exactly by its multivariance [1]. A discrete geometry is constructed as a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. But for such a generalization one needs to produce a logical reloading and to present $\mathcal{G}_{\mathrm{E}}$ in terms of the world function [10, 11]. A use of the discrete space-time geometry admits one to formulate the skeleton conception of elementary particles, where the particle state and all parameters of an elementary particle are described by the particle skeleton [12]. The skeleton is several space-time points, connected rigidly. Distances between the skeleton points determine parameters of the particle. World chain (4) with the two-point skeleton describes the simplest case of elementary particle. In this case there is only one parameter of the skeleton. It is the length $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}|$ of the world chain link. According to (5) the particle mass is a geometrical quantity. In other words, description of a particle motion is geometrized completely.

A generalization of two-point skeleton of a pointlike particle arises at consideration of the Dirac equation [13, 14, 15]. Analyzing the Dirac equation from the viewpoint of quantum mechanics, one meets abstract dynamic variables (γ-matrices), whose meaning is unclear. Analyzing the Dirac equation and using the united formalism of particle dynamics (without a use of quantum principles), one concludes that world line of the Dirac particle is a helix with timelike axis. Helical motion of a free particle is possible, if its skeleton contains three (or more) points [16]. Helical motion of a particle explains the particle spin and magnetic moment, whereas at the quantum approach spin and magnetic moment are simply quantum numbers, whose nature is unknown. Thus, the skeleton conception of elementary particle dynamics admits one to investigate structure and arrangement of elementary particles.

The skeleton conception is obtained as a direct corollary of physical principles without a use of artificial principles and hypotheses alike the quantum mechanics principles. It is the main worth of the skeleton conception. The skeleton conception is obtained as a result of correction mistakes in the conventional theory: (1) nonrelativistic concept of the particle state and (2) unfounded restriction by the continuous space-time geometry. Correction of these mistakes leads to the skeleton conception without any additional suppositions.

A use of the skeleton conception admits one to explain the dark matter as a tachyon gas and to explain impossibility of a single tachyon detection [3, 4]. These phenomena cannot be explained from the point of view of quantum approach.

A use of the logical reloading is followed by essential change of a mathematical formalism. This change of formalism is perceived hardly by people, using conventional formalism. For instance, the discrete geometry $\mathcal{G}_{\mathrm{d}}$ described by the world function (1) is uniform and isotropic. Indeed, the world function σ_{M} (2) of the geometry of Minkowski is invariant with respect to Poincare group of transformations. The world function σ_{d} (1) is a function of σ_{M}. It is also invariant with respect to Poincare group of transformations. It means that the discrete geometry (1) is uniform and isotropic. This fact contradicts to conventional approach to the discrete geometry which is considered as a geometry on a lattice. Geometry on a lattice cannot be uniform and isotropic. Besides, in the discrete geometry (1) there is no definite dimension (maximal number of linear independent vectors). At the conventional approach to geometry such a situation is impossible, because any construction of a geometry begins from a fixation of the geometry dimension in the form of a natural number.

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EInvestigation of the role of the irreducible parts of torsion propagating as plane waves in the Riemann-Cartan space

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The problem of the existence of plane waves of torsion in the Poincare gauge theory of gravity in the space of Riemann-Cartan U_4 with curvature and torsion was investigated, and the role of irreducible parts of the torsion in the propagation of a plane-wave was formulated.

Introduction

The study of exact solutions of the field equations in spaces endowed with a more difficult structure than Riemannian space of general relativity is considerable interest. A special place here is the search for wave solutions that has both theoretical and possible practical value. For example, in (1) gravitational waves were studied in space with a non-zero torsion in the theory with the Lagrangian of a special type, which consists of a linear Lagrangian of the Einstein-Cartan theory, one of six squares of the curvature tensor and all possible squares of the torsion tensor. In (2) torsion waves were studied against the background of the flat space in the theory with quadratic curvature Lagrangian. In (3), the authors considered the theory of plane waves with a Lagrangian quadratic in torsion and curvature without the linear part. Paper (4) was devoted to the study of torsion waves for 2-form of torsion algebraically special N-type. In paper (5) the structure of plane waves traceless part of torsion is studied, and in (6) the authors summarized the results of studies of the properties of plane waves as the trace and pseudotrace torsion.

The aim of this study is to investigate the structure of the irreducible parts of torsion in the propagation of a plane-wave in the Riemann - Cartan space.

Building a modern Poincare gauge theory of gravity is based on the significant use of non-linear Lagrangians in curvature and torsion (7)–(13). Most quadratic theories of gravity in Riemann - Cartan space can be described as special cases of the general 10-parameter Hayashi Lagrangian introduced as the sum of the linear Lagrangian of Einstein-Cartan theory and squares of the irreducible parts of curvature and torsion tensors. Using the theory of quadratic Lagrangians in the gravitational field is also stimulated by the construction of a renormalizable theory of gravitation in Riemann-Cartan space. Most quadratic theories of gravity in Riemann - Cartan space can be described as special cases of the general 10-parameter Hayashi Lagrangian introduced as the sum of the linear Lagrangian of Einstein-Cartan theory and squares of the irreducible parts of curvature and torsion tensors.

At the present stage the theory of gravity is described in terms of Cartan exterior differential forms. We're going to use variational formalism from the point of view of the external forms based on the lemma stated and proved in (5).

The basic concepts of the theory of gravity in a Riemann-Cartan space in the external form formalism

To start with, Riemann-Cartan space U_4 is a four-dimensional oriented differentiable manifold $\mathcal{M}$, endowed with the metric tensor $g_{ab}(a,b=1,2,3,4)$ of index 1, 4-form of volume η, a 1-form linear connection $\Gamma^a{}_b$, compatible with the metric, 2-form curvature $\mathcal{R}^a{}_b$ and 2-form torsion $\mathcal{T}^a$.

2-form of torsion and 2-form of curvature defined on the basis of the first and second Cartan structure equations:

$$\mathcal{T}^a = \frac{1}{2}\mathcal{T}^a{}_{bc}\theta^b \wedge \theta^c = \mathrm{D}\theta^a = d\theta^a + \Gamma^a{}_b \wedge \theta^b \,, \tag{1}$$

$$\mathcal{R}^a{}_b = \frac{1}{2}R^a{}_{bcd}\theta^c \wedge \theta^d = \mathrm{d}\Gamma^a{}_b + \Gamma^a{}_c \wedge \Gamma^c{}_b \,. \tag{2}$$

Here θ^a $(a=0,\ 1,\ 2,\ 3)$ – 1-forms co-basis of space (U_4, g),
$\wedge$ – operator of the exterior multiplication,
D – generalized exterior covariant differential: $\mathrm{D} = \mathrm{d} + \Gamma \wedge \ldots$ (d – operator of the exterior differential).

We shall use local nonholonomic vector basis $\mathbf{e}_b$ $(b=0,\ 1,\ 2,\ 3)$, where $\mathbf{e}_b \rfloor\, \theta^a = \delta^a_b$, $\rfloor$ – operation of the inner multiplication (convolution) and $g_{ab} = \breve{g}\,(\mathbf{e}_a,\ \mathbf{e}_b) = \mathrm{diag}(\ 1,\ -1,\ -1,\ -1)$.

It is convenient to use additional fields: 3-forms η_a, 2-forms η_{ab}, 1-forms η_{abc} and 0 -forms η_{abcd}, defined as follows (10):

$$\begin{aligned}
&\eta_a = \mathbf{e}_a \rfloor\, \eta = *\theta_a\,, \qquad \eta_{ab} = \mathbf{e}_b \rfloor\, \eta_a = *\,(\theta_a \wedge \theta_b)\ ,\\
&\eta_{abc} = \mathbf{e}_c \rfloor\, \eta_{ab} = *(\theta_a \wedge \theta_b \wedge \theta_c)\,,\\
&\eta_{abcd} = \mathbf{e}_d \rfloor\, \eta_{abc} = *(\theta_a \wedge \theta_b \wedge \theta_c \wedge \theta_d)\,,\\
&\theta^a \wedge \eta_b = \delta^a_b \eta\,, \qquad \theta^a \wedge \eta_{bc} = -2\delta^a_{[b}\eta_{c]}\,,\\
&\theta^d \wedge \eta_{abc} = 3\delta^d_{[a}\eta_{bc]}\,, \qquad \theta^f \wedge \eta_{abcd} = -4\delta^f_{[a}\eta_{bcd]}\,,
\end{aligned} \tag{3}$$

where $*$ – the operator of the dual conjugation by Hodge.

In the space of Riemann-Cartan U_4 2- form of curvature $\mathcal{R}^a{}_b$ and 2- form of torsion $\mathcal{T}^a$ can be broken down into parts that are irreducible representations of pseudo orthogonal transformations four-dimensional space-time.
In space-time U_4 2-form of curvature $\mathcal{R}_{ab}$ is antisymmetric:

$$\mathcal{R}_{ab} = \mathcal{R}_{[ab]}, \tag{4}$$

and has the following decomposition into irreducible parts (10):

$$\mathcal{R}^{ab} = \overset{(1)}{\mathcal{R}^{ab}} + \overset{(2)}{\mathcal{R}^{ab}} + \overset{(3)}{\mathcal{R}^{ab}} + \overset{(4)}{\mathcal{R}^{ab}} + \overset{(5)}{\mathcal{R}^{ab}} + \overset{(6)}{\mathcal{R}^{ab}}, \tag{5}$$

where,

$$\overset{(2)}{\mathcal{R}^{ab}} = -* \left(\theta_{[a} \wedge \Psi_{b]}\right), \quad \overset{(3)}{\mathcal{R}^{ab}} = -\tfrac{1}{12} * \left(\mathrm{X} \wedge \theta_a \wedge \theta_b\right),$$

$$\overset{(4)}{\mathcal{R}^{ab}} = -\theta_{[a} \wedge \Phi_{b]}, \qquad \overset{(5)}{\mathcal{R}^{ab}} = -\tfrac{1}{2}\theta_{[a} \wedge e_{b]} \rfloor \left(\theta^c \wedge W_c\right),$$

$$\overset{(6)}{\mathcal{R}^{ab}} = -\frac{1}{12} W \theta_a \wedge \theta_b, \qquad \overset{(1)}{\mathcal{R}^{ab}} = \mathcal{R}_{ab} - \sum_{n=2}^{6} \overset{(n)}{\mathcal{R}^{ab}}, \tag{6}$$

where,

$$W^a := e_b \rfloor R^{ab}, \quad W := e_b \rfloor W^b, \quad X^a := * \left(R^{ab} \wedge \theta_b\right),$$

$$X := e_a \rfloor X^a, \quad \Psi_a := X_a - \tfrac{1}{4}\theta_{a \wedge X} - \tfrac{1}{2}\, e_a \rfloor \left(\theta^b \wedge X_b\right),$$

$$\Phi_a := W_a - \frac{1}{4} W \theta_a - \frac{1}{2}\, e_a \rfloor \left(\theta^b \wedge W_b\right), \tag{7}$$

We will use decomposition of 2-torsion form into irreducible parts (1-form of traceless $\overset{(1)}{\mathcal{T}^a}$, 1-form of trace $\overset{(2)}{\mathcal{T}^a}$ and 1-form of pseudotrace $\overset{(3)}{\mathcal{T}^a}$):

$$\overset{(1)}{\mathcal{T}^a} = \mathcal{T}^a - \overset{(2)}{\mathcal{T}^a} - \overset{(3)}{\mathcal{T}^a}, \quad \overset{(2)}{\mathcal{T}^a} = \frac{1}{3}\theta^a \wedge \left(\mathbf{e}_b \rfloor \mathcal{T}^b\right), \quad \overset{(3)}{\mathcal{T}^a} = \frac{1}{3} * \left(\theta^a \wedge *(\mathcal{T}^b \wedge \theta_b)\right). \tag{8}$$

In this case, the irreducible terms of torsion have properties:

$$\overset{(1)}{\mathcal{T}^a} \wedge \theta_a = 0, \qquad \overset{(2)}{\mathcal{T}^a} \wedge \theta_a = 0, \qquad \mathbf{e}_a \rfloor \overset{(1)}{\mathcal{T}^a} = 0, \qquad \mathbf{e}_a \rfloor \overset{(3)}{\mathcal{T}^a} = 0. \tag{9}$$

$$e_a \rfloor \overset{(1)}{\mathcal{T}^a} = 0, \qquad e_a \rfloor \overset{(3)}{\mathcal{T}^a} = 0$$

$$\left(* \overset{(1)}{\mathcal{T}^a} \right) = * \left(\overset{(1)}{\mathcal{T}^a} \right), \quad \left(* \overset{(2)}{\mathcal{T}^a} \right) = * \left(\overset{(3)}{\mathcal{T}^a} \right), \quad \left(* \overset{(3)}{\mathcal{T}^a} \right) = * \left(\overset{(2)}{\mathcal{T}^a} \right) \tag{10}$$

We choose the following expression to define **Lagrangian density** in this theory:

$$\mathcal{L} = f_0 \mathcal{R}^a{}_b \wedge \eta_a^b + \sum_{i=1}^{6} \lambda_i \overset{(i)}{\mathcal{R}^{ab}} \wedge * \overset{(i)}{\mathcal{R}^{ab}} + \sum_{i=1}^{3} \chi_i \overset{(i)}{\mathcal{T}^a} \wedge * \overset{(i)}{\mathcal{T}^a} \tag{11}$$

Here $f_0 = 1/(2\kappa)$ ($\kappa = 8\pi G/c^4$), λ_i, χ_i – connection constants, and (i) – index, listing the irreducible components of the curvature and torsion.

Using expressions for the irreducible parts of curvature and torsion we rewrite our Lagrangian in a more convenient form for the variation procedure:

$$\begin{aligned}
\mathcal{L} = {} & f_0 \mathcal{R}^a{}_b \wedge \eta_a^b + \tau_1 \mathcal{R}^a{}_b \wedge * \mathcal{R}^b{}_a + \\
& + \tau_2 \left(\mathcal{R}^{ab} \wedge \theta_a \right) \wedge * \left(\mathcal{R}^c{}_b \wedge \theta_c \right) + \\
& + \tau_3 \left(\mathcal{R}^{ab} \wedge \theta_c \right) \wedge * \mathcal{R}^c{}_b \wedge \theta_a) + \\
& + \tau_4 \left(\mathcal{R}^a{}_b \wedge \theta_a \wedge \theta^b \right) \wedge * \left(\mathcal{R}^c{}_d \wedge \theta_c \wedge \theta^d \right) + \\
& + \tau_5 \left(\mathcal{R}^a{}_b \wedge \theta_a \wedge \theta^d \right) \wedge * \left(\mathcal{R}^c{}_d \wedge \theta_c \wedge \theta^b \right) + \\
& + \tau_6 \left(\mathcal{R}^a{}_b \wedge \theta_c \wedge \theta^d \right) \wedge * \left(\mathcal{R}^c{}_d \wedge \theta_a \wedge \theta^b \right) + \varrho_1 \mathcal{T}^a \wedge * \mathcal{T}_a + \\
& + \varrho_2 (\mathcal{T}^a \wedge \theta_a) \wedge * (\mathcal{T}^b \wedge \theta_b) + \varrho_3 (\mathcal{T}^a \wedge \theta_b) \wedge * (\mathcal{T}^b \wedge \theta_a)
\end{aligned} \tag{12}$$

where $\tau_1 - \tau_6$, $\varrho_1 - \varrho_3$ – interaction constants, related to the interaction constants of the Lagrangian (11) by the following relations:

$$\begin{aligned}
& \tau_1 = -\tfrac{1}{6} \left(2\lambda_1 + 3\lambda_4 + \lambda_6 \right), \quad \tau_2 = \tfrac{1}{2}(-\lambda_1 + \lambda_2 - \lambda_4 + \lambda_5), \quad \tau_3 = \tfrac{1}{3}(\lambda_1 - \lambda_6), \\
& \tau_4 = \tfrac{1}{12}(2\lambda_1 - 3\lambda_2 + \lambda_3), \qquad \tau_5 = \tfrac{1}{2}(-\lambda_1 + \lambda_2 + \lambda_4 - \lambda_5), \quad \tau_6 = -\tfrac{1}{12}(4\lambda_1 - 3\lambda_4 - \lambda_6), \\
& \varrho_1 = \tfrac{1}{3}(2\chi_1 + \chi_2), \quad \varrho_2 = \tfrac{1}{3}(-\chi_1 + \chi_3), \quad \varrho_3 = \tfrac{1}{3}(\chi_1 - \chi_2).
\end{aligned} \tag{13}$$

As you know, in order to get the gravitational field equations in the Riemann - Cartan space RC_4 in a vacuum we have to do independent variation of the Lagrangian density (12) on the basis form θ^a and connection 1-form $\Gamma^a{}_b$ via a variational formalism of the first order.

Γ–equation has the form:

$$
\begin{aligned}
&f_0 \mathrm{D}\eta_a{}^b + 2\tau_1 \mathrm{D} * \mathcal{R}^b_{\ a} + \tau_2 \mathrm{D}\left(\theta_a \wedge *\left(\mathcal{R}^{cb} \wedge \theta_c\right) - \theta^b \wedge *\left(\mathcal{R}^c_{\ a} \wedge \theta_c\right)\right) + \\
&+\tau_3 \mathrm{D}\left(\theta^c \wedge *\left(\mathcal{R}_c{}^b \wedge \theta_a\right) - \theta^c \wedge *\left(\mathcal{R}_{ca} \wedge \theta^b\right)\right) + \\
&+2\tau_4 \mathrm{D}\left(\theta_a \wedge \theta^b \wedge *\left(\mathcal{R}^{cd} \wedge \theta_c \wedge \theta_d\right)\right) + \\
&+\tau_5 \mathrm{D}\left(\theta_a \wedge \theta^d \wedge *\left(\mathcal{R}^c_{\ d} \wedge \theta_c \wedge \theta^b\right) - \theta^b \wedge \theta^d \wedge *\left(\mathcal{R}^c_{\ d} \wedge \theta_c \wedge \theta_a\right)\right) + \\
&+2\tau_6 \mathrm{D}\left(\theta_c \wedge \theta^d \wedge *\left(\mathcal{R}^c_{\ d} \wedge \theta_a \wedge \theta^b\right)\right) + \\
&+\varrho_1\left(\theta^b \wedge *\mathcal{T}_a - \theta_a \wedge *\mathcal{T}^b\right) + 2\varrho_2 \theta^b \wedge \theta_a \wedge *\left(\mathcal{T}^c \wedge \theta_c\right) + \\
&+\varrho_3\left(\theta^b \wedge \theta^c \wedge *\left(\mathcal{T}_c \wedge \theta_a\right) - \theta_a \wedge \theta^c \wedge *\left(\mathcal{T}_c \wedge \theta^b\right)\right) = 0\,.
\end{aligned}
\tag{14}
$$

After variation on the basis forms θ^a and accounting (8) and (9), we get the equation:

$$
\begin{aligned}
&f_0 \mathcal{R}^{bc} \wedge \eta_{abc} + \tau_1\left(\mathcal{R}^n_{\ b} \wedge *\left(\mathcal{R}^b_{\ n} \wedge \theta_a\right) + *\left(*\mathcal{R}^b_{\ n} \wedge \theta_a\right) \wedge *\mathcal{R}^n_{\ b}\right) + \\
&+\tau_2\left(2\mathcal{R}_{an} \wedge *\left(\mathcal{R}^{mn} \wedge \theta_m\right) - \left(\mathcal{R}^{rb} \wedge \theta_r\right) \wedge *\left(\mathcal{R}^m_{\ b} \wedge \theta_m \wedge \theta_a\right)\right) - \\
&-\tau_2\left(*\left(*\left(\mathcal{R}^{mn} \wedge \theta_m\right) \wedge \theta_a\right) \wedge *\left(\mathcal{R}^{rb} \wedge \theta_r\right)\right) + \\
&+\tau_3\left(-2\mathcal{R}^t_{\ n} \wedge *\left(\mathcal{R}^n_{\ a} \wedge \theta_t\right) - \left(\mathcal{R}^b_{\ l} \wedge \theta_m\right) \wedge *\left(\mathcal{R}^m_{\ b} \wedge \theta^l \wedge \theta_a\right)\right) - \\
&-\tau_3\left(*\left(*\left(\mathcal{R}^m_{\ b} \wedge \theta^l\right) \wedge \theta_a\right) \wedge *\left(\mathcal{R}^b_{\ l} \wedge \theta_m\right)\right) + \\
&+\tau_4\left(4\mathcal{R}_{ab} \wedge \theta^b \wedge *\left(\mathcal{R}^{ln} \wedge \theta_l \wedge \theta_n\right) + *\left(*\left(\mathcal{R}^l_{\ n} \wedge \theta_l \wedge \theta^n\right) \wedge \theta_a\right) \wedge *\left(\mathcal{R}^t_{\ b} \wedge \theta_t \wedge \theta^b\right)\right) + \\
&+\tau_5\left(4\mathcal{R}_{[a|b|} \wedge \theta^l \wedge *\left(\mathcal{R}^n_{\ l]} \wedge \theta_n \wedge \theta^b\right) + *\left(*\left(\mathcal{R}^n_{\ l} \wedge \theta_n \wedge \theta^b\right) \wedge \theta_a\right) \wedge *\left(\mathcal{R}^t_{\ b} \wedge \theta_t \wedge \theta^l\right)\right) + \\
&+\tau_6\left(4\mathcal{R}^n_{\ b} \wedge \theta^l \wedge *\left(\mathcal{R}_{al} \wedge \theta_n \wedge \theta^b\right) + *\left(*\left(\mathcal{R}^l_{\ n} \wedge \theta^t \wedge \theta^b\right) \wedge \theta_a\right) \wedge *\left(\mathcal{R}_{tb} \wedge \theta_l \wedge \theta^n\right)\right) + \\
&+\varrho_1\left(2\mathrm{D} * \mathcal{T}_a + \mathcal{T}_n \wedge *\left(\mathcal{T}^n \wedge \theta_a\right) + *\left(*\mathcal{T}^n \wedge \theta_a\right) \wedge *\mathcal{T}_n\right) + \\
&+\varrho_2\left(4\mathrm{D}\mathcal{T}_a \wedge *\left(\mathcal{T}^b \wedge \theta_b\right) - 2\theta_a \wedge \mathrm{D} *\left(\mathcal{T}^b \wedge \theta_b\right) - \left(\mathcal{T}^l \wedge \theta_l\right) \wedge *\left(\mathcal{T}^b \wedge \theta_b \wedge \theta_a\right)\right) - \\
&-\varrho_2\left(*\left(*\left(\mathcal{T}^b \wedge \theta_b\right) \wedge \theta_a\right) \wedge *\left(\mathcal{T}^l \wedge \theta_l\right)\right) + \\
&+\varrho_3\left(2\mathcal{T}^b \wedge *\left(\mathcal{T}_a \wedge \theta_b\right) + 2\mathrm{D}\left(\theta^b \wedge *\left(\mathcal{T}_b \wedge \theta_a\right)\right) - \left(\mathcal{T}_l \wedge \theta_n\right) \wedge *\left(\mathcal{T}^n \wedge \theta^l \wedge \theta_a\right)\right) - \\
&-\varrho_3\left(*\left(*\left(\mathcal{T}^n \wedge \theta^l\right) \wedge \theta_a\right) \wedge *\left(\mathcal{T}_l \wedge \theta_n\right)\right) = 0\,.
\end{aligned}
\tag{15}
$$

We note, that equations (14), (15) is special case of field equations conformal theory of gravity in the Cartan–Weyl space, developed in (14) – (18), if 1-form nonmetricity $\mathcal{Q}$ is zero, and for a scalar Deser - Dirac field to put $\beta = 1$ and replace $\varrho_2 \longleftrightarrow \varrho_3$.

Plane torsion waves in a Riemann–Cartan space

We shall be interested in the limitations on the gravitational Lagrangian (11), which may result from the existence of plane torsion waves, since the total gravitational Lagrangian (12) in the Riemann–Cartan space contains nine arbitrary parameters, and an important task of the theory of gravity with quadratic Lagrangians is to find criteria to reduce the number of free parameters.

It is known that it is convenient to consider the problem of plane waves using special coordinate basis formed by two zero vectors $\mathbf{e}_0 = \partial_{\mathrm{v}}$, $\mathbf{e}_1 = \partial_u$ and two space-like vectors $\mathbf{e}_2 = \partial_x$, $\mathbf{e}_3 = \partial_y$.

Vector $\mathbf{e}_0$ is chosen as covariant constant and directed along the wave path, wave surface $(u, \text{v}) = const$ is parametrized by coordinates x and y.
Plane wave metric has the form:

$$\breve{g} = 2H(x,y,u)du^2 + 2dudv - dx^2 - dy^2\,, \tag{16}$$

and is a special case of the plane front gravitational waves of metric with parallel rays (pp-waves) [1], [4], where the coordinate u has meaning the delayed time and is interpreted as the phase of the wave.

Basis of 1-forms has the form:

$$\theta^0 = H(x,y,u)\mathrm{d}u + dv\,, \quad \theta^1 = \mathrm{d}u\,, \quad \theta^2 = \mathrm{d}x\,, \quad \theta^3 = \mathrm{d}y\,, \tag{17}$$

they correspond to the metric tensor:

$$\breve{g} = g_{ab}\theta^a \otimes \theta^b\,, \qquad g_{ab} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}.$$

Riemann space R_4 with the plane wave metric allow for group of symmetries G_5, generated by the vector field X with the structure (1):

$$X = (a + b^{'}x + c^{'}y)\partial_v + b\partial_x + c\partial_y\,, \tag{18}$$

where $a = const$, $b(u)$, $c(u)$ – arbitrary functions and b', c' – their derivatives.

The group of motion G_5 leaves isotropic hypersurface in R_4 unchanged, which describs the wave front with constant amplitude (1). As pp-waves, plane waves of metric have zero shift, rotation and stretching.

Definition. We call the space of Riemann-Cartan U_4 as space U_4 type of a plane wave, and its metric and torsion - plane wave of metric and torsion, if the metric g_{ab} and the 2-form of torsion $\mathcal{T}^a$ of this space allow for group of symmetries G_5, which means next the conditions: $\mathrm{L}_X g_{ab} = 0$, $\mathrm{L}_X \mathcal{T}^a = 0$, where L_X – Li derivative along vector field X, generating symmetry group G_5.

The following theorem is proved [6].

Theorem 1. Torsion 2-form of a plane wave type of space U_4 has the structure: traceless part depended on two arbitrary functions $t_1\,(u)$ and $t_2\,(u)$ of the delayed argument u, and the trace and pseudotrace, depended on one arbitrary function of u, respectively, $t_3(u)$ and $t_4(u)$:

$$\overset{(1)}{\mathcal{T}}{}^0 = t_1(u)\,\theta^1 \wedge \theta^2 + t_2(u)\,\theta^1 \wedge \theta^3\,, \tag{19}$$

$$\overset{(2)}{\mathcal{T}}{}^0 = t_3(u)\theta^0 \wedge \theta^1, \quad \overset{(2)}{\mathcal{T}}{}^2 = -t_3(u)\theta^1 \wedge \theta^2, \quad \overset{(2)}{\mathcal{T}}{}^3 = -t_3(u)\theta^1 \wedge \theta^3\,, \tag{20}$$

$$\overset{(3)}{\mathcal{T}}{}^0 = t_4(u)\theta^2 \wedge \theta^3, \quad \overset{(3)}{\mathcal{T}}{}^2 = t_4(u)\theta^1 \wedge \theta^3, \quad \overset{(3)}{\mathcal{T}}{}^3 = -t_4(u)\theta^1 \wedge \theta^2\,. \tag{21}$$

To prove this, we substitute the vector field (18) in equation $\mathrm{L}_X\mathcal{T}^a = 0$. As a result, we get a set of equations. According to the arbitrariness of the functions b, c, b' and c', in the system all components of the torsion except those ones expressed by the formulas (19) - (21) disappear.

This theorem generalizes the result of (5), in which the plane waves of torsion depend only on two arbitrary functions that define its traceless part. In (1), and then in (5), analogy with the plane electromagnetic waves, introduced an additional condition $\Gamma^a{}_b = \Gamma^a{}_{b1}\theta^1$. The consequence of this condition was that only the traceless part of torsion (19), containing two arbitrary functions, could have the property of plane waves of torsion. Once in (6), according to (2), this condition no longer holds, trace (20) and pseudotrace (21) of torsion also would become to have the property of plane waves.

According to (10), connection in the space of RC_4 can be decomposed into a part of the Riemann and contortion, which linearly dependent on the torsion tensor:

$$\overset{C}{\Gamma}{}^a{}_b = \overset{R}{\Gamma}{}^a{}_b + \mathcal{K}^a{}_b\,, \qquad \mathcal{T}^a = \mathcal{K}^a{}_b \wedge \theta^\beta\,, \tag{22}$$

$$\mathcal{K}_{ab} = 2\mathbf{e}_{[a}\rfloor\mathcal{T}_{b]} - \frac{1}{2}\mathbf{e}_a\rfloor\mathbf{e}_b\rfloor(\mathcal{T}_c \wedge \theta^c)\,, \tag{23}$$

where $\overset{R}{\Gamma}{}^a{}_b$ is 1-form of Riemann connection and $\mathcal{K}^a{}_b$ – 1-form of contortion.

The decomposition of connection (22) induces a decomposition of curvature:

$$\overset{C}{\mathcal{R}}{}^a{}_b = \overset{R}{\mathcal{R}}{}^a{}_b + \overset{R}{\mathrm{D}}\,\mathcal{K}^a{}_b + \mathcal{K}^a{}_c \wedge \mathcal{K}^c{}_b\,, \tag{24}$$

where $\overset{R}{\mathcal{R}}{}^a{}_b$ – 2-form of Riemann curvature and $\overset{R}{\mathrm{D}}$ is an external covariant differential relative to the 1-form of Riemann connection $\overset{R}{\Gamma}{}^a{}_b$.

Curvature scalar decomposition is defined as:

$$\mathcal{R}^a{}_b \wedge \eta^b{}_a = \overset{R}{\mathcal{R}}{}^a{}_b \wedge \eta^b{}_a - \overset{R}{\mathrm{D}}\left(*\overset{(2)}{\mathcal{T}}_a \wedge \theta_a\right) + \overset{(1)}{\mathcal{T}}{}^a \wedge *\overset{(1)}{\mathcal{T}}_a - 2\overset{(2)}{\mathcal{T}}{}^a \wedge *\overset{(2)}{\mathcal{T}}_a - \frac{1}{2}\overset{(3)}{\mathcal{T}}{}^a \wedge *\overset{(3)}{\mathcal{T}}_a\,. \tag{25}$$

In this case, the non-zero components of the curvature and the Ricci tensor of the Riemann space are equal to (the other components are zero, $R_{ab} = R^{c}{}_{acb}$)

$$\stackrel{R}{\mathcal{R}}{}^{0}{}_{2} = \stackrel{R}{\mathcal{R}}{}^{2}{}_{1} = H_{xx}\theta^2 \wedge \theta^1 + H_{xy}\theta^3 \wedge \theta^1 \,, \tag{26}$$
$$\stackrel{R}{\mathcal{R}}{}^{0}{}_{3} = \stackrel{R}{\mathcal{R}}{}^{3}{}_{1} = H_{xy}\theta^2 \wedge \theta^1 + H_{yy}\theta^3 \wedge \theta^1 \,, \tag{27}$$
$$\stackrel{R}{R}_{11} = H_{xx} + H_{yy} \,, \tag{28}$$

where H_{xx}, H_{xy} and H_{yy} – second partial derivatives of the function $H(u, x, y)$ for the corresponding coordinates. Calculating contortion for the components of torsion (19) - (21) on the basis of representation (23), we find expressions for the components of the curvature in space RC_4:

$$\begin{aligned} \mathcal{R}^0{}_2 = \mathcal{R}^2{}_1 = \;& (H_{xx} - \partial_u t_3 + (t_3)^2 - (1/4)(t_4)^2)\theta^2 \wedge \theta^1 + \\ & +(H_{xy} + (1/2)\partial_u t_4 - t_3 t_4)\theta^3 \wedge \theta^1 \,, \end{aligned} \tag{29}$$
$$\begin{aligned} \mathcal{R}^0{}_3 = \mathcal{R}^3{}_1 = \;& (H_{xy} - (1/2)\partial_u t_4 + t_3 t_4)\theta^2 \wedge \theta^1 + \\ & +(H_{yy} - \partial_u t_3 + (t_3)^2 - (1/4)(t_4)^2)\theta^3 \wedge \theta^1 \,. \end{aligned} \tag{30}$$

Substituting these expressions in the θ-field equation (15), we find that the only non-zero component of this equation is u-component of the form:

$$\begin{aligned} &(-2f_0(H_{xx} + H_{yy}) + 2\partial_u t_3(u)(f_0 - \rho_1 + 2\rho_2) - 2(t_3(u))^2(f_0 - \rho_1 + 2\rho_2) + \\ &+(1/2)(t_4(u))^2(f_0 - 4\rho_1 - 4\rho_2 + 12\rho_3))\mathrm{d}u \wedge \mathrm{d}x \wedge \mathrm{d}y = 0 \,. \end{aligned} \tag{31}$$

In this case, all the quadratic terms in the curvature are equal to zero.

Due to the arbitrariness of functions $t_3(u)$, $\partial_u t_3(u)$, $t_4(u)$ from the the equation (31) the following values are equal to zero:

$$f_0 - \rho_1 + 2\rho_2 = 0 \,, \quad f_0 - 4\rho_1 - 4\rho_2 + 12\rho_3 = 0 \,, \quad f_0(H_{xx} + H_{yy}) = 0 \,. \tag{32}$$

Taking into account the relation of the constants (13), we find that the first two equations (32) indicate that the following relations are true:

$$2f_0 - \chi_2 = 0 \,, \qquad f_0 - 2\chi_3 = 0 \,. \tag{33}$$

But on the basis of (28), from the the last equation (32) it follows that

$$f_0 \stackrel{R}{R}_{ab} = 0 \,. \tag{34}$$

Let's substitute expressions (29) and (30) in Γ–equation (14) and write one of the non-zero components of this equation, for example, (u, x)-component:

$$
\begin{aligned}
&-t_4(u)(f_0 - 2\rho_1 - 6\rho_2 - 2\rho_3)\mathrm{dv} \wedge \mathrm{d}u \wedge \mathrm{d}x - t_3(u)(2f_0 - \rho_1 + 2\rho_3)\mathrm{dv} \wedge \mathrm{d}u \wedge \mathrm{d}y + \\
&+(2f_0(\tau_1 - \tau_3)(H_{xyy} + H_{xxx}) - t_1(u)(f_0 + \rho_1 + \rho_3))\mathrm{d}u \wedge \mathrm{d}x \wedge \mathrm{d}y = 0\,. \qquad (35)
\end{aligned}
$$

Due to the arbitrariness of function $t_1(u)$, we get the following equality from the second term in the second line of this equation:

$$
f_0 + \rho_1 + \rho_3 = 0\,. \qquad (36)
$$

Further calculations show, that in the equation (14) terms with coefficients τ_2, τ_4, τ_5 and τ_6 identically equal to zero, and the terms with the coefficient τ_1 equal to

$$
\begin{aligned}
2\tau_1 \mathrm{D} * \mathcal{R}^0{}_2 = 2\tau_1 \mathrm{D} * \mathcal{R}^2{}_1 = 2\tau_1(H_{xxx} + H_{xyy})\eta^1 = 0\,, \\
2\tau_1 \mathrm{D} * \mathcal{R}^0{}_3 = 2\tau_1 \mathrm{D} * \mathcal{R}^3{}_1 = 2\tau_1(H_{xxy} + H_{yyy})\eta^1 = 0\,.
\end{aligned}
$$

Here the vanishing is a consequence of the last equation (32). The remaining terms of the equation (14) are equal to zero because of the equalities (32) and (36). According to the equation (13), (36) is equivalent to:

$$
f_0 + \chi_1 = 0\,. \qquad (37)
$$

The results can be formulated as the following theorem:

Theorem 2. The Riemann-Cartan space of a plane wave type with four arbitrary functions is a solution of the field equations of this space, if and only if the following conditions on the onstants for the gravitational Lagrangian (11) are true. This means that (a) the waves of metric are flat, and (b) flat waves of torsion are massless.

$$
1) \quad f_0 \overset{R}{R}_{ab} = 0\,, \qquad (38)
$$

$$
2) \quad f_0 + \chi_1 = 0\,, \quad 2f_0 - \chi_2 = 0\,, \quad f_0 - 2\chi_3 = 0\,. \qquad (39)
$$

This theorem has an important consequence.

Consequence. In a Riemann-Cartan space, plane waves of irreducible parts of torsion (traceless part, trace and pseudotrace) can transfer information propagating at the speed of light.

The first part of this statement is based on the fact that the solution for the plane waves of the irreducible components of torsion contains arbitrary functions, so according to A. Trautman, it is possible to transfer information by this type of waves.

The second part of the statement follows from the fact that the rest mass of quanta of irreducible components of the torsion field of plane wave type equals zero. Because of substitution (25) in the original Lagrangian density (11), considering (39), all the terms with the squares of irreducible components of torsion in (11) are zero, which means that quanta of corresponding components of the torsion field are massless.

The conditions on the gravitational Lagrangian constants established in the Theorem 2 are more important in view of the fact, that under these conditions spherical symmetric solution for the torsion, generated by the island system of matter, has asymptotic $1/r$ (20). This asymptotic is necessary for a correct interpretation of the law of energy–momentum conservation of the gravitational field in the space of Riemann–Cartan (11). Found conditions on the constants of Lagrangian are also important from the quantum point of view, as one of the possible conditions of absence of ghosts and tachyons in the theory.

The author thanks B.N. Frolov for his active assistance and V.V. Zhitnikov for the opportunity to use his program of symbolic calculations on a computer GRG (21).

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The nature of gamma – ray bursts in scalar model of gravitation

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Within the framework of scalar model of gravitation (SMG) it is shown, that in a small vicinity of heavy collapser, the field comes nearer to homogeneous and there are conditions for very long existence of satellites. It is shown, that gamma – ray bursts can occur at collision of heavenly bodies with satellites of collapsers. Such assumption of gamma – ray bursts, allows explaining all observable characteristics.

In SMG it is supposed, that gamma – ray bursts [1] occur as a result of collisions of heavenly bodies with satellites of collapsers. We shall be convinced, that in a vicinity of collapsers there are conditions for long existence of satellites. In work [2] it is shown, that intensity of a field depends on effective radius

$$g = \frac{GM}{R_{eff}^2} = \frac{GM}{\left(R_0 + r_m\right)^2} \tag{1.1}$$

In area, where $R_0 << r_m$

$$g \approx \frac{GM}{r_m^{\ 2}}$$ **Ошибка! Источник ссылки не найден.**

A field is practically homogeneous, especial for massive black holes, as $r_m \square M$. Therefore the tidal mechanism is extremely small and does not prevent long existence of stars of satellites in deep orbits. Calculation has shown, that energy of satellites (the Appendix 1(A1-1))

$$E_{sat} = mc^2\sqrt{\exp\left(-\Phi\right)} = mc^2\sqrt{z} \tag{1.2}$$

The cycle time of the satellite T **on local time** for all orbits having $R_0 << r_m$ does not depend on radius of an orbit

$$T \approx \frac{2\pi r_m}{c} = 2\pi\frac{GM}{c^3} \tag{1.3}$$

$$\omega = c / r_m \tag{1.4}$$

So for a "black hole" in the next galaxy having weight about $1.4 \cdot 10^8$ of solar weights we shall receive $T \approx 4.3*10^3 \sec$; $\omega = 1,5*10^{-3}$. Gravitational radiation in these conditions allows existing to the satellite about t ~30 million years in local time or $t_0 = zt$ for the terrestrial observer. It is very big time. Therefore satellites should be much. Supervision of astronomers confirms this fact. [3].

Collision of a heavenly body with the satellite is accompanied by huge allocation of energy. Let potential in an orbit of the satellite

$$\Phi << -1 \tag{1.5}$$

At the moment of collision, energy of a heavenly body

$$E(\Phi) \approx mc^2 e^{-\Phi} >> mc^2 \tag{1.6}$$

With the big accuracy it is possible to admit, that all kinetic energy of a body

$$E_k(\Phi) \approx mc^2\left(e^{-\Phi} - 1\right) \tag{1.7}$$

will turn in energy of radiation

$$E_\gamma \approx E_k \tag{1.8}$$

At an output from a potential hole, radiation will receive red displacement, $z = e^{-\Phi}$ and its energy becomes equal

$$E_{\gamma,0} = E_k e^{\Phi} \tag{1.9}$$

$$E_{\gamma,0} \approx mc^2\left(e^{-\Phi} - 1\right)e^{\Phi} \approx mc^2 \tag{1.10}$$

Thus, there is a splash in radiation with the full energy equal, under the order of size, energy of rest of a heavenly body fallen on the satellite. The maximal energy of brake γ - quantums is equal to kinetic energy of elektrons.

$$h\nu_{\max} = m_e c^2\left(e^{-\Phi} - 1\right) \tag{1.11}$$

At an output from a potential whole energy γ - quantums will decrease according to red displacement $z = e^{-\Phi}$ and becomes equal

$$h\nu_{\max} = m_e c^2\left(e^{-\Phi} - 1\right)e^{\Phi} \approx m_e c^2 \approx 0.5MeV \tag{1.12}$$

The received energy of γ - quantums and energy of gamma – ray bursts will well be coordinated to characteristics of observable gamma – ray bursts. The brake mechanism of radiation explains absence of spectral lines.

It is possible to expect, that duration of gamma – ray bursts on local time, is comparable to duration of collision of the heavenly body having the speed close by speed of light, with a surface of satellite of collapser.

$$t \approx D / c \tag{1.13}$$

D - Diameter of a heavenly body; c - speed of light.
For the terrestrial observer this process will last longer

$$t_0 \approx te^{-\Phi} \approx e^{-\Phi} D / c \tag{1.14}$$

For an explanation of the most powerful with energy of gamma – ray bursts 10^{54} эрг, it is necessary to assume, that the neutron star in diameter about 10^4 meters has fallen to the satellite of collapser. According to (1.16) time of collision will make size $t \approx 3*10^{-5}$. Observable duration of gamma – ray burst equal $t_0 \approx 30$ seconds. It means, that $\frac{dt_0}{dt} = e^{-\Phi} \approx 10^6$. Such ratio corresponds to potential in an orbit of the satellite of collapser $\Phi \approx -13.8$
The size z can vary over a wide range $\sim 10^2 - 10^8$
Diameter of a heavenly body too can lay in limits $10^4 - 10^6 m$
So duration of splashes time [4] can differ 10^8

On a surface of collapser the potential can be much less and reach size $\Phi_0 \approx -20.7$
Falling of a heavenly body on itself collapserp the scale too can create gamma – ray burst, however duration scale of splash in this case will be much more, than at collision with the satellite of collapser, as

$$dt_0 / dt = e^{-\Phi} \approx 10^9 \tag{1.15}$$

GRB in the described nature are obliged to be accompanied by powerful radiation of gravitational waves. It is interesting to consider the scheme of registration of waves in which should be observed palpation of the waves generated before arrival of a wave with waves generated in the field of wave. For this purpose it is necessary to keep radiation in the resonator with high good quality during time of increase of front of a wave. After that it is necessary to deduce radiation in the scheme of addition and to register frequency of palpation. Frequency of radiation kept in the resonator will change when it will appear in the field of a wave so frequency of palpation will make size

$$\omega_2 = 2\pi |\nu - \nu_0| = -2\pi\nu\Phi \tag{1.16}$$

The difference of phases in this case grows in due course

$$\Delta\phi_2 = \omega_2 t = -2\pi\nu\Phi t \tag{1.17}$$

In the detector of waves VIRGO a difference of phases

$$\Delta\phi_1 = 2\pi L \Delta\Phi / \lambda \tag{1.18}$$

It is easy to see that the difference of phases in the offered scheme repeatedly exceeds a difference of phases in scheme VIRGO

$$\Delta\phi_2 / \Delta\phi_1 = \frac{\nu\lambda t\Phi}{L\Delta\Phi} = \frac{ct}{L}\frac{\Phi}{\Delta\Phi} \approx \frac{3\cdot 10^8 \cdot 10}{3\cdot 10^3} 10^{2-6} \approx 10^{7-12} \quad (1.19)$$

It is good stimulus for detailed study of the offered scheme (fig. 1)

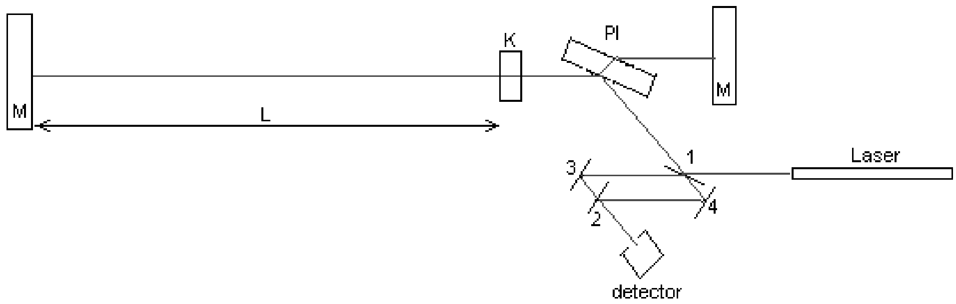

Fig. 1: The scheme of detecting of gravitational waves with a method of comparison of frequencies.

<u>The appendix 1. Energy of satellites.</u>

Let's determine energy of the satellite in a circular orbit. Centripetal acceleration is equal to intensity of a field $\upsilon^2 / R_{Eff} = MG / R_{Eff}^2$

Hence, $\upsilon^2 = c^2 \frac{r_m}{R_{Eff}}$ or $\frac{\upsilon^2}{c^2} = \frac{r_m}{R_0 + r_m}$. For energy of the satellite we receive expression

$$E_{sat} = \frac{mc^2}{\sqrt{1-\upsilon^2/c^2}} = \frac{mc^2}{\sqrt{\frac{R_0}{R_0 + r_m}}}.$$

Taking into account, that $e^{(-\Phi)} = 1 + \frac{r_m}{R_0} = \frac{R_0 + r_m}{R_0}$ we shall receive

$$E_{sat} = mc^2\sqrt{\exp(-\Phi)} = mc^2\sqrt{z} \quad \text{(A1-1)}$$

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The nature of quasars in scalar model of gravitation

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Within the framework of scalar model of gravitation the description of the possible nature of quasars is given. It is shown influence of quasars on acceleration of galaxies. The formula of acceleration of far galaxies is received.

Key words: a dark matter; dark energy; antigravitation; streams of a dust; heterogeneity of time and theorem Nether; infringement of the law of conservation of energy.

At conference PIRT-2009 I reported, that collapser, becomes a bright radiation source, if red displacement $z \approx 100T$ [1]. Radiation sources observable by astronomers in a meter range frequently have a bright temperature $T_{br} \approx 10^{16} K^0$ [2]. Such brightness turns out at temperature of a star $T = 10^7 K^0$ and displacement $z = 10^9$ (fig. 1).

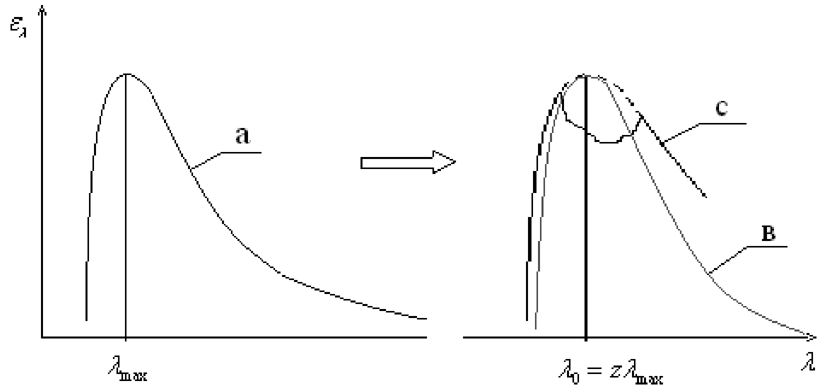

Fig. 1: An illustration of the possible mechanism of occurrence of radiation sources with « a thermal spectrum ».

á - a thermal spectrum; b - the thermal spectrum displaced in a radiorange; c - a characteristic spectrum of a radiation source. Difference of experimental spectra from settlement speaks the present mechanisms of expansion and absorption in an environment.
Corresponding displacement is reached at an output from a potential hole by depth $\Phi \approx -21$. Depth of a potential hole is a dimensionless potential $\Phi = \varphi / c^2$. Thus red displacement $z = e^{-\Phi} \approx 10^9$. Full energy of any particle falling in a potential hole, increases in $z \approx \gamma$ time.

Now we shall see, that same collapser, being in the center of a galaxy, becomes simultaneously a quasar. We shall assume that on central collapser galaxies the stream of substance which kinetic energy repeatedly exceeds energy of rest of this substance falls. Kinetic energy of this substance at collision with collapser is radiated in space. Collision at which the relativistic factor reaches size $\gamma \approx 10^9$ is probably accompanied by plural birth different, still unknown particles. After dump of energy the weight of substance decreases in trillion of times, coming nearer to weight of rest. Under pressure of radiation this substance is taken out back in area of low potential. It is important, that at falling the weight of substance repeatedly exceeds weight of the same substance at rather slow rise. For this reason force created by pressure of radiation is small in comparison with weight at falling. In other words, radiation does not prevent substance to fall and collect enormous kinetic energy, but is capable to lift the same substance in area of zero potential with the minimal expense of energy. Thus, in each motion cycle of substance huge energy of radiation is developed. The energy of radiation comparable to kinetic energy of falling substance in millions of time exceeds energy of rest of substance. Changeability of a falling stream it is shown as variable luminosity of quasars. The maximal energy brake γ - quantums is equal to kinetic energy of electrons [3].

$$h\nu_{\max} = m_e c^2 \left(e^{-\Phi} - 1\right) \tag{1.1}$$

At an output from a potential whole energy γ - quantums will decrease according to red displacement $z = e^{-\Phi}$ and becomes equal

$$h\nu_{\max} = m_e c^2 \left(e^{-\Phi} - 1\right) e^{\Phi} \approx m_e c^2 \approx 0.5 MeV \tag{1.2}$$

Let's consider more in detail the forces working on a particle, falling on collapser. Let D - diameter of a particle and $m = \gamma m_0$ - its weight; $\gamma \approx e^{-\Phi} \approx 10^9$

Besides force of an attraction

$$\vec{F}_g = m\vec{g} = \gamma m_0 g \frac{-\vec{R}}{R} \tag{1.3}$$

on a particle presses radiation

$$\vec{F}_\nu \approx \frac{\pi D^2}{4} \frac{I}{c} \frac{\vec{R}}{R} \tag{1.4}$$

I - Density of a stream of radiation $\left(W / m^2\right)$

Force of an attraction will be the basic at falling, when $\gamma \approx 10^9$

$$\gamma m_0 g \gg \frac{\pi D^2}{4} \frac{I}{c} \tag{1.5}$$

As against falling, emission of substance by radiation can occur only at small values of the factor γ. Pressure of radiation accelerates a particle up to such value γ at which equality is reached.

$$\gamma m_0 g = \frac{\pi D^2}{4} \frac{I}{c} \tag{1.6}$$

Let's take into account further, that intensity of a field - $\vec{g}$ and intensity of radiation I decrease with distance proportionally each other. Therefore the achieved value γ is kept. In this approximation of constant speed of a stream, a stationary stream

$$4\pi r^2 \upsilon\rho = C \tag{1.7}$$

From here we shall receive density of substance

$$\rho = \frac{C}{4\pi r^2 \upsilon} \tag{1.8}$$

The weight of substance in sphere which radius is equal r corresponds to the formula

$$M(r) = \int_0^r \rho dV = \int_0^r \frac{C}{4\pi r^2 \upsilon} 4\pi r^2 dr = \frac{C}{\upsilon} r \sim r \tag{1.9}$$

It provides observable distribution of tangential speeds of stars in a galaxy [4, with. 1129]. Really, district speed is determined from equality $\frac{V^2}{r} = g = \frac{GM(r)}{r^2}$. Therefore speed $V = \sqrt{\frac{GM(r)}{r}} = \sqrt{\frac{GC}{\upsilon}}$ as a first approximation. On the big distances it is necessary to take into account absorption of radiation by a thick layer of scattering substance. Besides the field created by a layer of scattering substance will start to play an essential role. In result, the particle will be slowed down and will depart back. Delay of a stream of particles increases their density and it is shown in growth of tangential speeds of stars (fig. 2) [5]. Delay of a stream and its condensation in the field of change of speed on return, makes a dust sphere sharply outlined. American telescope "Fermi» has found out sharply outlined areas disseminating x-ray radiation of central collapser (fig. 3) [6].

During the long travel the particle will incorporate to other same particles and will be part of a heavenly body which free again will gain the speed corresponding $\gamma \sim 10^9$ and once again converts kinetic energy in radiation.

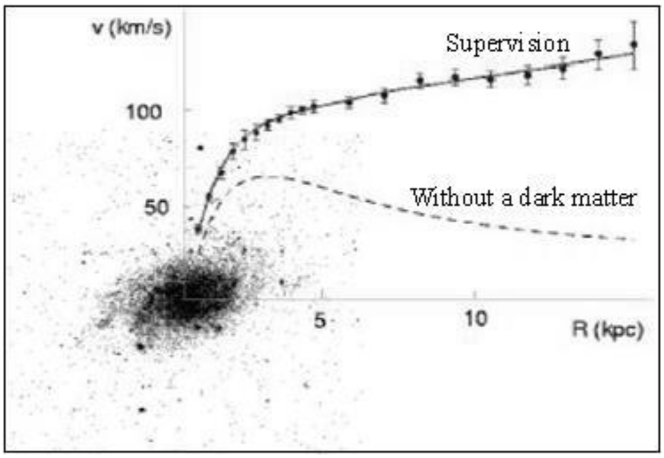

Fig. 2: District speeds of stars in a galaxy

Creation of energy and particles in a vicinity of a quasar increases weight and potential of the universe. Change of potential reduces speed time. Owing to heterogeneity of time the deviation from the law of conservation of energy does not contradict Noethers theorem [7].

According to the third postulate of scalar model of gravitation, the standard of length depends on potential. Process of creation of energy and a matter in a vicinity of quasars increases depth of a potential hole. The distances measured by constantly compressed standard grow with acceleration. Such acceleration does not characterize movement, and characterizes compression of the standard of length. We shall take into account, that the weight of the universe by modern estimations $M_U = 2.4 \cdot 10^{53} kg$ determines its minimal radius at a collapse $r_m = \frac{MG}{c^2} = 1.8 \cdot 10^{26} m$ or 18 billion light years. This size corresponds to the modern observable size. Hence, the universe is at a late stage of a collapse when its parameters are close to limiting parameters. The limiting radius is proportional to weight and grows owing to processes in a vicinity of quasars.

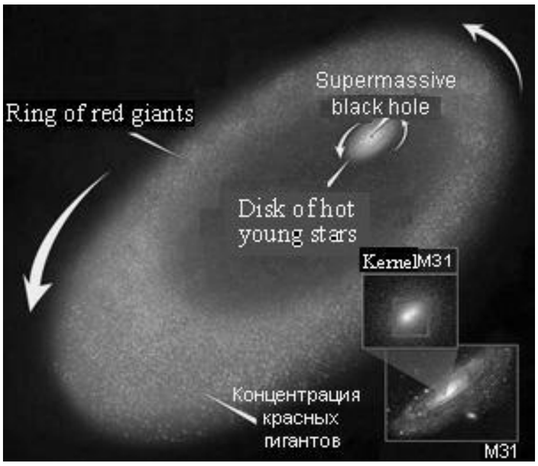

Fig. 3: Bubbles of scale - radiation from above and from below from a plane the Milky Way. An illustration NASA

Let's consider separately influence of quasars on Scattering of galaxies.
It is possible to assume, that the number of quasars and their total capacity are proportional to weight of the universe. Then

$$dM / dt_0 \approx M / \tau \tag{1.10}$$

$$M = M_0 e^{t/\tau} \tag{1.11}$$

Let W - total capacity of quasars, then from (1.10) characteristic time τ

$$\tau = \frac{M}{W / c^2} \tag{1.12}$$

Taking into account, that the minimal radius of the universe

$$r_m = \frac{GM}{c^2} \tag{1.13}$$

Let's calculate speed of expansion of external border as

$$\frac{dr_m}{dt_0} = \frac{G}{c^2}\frac{dM}{dt_0} = \frac{G}{c^2}\frac{M}{\tau} = \frac{r_m}{\tau} \tag{1.14}$$

Acceleration of external border will make

$$\frac{d^2 r_m}{dt_0^2} = \frac{G}{c^2}\frac{M}{\tau^2} = \frac{r_m}{\tau^2} \tag{1.15}$$

Taking into account, that speed can be written down according to Hubble's law

$$\frac{dr_m}{dt_0} = Hr_m \tag{1.16}$$

From (1.14) and (1.16) we shall find

$$H = \tau^{-1} \tag{1.17}$$

With the account (1.15) we shall write down

$$\boxed{\ddot{r}_m = H^2 r_m} \tag{1.18}$$

On any distance r acceleration will make

$$\boxed{\ddot{r} = H^2 r_m \frac{r}{r_m} = H^2 r} \tag{1.19}$$

Quasars make weight and increase the minimal radius of the universe. It results in growth of distances between galaxies due to reduction of the standard of length.
Acceleration connected to change of the standard of length does not demand application of forces, therefore and the hypothesis about antigravitation is not required. Movement connected to change of the standard of length has a number of differences from usual movement:
- Speed can be more speeds of light;

- It occurs as a result of change of potential in space between the observer and object and does not possess property of inertia. So in experiment on sounding Venus with use of a beam, taking place near to the Sun, the distance up to Venus increases, and at removal of a beam from the Sun the distance is restored.

- Acceleration connected to change of the standard of length is not caused by forces, and connected to respective alteration of the standard of length.

In it the possible answer to widely discussed question on the reason of the accelerated expansion of the universe [8, p. 267].

From (1.12) it is possible to estimate total capacity of quasars in the assumption that the weight of the universe grows only due to quasars.

$$W / c^2 = M / \tau = MH \tag{1.20}$$

Substituting values of weight of the universe and Hubble's constant according to modern representations $\mathrm{H} = 2.3*10^{-18} c^{-1}$; $M_U = 2.4*10^{53} kg$ we shall receive

$$dM / dt_0 \approx M / \tau \approx 5.52*10^{35} kg / c \tag{1.21}$$

Thus, the weight of the universe grows with speed $2.75*10^3$ of weights of the Sun in a second.
Capacity, developed quasars, makes accordingly size

$$W \approx 5.52*10^{35} * c^2 \approx 5*10^{52} J / c \tag{1.22}$$

The scale of a deviation from the law of conservation of energy is those.

The considered mechanism of manufacture of weight in a vicinity of quasars explains existence such over massive objects which weight exceeds weight of the Sun in billions and more time [9].

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Caianiello-based causal set theory

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In this paper we define causal set as a discretization of Caianiello-type ((1)) spacetime+velocity ("phase spacetime", (3)) as opposed to ordinary spacetime. This proposal will allow for relativistic covariance, compactness, and discretization to co-exist more peacefully than they do in ordinary causal set theory. This paper is meant to be an outline of key ideas; more detailed proposal of further constructions is found in (3).

Introduction

Most approaches to quantum theory of gravity rely on spacetime discretization. The idea of discrete spacetime, however, seems incompatible with relativity. For example, if spacetime is envisioned as cubic lattice, then "edges" and "diagonals" of that lattice seem to describe preferred directions. Similarly, any other discrete structure would likewise violate relativity on small enough scale. The one exception to this is causal set theory proposed by Rafael Sorkin. According to that model, our spacetime is a discrete, partially ordered set (known as causal set) in which partial ordering, $\prec$, is identified with lightcone causal relation. It seems that the very definition of $\prec$ rules out the violations of relativity by default. However, it comes with the price. Lorentz covariance demands that any given point is "linked" to any other point on the vicinity of its light cone. Due to the non-compactness of the latter, "most" of the "neighbors" of any given point are arbitrary "far away" coordinate-wise!

In this paper we attempt to avoid the above difficulties by using Caianiello's idea (see (1)) of attaching both "position" and "velocity" to any given event, and also by imagining that the signals between these events have finite acceleration (thus, they attain velocity "very close" to, but still distinct from, $c = 1$, within "very small", but finite, time). This would allow us to use reference frames of these events as "preferred coordinate system" with respect to which to define "compact neighborhoods". Furthermore, even the points within the said "neighborhood" are excluded if they move "too fast" relative to the point that defines said neighborhood. In light of the fact that "velocity coordinate" v^μ is attached to the event similarly to space coordinate x^μ, the violation of Lorentz covariance by the former is logically equivalent to violation of translational covariance by the latter. This implies that Lorentz covariance was not violated. In fact, the procedure of "taking into account" v^μ can be written down in Lorentz covariant terms.

This paper is limited to the most basic kinematical concepts. The constructions of dynamics as well as emergent coordinate system are found in (3).

Geometric constructions

Let us now proceed with the construction. As stated earlier, we have partial ordering $\prec$ such that $(x^\mu, v^\mu) \prec (y^\mu, w^\mu)$ if and only if we can start at location x^μ, moving with velocity v^μ and reach y^μ with final velocity w^μ, without exceeding some specified upper bound on acceleration. Now, if we view the events as the "building blocks" of spacetime, there is no "in between" spacetime that would define the said acceleration. Instead, we view $\prec$ as the "elementary" partial order (without any underlying geometry) but "secretly" aim at the "special case" of Poisson distribution on a tangent bundle; in that "special case" the partial order "would" coincide with the above "geometric" construction.

Now, we will define the "geodesic translation" of a given event. Intuitively, this means that we will "start out" *in the direction defined by* the "velocity" of said event and then move a specified distance along the geodesic. We would like to define it in turns of partial ordering alone. First, we define direct link, $\prec^*$; namely, $a \prec^* b$ if and only if $a \prec b$ and there is *no* c satisfying $a \prec c \prec b$. Now, a chain $a \prec^* r_1 \prec^* \cdots \prec^* r_{n-1} \prec^* b$ is visualized as a random walk. Reaching any given destination through random walk is "highly unlikely". But, at the same time, reaching another point on geodesic this way is "slightly less unlikely" (although still unlikely). Thus, we will define the "geodesic translation" as the "most likely" point that can be reached through random walk:

Definition Let p be an element of S and let n be an integer. Then the *past geodesic translation* and *future geodesic translation* of p by n are sets $G_{-n}(p) \subset S$ and $G_n(p) \subset S$, respectively defined as follows. If $a \in G_{-n}(p)$ while $a' \notin G_{-n}(p)$, then the number of chains $a \prec^* r_1 \prec \cdots \prec^* r_{n-1} \prec^* p$ is strictly greater than the number of chains $a' \prec^* r'_1 \prec \cdots \prec^* r'_{n-1} \prec^* p$. Likewise, if $b \in G_n(p)$ while $b' \notin G_n(p)$, then the number of chains $p \prec^* s_1 \prec \cdots \prec^* s_{n-1} \prec^* b$ is strictly greater than the number of chains $p \prec^* s'_1 \prec \cdots \prec^* s'_{n-1} \prec^* b'$. Finally, a "past (future) geodesic translation of p by zero" $G_0(p) = \{p\}$ is a one-element set consisting of p alone. Finally, the sets

$$G(p) = \cup_{n \in \mathbb{Z}} G_n(p) \ , \ G_-(p) = G(p) \cap J^-(p) \ , \ G_+(p) = G(p) \cap J^+(p) \tag{1}$$

are referred to "trajectory of p", "past trajectory of p" and "future trajectory of p", respectively, where

$$J^-(p) = \{p\} \cup \{r \in S | r \prec p\} \ , \ J^+(p) = \{p\} \cup \{s \in S | p \prec s\} \tag{2}$$

are past and future of p, respectively. The set $G(p)$ is also referred to as *geodesic passing through* p.

Since we have just defined the geodesic passing through a given point, we can "attach clock" to said geodesic and measure the "time" it takes to "send signal" to some outside event and "receive it back". We can use said "clock" in order to define the location and time of that other event. Our experience shows that the speed of signals approximates speed of light very closely. This implies that the spacetime scales we are dealing with are "much larger" than the inverse of upper bound of acceleration. We will define the spacetime displacements in such a way that they coincide with expected values on these larger scales, while expecting deviation on small scales. Thus, we will use the approximation that the speed of signal is lightlike:

Definition Let p and q be elements of S. Then *time displacement of q relative to p*, $t_p(q)$ and *space displacement of q relative to p*, $r_p(q)$ are defined as follows:

a)If $J^-(q) \cup G(p) \neq \emptyset$ and $J^+(q) \cup G(p) \neq \emptyset$, then

$$t_p(q) = \frac{1}{2}(\min\{m|G_m(p) \cap J^+(q) \neq \emptyset\} + \max\{n|G_n(p) \cap J^-(q) \neq \emptyset\}) \tag{3}$$

$$r_p(q) = \frac{1}{2}(\min\{m|G_m(p) \cap J^+(q) \neq \emptyset\} - \max\{n|G_n(p) \cap J^-(q) \neq \emptyset\}) \tag{4}$$

b)If $J^-(q) \cup G(p) \neq \emptyset$ and $J^+(q) \cup G(p) = \emptyset$, then $t_p(q) = +\infty$ and $r_p(q) = +\infty$.

c)If $J^-(q) \cup G(p) = \emptyset$ and $J^+(q) \cup G(p) \neq \emptyset$, then $t_p(q) = -\infty$ and $r_p(q) = +\infty$.

d)If $J^-(q) \cup G(p) = \emptyset$ and $J^+(q) \cup G(p) = \emptyset$, then $t_p(q) = i\infty$ and $r_p(q) = +\infty$.

Finally, we can define the "velocity" of the event q by passing geodesic *through* q and measuring "position" and "velocity" of some event displaced "far away" on the above geodesic; the "far away" part can be "enforced" by selecting "very large" n for the definition below:

Definition Let p and q be elements of S and let n be an integer. Then the *n-th Lorentz factor*, *n-th velocity*, *n-th angular velocity* and *n-th radial velocity* of q relative to p are, respectively, $\gamma_{p;n}(q)$, $v_{p;n}(q)$, $\omega_{p;n}(q)$ and $v^R_{p;n}(q)$, defined as follows:

$$\gamma_{p;n}(q) = \frac{\sum_{s \in G_n(p)} t_p(s)}{n\sharp G_n(q)} \ , \ v_{p;n}(q) = \frac{\sum_{s \in G_n(q)} r_p(s)}{n\sharp G_n(q)} \tag{5}$$

$$\omega_{p;n}(q) = \frac{v_{p;n}(q)}{(r_p(q))^2} \min\{r_p(s)|s \in G(q)\} \ , \ v^{Rad}_{p;n} = \sqrt{v^2_{p;n}(q) - r^2_p(q)\omega^2_{p;n}(q)} \tag{6}$$

Discussion

It is now important to address possible objections that might arize from (2). That paper argues that in case of sprinkling over compact space, any point has nearest neighbor that would define "preferred direction" (thus implying a "tradeoff" between "gaining" compactness and "sacrificing" relativity). In our case, we agree that the presence of v^μ in (x^μ, v^μ) is not the problem, but the "nearest neighbor" (y^μ, w^μ) might be. We address the above issue by pointing out that conceptual role of said preferred directions is very different. In particular, we have to *first* specify the "good' preferred velocity v^μ and only *then* we would obtain *two more* "preferred directions", namely $y^\mu - x^\mu$ as well as w^μ. At the same time, non-compactness of Lorentz group statistically implies that there are infinitely many choices of v^μ that are arbitrary close to any x^μ. Thus, there is no "preferred direction" corresponding to x^μ *alone*.

What we have just said is that there is *no* "single" preferred direction but there *is* a preferred *set* of directions. From the point of view of (2), the latter is not bothersome. After all, even in the absence of "nearest neighbors" we still have "direct neighbors defined (namely, $p \prec^* q$ if and only if $p \prec q$ and there is no r satisfying $p \prec r \prec q$). Thus, we can use *all* of "direct neighbors", none of which are "nearest", in order to define "set of" preferred directions. Since, according to (2), the absence of "one" preferred direction somehow alleviates that situation, the same should be true in our case as well.

One can countr-argue the above by pointing out that in order for the directions "towards neighbors" to be "preferred", there have to be "other" directions that the above are preferred "to". The "other" directions, however, are non-existent since the spacetime itself is discrete and is limitted to the scattered points. However, there is one conceivable way in which "preferred directions" can indeed be singled out. Namely, when one looks at "far away" points, one can define the "angle" more and more precisely. Conceivably, there might be some prescription of the above strictly based on the discrete structure alone, without reference to the supposed continuum it lives in. If so, everything we said about "infinitely many preferred directions" in non compact case would be true. And, in fact, we have to agree with the premise in order to stick to the conclusion of one preferred direction in non-compact case.

However, one way in which our version of "infinitely many preferred directions" is "worse" is that we can no longer view x^μ as as single element since (x^μ, v^μ) takes its place. This renders "infinitely many preferred directions" passing though x^μ irrelevent bringing us back to "one" preferred direction at (x^μ, v^μ). Nevertheless, as stated earlier, v^μ allows us to claim that the "one" preferred is not as problematic either. In order to "care" about that direction, we need a structure that somewhat resembles x^μ. Perhaps we could consider a hybrid between two extremes: a range of different (x^μ, v^μ) with some cutoff imposed on v^μ. This would imply that our answer to the question of relativistic covariance is some hybrid between the answers for x^μ and (x^μ, v^μ). Whether the ingredients would help each other or contradict each other is up to exact definitions of "Lorentz covariance" that are left for future debate. The claim of this paper is that we have successfully restored relativity from the point of view of one of the sides of the said debate.

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Lorentz transformations: discovery and interpretation

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The discovery of the relativistic transformations (Lorentz ones) occured in two directions – a research of the invariance of the wave equations (Voigt), Maxwell equations (Heaviside), and through the finding of transformation of coordinate and time to bridge the gap between theory and experiment of Michelson-Morley experiment (Fitzgerald, Lodge, Larmor, Lorentz). Larmor and Lorentz applied two-stage transformations with a different interpretation – Galilean transformation (kinematic interpretation) supplemented by transformations that were given electrodynamic interpretation. Poincaré was the first who combined these transformations in one, and wrote them in a letter to Lorentz in May 1905 and published in paper in June 1905, Poincaré also introduced four-dimensional description, but remained within the framework of dynamic interpretation. Einstein, in June 1905, independently from Poincare also proposed unified transformation and for the first time gave them a purely kinematic interpretation. Minkowski combined four-dimensional approach of Poincare and Einstein's interpretation in conception of the four-dimensional space-time.

1. Introduction

The transition from Galilean to relativistic transformations in mechanics and the whole physics underlay the relativistic revolution occurred in the beginning of XX century. At the same time Galilean transformations retained their importance as an ultimate case of the relativistic transformations for velocities much less than speed of light.

Now we have already understood that the relativistic transformations might have been derived purely mathematically without any experiments as early as in XIX century as a result of the invariance analysis of the wave equation of the form: $\Delta U = \frac{1}{c^2}\frac{\partial^2 U}{\partial t^2}$. Essentially, as early as in the very beginning of XIX century hidden transition occurred from Galilean invariance typical of classic corpuscular pattern to the relativistic invariance immanently inherent in the wave pattern of light simultaneously with the transition from corpuscular to wave pattern of light. Since 1860-s when J.C. Maxwell managed to create classic electrodynamics by joining electricity, magnetism and optics, light has become to be considered electromagnetic oscillations and the wave pattern has covered even broader range of phenomena. However the motion of massive bodies has still been described within classic mechanics. This disagreement in the foundations of the two theories – mechanics and electrodynamics lead to numerous attempts to find absolute motion relative to "luminiferous aether" that always had a negative result. Thus, contradictions between the two theories lead to conceptual crisis that became evident by the end of XIX century and was overcome in 1905 by the transition to new ideas of space and time underlain the generalization of classic mechanics and other physical theories.

Many papers are devoted to the history of special relativity theory (SRT) and basic principles are generally clear (W. Pauli, G. Holton, A. Bork, S. Brush, S. Goldberg, V.L. Ginzburg, I.Yu. Kobzarev, E. Whittaker, A.A. Tyapkin, A.A. Logunov, A. Miller, A.Pais etc. [1-17]) however Poincaré contribution has been quite differently interpreted. Formally historical reconstruction of the creation of special relativity theory as a reply of theorists to negative result of Michelson-Morley experiment has gained wide-spread acceptance. However it is clear now that the relativistic generalization of mechanics (and physics as a whole) was absolutely inevitable step of its development that would occur independently of any experiments and those experiments might play only testing role for the theory. Classic Euclidean geometry was similarly generalized in 1820-30s and isolated fundamental scale of curvature dimensionality appeared in Lobachevski geometry as in its generalization. Nevertheless, Euclidean geometry has retained its importance as an extreme case of Lobachevski geometry when curvature radius tends to infinity. Speed of light c plays the role of fundamental scale in SRT as Lobachevski geometry for velocity space.

Let us briefly describe the history of discovery of the relativistic transformations using unified modern designations for simplicity (Lorentz and Einstein denoted light speed by V, Larmor by C, Voigt by ω, etc.).

2. Elicitation of mathematical foundations of the wave model of light through the research of practical applications of Maxwell's electrodynamics (*Voigt*, *Heaviside*).

A transformations, for which the wave equation remains invariant, were proposed by W.Voigt in 1887 in the paper devoted to formula derivation for Doppler effect [18]. Voigt regarded light as a wave process spread in resilient incompressible medium (ether). In particular, W.Voigt showed in his paper that the wave equation of type $\frac{1}{c^2}\cdot\frac{\partial^2 U}{\partial t^2}=\Delta U$ maintains its shape with conversion to new space-time variables ($x'=x-\mathrm{v}t$, $y'=\gamma^{-1}y$, $z'=\gamma^{-1}z$, $t'=t-\frac{\mathrm{v}x}{c^2}$). These transformations within the accuracy of scale factor $\gamma^{-1}=\sqrt{1-\frac{\mathrm{v}^2}{c^2}}$ agree with modern relativistic transformations (γ factor in Voigt's paper was transferred to transverse coordinate transformations resulted in discrepancy with conversion to moving system and vice versa, i.e. Voigt transformations did not form a group). As seen, Voigt also dealt with time transformation later called local time that was usually associated with Lorenz. Consequently, Voigt was managed to derive classic Doppler effect in his paper.

Unfortunately, Voigt's paper was not accepted with due attention. It is notable that Voigt and Lorentz were in correspondence in 1887 with regard to Michelson-Morley experiment, however they appeared to skip the problem of motion transformations [17]. Voigt appeared to underestimate the universality of his transformations for the wave model too. Unfortunately, the importance of this Voigt's paper became clear only after the creation of special relativity theory. This fact was noted by H. Minkowski who published his papers on SRT in the same journal. In 1908 H. Minkowski noted the importance of Voigt paper at the meeting, in which Voigt took part too. Hereto Voigt answered that "some of the results later based on electromagnetic field theory had already been obtained by that time" (cited by [17]). H.A. Lorentz also accepted the importance of Voigt's paper. In the book "Theory of electrons" (1909) written on the basis of lectures given at Colombia University in 1906, he expressed regret at being unaware of this paper: "In a paper <...> published in 1887 ... and which to my regret has escaped my notice all these years, Voigt has applied to equations a transformation (wave equations – *K.T.*) <...> equivalent to… (the relativistic transformations – *K.T.*). The idea of the transformations <...>

might therefore have been borrowed from Voigt and the proof that it does not alter the form of the equations for the *free ether* is contained in his paper" [19]. Later on, in the paper of 1914 devoted to H. Poincaré works, Lorentz noted: "…the same transformation was already present in an article of Mr. Voigt published in 1887, and that I did not draw from this artifice all the possible parts" [20]. Another view was also expressed. In opinion of historian A. Miller, the knowledge of Voigt's paper would hamper Lorentz to advance in his way to hypothesis of longitudinal compression of moving bodies [16].

Important research was carried out by O. Heaviside in the field of theoretical electrodynamics at the boundary of 1880-90s. Based on Maxwell equations Heaviside particularly considered the motion of charged sphere and showed the sphere to be turned into ellipsoid with the axis compressed along motion direction in ratio of $1:\sqrt{1-\mathrm{v}^2/c^2}$ (issued in 1892) [21]. This question has also been discussed by J. Searle, J.J. Thomson and others (see *Whittaker*, vol.1). Later on H.A. Lorentz substantiated compression of moving bodies as dynamic effect caused by electromagnetic nature of electron mass based on similar pattern of compressed ellipsoid.

The path to relativistic transformations appeared more winding than it would have been if a direct task had been formulated to find those transformations that remain invariant the form of wave equations, and this path ran across a number of independent hypotheses: contraction of linear body dimensions (Lorentz-FitzGerald contraction), hypothesis of local time, etc. It should be noted that all of these hypotheses resulted from experiment interpretation with limited accuracy as any experiment, but results of Voigt and Heaviside were derived from Maxwell theory and therefore were better substantiated.

3. "Proliferation of hypotheses" as a reply to experimental challenge (*FitzGerald*, *Lodge*, *Lorentz*, *Larmor*)

Michelson and Michelson-Morley experiments for search of motion relative to ether set a challenge before theorists how to interpret their results. All the experiments starting from F. Arago argued for Fresnel's theory of ether (no ether entrainment or negligible ether entrainment by Earth). Negative result of Michelson experiment directly argued for Stokes theory – complete ether entrainment by Earth. This was the conclusion made by A. Michelson [22]. As it is clear now, all the experiments have in common no ether influence on any electrodynamic phenomenon that argues for applicability of relativity principle to both mechanical and electromagnetic phenomena. However that time theorists tried to save Fresnel's theory well confirmed experimentally using a number of additional hypotheses – "proliferation of hypotheses", in the words of H. Poincaré [23]. Historians of science single out up to 10 independent Lorentz hypotheses in order to eliminate contradiction between theory and experiment (*Miller*, [16]), we will give three basic hypotheses noted earlier by H. Poincaré: 1) all bodies are subjected to longitudinal contraction in the line of motion proportional to $\sqrt{1-\mathrm{v}^2/c^2}\approx 1-\frac{\mathrm{v}^2}{2c^2}$; 2) time quantity should be added with the corrective associated with motion – local time $t' = \frac{t - \frac{\mathrm{v}x}{c^2}}{\sqrt{1 - \frac{\mathrm{v}^2}{c^2}}} \approx t - \frac{\mathrm{v}x}{c^2}$; 3) all the forces including gravitation have electromagnetic origin or at least behave as electromagnetic forces at high velocities. All these hypotheses were proposed as ideological support of additional transformations required to restore invariance of Maxwell equations after the use of classic (Galilean) motion transformations.

The first hypothesis was stated by J. FitzGerald, O. Lodge and H.A. Lorentz. The history of discovery of Lorentz-FitzGerald contraction was studied in detail in the following papers: [5-7]. In 1889 J. FitzGerald submitted a note "The Ether and the Earth Atmosphere" to American journal "Science" where it was published [24] (letter reprinted in [6, 17]). The note contained only qualitative idea without any formulas and with verbal indication that body compression was proportional to expression depending on v^2/c^2. FitzGerald did not know about this issue himself as it was later revealed by historian of science S. Brush [6] and given in particular in his paper and in the book of Pais [17]. FitzGerald's idea became known due to the fact that FitzGerald and Lodge stated it in their lectures. FitzGerald himself told to Lodge about this idea when he visited Lodge at home in Liverpool in 1889 [25]. O. Lodge referred to FitzGerald's idea for the first time in the papers [26, 27]. In 1892 H.A. Lorentz independently came to similar idea [28]. As said by Lorentz he learned of FitzGerald's idea only from Lodge's paper (1893). In autumn of 1894 Lorentz was in communication with FitzGerald concerning this question and later on, starting from the book of 1895, Lorentz always mentioned FitzGerald with regard to this idea. According to FitzGerald and Lorentz, linear body dimensions are reduced due to the compression of distance between molecules as electrodynamic systems caused by their interaction with ether during motion. It is notable that FitzGerald and Heaviside were in friendly relations and apparently influenced each other scientifically.

As it became clear later, Lorentz-FitzGerald contraction combined with Galilean transformation gave *correct relativistic change of longitudinal coordinate*. All further attempts to harmonize theory with experiment concerned more precise definition of time transformation however it required another independent hypothesis.

Such a hypothesis was an idea to go over from usual time to so-called "local time" (Ortzeit), the most "sophisticated idea" as Poincaré noted [23]. W. Voigt was the first one who used time transformation $t' = t - \frac{\mathrm{v}x}{c^2}$ in 1887 although this approach became well-known owing to H.A. Lorentz. In 1895 H.A. Lorentz published comprehensive study of the influence of moving bodies on optical and electrical phenomena [29]. This book became a foundation for all further researchers including J. Larmor and A. Einstein. After Galilean transformations Lorentz used the following coordinate transformations: $x' = \gamma x$, $y'=y$, $z'=z$ (*Lorentz*) (longitudinal body compression) and then time transformation: $t' = t - \frac{\mathrm{v}_x}{c^2}x - \frac{\mathrm{v}_y}{c^2}y - \frac{\mathrm{v}_z}{c^2}z$ (ibid.). He called time t' as local time ("Ortzeit") and considered it as auxiliary quantity. These transformations provided invariance of Maxwell equations for degrees of v^2/c^2.

J. Larmor was the first one who discovered correct time transformation and published it in a book "Aether and Matter" [30] (the book was written in 1898 and was based on a number of his papers issued in 1890s). Larmor used the two-stage pattern – transition to moving system was described by Galilean transformations as it was generally accepted, and then he introduced additional coordinate and time transformations that lead to invariance of Maxwell equations.

In the lecture given at the meeting of Amsterdam Academy of Sciences on April, 23 of 1904 H.A. Lorentz [31] obtained invariance of Maxwell equations using the two-stage pattern mathematically equivalent to Larmor's pattern (1900) (the fact that Lorentz was acquainted with Larmor's book was confirmed in his paper [32]). As usual Lorentz initially applied Galilean transformations to Maxwell equations and then offered to go over to the following new variables: $x'=klx$, $y'=ly$, $z'=lz$, $t' = \frac{l}{k}t - kl\frac{w}{c^2}x$, где $k^2 = \frac{c^2}{c^2 - w^2}$ [31]. Lorentz *dynamically* interpreted additional transformation like Larmor: spheroid electron was compressed to ellipsoid along motion direction – "translational motion *produces* deformation ($1/kl$, $1/l$, $1/l$)" [31].

In his paper Lorentz also assigned the task of finding transformation formulas in the presence of sources. However Lorentz failed to derive correct transformation formulas of charge density and electron velocity that would lead to complete invariance of Maxwell equations because of unclear understanding of time relativity – transformed equations still contained unnecessary terms although they were negligible at high velocities (correct velocity and charge density transformations were written by H. Poincaré in his letter to Lorentz in 1905). In 1913 Lorentz commented on this in such a way: "These terms were too small to influence phenomena noticeably, and by this fact I could explain their independence of the Earth's motion, revealed by observations, but I did not establish the relativity principle as a rigorous and universal truth" [20].

As we could see, the way of "hypotheses invention" mathematically lead to correct objective however the very set of numerous hypotheses was bound to provoke discontent. H. Poincaré conceptually criticized the set of Lorentz's hypotheses in the course of lectures on electrodynamics given in Sorbonne in 1899 (published in 1901) and at International Physical Congress in 1900. In 1899 he spoke in support of relativity principle for optical phenomena for every degree of v/c [33, ,p.535-536]. In his lecture in 1904 H. Poincaré expressed even harder opinion by having qualified Lorentz's approach as "proliferation of hypotheses" [23].

4. Mathematical unification of Galilean and Larmor-Lorentz transformations (*Poincaré*).

When H. Poincaré was acquainted with Lorentz's paper (1904) he appeared unsatisfied with dynamic method of proving the equality l=1 for factor l included in Lorentz transformations. This was the reason for his own reflections on the subject and it made him write three letters to Lorentz in May of 1905 (unfortunately, Poincaré did not indicate an exact date of their writing, [34], photocopy in [16, p.81]). In the second short letter Poincaré proposed his evidence usefully different from the proof of Lorentz by its mathematical simplicity and elegance. The meaning of evidence reduced to requirement to the transformations to form a group. Poincaré gave the relativistic transformations *as unified transformation* $x' = kl(x + \varepsilon t)$, $y' = ly$, $z' = lz$, $t' = kl(t + \varepsilon x)$ in his letter to Lorentz for the first time; thereat time t was understood by Poincaré as a distance that the light passed for a certain time if c=1 is chosen (this is equivalent to the choice of ε=v/c and renaming of product ct by designation t). While arguing for his evidence Poincaré also derived the correct formula of velocity addition that he also mentioned in the letter: $\varepsilon'' = \dfrac{\varepsilon + \varepsilon'}{1 + \varepsilon\varepsilon'}$. As seen, Poincaré refused of Galilean transformations at all by direct joining them with additional Larmor-Lorentz transformations. Thereat both coordinates of space x and time t were symmetrically included by Poincaré in transformation formulas. Basic results set forth in this letter were published by H. Poincaré in Proceedings of French Academy of Sciences dated June 5, 1905 [35]. Correct unified relativistic motion transformations appeared in print format in this brief paper for the first time; however the designations used and specific features of their derivation were omitted.

Poincaré wrote in the beginning of his paper: "The results which I have obtained agree with those of Lorentz in all the principal points, and I have needed only to modify and augment them" [in the next paper: "modify and augment them in certain details" – *K.T.*]. "The essential point, established by Lorentz, is that the equations of the electromagnetic field are not altered by a certain transformation (which I will call by the name of Lorentz) of the form: $x' = kl(x + \varepsilon t)$, $y' = ly$, $z' = lz$, $t' = kl(t + \varepsilon x)$, where x, y, z are the coordinates and t the time before the transformation and x', y', z' and t' afterwards. Furthermore, ε is a constant that depends the transformation: $k = 1/\sqrt{1-\varepsilon^2}$, and l is an arbitrary function of ε" [35]. Then Poincaré pointed out: "The sum of all these transformations, together with the set of all rotations of space, must form a group; but for this to occur, we need l=1" (ibid.). After that Poincaré derived charge density transformations and noted a certain difference of them from formulas obtained by Lorentz. Thus, Poincaré joined together for the first time the two transformations used individually and different by nature in Lorentz work, simplified the method of their derivation using group considerations and also corrected some inaccuracy in charge density and electron velocity transformation made by Lorentz.

On July 23 of 1905 Poincaré submitted his large paper "On the Dynamics of the Electron" to Italian Journal of Palermo Mathematical Society that contained a broad range of questions concerning interaction of matter and ether: gave correct relativistic transformations (as earlier called them by the name of Lorentz), Poincaré entered the term "Lorentz group" and studied its invariants, considered body deformation according to Lorentz and Langevin and also hypotheses of the change of Newton law of gravitation in view of the finiteness of interaction velocity [36]. Especially valuable in the paper were Poincaré discovery of invariant x^2+y^2+z^2–c^2t^2=0, transition to four-dimensional space-time (with imaginary time) and interpretation of "Lorentz transformation" as a rotation in this four-dimensional space (ibid.). However Poincaré has still adhered to dynamic interpretation of the transformations. The paper of Poincaré has remained almost unnoticed, with the exception of H. Minkowski who combined an idea of Poincaré's four-dimensional description with Einstein's kinematic interpretation in the concept of four-dimensional space-time in 1907 [37, 38]. However Einstein did not know about this paper of Poincaré even in 1908 as followed from his correspondence with I. Stark concerned writing the review on this subject. The importance of the two papers of Poincaré was recognized by H.A. Lorentz in 1914 (although he has not mentioned Poincaré's letter stored in his archives) and then by W. Pauli in 1921; these articles of Poincaré were included in the digest of original texts together with the articles of Lorentz and Einstein known as “Relativity Principle” (1935). To the contrary, in 1953 E. Whittaker rewrote the history of SRT by representing Poincaré as its discoverer [2]. The same direction was developed later by A.A. Tyapkin and A.A. Logunov [12-15]. However this approach is based on looking at the works of Poincaré and Lorentz with post-Einstein eyes and ignoring their actual physical views, first of all, ether-electrodynamic program that they followed up to the bitter end (e.g., see [17]).

The notion of the identity of Lorentz (1904) and Poincaré-Einstein (1905) transformations is generally accepted and it is mathematically true, indeed: transformations given by Poincaré in his letter to Lorentz and in the two papers “On the Dynamics of the Electron” and then by Einstein in his paper (1905) were *mathematically* equivalent to the two-stage transformations (Galilean + additional Larmor and Lorentz transformations) used by Larmor (1900) and Lorentz (1904). However each of the two consecutive transformations was attributed by Lorentz and Larmor to different physical meaning (the first one had kinematic and the second one dynamic nature). The two-stage character of Lorentz transformations was pointed out by H. Poincaré [34], W. Pauli [1] and historians of physics A. Miller [16], and I.Yu. Kobzarev [11]. However mathematical reality was considered primary by Poincaré whereas physical interpretation seemed secondary and conventional to him. He might have underestimated the result he obtained and considered it mathematically and – in his opinion – physically equivalent to the result of Lorentz. The approach of Poincaré has remained ambiguous and contradictory: although he has obtained the unified transformations he has still adhered to their *dynamic* interpretation considering the compression of moving bodies as a real mechanical phenomenon. It was particularly reflected in his term “deformation” and the use of Lorentz value of *Earth’s compression* towards its motion of 1/200 000 000 [28] in the papers issued both before and after 1905 [23, 39, 40]. He surprisingly combined in the same paper [36] the unified relativistic transformations, space-time four-dimensionality and mechanical deformation of electrons moving in the ether.

5. Derivation of the relativistic transformations from the simple postulates, refusal of the ether and transition to new ideas of space and time (*Einstein*)

A. Einstein started as a scientist well after the majority of scientists involved in the problems of matter and ether interaction. In 1895 when Lorentz issued his monograph Einstein just finished school and failed an entrance examination at Zurich Polytechnic School. By that time H.A. Lorentz, H. Poincaré and J. Larmor had been recognized scientists well-known in the world with their own developed notions of physical reality consisted in electromagnetic view of the world. They aimed to create the “theory of everything” based on electromagnetism and ether as a medium of the electromagnetic interaction. In 1895 Einstein just attempted to understand the structure of physical reality and carried out his mental experiments, in particular, tried to comprehend the view of the world moving with the velocity of light.

As known, during his studies Einstein has been greatly influenced by the books of H. Poincaré and early papers of H. Lorentz especially the book “Science and Hypothesis” by Poincaré [41] and Lorentz monograph on the optics [29]. However he did not know the papers of Lorentz and Poincaré (1904-1905) that contained correct transformations and moreover their private letters, he independently derived the relativistic transformations. Einstein’s independent way was proved by G. Holton by comparative analysis of the papers of Einstein and Lorentz [3, 4]. As Einstein recollected later, he has initially followed the ether-electromagnetic path of Lorentz for about a year and only in the end of spring of 1905 he managed to discover the right way to new ideas of space and time. On June, 30 of 1905 Einstein submitted his paper "On the Electrodynamics of Moving Bodies" to journal "Annalen der Physik" that was published on September, 26 of the same year [42].

In this fundamental paper Einstein did not cite his predecessors (probably due to deductive style of the paper) and devoted to motion transformations a special paragraph 3 "Theory of the Transformation of Coordinates and Times from a Stationary System to another System in Uniform Motion of Translation Relatively to the Former". The concept of watches synchronization for two spatially separated objects by the arithmetic mean of the light emission time and the time of reflected signal return to initial point preceded derivation of the transformations. Here Einstein used the principle of light speed consistency in different directions [42]. With regard to group requirements to the transformations (this idea was contained in the letter of Poincaré to Lorentz and was mentioned by him without detailed derivation in the abstract dated June 5 of 1905) Einstein derived as a result the correct relativistic transformations: $t' = \beta(t - \frac{\mathrm{v}}{c^2}x)$, $x' = \beta(x - \mathrm{v}t)$, $y' = y$, $z' = z$ where $\beta = \frac{1}{\sqrt{1-(\mathrm{v}/c)^2}}$ and c is speed of light in vacuum. As a result of these transformations in the next paragraph Einstein derived the compression of linear dimensions of moving bodies observable from stationary reference system that had been introduced by Lorentz and FitzGerald as a real dynamic effect; additionally Einstein obtained the corresponding effect of slow run of moving watches (ibid.). In the next paragraphs Einstein obtained the relativistic formula of velocity addition and the field transformation formulas resulted in the relativistic formulas of Doppler effect and aberrations turned into well-known formulas at low velocities.

Despite many ideas (e.g., validity of relativity principle) had been proposed even before Einstein, this was the paper that contained an integral concept based on fundamental principles (the principles of relativity and light speed consistency) from which the author constructed a direct logic chain to the relativistic transformations and then to experimentally observable results. Thus, this was the paper that became fundamental one that underlay new concepts of space and time. Einstein's paper had some inevitable shortcomings – formulas for longitudinal and transverse masses appeared obvious dynamic vestige in the world of new kinematics. The formula of rest energy $E_o = mc^2$ that later became a symbol of the relativity theory might have been obtained even in the first published paper as well as Fresnel's formula that was substantiated by M. Laue in 1907 on the basis of Einstein's approach [43].

In 1910, after the discovery by H. Poincaré and H. Minkowski of four-dimensional space-time as a mathematical content of the special relativity theory, V. Varičac simplified SRT formulas by rewriting them in the form of hyperbolic geometry (Lobachevski geometry) for the space of velocities: $l' = -x\sinh u + l\cosh u$, $x' = x\cosh u - l\sinh u$, $y'=y$, $z'=z$ where $l=ct$ [44, S.93]. Varičac named the relativistic transformations in his papers as Lorentz-Einstein transformations [44, 45]. It is historically evident that additional transformations introduced by J. Larmor in 1900 and H. Lorentz in 1904 and interpreted them for purely electrodynamic reasons should be called Larmor-Lorentz transformations and the unified transformations proposed by H.

Poincaré and A. Einstein in 1905 should be called Poincaré-Einstein or the relativistic transformations. It is significant that Lorentz has come to the unified transformations (named as “Lorentz transformations” at the suggestion of Poincaré in 1905) not at once; for example, even in the book “Theory of Electrons” [19] published in 1909 he has still adhered to the two-stage pattern of transformations by only noting that it might be expressed as a single transformation. In the paper written in 1914 (published in 1921) devoted to the works of H. Poincaré (1905-1906) Lorentz finally used the unified transformations but noted that he had derived them in another way [20].

Conclusion

Thus transformations and interpretations used until 1905 can be expressed in next scheme:

Galilean transformations (*kinematic interpretation*)	+	additional transformations introduced by Larmor (1900) and Lorentz (1904) (*electrodynamical interpretation*)

Relativistic transformations (named nowadays as Lorentz transformations)
(*dynamical interpretation* – Poincaré, 1905)
(*kinematic interpretation* – Einstein, 1905; also Voigt, 1887, close to RT)

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Nonlocal electrodynamics and relativistic invariants

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We have considered the account relativistic invariants on correlation between the charged particles when they are current carriers. Such correlation takes place at abnormal skin-effect and at superconductivity. Dependence of conductivity on frequency, a wave vector and parameters of correlation function is received.

Introduction

By working out of theories of type of abnormal skin-effect there is a problem of the account of not local interaction, i.e. the mathematical description of that fact, that the current density is defined not only value of intensity of electric field in this point, and also in the next points. Usually it is supposed, that the current density in the given point is defined by kind integral [1, 2]

$$j_l = \int K\left|r_i - r_i'\right| E_l(r_i')\, d^3r', \tag{1}$$

where $K\left|r_i - r_i'\right| -$ is the function of the influence depending on the module of distance of the allocated point of space and the next points of space, $E_l(r_i') -$ intensity of electric field in the next point of space; $l, i = x, y, z.$ In these theories electrodynamics which is the relativistic theory is used. However, in the formula (1) dependence of function of influence on coordinates and time is not relativistic invariant.

In theories of abnormal skin-effect and superconductivity influence relativistic invariants on electronic correlations has not been studied [1-3]. In connection with continuation of development of these theories such research is necessary also it is the purpose of the given work.

Choice of argument of function of influence

Relativistic invariant function of coordinates and time should be argument of function of influence : $K(r_i' - r_i, t' - t)$. Such functions are an interval between events and a phase of a flat wave [4]. For both functions it is easy to formulate a causality condition. On this condition they are equivalent. In quantum electrodynamics the interaction carrier has structure of a flat wave [5]. Considering its structure, as argument of function of influence the phase of a flat wave has been chosen, and function of influence has been chosen in the form of superposition of flat waves.

Problem geometry

The problem of the analysis of the account of function of influence strongly becomes simpler by consideration of spending semi-space in case of distribution of an electromagnetic wave along a normal to a boundary flat surface. This geometry of a problem is illustrated on fig. 1.

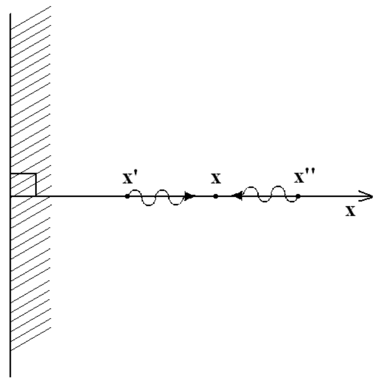

Fig.1: An interference of components of function of influence

On fig. 1 are shown components of function of influence.

The relativistic invariant formula for current density

In case of one type of not local interaction extending lengthways x. we have the formula for current density

$$j_1 = \iint K_0(\exp(-i\omega_L(t'-t))F(k_L, x', x)E_{01}(\exp(-i(\omega t' - kx'))icdt'dx', \tag{2}$$

$$F(k_L, x', x) = (0, x' < x; \exp(-ik_L(x'-x)), x' > x),$$

where K_0- a constant, ω_L- correlation parameter, dimensional frequencies; k_L- correlation parameter, dimensional a wave vector; E_{01}, ω and $k-$ amplitude of intensity of electric field, circular frequency and a wave vector of the flat electromagnetic wave extending along an axis *x*; $icdt'dx'-$ elementary relativistic invariant volume in a two-dimensional existential continuum, $c-$ is velocity of light, $i = (-1)^{1/2}$. At record of the formula (2) it was supposed, that the coordinate *x* belongs to an interval $[0, \infty]$. Coordinate x' also belongs to this interval. The integration area on coordinate x' in the formula (2) is broken into two intervals: $x' \in [0, x], x' \in [x, \infty]$ and the integration area on time t' has one interval: $t' \in [-\infty, t]$. As a result of integration performance in the formula (2) the following result has been received

$$j_1 = K_0 \frac{\tau}{1 - i\omega\tau}\left[\frac{1}{k_L - k}\right]E_{01}, \tag{3}$$

where the convenient designation is entered: $\tau = \dfrac{i}{\omega_L}$.

The formula for conductivity

If to present while an unknown constant K_0 in the following form:

$$K_0 = \frac{{\omega_{PL}}^2}{4\pi} k_L \tag{4}$$

where ω_{PL} – plasma frequency the required formula for conductivity turns out

$$\sigma(\omega,k) = \frac{{\omega_{PL}}^2\tau}{4\pi}\frac{1}{1-i\omega\tau}\left[\frac{1}{1-\left(\frac{k}{k_L}\right)}\right] \quad , \tag{5}$$

More general kind of formula (5) is connected with additive forms

$$\omega_L = \sum_K \alpha_K \omega_{LK}, \qquad k_L = \sum_K \beta_K k_{LK}, \ , \tag{6}$$

where α_K, β_K are c-integer numbers and $|.\alpha_K|,.|\beta_K| = 1$. Using forms (6) we have produced circular components of conduction

$$\sigma_\pm(\omega,k) = \frac{{\omega_{PL}}^2\tau}{4\pi}\frac{1}{1-i\omega\tau \pm i\omega_C\tau}\left[\frac{1}{1-\left(\frac{k}{k_{L\pm}}\right)}\right], \tag{7}$$

where ω_C is cyclotron frequency, $k_{L\pm} = -\frac{i}{\langle l\rangle} \mp \frac{1}{2\pi R_C}$, $\langle l\rangle$ is length of free path, R_C is cyclotron radius. If electrons form spontaneous magnetic moment then we have

$$\omega_C \Rightarrow \omega_C + \omega_S, \quad k_{L\pm} = -\frac{i}{\langle l\rangle} \mp \frac{1}{2\pi R_C} \mp \frac{1}{2\pi R_{CM}}, \tag{8}$$

where ω_S, R_{CM} are supplements of cyclotron frequency and cyclotron radius.. These values are proportional to magnetization. If there are some electron groups then formula (7) may be more general so current density is additive value.

The analysis of the formula for conductivity

In the formula (5) the first factor is conductivity on a direct current in Drude's model. A second factor defines frequency dependence of conductivity in the same model. A multiplier in square brackets defines dependence of conductivity on a wave vector or a spatial dispersion.

To analyze this result, we will consider a conductivity case on a direct current. If we have superconductor then equality is $\langle l\rangle = \infty$. Current density is equal zero if magnetic field is equal zero also.

And to abnormal skin-effect there corresponds a strong inequality

$$\left|\frac{k}{k_L}\right| >> 1, \tag{9}$$

The formula (7) defines a case of extremely abnormal skin-effect when mirror reflection takes place at Fermi's spherical surface.

Discussion

This method is effective for such kinetic coefficients as conductivity, dielectric function. We have found these coefficients without solution of kinetic equation.

So interband transitions are not considered this method is effective in microwave and infrared we may study such effects as subsurface waves [6] in area where spectral crossovers form [7-10] ,if reflection of carriers in metals and semiconductors is mirror.

Conclusions

With superposition use relativistic invariant functions of influence the formula for current density in a wide range of frequencies and wave vectors is received. On the basis of the received formula for current density the formula for conductivity as dependences on frequency and a wave vector is found. During the analysis of the formula for conductivity conditions are revealed: abnormal skin-effect, superconductivity. Efficiency of use in not local electrodynamics relativistic инвариантов is proved at construction of relativistic functions of influence for research of correlation of carriers of an electric current.

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Statistical method of anisotropy estimation of distribution of Hubble's parameter of celestial sphere according to astro- and photometry of QSO's data

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The present research is devoted of working and experimental approbation of original method of an estimation of values of Hubble's parameter in various directions of celestial sphere. This method is based on the statistical analysis of astro- and the photometric data of QSO's along these directions.

1. Idea of use of astro-and the photometric data of QSO's for estimation of anisotropy of distribution of Hubble's parameter

Are known (figure 1) results of an experimental estimation of anisotropy of distribution of Hubble's parameter on heavenly sphere [1, 2]. The estimation, for example, is based, on division of components of movement of galaxies of local group on own and cosmological movement. The disorder of estimations of numerical values of Hubble's parameter has thus made a range 55 … 85 $km{\cdot}s^{-1}{\cdot}Mpc^{-1}$ /

The present research contains estimations of distribution of values of Hubble's parameter in directions of the celestial sphere, received by an indirect method by the analysis of distributions of quasars on their luminosity in various directions.

The thirteenth edition [3] catalogues of quasars and active kernels of observatory of High Provence contains 168941 object among which there are 133336 quasars, 34231 active galaxies (including 16517 Seyfert 1s type galaxies), and also 1374 *BL*-lacertidae.. Thus starlike objects, or objects with the star-shaped kernel, surpassing in absolute luminosity value M_B=+22,25 are carried to quasars.. The similar objects possessing smaller luminosity, are carried to active galactic kernels.

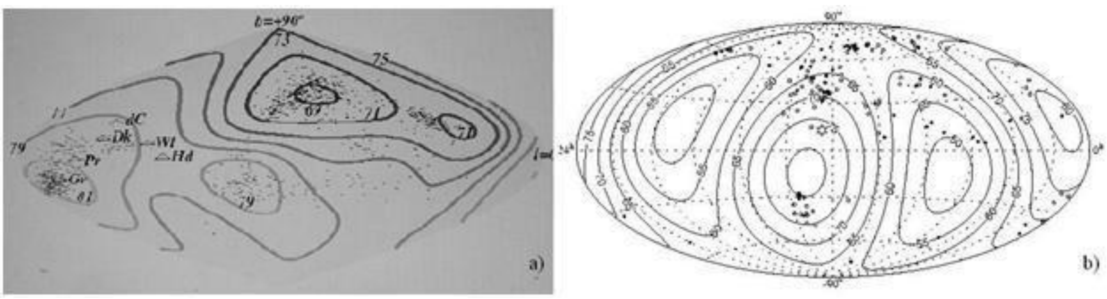

Fig. 1: Anisotropy of distribution of values of Hubble's parameter on celestial sphere it agree to data [1] (a) and [2] (b)

It is necessary to notice, that in the previous editions of the catalogue boundary value of absolute luminosity has been established [2] in magnitude $+23{,}0$. It is connected with processing of the previous edition of the catalogue and executed at this specification of value of Hubble's parameter used for calculation of absolute magnitude M_B of QSO's. It has been calculated by cataloguers under the formula [3; 4]:

$$M_B = B + 5 - 5\lg D - k + \Delta m(z) \ , \tag{1}$$

where B - relative magnitude on Johnson's scale; D - luminosity distance [6]

$$D = \frac{\mathrm{c}}{H_0}(1+z)\int_0^z \frac{\mathrm{d}z}{\sqrt{(1+\mathrm{z})^3\Omega_\mathrm{M}+\Omega_\Lambda}} \ , \tag{2}$$

defined by cosmological values: $H_0 = 71$ $km{\cdot}s^{-1}{\cdot}Mpc^{-1}$ (in the previous editions of the catalogue value $\mathrm{H}_0 = 50$ $km{\cdot}s^{-1}{\cdot}Mpc^{-1}$ was used [2]); $\Omega_\mathrm{M} = 0{,}29$, $\Omega_\Lambda = 0{,}71$ [3; 4]; z – red shift (For a wide range of values z relative magnitude B can be calculated on the basis of resulted in the catalogue photo-electric, or photographic relative magnitude V Taking into account the formula $\langle B - V \rangle = 0{,}40$); k – the amendment:

$$k = -2{,}5\lg(1+z)^{1-\alpha} \ , \tag{3}$$

$\alpha = 0{,}3$ – optical spectral index [5]; $\Delta m(z)$ – the amendment to the size k [6], caused by features of spectra of QSO's.

Taking into account (1) - (3), the absolute magnitude of QSO's of objects can be described the kind formula:

$$M_B = B + 5\lg H_0 + f(z) \ . \tag{4}$$

where $f(z)$ – the function of red shift containing in a kind of constant factors of size c; Ω_M; Ω_Λ.

Let in some direction on heavenly sphere there is an area in which limits it is observed n_1 quasars. Then with reference to this area it is possible to summarize and average on n_1 quasars of size M_{B_i}, B_i, $\mathrm{f_i(z)}$ according to formula (4):

$$\frac{1}{n_1}\sum_{i=1}^{n_1} M_{B_i} = \frac{1}{n_1}\sum_{i=1}^{n_1} B_i + 5\lg H_0 + \frac{1}{n_1}\sum_{i=1}^{n_1} f_i(z) \ . \tag{5}$$

Similarly, let within other area, in other direction, are observed n_2 quasars for which it is possible to write down:

$$\frac{1}{n_2}\sum_{i=1}^{n_2} M_{B_i} = \frac{1}{n_2}\sum_{i=1}^{n_2} B_i + 5\lg H_0 + \frac{1}{n_2}\sum_{i=1}^{n_2} f_i(z) \ . \tag{6}$$

If to admit, that in these directions of value of Hubble's parameter are identical, the difference of formulas (5) and (6) appears free from Hubble's parameter:

$$\frac{1}{n_1}\sum_{i=1}^{n_1} M_{B_i} - \frac{1}{n_2}\sum_{i=1}^{n_2} \mathrm{M}_{\mathrm{B_i}} = \left(\frac{1}{n_1}\sum_{i=1}^{n_1} B_i - \frac{1}{n_2}\sum_{i=1}^{n_2} B_{\mathrm{i}}\right) + \left(\frac{1}{n_1}\sum_{i=1}^{n_1} f_i(z) - \frac{1}{n_2}\sum_{i=1}^{n_2} f_i(z)\right) \ . \quad (7)$$

Let's admit, that both samples of quasars are powerful enough realizations of the general statistical distribution. Differently, let us assume, that statistical properties of quasars do not depend on a direction on celestial sphere, and also from the relative distance of luminosity *D*, corresponding at least to a range of red shift $0 \le z \le 6$ of their most mass supervision/ Then the left part of (7) appears equal to zero:

$$0 = \left(\frac{1}{n_1}\sum_{i=1}^{n_1} \mathrm{B_i} - \frac{1}{n_2}\sum_{i=1}^{n_2} B_{\mathrm{i}}\right) + \left(\frac{1}{n_1}\sum_{i=1}^{n_1} f_i(z) - \frac{1}{n_2}\sum_{i=1}^{n_2} f_i(z)\right) \ . \qquad , (8)$$

Expression (8) contains differences of average values of relative magnitude *B*, and also differences of average values of functions $f(z)$ of red shift..

Let's notice, that if the first of sample realizations is, for example, on the average on more considerable removal from the observer, than the second its average red shift *z*, and together with it and its average value of function $f(z)$ appears big, than at other sample, that does positive expression in the second bracket. Thus observable average values of relative star size *B* of the first sample appear smaller, than at the second sample that does negative expression in the first bracket. The sum of the negative and positive differences bracketed, in a considered case appears equal to zero.

If now to admit, that two various directions on heavenly sphere are characterized by differing values H_1 and H_2 Hubble's parameter in expression (7) their logarithm of the private is added:

$$\frac{1}{n_1}\sum_{i=1}^{n_1} M_{B_i} - \frac{1}{n_2}\sum_{i=1}^{n_2} M_{B_i} = \left(\frac{1}{n_1}\sum_{i=1}^{n_1} B_i - \frac{1}{n_2}\sum_{i=1}^{n_2} B_i\right) + \left(\frac{1}{n_1}\sum_{i=1}^{n_1} f_i(z) - \frac{1}{n_2}\sum_{i=1}^{n_2} f_i(z)\right) + 5\lg\frac{H_1}{H_2} \ . \quad (8)$$

Considering (7) in (8), we receive:

$$\frac{1}{n_1}\sum_{i=1}^{n_1} M_{B_i} - \frac{1}{n_2}\sum_{i=1}^{n_2} M_{B_i} = 5\lg\frac{H_1}{H_2} \ ; \quad (9)$$

Expression (9) contains two unknown sizes H_1 и and H_2. However, if to put, that for one of выборок, for example, as value H_2 it is possible to take advantage of an estimation of value of Hubble's constant H_0= 71 $km{\cdot}s^{-1}{\cdot}Mpc^{-1}$ [5; 6], accepted for all general totality in quality most precisely established now expression (9) will be transformed to a kind:

$$H_1 = H_0 \cdot 10^{\frac{\frac{1}{n_1}\sum_{i=1}^{n_1} M_{B_i} - \frac{1}{n_2}\sum_{i=1}^{n_2} M_{B_i}}{5}} \ . \quad . \quad (10)$$

At last, if as value of absolute magnitude of QSO's subtracted in an exponent to take M_{B_0}– average value on all general totality of QSO's, last expression appears equal:

$$H_1 = H_0 \cdot 10^{\frac{M_{B_1} - M_{B_0}}{5}} \quad , \tag{11}$$

where M_{B_1}– average value of absolute magnitude of QSO's in a considered direction.

Direct averaging of absolute magnitude of QSO's on all n_1=168941 [1] gives to objects of the catalogue value M_{B_1}= –23,73 which is close enough to boundary value M_B = –23,25, conditionally separating actually quasars from active galactic kernels, i.e.:

$$H_1 = 71 \cdot 10^{\frac{M_{B_1} - 23,73}{5}} \quad , \quad km \cdot s^{-1} \cdot Mpc^{-1}. \tag{12}$$

Thus, according to the developed method of an estimation of anisotropy of speed of expansion of the Universe of its direction on celestial sphere break into sample windows, for each window estimate average absolute luminosity of quasars then under the formula (12) count value of Hubble's parameter in a corresponding direction.

It is necessary to stipulate, that the offered method of calculation of anisotropy of distribution of Hubble's parameter on celestial sphere allows to establish, thus, in how many time value of Hubble's parameter in any direction differs from considered as cataloguers most precisely established. Possible revision eventually valid value H_0 of Hubble's constant, as well as possible change of value M_{B_0} owing to recalculation of absolute luminosity of quasars with new value of Hubble's constant, or owing to catalogue addition with new objects, will demand updating of values entering in (12) constants.

2. Preparation of data taking into account minimization of errors of a method

Equality to zero of the left part of the equation (7) at an invariance of values of Hubble's parameter in two various directions of celestial sphere, i.e. for two compared files of quasars, is realised at an assumption that these files are the realizations which statistical characteristics a little differ from characteristics of a general totality of quasars. Such assumption is a condition of that according to formulas (11) - (12) various average values of absolute magnitude of quasars appear corresponding to two differing owing to anisotropy of expansion of the Universe to values of Hubble's parameter.

Thus the approached character of the accepted assumption can lead to that analyzed distinctions between average values of absolute magnitude of quasars can appear occurring casually fluctuations of function of distribution of density of quasars on absolute luminosity in various directions of celestial sphere. For reduction of influence of casual fluctuations it is necessary, that for a difference of absolute values of the luminosity, entering into an exponent (11) − (12), were essentially smaller corresponding estimations of standard deviations of average values of absolute luminosity in directions of celestial sphere. This condition can be satisfied by a special selection of distribution of available data file about quasars on windows of sample with various directions on celestial sphere.

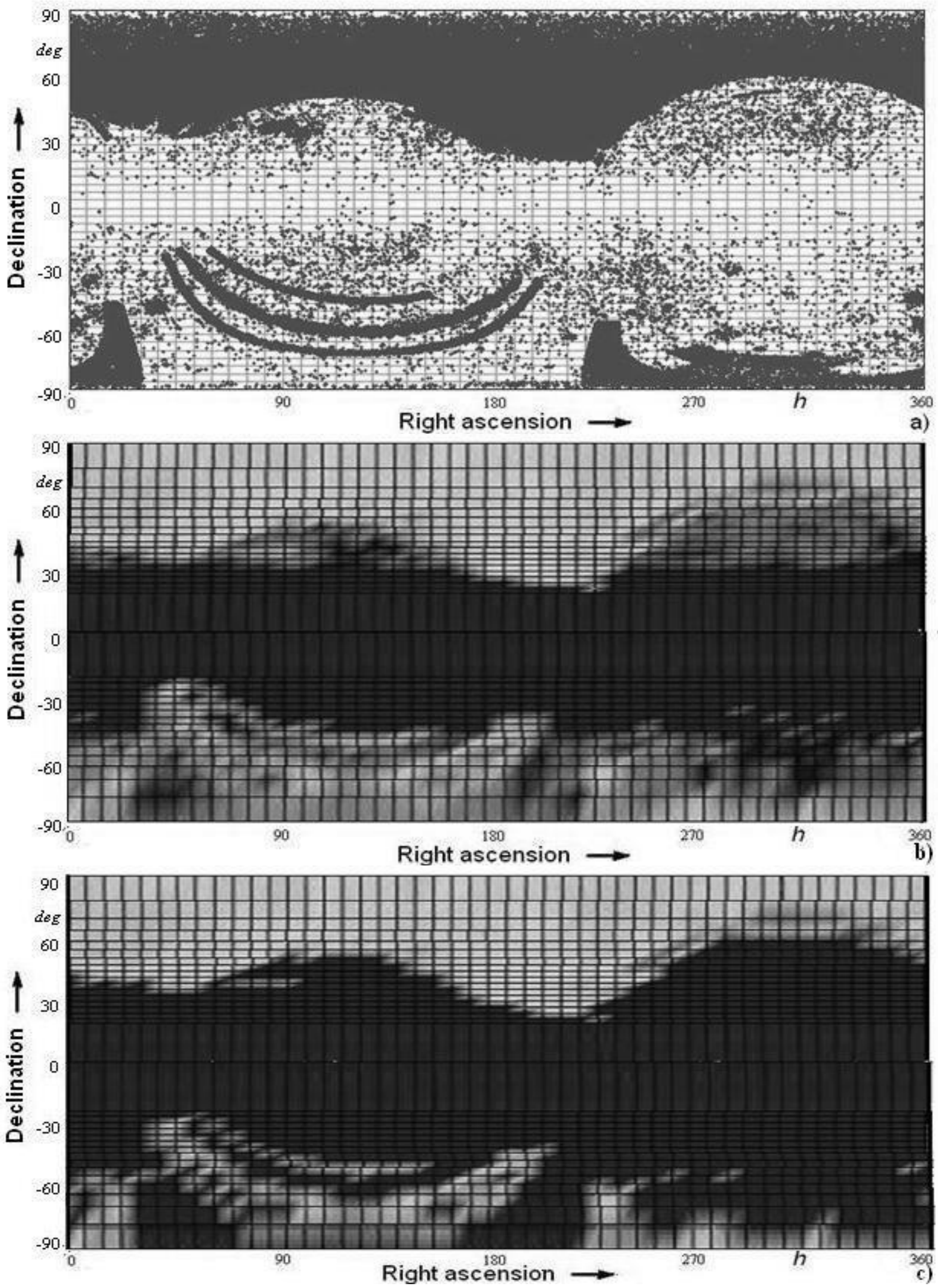

Fig. 2: Density of placing of QSO's on sectors of heavenly sphere before truncation of data (a; b) and after it (c)

In figure 2 (a) distribution of QSO's on celestial sphere according to the catalogue [3] in galactic coordinates is presented. The greatest density of quasars is reached in vicinities of galactic northern pole, and also on small sites on the area in vicinities of galactic South Pole.

In vicinities of galactic equator observant data practically are absent owing to influence of a luminosity by an edge of Our galaxy. Considering requirements to a standard deviation of average luminosity of quasars, and also a number of other features, for formation of statistical windows the galactic celestial sphere has been divided on coordinate of a right ascension into 48 equal pieces with step to 7,5 angular degrees. The coordinate of galactic declination also has been divided into 48 pieces. Variability of a step of such division provided splitting of a surface of heavenly sphere by a grid of the areas from 48×48 sectors equal on the area. Thus the step of a grid of sectors essentially surpasses a characteristic step of heterogeneity filaments and the voids forming large-scale structure of the Universe in a range of intergalactic distances. This structure, according to a number of assumptions, can be extended also to heterogeneity of superficial density of distribution QSO's on celestial sphere.

On a figure 2 (b) by various shades of grey color distribution on heavenly sphere of estimations of a standard deviation of average luminosity of quasars on a grid of sectors is shown.. Thus to the big saturation of grey color there correspond the big standard deviations. Black color notes the areas for which an estimation it was not made in view of absence of observant data. The analysis (12) shows, that at possible changes of Hubble's parameter because of imbalance of expansion of the Universe in limits ±10 $km \cdot s^{-1} \cdot Mpc^{-1}$ as it follows from sources [1, 2], corresponding increments of average absolute luminosity of quasars can reach ±0,4 magnitude.. Thus it is necessary to limit decision-making area to sectors in which estimations of a standard deviation of average luminosity do not surpass magnitude 0,10 … 0,15. In figure 2 (c) sectors in which limits this requirement is fulfilled are kept.

It is necessary to notice also, that essential distinctions between statistical characteristics of statistical databases of QSO's on sectors of celestial sphere, and also similar characteristics of a general totality of quasars can be caused also features of supervision of the QSO's having regular character which results have been taken as a principle the given catalogues. Considered features are that, that increase in number of the quasars falling to any window of sample, in comparison with their number in other windows, is, as a rule, the caused application of sensitivity of equipment. It allows to observed the quasars being on considerable removal from the observer and characterized by the big red shift z, reaching values 3 … 6.

Thus on the big removals it was possible to fix only the brightest quasars owing to what estimations of average value of absolute luminosity in sample windows appeared correlating with number of quasars in these windows. For reduction of influence of this correlation by a difference of estimations of average luminosity it is necessary to enter corresponding amendments into estimations of average luminosity. On a figure 3 (a) dependence of estimations of average luminosity of quasars on their number in the windows of sample represented in drawing 2 (c) is illustrated. Drawing contains also describing this dependences a square-law curve of regression which has been used for amendment calculation.

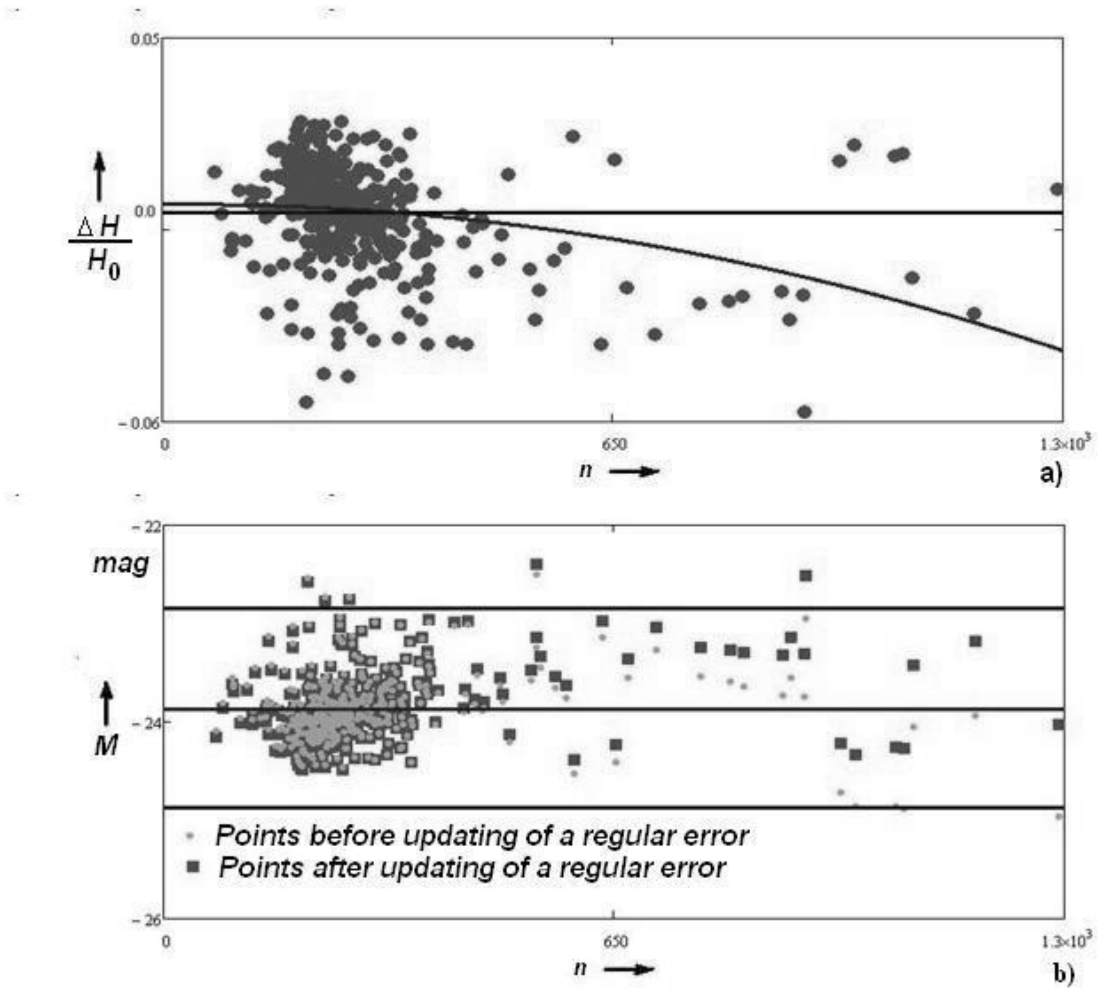

Fig. 3: Illustration of dependence of average values of luminosity of quasars from quantity of observable quasars (a) and results of corrective action (b) to luminosity

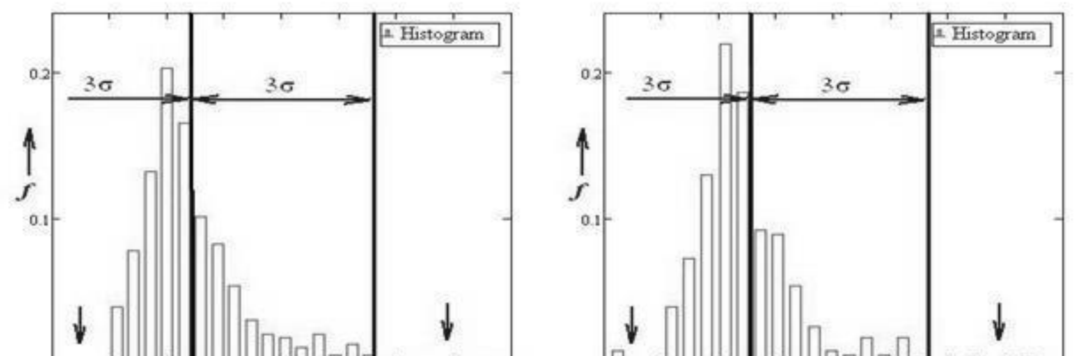

Fig. 4: Communication of absolute luminosity of quasars with number of measurements (a), correction and sample restriction (b)

The regression equation is picked up so that the luminosity of quasars corresponding to its average estimation on all general totality in absolute magnitude −23,73 was not subject to correction by corrective action.

On a figure3 (b) in the form of circles points not corrected, and in the form of rectangles - points of the corrected dependence are represented. Besides, in drawing confidential intervals of average luminosity in absolute magnitude −23,73 removed from it on distance in one standard deviation are represented. Corrective action to average luminosity corresponds also to correction of estimations of Hubble's parameter. On a figure 4 (a) the histogram of distribution of estimations of Hubble's parameter before correction, and in drawing 4 (b) - after it is represented. Correction has led to reduction of number of emissions of estimations for limits of a confidential interval in width in three standard deviations at the expense of redistribution of estimations in a confidential interval.

Thus also it has appeared, that emissions are characteristic, basically, for sectors of the celestial sphere, located in a southern galactic hemisphere. Therefore for the further analysis northern hemisphere sectors have been kept only on which restrictions according to an standard deviations of average luminosity did not extend, estimations of which average luminosity have been corrected according to drawing 3 (b), and also estimations of Hubble's parameter in which have not exceeded three standard deviations from level of a constant of Hubble.

It is necessary to notice also, that equality to zero of the right part of expression (7) is carried out at an assumption that measurements of relative magnitude of quasars B_i, and also their red shift z_i are executed with very small errors.

As a result of the preparation of sample described here for the further analysis the sectors of celestial sphere closing in aggregate about 23 % of the area of all celestial sphere have been kept.

3. The analysis of anisotropy of distribution of Hubble's parameter on celestial sphere

Procedure of reception of distribution of estimations of Hubble's parameter on celestial sphere is illustrated by a figure 5. From the sectors of northern galactic hemisphere of the celestial sphere kept for the further analysis, represented on a figure 2 (c), ring belts have been generated. Along which average values of luminosity for each of sectors have been estimated. Typical distribution of estimations on one of ring belts is represented on a figure 5 (a). The figure contains a population mean estimation (a line 1), borders of a confidential interval in one standard deviation (lines 2, 3), and also a straight line (a line 4) luminosity of the quasars, corresponding to its average estimation on all general totality in magnitude −23,73.

The figure 5 (c) illustrates an error of an estimation of Hubble's parameter. Besides a straight line 5 and a curve 9 on it results of calculations under the formula (12) in the form of an estimation of Hubble's parameter on sectors of a ring corbel (a line 10), and also the top and bottom borders of confidential intervals (are put a line 11, 12) by width in one standard deviation.

The figure 6 contains Fourier-decomposition of distribution a resultant of function of luminosity along family of ring corbels of northern galactic hemisphere for which statistically significant estimations of average luminosity have been received.

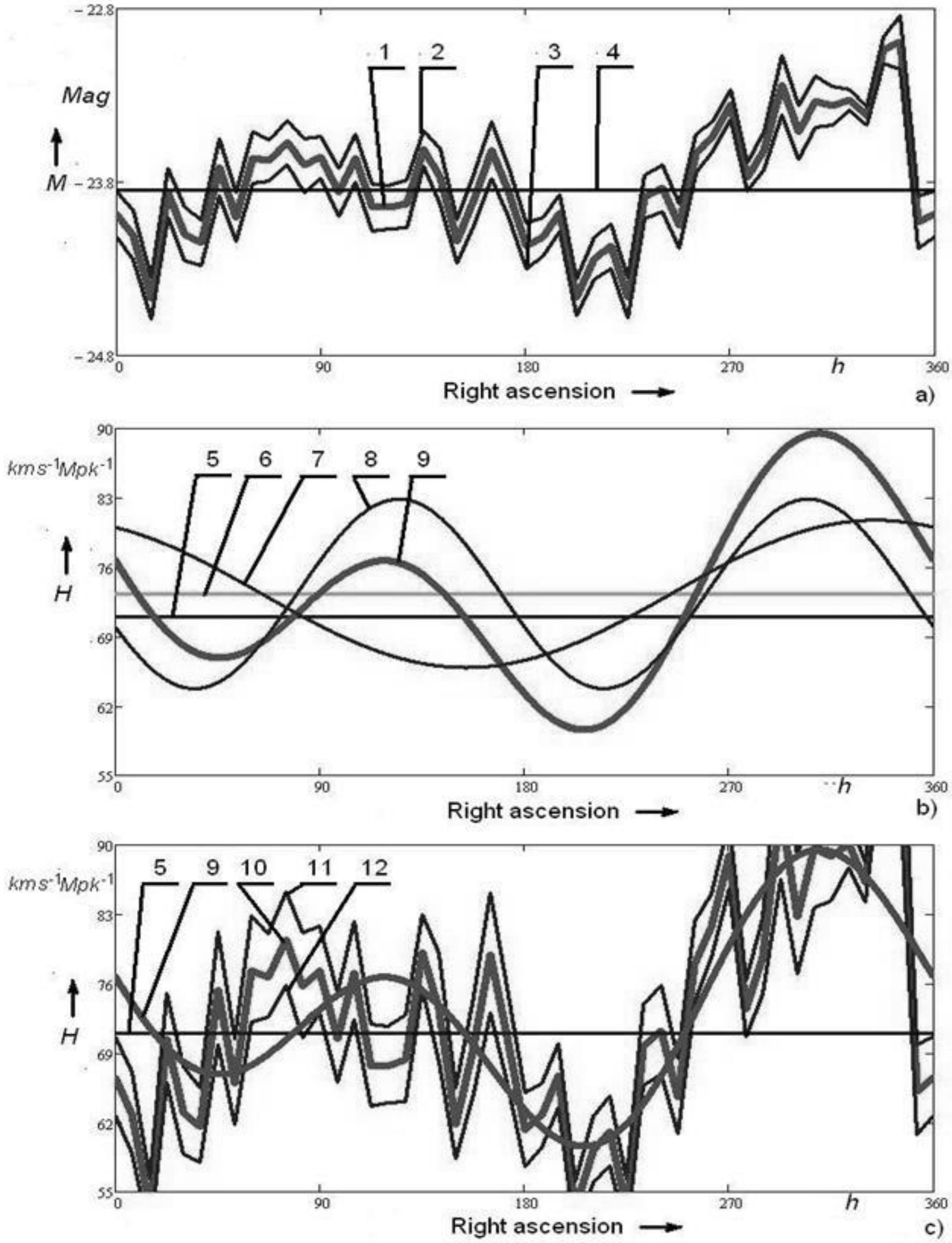

Fig. 5: Sequence of calculation of distribution of estimations of Hubble's parameter on a direct ascension of a galaxy for a range of declinations 66,4 … 61,0 ^{O}.

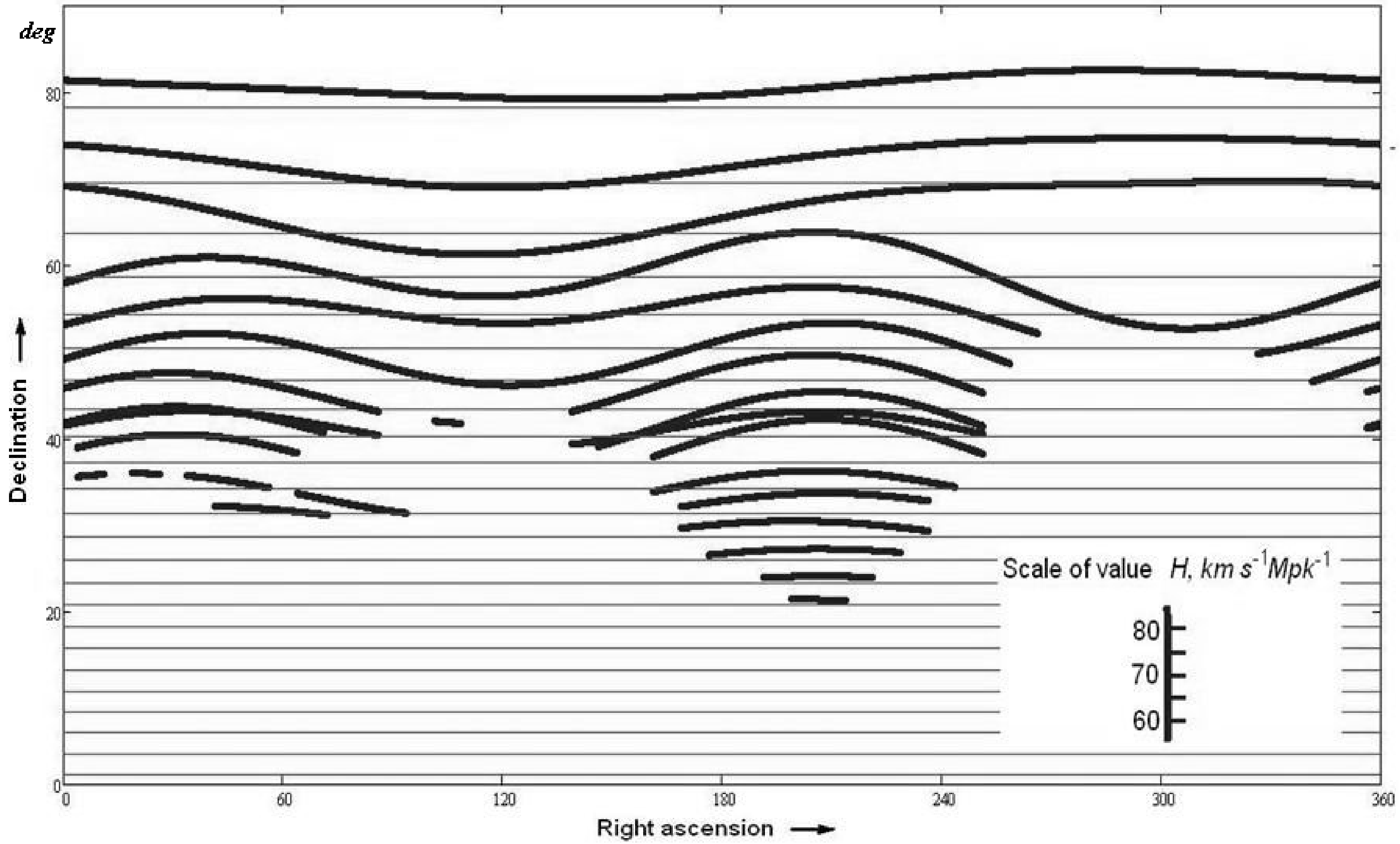

Fig. 5: Distribution of estimations of Hubble's parameter on northern galactic hemisphere

Conclusion

If to admit, that statistical properties of luminosity of QSO's do not depend on their direction on celestial sphere in relation to the ground observer the difference between two average values of luminosity of QSO's, allows to estimate the relation of values of Hubble's parameter, one of which characterizes a corresponding direction, and another is valid value of a Hubble's constant who has been put in pawn in catalogue calculation of absolute luminosity of QSO's. Thus one of values of average luminosity pays off for sample of QSO's in a direction of heavenly sphere, and another - for all their known set.

Available catalogue data allow to cover with statistical windows about 23 % of heavenly sphere near to northern pole of our Galaxy with number of windows 250 … 1100 depending on a way of splitting, in each of which the estimation of Hubble's parameter is realized.

Distribution of estimations of Hubble's parameter along a galactic longitude contains two harmonious making which amplitudes surpass corresponding average quadratic deviations of an average. Total amplitude of estimations anisotropy is equal 4… 8 (in a maximum to 12) $km \cdot s^{-1} \cdot Mpc^{-1}$.

The developed method can be used also in the field of the red displacement corresponding to elements of large-scale structure of the Universe.

The developed method can be used at discrimination astro- and the photometric data of QSO's on red shift for reception of estimations of anisotropy of parameters of acceleration of the Universe, and also parameters of density Ω_M и Ω_Λ.

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The century of tensor geometrical conception of gravitation as a foundation for general relativity.

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This article is devoted to the centenary of the tensor geometrical conception of gravitation. It analyzes the developing by Einstein (partly with M.Grossmann) of this conception that is a core of the general theory of relativity. Einstein's equivalence principle and Minkowski's four-dimensional geometrical formulation of the special theory of relativity are shown to be the two main prerequisites of the conception. It is also considered how the construction of the scalar theories of gravitation and their discussion led Einstein to the first preliminary formulation of this conception in the summer of 1912 and then, in final form stated by Einstein and M.Grossmann, in the spring of 1913. The methodological tools used by Einstein in this case are described in the conclusion.

1. Introduction

Contemporary physics of gravitational interaction, relativist astrophysics and cosmology are based on general relativity theory (GRT) accomplished in November of 1915. The foundation of this theory is tensor-geometrical concept of gravitation (TGCG) developed by A. Einstein in cooperation with M. Grossman and published at the latest in June of 1913. These days just a hundred of years have passed since this event.

The essence of the concept is that gravity potential describing gravitational field is symmetric 10-component tensor of the 2nd rank that completely coincides with metric tensor of four-dimensional Riemann (or more exactly, pseudo-Riemann) space-time. It clears the way to very uncommon understanding of gravitation as geometry or more exactly as space-time curvature. TGCG was highly appraised by leading theorists. It was exactly and emotionally noted by the authors of the first GRT monographs, recognized masterpieces or relativist classics , H.Weyl and W.Pauli. H.Weyl (1923) noted describing Einstein's way of thought: "The arguments mentioned above allow to clearly understand the physical meaning of Einstein's fusion of metric geometry ("Massgeometrie") and gravitation, the two realms of knowledge which have hitherto been developed fully independently of one another; this synthesis may be indicated by the scheme:

Pythagoras Newton
Einstein……» [1.P.225]

Pauli gave his opinion more briefly but equally expressively: "This fusion of two previously quite disconnected subjects – metric and gravitation – must be considered as the most beautiful achievement of the general theory of relativity" [2, p.708].

As a result basic – geometric – framework of the theory was created in 1913 but the authors failed to derive correct generally covariant equations of gravitational field although they closely came to ascertainment of these equations. Main steps of the way to TGCG and some procedural means used herein are discussed below.

2. First statement of TGCG

The paper of Einstein and Grossman where the theory was stated called "Entwurf einer verallgemeinerten Relativitätstheorie und einer Theorie der Gravitation" ("Outline of a generalized theory of relativity and of a theory of gravitation") was submitted to print at the latest on May 28, 1913 and issued in June [3].

Equivalence principle (EP) and its application to static fields have already lead to variability of light speed in space and when using four-dimensional Minkowski world it has lead to generalization of special relativist metric:

$$ds^2 = c(x,y,z)^2 dt^2 - dx^2 - dy^2 - dz^2$$

The following step on assumption of arbitrary transformations ("generalized relativity") was transition to Riemannian metric:

$$ds^2 = g_{ik} dx_i dx_k.$$

Then equations of particle motion in gravitational field may be searched in the form previously found for relativist mechanics by M.Planck and H. Minkowski:

$$\delta \int ds = 0$$

And then the components of metric tensor had meaning of gravity potential. This actually meant unification of gravitation and geometry that gave an opportunity to derive generally covariant equations characterizing influence of gravitation on every material process. In Einstein's and Grossman's paper this "synthesis" (Weyl) or "fusion" (Pauli) is stated in the following way: "…We thus arrive at the view *that in the general case the gravitational field is characterized by ten space-time functions*

$$\begin{matrix} g_{11} & g_{12} & g_{13} & g_{14} \\ g_{21} & g_{22} & g_{23} & g_{24} \\ g_{31} & g_{32} & g_{33} & g_{34} \\ g_{41} & g_{42} & g_{43} & g_{44} \end{matrix} \qquad g_{\mu\gamma} = g_{\gamma\mu}$$

wich in the case of the customary theory of relativity reduce to

$$\begin{matrix} -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & +c^2, \end{matrix}$$

where *c* denotes a constant" that is variable in static gravitational field, as Einstein added, and g_{44} = c $(x,y,z)^2$. And further: "From this one sees that, for given dx_1, dx_2, dx_3, dx_4 , the natural distance that corresponds to this differentials can be determined only if one knows the quantities $g_{\mu\gamma}$ that determine the gravitational field. This can also be expressed in the following way: the gravitational field influences the measuring bodies and clocks in a determinate manner". [3, p.155,157].

The use of Riemannian geometry also directly suggested how to build equations of gravitational field based on generally covariant tensors of curvature.

3. *Two main prerequisites of TGCG (1907)* [4]

Both prerequisites were created almost at the same time in the end of 1907. First of all, it concerns equivalence principle (EP) – “complete physical equivalence of gravitational field (uniform field – V.V.) and respective reference frame acceleration” postulated by Einstein in attempt to spread the theory of relativity on gravitation [5, p.476]. In 1911 he called this principle “a hypothesis on physical nature of gravitational field” and the name of “equivalence principle” has established since 1912.

Saying on the second prerequisite we keep in mind four-dimensional theoretical invariant formulation of the special relativity theory (SRT) presented by Minkowski in public lecture at the conference of Goettingen Mathematical Society for the first time on November 5, 1907 [6] (not in his famous lecture in Cologne in September of 1908 [7] as quite often considered).

Actually the EP has already contained an idea of kinematization of gravitation. Kinematization combined with SRT four-dimensional concept meant, in essence, geometrization of gravitational field. However it took over 5 years to realize these prerequisites and to arrive at TGCG. This way is described in the following section. In this section we consider some prescience of both prerequisites by H.Hertz (1857 – 1894) another genius at the turn of classic and non-classic physics died at the age of A.S. Pushkin. The case is equality of inert and gravitational mass as empiric base of the EP, which fundamental character was noted by Hertz, as well as the possibility of presentation of the motion law of mechanical system as a motion along a geodesic line in multidimensional configurational Riemannian space, i.e. in the form:

$$\delta \int \sqrt{\sum a_{ik}\, dx_i\, dx_k} = 0$$

That is how Hertz said in his lecture “On the structure of matter” (1884) about the importance of the fact of mass equality that required explanation: “In textbooks, too, it is usually, put forth as some-what obvious, although not particularly emphasized, that weight, which is proportional to the mass of a body, is fully independent of the material of which is consists. And yet we still have in essence two main characteristics of the material before us, that could be regarded as completely independent of each other, and that through experience, and only through experience, prove to be fully equal. This agreement is much more than a marvelous mystery to be wondered at: it demands explanation” (cited by [8,p.175]). As we know Einstein had never cited Hertz neither in view of the EP nor considering geodesic principle.

4. *Decisive steps on the way to TGCG and involvement of M.Grossman (summer of 1912)*

During 1912 scalar theories of gravitation (i.e. theories taking into account the EP and relativism with scalar potential) were intensively developed. There was a number of theories: M.Abraham (2 variants), G.Nordström – also two theories, G.Mie and also Einstein’s theory though he initially tried to built the theory of static non-uniform fields (also 2 variants). During construction and discussion of these theories, which we will not consider here , some important ideas have formed that contributed to the development of TGCG. Let us list them.

1. Variability of light speed in gravitational fields and necessity of going beyond SRT Einstein considered as a need for expansion of relativism rather than its failure (unlike Abraham) but a character of expansion was not clear to him.

2. Earlier he had understood that coordinates lost direct physical sense in gravitational fields. This lead to the use of four-dimensional concept of Minkowski that he had previously underestimated, in which metric played a key role ($ds^2 = c^2dt^2 - dx^2 - dy^2$- dz^2).

3. Einstein had still considered (in summer of 1912) that relativist program would not fail in the presence of gravitation though SRT remained valid only locally and non-Euclidean geometry might be realized thereat.

4. Ideas of general relativity associated with Mach's denial of space-time absolutes remain also important.

5. Certain exhaustiveness of vector and scalar approaches.

6. Idea of nonlinearity of gravitational field equations.

Schematically the development of the problem of gravitation from Lorentz-covariant theories and the principle of equivalence to the TGCG and GRT is given in the author's article [9, p.1287]. This scheme notes the scalar theory of gravitation and the fundamental premises of the TGCG created in the course of their discussion, as well as some important methodological tools used by Einstein. These include the so-called methodological principles of physics (such as principles of simmetry, conservation, correspondence and others) and two "empirical laws of epistemology" (expression of Wigner): unreasonable effectiveness of mathematics and analytical mechanics in physics [10, 11].

As early as in spring of 1912 Einstein appeared to find the key idea that finally becomes decisive in transition to TGCG. It consisted in the expansion of variational formulation of particle motion law in SRT to static fields (initially) and then to arbitrary fields. It was in Prague period when Einstein (as he wrote in his memoirs) understood that it might require Gaussian theory of surfaces and, respectively, new generalization of geometry [12]. Meanwhile (in the beginning of August) he left for Zurich where M.Grossman, his studentship friend, mathematician and geometrician, came to help.

Let us cite several pieces of Einstein's papers related to spring and summer of 1912 to confirm the above said and also fragments of his papers,letters and memoirs. E.g., in "Addition" to the paper gone to the press on March, 1912 at proofreading he has already found the decisive link that lead him to TGCG in future: "For a material that moves in a static gravitational field without being acted upon by external forces we have accordingly

$$\delta \int H dr = 0, \text{ или } \quad \delta \int \sqrt{c^2dt^2 - dx^2} \text{ - } dy^2 \text{-} dz^2 = 0$$

This shows too –as has been shown for the ordinary theory of relativity by Planck – that the equations of analytical mechanics have a significance reaching far beyond Newtonian mechanics. The Hamiltonian equation, which was the last one written down, gives an idea about how equations of motion of material point in a dynamic gravitational field are constructed." [13,p.120].

Nevertheless in the beginning of June of 1912 Einstein, despite being close to the right decision, still spent in doubts and reflections. In the paper gone to press on July, 4 he wrote:

"…The equivalence principle opens up for us interesting perspective according to which the equations of a relativity theory that would also include gravitation may also be invariant with respect to acceleration (and rotation) transformations. In any case, the road to this goal seems to be a quite difficult one. One can already see from the previously treated, highly specialized case of the gravitation of rest masses, that the space-time coordinates will lose their simple physical meaning, and it is not yet possible to foretell the form that the general space-time transformation equations can have. I would like to ask all of my colleagues to have a try at this important problem!" [14,p.133].

In lecture given by Einstein at Kyoto University on December 14, 1922 and written down by J. Ishiwara he told about situation emerged by August of 1912 when he left from Prague for Zurich: " If all system are equivalent, then Euclidean geometry cannot hold in all of them. To throw out geometry and keep laws is equivalent to describing thoughts without words. We must search for words before we can express thoughts. What must we search at this point? This problem remained insoluble to me until 1912, when I suddenly realized that Gauss's theory surfaces holds the key for unlocking this mistery. I realized that Gauss's surface coordinates had a profound significance. However, I did not know at that time that Riemann had studied the foundations of geometry in even more profound way. I suddenly remembered that Gauss's theory was contained in the geometry course given by Geiser when I was a student…I realized that the foundations of geometry have physical significance. My dear friend the mathematician Grossmann was there when I returned from Prague to Zürich. From him I learned for the first about Ricci and later about Riemann. So I asked my friend whether my problem could be solved by Riemann's theory, namely, whether the invariants of the line element could completely determine the qualities I had been looking for"(cited by [15,p.211-212]).

Einstein's letter to L.Hopf dated August 16, 1912 indicates that he became confident in the chosen path partially outlined in Prague and confirmed by Grossman at latest in the first half of August:"The work on gravitation is going splendidly. Unless I am completely wrong, I have now found the most general equations" [16, p.321].The work under new theory of gravitation had appeared to be greatly advanced by October of 1912 and Einstein highly appreciated constructive power of mathematics.He wrote to A.Sommerfeld on October,29: "I am working exclusively on the gravitation problem and believe that can overcome all difficulties with the help of a mathematician friend of mine here. But one thing is certain: never before in my life have I troubled myself over anything so much, and I have gained enormous respect for mathematics, whose more subtle part I considered until now, in my ignorance, as pure luxury! Compared with this problem, the original theory of relativity is child's play." [16, p.324].

The same follows from his October letter to astronomer E. Freundlich in which he said that theory of gravitation and fundamentals of TGCG have been near completion but the equations of gravitational field have not been obtained yet: "My theoretical studies are progressing briskly after undescribably painstaking research, so that chances are good that equations for general dynamics of gravitation will be set up soon. : The beauty of the thing one can keep clear of arbitrary assumptions, so that there is nothing to be "patched up"; instead, the whole thing will be either true or false" [16,p.323].

6. "*The second birth" of TGCG (spring and June of 1913)*

Decisive steps to TGCG have appeared to be done after torturing doubts and "intolerable search". Why did it take almost another year to decide to publish new theory of gravitation? We can find answer to this question in the text of "Entwurf" published at the latest in the beginning of June of 1913. After the statement of TGCG and its application to obtain motion equations in gravitational fields, the authors passed on to gravitational field equations. The requirement of general covariance suggested the correct solution, but the authors had to reject it because they could not harmonize generally covariant equations of gravitation with such important general physical and methodological directives as principles of correspondence, energy-momentum conservation and casuality. As a result they went over to non-generally covariant field equations that delayed the appearance of general relativity theory for two and a half years.

"The second birth" of TGCG is fallen on the end of March of 1913. Einstein wrote in his letter to Elsa Löwenthal on March, 23: "I worked more strenuously during the past half year than ever before in my life, and I finally solved the problem a few weeks ago. In is a bold extension of the theory of relativity, together with a theory of gravitation" [16.P.331]. In the end of May when "Entwurf" was prepared to print he expressively and exactly described his choice in the letter to P.Ehrenfest. To explain his long silence he noted: "…But my excuse is the frankly superhuman effort I have invested in the gravitation problem. I am now deeply convinced that I have gotten the thing right, and also, of course, that a murmur of indignation will spread through the ranks of our colleagues when the paper appears, which will take place in a few weeks. Naturally, I will then send you a reprint immediately. The conviction to which I slowly forced my way through is that *privileged coordinate systems do not exist at all*. However, I succeeded only partly in working my way through to this position formally as well" [16. P.334-335].

Einstein's letter to E.Mach dated June 25, 1913 may be considered as a result of this way to GRT: "You have probably received a few days ago my new paper on relativity and gravitation, which I now finally completed after unceasing toil and tormenting doubts. Next year, during the solar eclipse, we shall learn whether light rays are deflected be the sun, or in other words, whether the underlying fundamental assumption of the equivalence of the acceleration of the reference system, on the one hand, and the gravitational field, on the other hand, is really correct.

If yes, then – in spite of Planck's unjustified criticism – your brilliant investigations on the foundations of mechanics will have received a splendid confirmation …" [16. P.346]. The letter contains "unceasing toil and tormenting doubts" completed by publication of "Entwurf", which, in acknowledgment, was immediately sent to the person whom Einstein concerned his main forerunner, and hopes of near experimental confirmation of the new theory, and a special piety he felt to Mach.

Concerning the reasons for transition to TGCG we several times noted them before. Let us repeat the corresponding chain of reasoning: relying on four-dimensional Minkowski geometry – equations of particle motion in variation form here leads to its expression via metric ds^2 $(\delta \int ds = 0) -$ equivalence principle actually leads to metric beyond SRT – according to ideas of general relativity this gives Riemannian metric: $ds^2 = g_{ik} dx_i dx_k$, where g_{ik} are components of both metric tensor and gravity potential. We should add that our reconstruction mainly agree with the picture of this period of GRT formation described in one of the best Einstein's biographies by A.Pais [17].

7. *"Einstein's arc" and factors that determine it in case of TGCG*

GRT and its conceptual core, TGCG, as we saw, were not derived from any general propositions or equations. They have been constructed. The process of construction lasted for 6 years (for TGCG) and 8 years (for final formulation of GRT) was a *nonlogical way* from Newtonian theory of gravitation, SRT and empirical facts (first of all, equality of inert and gravitational mass) to TGCG and later on – to GRT.

It will be recalled that Einstein wrote about this "nonlogical way" in his letter to M.Solovin on May 7, 1952 where he showed it as a curve from empirism to axiom system of the constructed theory [18]. Let us call this curve as "Einstein's arc" and note several factors that determine it [10,11].

They may primarily include the two *"empirical laws of epistemology"*: *"unreasonable effectiveness of mathematics" and "unreasonable effectiveness of analytical mechanics"*, in particular, Lagrangians and variation principles related to them. We also saw that both these "laws" were very important for Einstein.

The efficiency of mathematics was enthusiastically described by Einstein in his letter to Sommerfeld cited above (in October of 1912). This factor certainly played a key role: this includes serious adoption of Minkowski four-dimensional world, Gaussian theory of surfaces, Grossman's role and Riemannian geometry.

Analytical mechanics also appeared useful: variation formulation of particle motion equation discovered by M.Planck and Minkowski for SRT was initially extended to static and then to arbitrary gravitational fields that as we saw played paramount role in the genesis of TGCG.

Einstein also thoroughly used such principles (known as *methodological principles of physics* now) as symmetry, conservation, correspondence, causality, observability, simplicity and unity of knowledge. The concept of methodological principles of physics was developed in detail under the leadership of N.F.Ovchinnikov [11].

Almost each of these principles was directly or implicitly used by Einstein. In particular, the principle of symmetry meant Einstein's firm confidence in the correctness of relativist program that allowed expansion of special relativist symmetry. Conservation principle (namely, energy-momentum) was also important and particularly used for the search of equations of gravitational field. Correspondence principle required to seek for such generalizations of theory that would give Newtonian and special relativist approximations in extreme case of weak fields. Observability principle was actually used by Einstein to discover the equivalence principle. The principle of classic causality also underlay Einstein's construct, etc.

Nevertheless, based on some of these principles "Entwurf" authors arrived at TGCG refused of general covariant approach to the equations of gravitational field. It seemed to them (first of all, to Einstein) that general covariant equations came into conflict with the principles of correspondence, energy-momentum conservation and causality. It took about two and a half years to overcome these objections and come to great equations (in November of 1915)

$$R_{ik} - \frac{1}{2} g_{ik} R = -\chi T_{ik}.$$

But this is another history concerning completion of GRT, 100-year anniversary of which will be celebrated in two years (see [8, 11]).

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THE TESTS IN THE KALUZA-KLEIN GRAVITATIONAL THEORY UNDER PRESENCE OF THE ELECTROMAGNETIC AND REACTIVE POTENTIALS

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Abstract

The tests to check the possible existence of the additional dimensions to the space-time on the base of the electromagnetic and reactive potentials in generalised Gross- Perri metrics and Kerr metrics are offered in compliance with cylindricity on a fifth coordinate; the tests on the light deflection in relation to the massive bodies, on time delay of signals, on deflection of the massive particles from radial movement are considered .

The Keywords: variable mass, fifth coordinate, five velocity, reactive and electromagnetic potentials, aiming parameter, the light deviation angle, GTR, dark energy.

1.Introduction

As is noted in work [1] in principle the five-demensional theory is capable to explain the effects, not covered by the general theory of relativity (GTR) and must be tested by observations. Further to work [1], [2], [3] the analysis of the light deflection with the massive body is performed on base of the generalised Gross- Perri and Kerr metrics [4]. In works [5],[6] the physical and technological problems of the observant tests of gravitational and cosmological theories in Solar system in 21 century are considered, in particular, an unification of Kaluza-Klein theory and investigations on model of the dark energy is mentioned. The Present investigation is directed to one of the decisions of the similar problem. Having continue works [1] -[3], we consider a problem of testing on the base of generalization of the Gross- Perri and Kerr metrics including the Y. Rogozhin reactive field [7], [8], that also has property of the antigravitation and introduced for description of the dark energy (in literature she is postulated as a particle-like excitement of the scalar field- the quintessence with a big length of the Compton wave, with linear size l≈ 85 mkm [6], we shall name her as "T-particle"; also taking into account the electromagnetic potentials; the influence of massive body rotation, parameter of the magnetic monopole and dark energy. The tests on deflection of the light relatively to the massive bodies , on time delay of signals, on deflection of the radial movement of the massive particles in consequence of disturbance from the scalar reactive potential are considered in this work. The presence curvatures of the space- time in 4D influences upon increase time of " run" of the photon between any two chosen points in comparison with time of "run" for event, when the curvature of surface is zero. In our event components of curvatures in 4 D not equal zero also participate in 5D. In article [4] is noted that " the theory, where additional dimensions are added to the GTR, becomes much rich with the physical consequences, but also more mathematically complex.."

2. Generalization of the Gross-Perri metrics and Kerr metrics

It is known, the Kaluza-Klein theory unifies gravitation and electromagnetism [9] , [10]. So, when we speak about inclusion of the Y.ROGOZHIN reactive field, it is impossible to forget about presence of the charged particles. In Kaluza-Klein theory the fifth space dimension is rolled into a circle, which radius is too small to be experimental accessible. In the generalised five-dimensional metrics besides potential of the charged particle CA^{e}_{ν} , $C=(16\pi G_k)^{1/2}/c$, G_k–

gravitational constant in D5, c-velocity of light, is entered, for instance, in the event of particles emission, the reactive potential $A_\nu=(\mu^* V_{ucm\nu}+\mu u_\nu)$ type, $\mu= m/M_0$, $\mu^*=m^*/M_0$, $\mu^* V_{ucm\nu}$ - pulse of the droppable particles [4], being operator of the "*T- particle*" model configuration; A_ν is defined from the induced tensor of energy-pulse in the Kaluza-Klein theory in the event of neutral particles [6]:

$$8\pi T^\sigma{}_\beta= \gamma^{\alpha\sigma}\{\Phi_{\alpha,\beta}-\gamma^{k\delta} (\gamma_{\delta\alpha,\beta}+\gamma_{\delta\beta,\alpha}-\gamma_{\alpha\beta,\delta})\, \Phi_k/2\}/ \Phi,\ \alpha,\beta,\sigma,\nu=0,1,2,3, \quad (1)$$

and equations of the "*T- particle*" state $p= - w\,\rho$; and in the event of the charged particles from the induced tensor of energy - pulse if the curvature tensor component $R_{5\alpha}$ is equal to zero [3]:

$$R_{5\alpha} =F^\lambda{}_{\alpha;\lambda} +3\Phi^{-1}\Phi^\lambda F_{\lambda\alpha} =0, \quad (2)$$

$$T^\delta{}_\beta =\gamma^{\alpha\delta}\{-\Phi^2(F_{\alpha\lambda}F^\lambda{}_\beta)/2 +\Phi_{\alpha;\beta}/\Phi\}, \quad \alpha,\beta,\delta,\lambda=0,1,2,3, \quad (3)$$

where $F^\lambda{}_\beta =\gamma^{\lambda\sigma}F_{\alpha\sigma}$, $\Phi_{\alpha;\beta}=\Phi_{\alpha,\beta}-\Gamma_{\alpha\beta}{}^\kappa\Phi_\kappa$, $F^\lambda{}_{\alpha;\lambda} =(F_\alpha{}^\lambda)_{,\lambda} -\Gamma_{\alpha\lambda}{}^m\Gamma_m{}^\lambda+\Gamma_{m\lambda}{}^\lambda\Gamma_\alpha{}^m$,

$F_{\alpha\lambda}-$ Maxwell tensor, $\Phi=1-\alpha/2r$, $\alpha/r-$ gravitational potential.

For example let consider generalization of the Gross- Perri metrics [4] ($\theta=\pi/2$):

$dS^2=(1-\alpha/r)^a(dx^0)^2- (1-\alpha/r)^{-(a+b)}(dr)^2-r^2 (1-\alpha/r)^{1-(a+b)}(d\psi)^2-(1-\alpha/r)^b\{dx^5+ A^*_\nu dx^\nu\}^2$, $(0\le x^5 <2\pi r_c)$ or

$$L = (1-\alpha/r)^a(dx^0 /dS)^2 - (1-\alpha/r)^{-(a+b)}(dr/dS)^2-r^2 (1-\alpha/r)^{1-(a+b)}(d\psi/dS)^2-(1-\alpha/r)^b\{(dx^5/dS) + A^*{}_\nu (dx^\nu/dS)\}^2 = \eta^*, \quad (4)$$

$\eta^*=1$ for a particle, $\eta^*=0$ for a photon.

Here $A^*_\nu =A_\nu +CA_\nu^e$, CA_ν^e electromagnetic potential, $a(b)-$constant parameter, r_c- the fifth dimension compactification radius, $r_c =2(\hbar G_k\alpha/c^3)^{1/2}=3.7.10^{-32}cm.$, $\alpha=q^2/4\pi\hbar c$, $q-$ test particle charge.

Enter the scalar function n, does not depend from the fifth coordinate

$$n= \Phi^2((dx^5/dS)+ A_\alpha^* (dx^\alpha /dS)), \quad \Phi^2= (1-\alpha/r)^b; \quad (5)$$

$n-$ value accepted in literature of Kaluza-Klein theory proportional to ratio of the electric charge to mass [3],

$$Cq/m=n/ (1+ n^2 \Phi^2)^{1/2}. \quad (6)$$

Consider hereinafter generalization of the Kerr metrics [4]:

$$dS^2=ds^2 -\Phi^2{}_1(dx^5-A_3Pd\psi)^2-\Phi^2{}_2(dx^5-(Q/a)\, d\psi)^2,\ A_\nu= A_3,$$

$$\Phi^2{}_1=\Delta/\Sigma,\ \Phi^2{}_2=a^2/\Sigma,\ \theta= \pi/2\ d\theta/dS=0, \quad (7)$$

$a-$ the central body rotation parameter, and $ds^2=(dx^0)^2- \Sigma (dr)^2 /\Delta$.

Then, in analogy with (4) we introduce the amount in metrics, consisting of reactive potential $(A_\nu A^\nu)^{1/2} = A_0$ and the monopole "magnetic" mass $N=m^*$ [4]; this implies $\Delta= r^2 -\alpha r-a^2 - A_0^2 r^4-N^2$, $\sum= r^2 - A_0^2 r^4- N^2$,

$P = a-a(A_0^2 r^4+ N^2)/(A_0^2 r^4+ N^2-a^2)$, $Q = r^2 -a^2- a^2(A_0^2 r^4 + N^2)/(A_0^2 r^4 + N^2- a^2)$;

$$L= (dx^0/ dS)^2- \Sigma (dr/ dS)^2 /\Delta- \Sigma (d \theta/ dS)^2 +\varepsilon\{(A^2_3 \Delta P^2+ Q^2\}/ \Sigma)\} (d\psi/ dS)^2+2 \varepsilon \{\{ A_3 P \Delta + a Q\}/ \Sigma\}(dx^5/ dS) (d\psi/ dS)+ \varepsilon \{\Delta + a^2\}/ \Sigma(dx^5/ dS)^2 = \eta^*, \varepsilon=\pm 1, \quad (8)$$

and the metrics tensor components are $\gamma_{00}=1$, $\gamma_{11}= -\sum/ \Delta$, $\gamma_{33}= - (A^2_3 \Delta P^2+ Q^2)/ \Sigma$, $\gamma_{55}= - (\Delta + a^2)/ \Sigma= \Phi^2$,

$\gamma_{35} = - (A_3 P \Delta + a Q)/ \Sigma$, $\gamma^{00}=1/ \gamma_{00}$, $\gamma^{11}= - (1/ \gamma_{11})$, $\gamma^{33}= \gamma_{55}/(\gamma_{33} \gamma_{55}- \gamma_{35}^2)$,

$\gamma^{35} = - \gamma_{35}(\gamma_{33} \gamma_{55}- \gamma_{35}^2)$, $\gamma^{55} = \gamma_{33}/(\gamma_{33} \gamma_{55}- \gamma_{35}^2)$.

If accept $n_1= \Phi^2_1 ((dx^5/dS) -A_3P(d\psi/dS))$, $n_2= \Phi^2_2((dx^5 /dS)-(Q/a)(d\psi/dS))$, then from (6) is possible to define the dependency dS from ds in D 4:

$$dS = ds/[1+ \Lambda^2 /(\Phi^2_1 \Phi^2_2)]^{1/2}; \quad \Lambda^2= n_1^2 \Phi^2_2+ n_2^2 \Phi^2_1. \quad (9)$$

The following first integrals from (8)

$$((\Delta + a^2)/ \Sigma (dx^5 /dS) + ((A_3 P \Delta + a Q)/ \Sigma)(d\psi/dS) = k, \quad (10)$$

$$((A^2_3 \Delta P^2+ Q^2)/ \Sigma) (d\psi/dS)+ ((A_3 P \Delta + a Q)/ \Sigma)(d\psi/dS) = h, \quad (11)$$

give possibility to express the functions n1, n2 through constants k and h. In analogy with (6) the expression $\mu_m/m= \Lambda/(1+ \Lambda^2 / \Phi^2_1 \Phi^2_2)^{1/2}$ (12)

is accepted as a ratio of the magnetic charge μ_m to mass m.

Thereby, parameter Λ become proportional to the value μ_m/m and parameters n1 ,n2 must satisfy this condition.

So, metrics (4), (7) at presence of the reactive power, electromagnetism and magnetic monopole allow to refine testing of the deflection of the light beams during passing near the central body, to specify time of the signal spreading in process of radiolocation from the Earth, the Mars, the Moon.

3. The light beam deviation in the generalised Gross- Perri metrics

We return to the Lagrangian functions (4); *for* $\{dx^5+ A^*_\nu dx^\nu\}^2 =0$ and $a=1$ instead of (4) we have a Schwarzschild metrics, for which the beam orbit will have "point of the rotation" [11] by maximal radius R_0, equal :

$$R^3_0 - (R_0- \alpha) b^{*2} =0. \quad R_0=2b^* \cos [\arccos((-3^{3/2} \alpha/2b^*))/3]/3^{1/2}, \quad (13)$$

where b^* - an aiming parameter. The angle of rotation is defined by the formula

$\Delta\psi=2\int_0^{1/Ro} du/(R^{-2}{}_o - \alpha R^{-3}{}_0 - u^2 + \alpha u^3)^{1/2}), u=1/R_0;$ (14)

Differentiating (14) on $\alpha/2$ and calculating result for $\alpha=0$, we shall get the angle of the light deflection for the Sun ($b^*=695510$ km, $R_0=695509$ km), $\delta\psi= 2\alpha G/b^* c^2 =1.75''$.

For equation of the light beam $dS^2=0$ from (2) we have: $£= (1-\alpha/r)^a (dx^0/d\lambda)^2-$

$-(1-\alpha/r)^{-(a+b)} (dr/d\lambda)^2 - r^2(1-\alpha/r)^{1-(a+b)} (d\psi/d\lambda)^2 - (1-\alpha/r)^b \{dx^5/d\lambda + +\overset{*}{A}_v (dx^v/d\lambda)\}^2=0,$ (15)

$\lambda-$ affine parameter; from (15) follows the first integrals:

$[(dx^0/d\lambda) - k\overset{*}{A}_0]/(1-\alpha/r)^a = l = const,$ (16)

$r^2(1-\alpha/r)^{1-a-b}(d\psi/d\lambda) + k\overset{*}{A}_3 = h = const,$ (17)

$(1-\alpha/r)^b \{dx^5/d\lambda + \overset{*}{A}_v(dx^v/d\lambda)\} = k = n = const;$ (18)

Considering (16), (18), we have for (15)

$£= (l+k\overset{*}{A}_0)^2/(1-\alpha/r)^a - \{(1-\alpha/r)^{-(a+b)} + 2k\overset{*}{A}_1\}(dr/d\lambda)^2 - r^2(1-\alpha/r)^{1-(a+b)}(d\psi/d\lambda)^2 - k^2/(1-\alpha/r)^b=0.$ (19)

Dividing (19) on $(d\psi/d\lambda)^2$ from (15) and considering $\overset{*}{A}_v \ll 1$, $\alpha/r \ll 1$ as a small size, we shall get for the Kaluza-Klein theory main expression:

$(dr/d\psi)^2 = r^4 F - r^3 p\,\alpha - r^2 + r\,\alpha + r^4 p_1\,\alpha\,(A_0+A_0^э) + r^5 p_2(A_3+A_3^э);$ (20)

Here $F=(l^2-k^2)/h^2$, $p= k^2(2-a-2b)/h^2 - l^2(2-2a-b)/h^2$, $p_1=2kl/h^2$, $p_2=2k(l^2-k^2)/h^3$, $h^2/l^2=b^*$, $A_0^э$ – a Coulomb potential of the field, $A_0^э=C\varphi u$; $A_3=\alpha f(u)$. The potential $\overset{*}{A}_1=A_1+A_1^э$ will be introduced into parameter k (18). The analysis of the additional deflection of the beam (its reduction) in D5 under $k=0$ and zero potential $\overset{*}{A}_v=0$ is organized in work [1] - [3]. Our problem is to get the algorithms for additional deflections under $k\neq 0$ at presence of reactive and electric potentials.

For $dr/d\psi=0$ in (20) we shall get:

$R^4{}_0 F - R^3{}_0\,p - R^2{}_0 + R_0\,\alpha + R^4{}_0\,p_1(A_0+A_0^э) + R^5{}_0\,p_2(A_3+A_3^э)=f(R_0)=0.$ (21)

Equation (21) for $A_3+A_3^э=0$ has the different in 5D from (13) type

$R^3{}_0(1-k^2+2k\,\alpha\,A_0/b^*) - R^2{}_0\sigma\,\alpha - (R_0-\alpha)\,b^*=0.$ (22)

Here $\sigma = k^2(2-a-2b) - l^2(2-2a-b)$; we assume $k \ll 1$, $\overset{*}{A}_1+\overset{*}{A}_3=0$, $\overset{*}{A}_1 dx^5/d\lambda=0$, $k\approx(1-\alpha b/r)A_0$ then for passing of the light beam beside of the Sun ($b^*=R$) we have a decision (22) $R_0^*\approx 968740$ km. . It is reasonable in (20) to go to new variable $u=1/r$. We have:

$$(du/d\psi)^2 = F - \varepsilon up - u^2 + \varepsilon u^3 + p_1(A_0\,\varepsilon + A_0^{\,\circ}) + p_2(A_3\varepsilon + A_3^{\,\circ})/u; \text{ here } \varepsilon = \alpha. \quad (23)$$

Then for the "point of the rotation":

$$(du/d\psi)^2 = 0,\ F = \varepsilon u_0\, p + u_0^{\,2} - \varepsilon\, u_0^{\,3} - p_1(A_0\,\varepsilon + A_0^{\,\circ}(0)) - p_2(A_3(0)\varepsilon + A_3^{\,\circ}(0))/u_0; \quad (24)$$

Due to the mirror symmetry of the zero geodetic orbit the contributions in $\Delta\psi$, , which are defined on two parts of the way before the point of rotation and after are

$$\Delta\psi = 2\int_0^{1/Ro} du/\{F - \varepsilon up - u^2 + \varepsilon u^3 + p_1(A_0\,\varepsilon + A_0^{\,\circ}) + p_2(A_3\varepsilon + A_3^{\,\circ})/u;\}^{1/2}. \quad (25)$$

Differentiating (24) on $\varepsilon/2$ for fixed u_0 and presenting result for $\varepsilon/2=0$, we have:

$$2\partial\Delta\psi/\partial\varepsilon|_{\varepsilon/2=0} = 2\int\{[u^3{}_0 - u^3 + pu + pu_0 + p_1 A_0 + p_2 A_3(u)/u - p_1(A_0(0) - p_2(A_3(0)\}du/\{u^2{}_0 - u^2 - p_1A_0^{\,\circ}(0) - p_2A_3^{\,\circ}(0)/u_0 + p_1A_0^{\,\circ}(u) + p_2A_3^{\,\circ}(u)/u\}^{3/2} = I_{\varepsilon/2=0}\ . \quad (26)$$

We shall take the expression

$$[\,p_1A_0^{\,\circ}(0) + p_2A_3^{\,\circ}(0)/u_0 - p_1A_0^{\,\circ}(u) - p_2A_3^{\,\circ}(u)/u]\,/(\,u^2{}_0 - u^2) \ll 1; \quad (27)$$

as small in comparison with unit. Then we have

$$2\partial\Delta\psi/\partial\varepsilon|_{\varepsilon/2=0} = 2\int [u^3{}_0 - u^3\,]\,/\,[u^2{}_0 - u^2]^{3/2}\,du + 2\int\{pu + pu_0\}/\,[u^2{}_0 - u^2]^{3/2}\,du + 2\int(p_1 A_0(u) - p_1(A_0(0))/\,[u^2{}_0 - u^2]^{3/2}du + 2\int(\{p_2 A_3(u)/u - p_2(A_3(0)\}/\,[u^2{}_0 - u^2]^{3/2})du + 3\int[u^3{}_0 - u^3\,]\,\delta^*/[u^2{}_0 - u^2]^2\,du + 3\int\{pu + pu_0\}\cdot\delta^*/\,[u^2{}_0 - u^2]^2\,du + 3\int(p_1 A_0(u) - p_1(A_0(0))\,\delta^*/\,[u^2{}_0 - u^2]^2 + 3\int(\{p_2 A_3(u)/u - p_2(A_3(0)\}\,\delta^*/\,[u^2{}_0 - u^2]^2)du; \quad (28)$$

Here $\delta^* = [\,p_1A_0^{\,\circ}(0) + p_2A_3^{\,\circ}(0)/u_0 - p_1A_0^{\,\circ}(u) - p_2A_3^{\,\circ}(u)/u]$. The upper limit of the first integral is u_0; The integration gives a classical Schwarzschild's result, equal to $\delta\,\psi = 1.75''$. The upper limit of the following integrals in D5 $u^*_0 = 1/R^*_0$ is possible to define from the fourth order equation (21) for the known reactive and electromagnetic potentials [3] , defined from more general , than (1) , expression for the induced tensor of the energy-pulse by the account of electromagnetism and an equation of state of the small charged volume $p = -w\,\rho$:

$$8\pi T^\beta{}_\delta = 8\pi\gamma_{\alpha\delta}(T^{\alpha\beta}{}_e + T^{\alpha\beta}{}_n),\ T^{\alpha\beta}{}_e = -\Phi^2(F_{\alpha\lambda}\,F_\beta{}^\lambda)/2;\ T^{\alpha\beta}{}_n = \Phi^{-1}\,\Phi_{\alpha;\beta}\,,\ F^\lambda{}_{\alpha;\lambda} = -3\,\Phi^{-1}\,\Phi^\lambda F_{\lambda\alpha}\,,$$
$$\Phi^\alpha{}_{;\alpha} = -\Phi^3\,F_{\mu\nu}\,F^{\mu\nu},\ \Phi = 1 - \alpha b/2r, \qquad if\ R_{55}=0, R_{5\alpha}=0. \quad (29)$$

In work [1] for Solar system it is recommended that $b = 7.5.10^{-3}$, $a=0.9962289$;

So the influence of the second integral (27) under the condition $k \neq 0$ (with provision for A_1) is refined; the third integral defines the influence of the zero component of the reactive potential A_0; the fourth integral defines the influence of the reactive potential A_3 (as same as A_1) , defined from the system of equations (1); the following integrals take into account the influence of electromagnetic potentials (29). Since the small parameter δ^* is entered in the integrals, results of influence of the scalar electric potential of the body on deflection of the light beam will be considered below. Thereby the second integral in (28) with provision A_0 value is

$$2\int\{pu + pu_0\}/\,[u^2{}_0 - u^2]^{3/2}\,du = 2p\{[(u_0 + u^*_0)/(\,u_0^{\,2} - u_0^{*2}\,)^{1/2}] - 1\},\ \{[(u_0 + u^*_0)/(\,u_0^{\,2} - u_0^{*2}\,)^{1/2}] - 1\} \approx \alpha = 2.94.$$

It is seen that a sign of the integral $2\int\{pu + pu_0\}/\,[u^2{}_0 - u^2]^{3/2}\,du$ depends on correlation k^2/h^2 and l^2/h^2, included into p parameter; if $k^2/h^2 > l^2/h^2$, then we have a gain of the light deflection angle; for $k^2/h^2 < l^2/h^2$ −reduction of the angle; if k is considered as negligible and equal 0 we have $2\int(\{pu+$

$pu_0 \}/[u^2_0 - u^2]^{3/2})du < 0$, i.e. the deflection angle in this case decreases [1]. The third integral very small and has a positive value:

$$2\int (p_1 A_0(u) - p_1(A_0(0)) / [u^2_0 - u^2]^{3/2} du > 0 . \qquad (30)$$

4. The influence of the scalar electric potential on deflection of the light beam.

First of all we shall define the scalar electric potential in 5D. We shall consider the results of equality to zero of the curvature tensor component $R_{5\alpha}=0$ and the induced energy - pulse tensor T^5_β (2), (3) in 5D [3];

From [1] we remind the Gross- Perri metrics (under $\theta=\pi/2$):

$$dS^2=(1-\alpha/r)^a(dx^0)^2-(1-\alpha/r)^{-(a+b)}(dr)^2-r^2(1-\alpha/r)^{1-(a+b)}(d\psi)^2-(1-\alpha/r)^b\{dx^5+\overset{*}{A}_\nu dx^\nu\}^2, \quad (0\leq x^5<2\pi r_c) \qquad (31)$$

(31)

where $a^2+ab+b^2=1$, $a(b)$−constant parameter, showed above , $\overset{*}{A}_\nu = A_\nu + CA_\nu^e$, A_ν -reactive potential,

CA_ν^e - electromagnetic potential [1], $CA_0^e = \phi$ - its scalar component. From (2), (3) we search the decisions

for $\phi=\phi(r, x_0, \psi)$, neglecting the vector potential components $CA^э_\nu=0$, $\nu=1,2,3$ and reactive potential A_0.

In view of small size $\alpha(a+b)/r \ll 1$, $\alpha(a+b)\varphi/r \ll 1$ metric (31) takes the form

$$dS^2=(1-\alpha a/r)(dx^0)^2-(1+\alpha(a+b)/r)(dr)^2-r^2(1-\alpha[1-(a+b)]/r)(d\psi)^2-(1-\alpha b/r)\{dx^5+\phi\, dx^0\}^2. \qquad (32)$$

(32)

We set the value of scalar potential as small, $\phi(r, x_0, \psi) < 1$; then the components of the metric tensor

$$\gamma_{00} = 1-\alpha b/r-\phi^2,\ \gamma_{11}=-(1+\alpha(a+b)/r),\ \gamma_{55}=-(1-\alpha b/r),\ \gamma_{33}=-r^2(1-\alpha[1-(a+b)]/r),\ \gamma_{05}=-\phi,$$

$$\gamma^{00} = 1+\alpha a/r+\phi^2,\ \gamma^{05}=-\phi,\ \gamma^{11}=-(1-\alpha(a+b)/r),\ \gamma^{55}=-(1+\alpha b/r+\phi^2). \qquad (33)$$

(33)

Then we have from (1) when $\alpha=0$, allowing independence ϕ from the tangential direction ($\phi_{,\psi}=0$), an equation $d\xi/d\eta - [1/\eta + e^{\nu 1-\nu 2}/\eta^2]\xi = 0.$ (34)

Here $\eta=r/r_0$, $\xi = d\phi/d\eta$, $\nu_1=[1-(4a+2b)\alpha/r_0$, $\nu_2=3\alpha b/2r_0$;

the decision (34) is defined $\phi \approx \phi_0 + (d\phi/d\eta)_{\eta=1}\ \eta[\eta-1]e^{\nu 1-\nu 2}/2$, $e^{\nu 1-\nu 2} \rightarrow 1.$ (35)

To find the derivative $(d\phi/d\eta)_0$, we write the Lagrangian on base of the metrics (4) :

$$L=(1-\alpha a/r-\phi^2)(dx^0/dS)^2-(1+\alpha(a+b)/r)(dr/dS)^2-r^2(1-\alpha[1-(a+b)]/r)(d\psi/dS)^2-$$
$$-(1-\alpha b/r)(dx^5/dS)^2-2(1-\alpha b/r)(dx^5/dS)(dx^0/dS)=1 \qquad (36)$$

Thence the first integrals follow:

$$(1-\alpha b/r)(dx^5/dS)+\phi(dx^0/dS)=k=const , \qquad (37)$$
$$(1-\alpha a/r-\phi^2)(dx^0/dS)-\phi(dx^5/dS)=I=H=const ; \qquad (38)$$

$l=H$ - a value , representing the total energy. With provision for (11) parameter is defined in (8):

$$\xi_0 = (d\phi/dr)_{r=r0}\, r_0 = ((H+k)/k)(\alpha a/r_0)\phi_0 . \qquad (39)$$

Thereby the decision (8) is defined completely.

From (1) also we have

under $\alpha=1$, $(\phi_{,1})_{,0}=0$, (40)

under $\alpha=3$, $(\phi_{,0})=0$.

So , from (1) , (11) we get the radial decision (35): $\phi(r, x_0, \psi) \equiv \phi(r)$. (41)

We shall consider hereinafter the integral component J [1, (27) , where the scalar potential (35) is included;

supposing $u=1/r$, we have $J=3\int_0^{1/R^*}[u^3{}_0-u^3](\phi_0-\phi)\,p_1/[u^2{}_0-u^2]^2\,du$; (42)
parameter $u_0{}^*=1/R^*$ is defined from the Sun features from equation, that was got from [1]:
$R^4{}_0 F-R^3{}_0\,p-R^2{}_0+R_0\,\alpha+R^4{}_0 p_1(A_0+A_0{}^3) = f(R_0)=0$. (43)
Here $F=(l^2-k^2)/h^2$, $p= k^2\,(2-a-2b)/h^2-l^2(2-2a-b)/h^2$, $p_1=2kl/h^2$, l, k, h ; a – parameter of the metrics (4). For the Sun we have $u_0=1.438228\cdot 10^{-6}$, from [1, (21)] follows $u_0{}^*\approx 1.4380548\cdot 10^{-6}$, $u_0-u_0{}^* > 0$.
Thereby, according to [1] if $J > 0$, the light beam deflection increases. Writing (35) under $r=1/u$ we have : $\phi_0-\phi=(d\phi/du)\,(u_0-u)/2;$ substituting in (42) , we shall get:
$J= 3p_1\,(d\phi/du)_{u=u^*}\{((3uu_0+2u^2-3u_0{}^2)/2(u+u_0))-$
$-\ u_0\ln(0.5(1+u/u_0))\}\rightarrow 3p_1\,(d\phi/du)_{u=u^*}u^*>0$; (44)
So, for chosen scalar potential ϕ $J> 0$, consequently , deflection of the light beam grows.

5. Deflection of the light beam in the generalised Kerr metrics.

In analogy with (15) for Lagrangian from (8) we have
$L^2= (dx^0/d\lambda)^2-\Sigma\,(dr/d\lambda)^2/\Delta-\Sigma\,(d\theta/d\lambda)^2+\varepsilon\{(A^2{}_3\,\Delta\,P^2+Q^2\}/\Sigma)\}\,(d\psi/d\lambda)^2+2\,\varepsilon\,\{\{A_3\,P\,\Delta+$
$+\,a\,Q\}/\Sigma\}(dx^5/d\lambda)\,(d\psi/d\lambda)+\varepsilon(\{\Delta+a^2\}/\Sigma)(dx^5/d\lambda)^2$; (45)

Supposing that
$n_1=\Phi^2{}_1\,((dx^5/d\lambda)-A_3P(d\psi/d\lambda)),\ n_2=(\Phi^2{}_2\,(dx^5/d\lambda)-(Q/a)(d\psi/d\lambda)$. (46)

Then we have $£= (dx^0/d\lambda)^2-\Sigma\,(dr/d\lambda)^2/\Delta+\varepsilon\,n_1{}^2/\Phi^2{}_1+\varepsilon\,n_2{}^2/\Phi^2{}_2=0,\ \varepsilon=-1,$ (47)

$(d\psi/d\lambda)^2=[\,n_2\Phi^2{}_1-n_1\Phi^2{}_2]^2/(\Phi^2{}_1\Phi^2{}_2)^2(A_3P-Q/a)^2;$ (48)

$\partial L/(dx^0/d\lambda)=(dx^0/d\lambda)=l=const$ (49),
Dividing (47) on (48) and substituting integral (49),we shall get the equation
$(dr/d\psi)^2=(\Delta/\Sigma)\{(\Phi^2{}_1\Phi^2{}_2)\,(A_3P-Q/a)^2/[n_2\Phi^2{}_1-n_1\Phi^2{}_2]^2)-1\}[\,l^2\,\Phi^2{}_1\Phi^2{}_2-(n^2{}_2+n^2{}_1\,\Phi^2{}_1\,\Phi^2{}_2)]$.
(50)
For the point of rotation $(dr/d\psi)^2=0$ and $l^2\,\Phi^2{}_1\Phi^2{}_2=(n^2{}_2\,\Phi^2{}_1+n^2{}_1\,\Phi^2{}_1\,\Phi^2{}_2)]$,
we have $a_0R_0{}^4+a_1R_0{}^3+a_2R_0{}^2+a_3R_0+a_4=0,\ a_0=n_2{}^2,\ a_1=-n_2{}^2\,\alpha,$
$a_2=-(1+n_2{}^2+n_1{}^2)\,a^2,\ a_3=-\alpha\,a^2,\ a_4=-a^4.$ (51)

For decision (51) when referencing to Kerr theory in 4D [11,446], it is reasonable to use the point of rotation as the initial value for radius R_{00} which brings to the following expression:
$R^3{}_{00}-b^{*2}(R_{00}-\alpha)-4a\,\alpha b^*=0,\ R_{00}=2b^*\cos(60^0-\varphi/3),\ \cos\varphi=3^{3/2}\alpha(1-4a/b^*)/b^*$. (52)

If rotation of the central body $a=0$ we shall get the consequence from the Schwarzschild decision (13). So, we define from (50) maximum value of radius. Taking into account the small $A_3 \ll 1$, $A_3(a^2/r^2)\ll 1$, $a^4/r^4\ll 1$, $N^2/r^2\ll 1$ size of the values and supposing $u=1/r$ we shall get a decision after similar with (28) integration (50):
$\delta\psi=2\alpha_0\{1-A_3{}^2k^2+2N^2/b^{*2}\}/b^*$, (53)

$A_3\approx(\alpha/b^*w)^{1/2},\ k=k\,(dx^5/dS)$. (54)

6. The time delay of a signal

We consider hereinafter effect, concerned to behavior of the geodetic data in Kaluza-Klein geometry.

Considering equations, presented in work [1]

$(du/d\psi)^2 = F-\varepsilon up-u^2+ \ \varepsilon u^3+ p_1(A_0\,\varepsilon+A_0^3)+ p_2(A_3\varepsilon+A_3^3)/u; \ ; \ \varepsilon= \alpha.$ (55)

where $F= \varepsilon u_0\, p+ u_0^2- \varepsilon\, u_0^{\,3}- p_1(A_0\,\varepsilon+A_0^3\,(0))$ and considering that curvature of the way will be as a small contribution for the time delay, in the first approximations from (19) we have $u(\psi)=u_0 sin\ \psi$, whence it follows

$r^{\,2}\,(d\,\psi)^2 = r_0^2 dr^2/(\,r^2- r_0^2).$ (56)

Lagrangian at presence of the electric potential ϕ only for generalised Gross- Perri metrics with provision for (56) and (33) is

$L = A^{\,a}\,(dx^0\,/dS)^2-[A^{\,-(a+b\,)}\,+A^{1-(a+b\,)}r_0^2/(\,r^2- r_0^2)](dr\,/dS)^2- A^b\,[(dx^5/\,dS)^2\,-2\,\phi\,A^b\,(d\,x^0\,/dS)(\,dx^5/\,dS)]=0,$

$A= (1-\alpha\,/r).$ (57)

We have the integrals from (57) ($\phi^2 \ll 1$): $(d\,x^0\,/dS) =lA^{-\,a}\,+\phi\,kA^{\,-(a+b)}$, $l=const$, $l=\hbar H-$ the photon total energy; (58)

$(dx^5/\,dS) =kA^{-b}- \phi I\,A^{\,-(a+b)},$ $k=const,$ (59)

We divide (57) in $(dx^0/dS)^2$ and have

$A^{\,a} = [A^{\,-(a+b)}\,+A^{1-(a+b)}r_0^2/(\,r^2- r_0^2)]\ (dr\,/dx^{\,0})^2+ (k/l)^2\,A^{2a-b} - \phi(k/l)A^{2a+b}(1-(k/l)^2\,),$ (60)

Then from (60) it follows $(dr/\,dx^{\,0}) =[\,1- A^{a-b}\,(k/l)^{\,2} + \phi(k/l)A^{a+b}\,(1-(k/l)^2\,]^{1/2}/\,[\,A^{\,-(2a+b\,)} +A^{1-(2a+b)}\,r_0^2/(\,r^2- r_0^2)]^{1/2}$, (61)

or $(dx^{\,0}\,/dr) =\{[\,A^{\,-(2a+b\,)}\,+A^{1-(2a+b)}r_0^2/(\,r^2- r_0^2)]^{1/2}\,/\,\ [1-A^{a-b}\,(k/l)^{\,2}]\}[1-\lambda]\,,\lambda =\phi\,(k/l)A^{a+b}\,(1-(k/l)^2\,]/2$. (62)

The radar signal is emitted from the Earth, that has a Schwarzschild radial coordinate r_e; the signal passes near the Sun , having parameter of the minimum distance r_Θ , and then is reflected from the planet, having also the Schwarzschild coordinate r_p. The problem of determination of the time lag $\Delta\tau$ by the timer of the terrestrial watcher between the signal radiation and receiving with provision for ϕ influence is tasked.

For this purpose we carry out integration (62) and then differentiation on α (leaving r_Θ fixed) in analogy with conclusion of the effect of the light beam deflection refer to [11,p.549]:

$\Delta t =\Delta t^{\,*}+ \delta\,\Delta x^0(\phi)\,/c\, =\, 2\{[1+\, (k/l)^2](\,r_p^{\,2}- r_\Theta^{\,2})^{1/2}\, +(\,r_e^{\,2}- r_\Theta^{\,2})^{1/2}\, +(\,\alpha[(2a+b)\, +3b(k/l)^2/2]/2)[ln((r_p\, + (r_p^{\,2}- r_\Theta^{\,2})^{1/2})/\,r_\Theta)\, +ln((r_e\, +\,(r_e^{\,2}\, - r_\Theta^{\,2})^{1/2})/\,r_\Theta)] - \alpha[(1+\, (k/l)^2][\,(\,r_p^{\,2}- -r_\Theta^{\,2})^{1/2}/\,r_\Theta\,)+\,(\,r_e^{\,2}\, - r_\Theta^{\,2})^{1/2})/ r_\Theta]/2\}/c\, +\delta\,\Delta x^0(\phi)\,/c,$ (63)

The sought additional incrementation to time lag taking into account the scalar potential influence in 5D is

$\delta\,\Delta x^0\ \ (\phi)\,/c\, =\, -\,(\phi(k/l))\,\Delta t-\chi_1[[1+(k/l)^2][(2\,r_\Theta^{\,2}+\,r_p^{\,2})(\,r_p^{\,2}- r_\Theta^{\,2})^{1/2}+(2\,r_\Theta^{\,2}+\,r_e^{\,2})(\,r_e^{\,2}- r_\Theta^{\,2})^{1/2}\ ]+[1+ (k/l)^2/2+\alpha(((2a+b)+3b\,(k/l)^2/2)/4\,r_\Theta)][\,r_p\,(\,r_p^{\,2}- r_\Theta^{\,2})^{1/2}+\,r_e\,(\,r_e^{\,2}- r_\Theta^{\,2})^{1/2}\, +\,r_\Theta^{\,2}\,ln((r_p+(\,r_p^{\,2}-r_\Theta^{\,2})^{1/2})/\,r_\Theta))+ r_\Theta^{\,2}\,ln((r_e+\,(\,r_e^{\,2}- r_\Theta^{\,2})^{1/2})/\,r_\Theta))]-\alpha[(2a+b)+\,3b\,(k/l)^2/2](\,(\,r_p^{\,2}- r_\Theta^{\,2})^{1/2}\, +(\,r_e^{\,2}-r_\Theta^{\,2})^{1/2}\,)-[\,(1+\,(k/l)^2)\alpha\, r_\Theta/2](arcos(r_\Theta/\,r_p)+\,(arcos(r_\Theta/\,r_e)\,)]/c,\ \chi_1=(r/l)(d\,\phi\,/dr)_{r=r0}.$ (64)

In the event of the Schwarzschild metrics under $k=0, \phi=0$ we have the classical decision from (63) [11, p.217].

The fixed time $\Delta\tau$, observed with the terrestrial timer and depended from the change of the coordinate time Δt, is defined by formula

$$\Delta\tau =(1-\alpha/r)^{1/2}\Delta t \quad (65)$$

So, with the first-order accuracy on α the reflected signal of the radar will be fixed on the Earth after $\Delta\tau=-2(\alpha/ r_e c)[(r_e^2 - r_{\odot}^2)^{1/2}+(r_p^2- r_{\odot}^2)^{1/2}] +\Delta t$. (66)

7. Radial movement of the neutral massive particles.

We shall consider hereinafter the radial moving of the particle relative to the massive body at presence of the reactive potential A_0 that also can be considered as a test check in 5D in Kaluza-Klein theory. Let's write the generalised metrics (5) under $\phi= 0$: $dS^2=A^a(dx^0)^2- A^{-(a+b)}(dr)^2- A^b\{dx^5+ A_0\, dx^0\}^2$; (67)

Then for the metrics coefficients (31) we have (taking into account the small value of the gravitational potential):

$\gamma_{00}=A^a-A_0^2$, $\gamma_{11} =-A^{-(a+b)}$, $\gamma_{55} =-A^b$, $\gamma_{05} =- A^b A_0$; $\gamma^{00} = A^{-a} +(\alpha b/r) A_0^2$, $\gamma^{11} =- A^{(a+b)}$,

$\gamma^{05}= - A_0 (A^{-a}+ A_0 +(\alpha b/r))$, $\gamma^{55} = -(A^{-b}- A^{-a} A_0^2)$; (68)

Here A_0 -a reactive potential. On base (67) with provision for (68) we have a Lagrangian in the event of a radial motion

$L= (1-\alpha a/r-A_0^2) (dx^0/ dS)^2- (1+\alpha(a+b)/r) (dr\, dS/)^2- (1-\alpha b/r) (dx^5/ dS)^2 -2A_0(1-\alpha b/r) (dx^5/ dS)(dx^0/ dS) =1$ (69)

Thence the first integrals are follow:

$[(H+ A_0 k) A_0 (1+\alpha a/r) + (dx^5/ dS)] (1-\alpha b/r) =k = const,$ (70)

$(1-\alpha a/r) (dx^0/ dS) = H+ A_0\, k =H^*=const$, (71)

H^* - a value, presenting the total energy (including H and initial reactive energy $A_0 k$ on unit of the inertial rest mass of a particle, moving on the considered geodetic, to the steady-state watcher on infinity). $A_0(0)k$ value is possible to associate with parameter n for dark energy [6] , considering its as the non-dimensional velocity of the "relativistic gas" outflow, proved in work K.P.STANYUKOVICH [12] :

$n=(2 A_0 (0)k (1- A_0(0)k/2))^{1/2}$, $A_0(0)k =1-(1- n^2)^{1/2}$; $n = u_d/c\rho_k 0.73$, (72)

where u_d - density of the energy equal $u_d\approx 3.8kev/cm^3$ at scale of the dark energy length equal $\lambda_0\approx 85mkm$, c- the light velocity, ρ_k - critical density in standard model of Universal, , $\rho_k =10^{-29}g/ cm^3$; then we have definitively $n \approx 9.75\cdot 10^{-6}$; thence follows that $A_0 (0)k \approx 4.8\cdot 10^{-11}$.

From the induced tensor energy- pulse [3] with an allowance for equation of the dark energy state

$8\pi T^\sigma{}_\beta=\gamma^{\alpha\sigma}\{\Phi_{\alpha,\beta}-\gamma^{k\delta} (\gamma_{\delta\alpha,\beta}+\gamma_{\delta\beta,\alpha}-\gamma_{\alpha\beta,\delta}) \Phi_k/2\}/ \Phi$, $\alpha,\beta,\sigma,\nu=0,1,2,3$, $p= - w\rho$, (73)

and from (32) it is possible to write the approximate linear equation , which decision gives dependency $A_0(r)$:

$$d(A_0^2)/dr =2(1- w)\, \alpha a/r^2 , \quad (74)$$

$$A_0 =\{ A_0^2-2(1- w)\, \alpha a[(1/r)-(1/r_0)]\}^{1/2} ; \quad (75)$$

$A_0^2 \geq | 2(1- w)\, \alpha a[(1/r)-(1/r_0)] |$, $A_0(0)k\approx 4.8\cdot 10^{-11}$,find the estimation $k\approx 4.8\cdot 10^{-11} /| 2(1- w)\, \alpha a[(1/r)-(1/r_0)] |^{1/2}$.

Let's write (69) in the manner of $(dr/dS)^2=A^{2a+b} (dx^0/dS)^2-$

$-A^{a+2b}\{(dx^5/dS)^2+2A_0 A^{(a+2b)} \ (dx^0/dS)(dx^5/dS)+A^{(a+b)} =0$ (76)

The velocities (dx^5/dS) in (70) and ($dx^0 \ dS$) in (71) and their squares is possible to present in the manner of
$(dx^0/dS) \approx HA^{-a} + A_0 A^{-a}k$, $(dx^5/dS) \approx kA^{-b} - A_0 A^{-a}[H+A_0 k]$, $(dx^0/dS)^2 \approx (HA^{-a})^2 + A_0 \ kA^{-2a}(k+2H)$, $(dx^5/dS)^2 \approx (kA^{-b})^2 + A^{-a}(H^2-2k)A_0^2$, $(dx^0/dS)(dx^5/dS) \approx HkA^{-3a} + (k^2-H^2)A_0$; (77)

Substituting (77) in (76), we have $(dr/dS)^2 \approx H^2A^b - k^2A^a - A^{a+b} + (2-k)kA_0^2 - 4HkA_0^3$; (78)

We assume under $r=r_0$ $dr/dS=0$; from (78) we have $H^2 \approx k^2 A^{a-b}(0) + A^a(0) - ((2-k)kA_0^2 - 4HkA_0^3)A^{-b}(0)$; (79)
Now (79) substitute in (78): $(dr/dS)^2 \approx \{(A^{a-b}(0)A^b - A^a)k^2 + (A^a(0) - A^a)A^b + \sigma$,
$\sigma = [(2-k)kA_0^2 - 4HkA_0^3](1-A^{-b}(0)A^b]$. (80)

Expression (80) complies in accuracy with results in work [1] , except member σ, reflecting an influence of the initial reactive potential; it can be named "energy equation" with provision of the reactive potential energy. It is seen that under k=0, a=1, b=0 this is reduced to formula $(dr/dS)^2/2 = \alpha(1/r - 1/r_0)/2$, as a relation in the Schwarzschild metrics and also corresponds to energy equation for the vertical free drop in the classical Newton's theory.
For our fifth- dimensional interval of the velocity component in the radial direction there is
$dr/dx^0 = (dr/dS)(dS/dx^0)$; (81)

From (77) we have $(dx^0/dS) \approx HA^{-a} + A_0 A^{-a}k$, (82)

from (79) follows
$H_0 \approx [A^a(0) + k^2 A^{a-b}(0)]^{1/2}(1+\delta)$, $\delta = \{-((2-k)kA_0^2 - 4HkA_0^3)A^{-b}(0)\}/[A^a(0)+k^2 A^{a-b}(0)]$; (83)

Substituting (83) in (82), we shall get
$dS/dx^0 \approx 1/(HA^{-a} + A_0 A^{-a}k) = 1/\{([A^a(0) + k^2 A^{a-b}(0)]^{1/2}(1+\delta))A^{-a} + A_0 A^{-a}k\}$. (84)

From (80) we have $dr/dS = [(A^b A^{a-b}(0) - A^a)k^2 + A^b \ A^a(0) - A^{a+b} + \sigma]^{1/2}$;
In total follows $dr/dx^0 = -[(A^b A^{a-b}(0) - A^a)k^2 + A^b \ A^a(0) - A^{a+b} + \sigma]^{1/2} / \{([A^a(0) + k^2 A^{a-b}(0)]^{1/2}(1+\delta))A^{-a} + A_0 A^{-a}k\}$. (85)
So, the radial velocity of the particle will be depended from its velocity component along fifth dimension and additional member, associated with the initial value of the reactive potential through parameter к, δ and σ.

Conclusion.

1. The Gross- Perri and Kerr metrics are complemented with introduction of the electromagnetic and reactive fields (the dark energy models) in the Kaluza-Klein gravitational theoryю.
2. The additional (to classical results in 4D) tests of the light deflection angle in the massive body field and the additional time delays of the signal for location of the planets of the Solar system are obtained in 5D.
3.The radial movement of the massive particles at presence of the reactive scalar potential, that was got from decision of the induced energy- pulse tensor in the fifth-dimensional space is considered as test. The specified potential was found as depended from the first integral for the fifth-

dimensional velocity, the gravitational potential and the coefficient of proportionality of the pressure to density for the equation of state of the dark energy.

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Redshift for a weak gravitational field in finsler space of events berwald-moor

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The weak gravitational field in curved Finsler space of events Berwald-Moor is considered. From the equations of a geodesic line classical equations of motion of a particle in a limiting case for non-Newtonian three-dimensional space in a gravitational field are given. Linear equations for a metric tensor and their solutions are reduced. The problem on red-shift is considered.

Introduction

Now there is a creation of the theory of gravitation in Finsler space of events of Bervald-Moor. Various approaches (see for example, [1-3]) are considered. Outside of a gravitational field metric function of local four-dimensional space-time of Bervald-Moor has forms-invariant expression [4]

$$F = ds = \left(g_{ijkl} dx^i dx^j dx^k dx^l \right)^{1/4} = \left[\prod_m^4 \left(cdt + \boldsymbol{\varepsilon}^m d\mathbf{x} \right) \right]^{1/4}. \tag{1}$$

for inertial systems. The element of physical time is defined by the summand

$$\begin{aligned} dT &= \left[dt^4 - 2\, dt^2 d\mathbf{x}^2 / c^2 + 4dt\left(d\mathbf{x}\{d\mathbf{x}d\mathbf{x}\}\right)/3c^3 \right]^{1/4} = \\ &= \left[dt^4 - 2\left(dt^2dx^2 + dt^2dy^2 + dt^2dz^2 \right) + 8dtdxdydz \right]^{1/4} \end{aligned} \tag{2}$$

with coordinate time.

Known values of components of vectors $\boldsymbol{\varepsilon}^1 = (1,1,1)$, $\boldsymbol{\varepsilon}^2 = (-1,1,-1)$, $\boldsymbol{\varepsilon}^3 = (1,-1,-1)$, $\boldsymbol{\varepsilon}^4 = (-1,-1,1)$ of the allocated directions in three-dimensional space satisfy to equalities [5]

$$\begin{gathered} \sum_m^4 \varepsilon_\alpha^m = 0,\ \ \frac{1}{4}\sum_m^4 \varepsilon_\alpha^m \varepsilon_\beta^m = \delta_{\alpha\beta},\ \ \frac{1}{4}\sum_m^4 \varepsilon_\alpha^m \varepsilon_\beta^m \varepsilon_\gamma^m = \varepsilon_{\alpha\beta\gamma}, \\ 1 + \left(\boldsymbol{\varepsilon}^m \right)^2 = 4,\ \ 1 + \left(\boldsymbol{\varepsilon}^m \boldsymbol{\varepsilon}^r \right) = 0\ \ (m \neq r), \end{gathered} \tag{3}$$

where $\delta_{\alpha\beta}$ is Kronecker delta, $(d\mathbf{x}d\mathbf{x}) = d\mathbf{x}^2 = \delta_{\alpha\beta}dx^\alpha dx^\beta$ – is a scalar product for $d\mathbf{x} = \{dx, dy, dz\}$, $\{d\mathbf{x}d\mathbf{x}\}_\alpha = \varepsilon_{\alpha\beta\gamma}dx^\beta dx^\gamma$ are components of a new vector product [5] and $\varepsilon_{\alpha\beta\gamma}$ is a three-dimensional absolutely symmetric symbol with property $\varepsilon_{\alpha\beta\gamma} = 1$ at $\alpha \neq \beta \neq \gamma$, and remaining values are zero, m is vector number, and α, β and γ run values 1, 2, 3. It is important to notice that, according to (3), takes place only three independent allocated directions and their number equals to four.

Metric function (1) in known forms will be written so

$$
\begin{aligned}
F = ds = \Big[& c^4 dt^4 + dx^4 + dy^4 + dz^4 - \\
& -2\left(c^2dt^2dx^2 + c^2dt^2dy^2 + c^2dt^2dz^2 + dx^2dy^2 + dy^2dz^2 + dz^2dx^2\right) + 8cdtdxdydz\Big]^{1/4} = \\
& = \left\{\left[\left(cdt+dx\right)^2 - \left(dy+dz\right)^2\right]\left[\left(cdt-dx\right)^2 - \left(dy-dz\right)^2\right]\right\}^{1/4} = \\
& = \left[\left(cdt+dx+dy+dz\right)\left(cdt-dx+dy-dz\right)\left(cdt+dx-dy-dz\right)\left(cdt-dx-dy+dz\right)\right]^{1/4} = \\
& = \begin{vmatrix} cdt & dx & dy & dz \\ dx & cdt & dz & dy \\ dy & dz & cdt & dx \\ dz & dy & dx & cdt \end{vmatrix}^{1/4} = \\
& = \left(\varepsilon_{1234} H_i^1 H_j^2 H_k^3 H_l^4 dx^i dx^j dx^k dx^l\right)^{1/4} = \left(\frac{1}{4!}\varepsilon_{abcd} H_i^a H_j^b H_k^c H_l^d dx^i dx^j dx^k dx^l\right)^{1/4},
\end{aligned} \tag{4}
$$

where the symbol ε_{abcd} is absolutely symmetrical symbol with property $\varepsilon_{abcd} = 1$ if $a \neq b \neq c \neq d$, remaining values are zero, $dx^i = \left(cdt, d\mathbf{x}\right)$, H_i^a is a normalized Hadamard matrix of order four

$$
\mathbf{H}_4 = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & -1 \\ 1 & 1 & -1 & -1 \\ 1 & -1 & -1 & 1 \end{pmatrix} = \begin{pmatrix} \mathbf{H}_2 & \mathbf{H}_2 \\ \mathbf{H}_2 & -\mathbf{H}_2 \end{pmatrix},\ \mathbf{H}_2 = \begin{pmatrix} \mathbf{H}_1 & \mathbf{H}_1 \\ \mathbf{H}_1 & -\mathbf{H}_1 \end{pmatrix},\ \mathbf{H}_1 = 1,\ \mathbf{H}\mathbf{H}^T = 4\mathbf{I}, \tag{5}
$$

with property $H_i^a H_a^j = 4\delta_i^j$ and a four-dimensional Kronecker delta δ_i^j, $\mathbf{I}$ is an unit four-dimensional matrix, also the determinant is a determinant of a characteristic matrix of algebra quadranumber (hyperbolic number). The interval ds is interpreted as a seminorm of a four-dimensional vector dx^i.

Thus, from (4) expression for a metric tensor follows

$$
g_{ijkl} = \frac{1}{4!}\varepsilon_{abcd} H_i^a H_j^b H_k^c H_l^d . \tag{6}
$$

Let's rewrite the differential form (1) in the form

$$
\begin{aligned}
\left(ds\right)^4 = g_{ijkl} dx^i dx^j dx^k dx^l &= g_{0000}\left(dx_0\right)^4 + g_{00\alpha\beta}\left(dx_0\right)^2 dx^\alpha dx^\beta + \\
&+ g_{0\alpha\beta\gamma} dx_0 dx^\alpha dx^\beta dx^\gamma + g_{\alpha\beta\gamma r} dx^\alpha dx^\beta dx^\gamma dx^r
\end{aligned} \tag{7}
$$

with $g_{0000} = 1$, $g_{00\alpha\beta} = -\delta_{\alpha\beta}$, $g_{0\alpha\beta\gamma} = 4\varepsilon_{\alpha\beta\gamma}/3$, $g_{\alpha\beta\gamma r} = \frac{1}{2!}\left[\delta_{\alpha(\beta}\delta_{\gamma)r} - \varepsilon_{m\alpha(\beta}\varepsilon^{m}_{\gamma)r}\right]$.

In the presence of the gravitational field of the components of the metric tensor depend on the time and coordinates. Therefore at first it is necessary to consider the equation of the geodesic line. It should be such that for a weak field when components of a metric tensor differed from values in (7) a little, corresponded in a limit of $c \to \infty$ classical mechanics with the equation of motion in non-Newtonian three-dimensional space. Further we will pass to the equations of a gravitational field in linear approach.

The equations of motion of a particle in a non-Newtonian classical mechanics for proper space

In relativistic mechanics of action is connected with metric function

$$S=-\alpha\int_a^b ds=\int_{t_1}^{t_2} L dt=-\int_{t_1}^{t_2}\alpha c\sqrt[4]{g_{ijkl}\frac{dx^i}{dt}\frac{dx^j}{dt}\frac{dx^k}{dt}\frac{dx^l}{dt}}dt\,. \tag{8}$$

Let's consider the transition from relativistic mechanics, in which the action has the form

$$S=\int_{t_1}^{t_2} L dt=-\int_{t_1}^{t_2}\alpha c\sqrt[4]{\left(\frac{dT}{dt}\right)^4+\frac{(\mathbf{vv})^2-\{\mathbf{vv}\}^2}{c^4}}dt \tag{9}$$

to classical mechanics. In a transition to the limit $c\to\infty$ coordinate time, according to (2), approximately equals physical, that is, we have $dT\approx dt$. Then the Lagrangian L at the expansion on degrees $\mathbf{v}/c=d\mathbf{x}/cdt$ can be written as

$$L\approx-\alpha c\sqrt[4]{1+\left[\frac{(\mathbf{vv})^2-\{\mathbf{vv}\}^2}{c^4}\right]}\approx-\alpha c+\frac{\alpha}{4c^3}\left[(\mathbf{vv})^2-\{\mathbf{vv}\}^2\right], \tag{10}$$

where the coordinate velocity is approximately equal to the physical. Action have dimension of energy multiplied by time that leads to the value $\alpha=m_0 c$. Removing the constant term $\left(-m_0c^2\right)$ in the Lagrangian, we obtain the expression

$$L=\frac{m_0}{4c^2}\left[(\mathbf{vv})^2-\{\mathbf{vv}\}^2\right]=\frac{m_0}{4c^2}\prod_m^4\left(\varepsilon^m\mathbf{v}\right), \tag{11}$$

different from $L=m_0(\mathbf{vv})/2$ in Newtonian mechanics. This was expected, as for Newtonian mechanics isotropic Euclidean geometry takes place. In our case, the starting point is a non-isotropic geometry of events with allocated directions which are remaining and in case of a new classical mechanics. In this mechanical motion of a particle in a gravitational field is determined by means of a Lagrangian

$$L=\frac{m_0}{4c^2}\left[(\mathbf{vv})^2-\{\mathbf{vv}\}^2\right]-\frac{m_0}{c^2}\varphi\,. \tag{12}$$

According to the Euler-Lagrange equations

$$\frac{d}{dt}\frac{\partial L}{\partial\mathbf{v}}=\frac{\partial L}{\partial\mathbf{r}}, \tag{13}$$

we will receive impulse of a particle, the equation of motion and energy

$$\mathbf{p}=\frac{\partial L}{\partial\mathbf{v}}=\frac{m_0}{c^2}\left[\mathbf{v}(\mathbf{vv})-\{\mathbf{v}\{\mathbf{vv}\}\}\right], \tag{14}$$

$$m_0\frac{d\left[\mathbf{v}(\mathbf{vv})-\{\mathbf{v}\{\mathbf{vv}\}\}\right]}{dt}=-m_0\frac{\partial\varphi}{\partial\mathbf{r}}\,. \tag{15}$$

$$E=(\mathbf{pv})-L=\frac{3m_0}{4c^2}\left[(\mathbf{vv})^2-\{\mathbf{vv}\}^2\right]+\frac{m_0}{c^2}\varphi\,. \tag{16}$$

In the Lagrangian (12) before the potential introduced the multiplier $\left(1/c^2\right)$ on purpose that in the equations of motion of the particle (15) did not have value of velocity of light. In the coordinate form of the equations of motion (15) can be written as

$$m_0 g_{\alpha\beta\gamma r} \frac{d}{dt}\left(v^{\alpha} v^{\alpha} v^{\beta}\right) = m_0 \frac{\partial \varphi}{\partial x^{\alpha}} . \tag{17}$$

The gravitational potential φ depending only on the coordinates, satisfies the differential equation of the fourth order

$$\Delta_4 \varphi = \beta\rho \tag{18}$$

with the scalar operator [6]

$$\begin{gathered}\Delta_4 = \Delta\Delta - \left(\{\nabla\nabla\}\{\nabla\nabla\}\right) = \\ = \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + \frac{\partial^4}{\partial z^4} - 2\left(\frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^2 \partial z^2} + \frac{\partial^4}{\partial z^2 \partial x^2}\right) = \prod_m^4 \left(\boldsymbol{\varepsilon}^m \nabla\right).\end{gathered} \tag{19}$$

Quantity ρ is the mass density of source, and β is not yet defined constant factor, Δ is Laplacian, ∇ is the gradient operator.

The equation for the gravitational potential in the presence of a source in the classical mechanics is obtained by the variational method. For what we consider the action

$$\begin{gathered}I = \int L dx dy dz = \\ = \int \left\{ \frac{1}{2}\left[\left(\frac{\partial^2 \varphi}{\partial x^2}\right)^2 + \left(\frac{\partial^2 \varphi}{\partial y^2}\right)^2 + \left(\frac{\partial^2 \varphi}{\partial z^2}\right)^2 - 2\left(\frac{\partial^2 \varphi}{\partial x^2}\frac{\partial^2 \varphi}{\partial y^2} + \frac{\partial^2 \varphi}{\partial y^2}\frac{\partial^2 \varphi}{\partial z^2} + \frac{\partial^2 \varphi}{\partial z^2}\frac{\partial^2 \varphi}{\partial x^2}\right)\right] - \beta\rho\varphi \right\} dx dy dz\end{gathered} \tag{20}$$

with the Lagrangian density L. By varying the action on a potential $\delta\varphi$, we obtain the equation

$$\frac{\partial^4 \varphi}{\partial x^4} + \frac{\partial^4 \varphi}{\partial y^4} + \frac{\partial^4 \varphi}{\partial z^4} - 2\left(\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi}{\partial y^2 \partial z^2} + \frac{\partial^4 \varphi}{\partial z^2 \partial x^2}\right) = \beta\rho , \tag{21}$$

coinciding with the equation (2.11).

Geodesic lines

Consider the problem of geodesic lines in Finsler space which are extremal length curves. For what we find the extremum of the integral

$$\delta \int_{P_1}^{P_2} ds = \delta \int_A^B F\left(x^i, \frac{dx^i}{d\tau}\right) d\tau = \delta \int_A^B \left[\frac{\partial F}{\partial x^i} - \frac{d}{d\tau}\frac{\partial F}{\partial\left(dx^i/d\tau\right)}\right] \delta x^i d\tau \tag{22}$$

and will obtain the Euler-Lagrange equations a

$$\frac{d}{d\tau}\frac{\partial F}{\partial\left(dx^i/d\tau\right)} - \frac{\partial F}{\partial x^i} = 0,\ F = \left(g_{ijkl}\frac{dx^i}{d\tau}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\right)^{1/4}, \tag{23}$$

where τ is arbitrary parameter. For derivatives we have expressions

$$\frac{\partial F}{\partial x^i} = \frac{1}{4}\frac{\partial g_{jklm}}{\partial x^i}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{dx^m}{d\tau}, \tag{24}$$

$$\frac{d}{d\tau}\frac{\partial F}{\partial\left(dx^i/d\tau\right)}=\frac{d}{d\tau}\left[g_{ijkl}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\left(\frac{ds}{d\tau}\right)^{-3/4}\right]=\frac{\partial g_{ijkl}}{\partial x^m}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{dx^m}{d\tau}\left(\frac{ds}{d\tau}\right)^{-3/4}+$$
$$+3g_{ijkl}\frac{d^2x^j}{d\tau^2}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\left(\frac{ds}{d\tau}\right)^{-3/4}-\frac{3}{4}g_{ijkl}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{d^2s}{d\tau^2}\left(\frac{ds}{d\tau}\right)^{-7/4} \quad (25)$$

and, respectively, the equations (23) in the form of

$$3g_{ijkl}\frac{d^2x^j}{d\tau^2}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}+\left[\frac{\partial g_{ijkl}}{\partial x^m}-\frac{1}{4}\frac{\partial g_{jklm}}{\partial x^i}\right]\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{dx^m}{d\tau}-$$
$$-\frac{3}{4}g_{ijkl}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{d^2s}{d\tau^2}\left(\frac{ds}{d\tau}\right)^{-1}=0, \quad (26)$$

which after symmetrization on the indices j and m, k and m, j and l in the square bracket will definitively written as

$$g_{ijkl}\frac{d^2x^j}{d\tau^2}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}+\Gamma_{i,jklm}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{dx^m}{d\tau}-\frac{3}{4}g_{ijkl}\frac{dx^j}{d\tau}\frac{dx^k}{d\tau}\frac{dx^l}{d\tau}\frac{d^2s}{d\tau^2}\left(\frac{ds}{d\tau}\right)^{-1}=0. \quad (27)$$

System of functions in the equation of geodesic (27)

$$\Gamma_{i,jklm}=\frac{1}{12}\left(\frac{\partial g_{klmi}}{\partial x^j}+\frac{\partial g_{lmij}}{\partial x^k}+\frac{\partial g_{mijk}}{\partial x^l}+\frac{\partial g_{ijkl}}{\partial x^m}-\frac{\partial g_{jklm}}{\partial x^i}\right). \quad (28)$$

is a geometrical object that is the analogue of the Christoffel symbol. The equations a geodesic and the 5-indices object are considered in [7-9]. Let the parameter is canonical and has a value $\tau=s$. Then the equation (27) with $dx^i/ds=u^i$ transformed to form

$$g_{ijkl}\frac{du^j}{ds}u^ku^l+\Gamma_{i,jklm}u^ju^ku^lu^m=0 \quad (29)$$

and coincides with the equation a geodesic from work [8, 9], which is to be expected. We write equation geodesic (29) in the following form

$$g_{ijkl}\frac{du^ju^ku^l}{ds}+\Gamma_{i,jklm}u^ju^ku^lu^m=0 \quad (30)$$

at generalization non-relativistic analogue (29). Here the object (28) is the multiplier of $(1/4)$.

We give some known and unknown properties:

а) symmetry property on the indices j, k, l and m;

б) property objects in the summation

$$\Gamma_{i,jklm}+\Gamma_{j,klmi}+\Gamma_{k,lmij}+\Gamma_{l,mijk}+\Gamma_{m,ijkl}=\frac{1}{4}\left(\frac{\partial g_{iklm}}{\partial x^j}+\frac{\partial g_{ijlm}}{\partial x^k}+\frac{\partial g_{ijkm}}{\partial x^l}+\frac{\partial g_{ijkl}}{\partial x^m}+\frac{\partial g_{jklm}}{\partial x^i}\right); \quad (31)$$

в) solution of (28), taking into account (31), relative to derivative

$$\frac{\partial g_{jklm}}{\partial x^i}=2\left(\Gamma_{j,klmi}+\Gamma_{k,lmij}+\Gamma_{l,mijk}+\Gamma_{m,ijkl}\right)-4\Gamma_{i,jklm}; \quad (32)$$

г) identity

$$\frac{\partial \Gamma_{j,klmi}}{\partial x^r}+\frac{\partial \Gamma_{k,lmij}}{\partial x^r}+\frac{\partial \Gamma_{l,mijk}}{\partial x^r}+\frac{\partial \Gamma_{m,ijkl}}{\partial x^r}-2\frac{\partial \Gamma_{i,jklm}}{\partial x^r}=$$
$$=\frac{\partial \Gamma_{j,klmr}}{\partial x^i}+\frac{\partial \Gamma_{k,lmrj}}{\partial x^i}+\frac{\partial \Gamma_{l,mrjk}}{\partial x^i}+\frac{\partial \Gamma_{m,rjkl}}{\partial x^i}-2\frac{\partial \Gamma_{r,jklm}}{\partial x^i}; \tag{33}$$

д) property with antisymmetrization of indices i and j

$$\Gamma_{i,jklm}-\Gamma_{j,iklm}=\frac{1}{6}\left(\frac{\partial g_{klmi}}{\partial x^j}-\frac{\partial g_{jklm}}{\partial x^i}\right); \tag{34}$$

е) property of convolution

$$\Gamma_{i,jklm}u^iu^ju^ku^lu^m=\frac{1}{4}\frac{\partial g_{jklm}}{\partial x^i}u^iu^ju^ku^lu^m=\frac{1}{4}\frac{dg_{jklm}}{ds}u^ju^ku^lu^m; \tag{35}$$

ж) seminorm vector dx^i/ds is conserved on the geodesic line which leads to the equation

$$\frac{d}{ds}g_{ijkl}u^iu^ju^ku^l=0. \tag{36}$$

Differentiation of the left part in (36) gives expression

$$\frac{d}{ds}g_{ijkl}u^iu^ju^ku^l=\frac{1}{4}\frac{dg_{ijkl}}{ds}u^iu^ju^ku^l+g_{ijkl}\frac{du^j}{ds}u^iu^ku^l=$$
$$=g_{ijkl}\frac{du^j}{ds}u^iu^ku^l+\frac{1}{12}\left(4\frac{\partial g_{ijkl}}{\partial x^m}-\frac{\partial g_{ijkl}}{\partial x^m}\right)u^iu^ju^ku^lu^m. , \tag{37}$$

After symmetrization of on the indices in parentheses have

$$\frac{d}{ds}g_{ijkl}u^iu^ju^ku^l=\left(g_{ijkl}\frac{du^j}{ds}u^ku^l+\Gamma_{i,jklm}u^ju^ku^lu^m\right)u^i. \tag{38}$$

In brackets we have the equation geodesic and owing to randomness dx^i/ds we will definitively receive equality to zero of expression (38) that gives equality (36).

з) transformation law

From the invariance $ds=ds'$ we will obtain the equality

$$g_{ijkl}dx^idx^jdx^kdx^l=g_{i'j'k'l'}dx^{i'}dx^{j'}dx^{k'}dx^{l'}, \tag{39}$$

which under coordinate transformation

$$dx^{i'}=\frac{\partial x^{i'}}{\partial x^m}dx^m,\ \frac{\partial x^{i'}}{\partial x^m}\frac{\partial x^m}{\partial x^{j'}}=\delta^{i'}_{j'},\ \frac{\partial x^i}{\partial x^{m'}}\frac{\partial x^{m'}}{\partial x^j}=\delta^i_j \tag{40}$$

leads to the transformation of the metric tensor

$$g_{i'j'k'l'}=g_{ijkl}\frac{\partial x^i}{\partial x^{i'}}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^k}{\partial x^{k'}}\frac{\partial x^l}{\partial x^{l'}}. \tag{41}$$

Differentiating (41), we find the derivative

$$\frac{\partial g_{i'j'k'l'}}{\partial x^{m'}}=\frac{\partial g_{ijkl}}{\partial x^m}\frac{\partial x^i}{\partial x^{i'}}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^k}{\partial x^{k'}}\frac{\partial x^l}{\partial x^{l'}}\frac{\partial x^m}{\partial x^{m'}}+4g_{ijkl}\left(\frac{\partial^2 x^i}{\partial x^{i'}\partial x^{m'}}\frac{\partial x^j}{\partial x^{j'}}\frac{\partial x^l}{\partial x^{l'}}\frac{\partial x^k}{\partial x^{k'}}\right) \tag{42}$$

and taking into account (28) obtain the law of transformation of the object

$$\Gamma_{i',j'k'l'm'} = \Gamma_{i,jklm} \frac{\partial x^i}{\partial x^{i'}} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^k}{\partial x^{k'}} \frac{\partial x^l}{\partial x^{l'}} \frac{\partial x^m}{\partial x^{m'}} + \frac{1}{3}\left(g_{iklm} \frac{\partial x^i}{\partial x^{i'}} \frac{\partial x^k}{\partial x^{k'}} \frac{\partial x^l}{\partial x^{l'}} \frac{\partial^2 x^m}{\partial x^{m'} \partial x^{j'}} + \right.$$
$$+ g_{ijlm} \frac{\partial x^i}{\partial x^{i'}} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^l}{\partial x^{l'}} \frac{\partial^2 x^m}{\partial x^{m'} \partial x^{k'}} + g_{ijkl} \frac{\partial x^i}{\partial x^{i'}} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^k}{\partial x^{k'}} \frac{\partial^2 x^m}{\partial x^{m'} \partial x^{l'}} + \tag{43}$$
$$\left. + g_{ijkl} \frac{\partial x^i}{\partial x^{i'}} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^k}{\partial x^{k'}} \frac{\partial^2 x^l}{\partial x^{l'} \partial x^{m'}} - g_{ijlm} \frac{\partial x^j}{\partial x^{j'}} \frac{\partial x^k}{\partial x^{k'}} \frac{\partial x^l}{\partial x^{l'}} \frac{\partial^2 x^m}{\partial x^{m'} \partial x^{i'}} \right),$$

from which it follows that the functions (28) are not components of 5-indices tensor.

The gravitational field in the linear approximation

Let's consider metric function in Finsler space of Bervald-Moor

$$ds = \left[g_{0000} \left(dx^0\right)^4 - 2\left(dx^0\right)^2 \left(dx^2 + dy^2 + dz^2\right) + dx^4 + dy^4 + dz^4 - \right.$$
$$\left. -2\left(dx^2 dy^2 + dy^2 dx^2 + dz^2 dx^2\right) + 8 dx^0 dx^1 dx^3 dx^4 \right]^{1/4}, \tag{44}$$

in which only one quantity g_{0000} poorly differs from unit. In approach to classical mechanics is summand with $\left(dx^0\right)^4$ which leads function (44) in the form

$$ds = g_{0000}\left(dx^0\right)^4 + dx^4 + dy^4 + dz^4 - 2\left(dx^2 dy^2 + dy^2 dx^2 + dz^2 dx^2\right). \tag{45}$$

Then the non-zero values of the components of the object (28) have the form

$$\Gamma_{i,0000} = -\frac{1}{12} \frac{\partial g_{0000}}{\partial x^i}. \tag{46}$$

The equation geodesic (30), presented in the form of

$$\frac{d\left[\left(\mathbf{v}(\mathbf{vv})\right) - \left\{\mathbf{v}\{\mathbf{vv}\}\right\}\right]}{dt} = \frac{c^2}{4} \frac{\partial g_{0000}}{\partial \mathbf{r}}, \tag{47}$$

allows, taking into account (15), to receive force expression

$$\mathbf{F} = \frac{c^2}{4} \frac{\partial g_{0000}}{\partial \mathbf{r}} = -\frac{\partial \varphi}{\partial \mathbf{r}}. \tag{48}$$

From (48) it follows value of the component

$$g_{0000} \approx 1 - \frac{4\varphi}{c^2}, \quad \left(0 < \frac{4\varphi}{c^2} < 1\right) \tag{49}$$

and after substituting it into (45) we have

$$ds = \left(1 - \frac{4\varphi}{c^2}\right) c^4 dt^4 + dx^4 + dy^4 + dz^4 - 2\left(dx^2 dy^2 + dy^2 dx^2 + dz^2 dx^2\right). \tag{50}$$

Thus g_{0000} is the gravitational potential, and $\Gamma_{i,0000}$ is strength of the gravitational field. At last, we write the equation for g_{0000}

$$\Delta_4 g_{0000} = -\frac{4}{c^2} \beta\rho. \tag{51}$$

In general case, when the components g_{ijkl} are depend on time, the field equations are non-linear terms

$$\square_4 g_{ijkl} + \text{"non-linear terms"} = \frac{4}{c^2}\beta T_{ijkl}\,, \tag{52}$$

obtaining of which requires special consideration. Here, $T_{ijkl} = \rho c^2 u^i u^j u^k u^l$ is 4-indices energy-impulse tensor and the scalar operator [7]

$$\begin{aligned}\square_4 &= \frac{\partial^4}{c^4\partial t^4} - 2\frac{\partial^2}{c^2\partial t^2}\Delta + \frac{4}{3}\frac{\partial}{c\partial t}\left(\nabla\{\nabla\nabla\}\right) + \Delta\Delta - \left(\{\nabla\nabla\}\{\nabla\nabla\}\right) = \\ &= \frac{\partial^4}{c^4\partial t^4} + \frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial y^4} + \frac{\partial^4}{\partial z^4} - 2\left(\frac{\partial^4}{c^2\partial t^2\partial x^2} + \frac{\partial^4}{c^2\partial t^2\partial y^2} + \frac{\partial^4}{c^2\partial t^2\partial z^2} + \frac{\partial^4}{\partial x^2\partial y^2} + \right. \\ &\left. + \frac{\partial^4}{\partial y^2\partial z^2} + \frac{\partial^4}{\partial z^2\partial x^2}\right) + 8\frac{\partial^4}{c\partial t\partial x\partial y\partial z} = \prod_m^4\left[\frac{\partial}{c\partial t} + \left(\boldsymbol{\varepsilon}^m\nabla\right)\right].\end{aligned} \tag{53}$$

The solution of equations for the potential

Consider the case of fourth-order equation with an operator (53) for the potential $\varphi = \varphi(s)$

$$\square_4\varphi = 0\,, \tag{54}$$

which depends only on s

$$\begin{aligned} s &= \left[(ct+x+y+z)(ct-x+y-z)(ct+x-y-z)(ct-x-y+z)\right]^{1/4} = \\ &= \left[c^4t^4 + x^4 + y^4 + z^4 - 2\left(c^2t^2x^2 + c^2t^2y^2 + c^2t^2z^2 + x^2y^2 + y^2z^2 + z^2x^2\right) + 8ctxyz\right]^{1/4}.\end{aligned} \tag{55}$$

We substitute $\varphi(s)$ in (54) and obtain the following equation

$$\frac{\partial^4\varphi}{\partial s^4} + \frac{6}{s}\frac{\partial^3\varphi}{\partial s^3} + \frac{7}{s^2}\frac{\partial^2\varphi}{\partial s^2} + \frac{1}{s^3}\frac{\partial\varphi}{\partial s} = 0\,. \tag{56}$$

Solving it, we find the explicit form of function

$$\varphi = A\ln\frac{s}{s_0}\,, \tag{57}$$

where A и s_0 is constant quantities. At $s = s_0$ have the equality $\varphi(s_0) = 0$.

Then we move on to the case of the initial equation

$$\Delta_4\varphi = 0\,, \tag{58}$$

which follows from (21) with $\rho = 0$. According to (55), we write

$$s = \left[(x+y+z)R^3\right]^{1/4}, \tag{59}$$

where function R, according to (3), depends on three independent vectors $\boldsymbol{\varepsilon}^2 = (-1,1,-1)$, $\boldsymbol{\varepsilon}^3 = (1,-1,-1)$, $\boldsymbol{\varepsilon}^4 = (-1,-1,1)$ of allocated directions and has the form

$$\begin{aligned} R &= \left[(-x+y-z)(x-y-z)(-x-y+z)\right]^{1/3} = \\ &= \left[x^3 + y^3 + z^3 - \left(x^2z + z^2x\right) - \left(y^2x + x^2y\right) - \left(z^2y + y^2z\right) + 2xyz\right]^{1/3}.\end{aligned} \tag{60}$$

Therefore, the equation (58) can be written as

$$\left(\frac{\partial}{\partial x}+\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\left(\frac{\partial^3\varphi}{\partial R^3}+\frac{3}{R}\frac{\partial^2\varphi}{\partial R^2}+\frac{1}{R^2}\frac{\partial\varphi}{\partial R}\right)=0 \tag{61}$$

and its solution is the function

$$\varphi = A\ln\frac{R}{B}+C_1x+C_2y+C_3z,\ \left(C_1+C_2+C_3=0\right), \tag{62}$$

where A, C_1, C_2, C_3 и B is constant quantities. Equality $\varphi(R_0)=0$ takes place at $C_1=C_2=C_3=0$ and $B=eR_0$. Then, from (62), according to (49), we obtain the following expression for the potential

$$\frac{4\varphi}{c^2}=\ln\frac{eR_0}{R},\ \left(0<R<eR_0\right). \tag{63}$$

Function R_0 is function of the distant horizon in proper space.

Similarly, we have the expression

$$s=\left[\left(-x+y-z\right)\xi^3\right]^{1/4}=\left[\left(x-y-z\right)\eta^3\right]^{1/4}=\left[\left(-x-y+z\right)\nu^3\right]^{1/4}, \tag{64}$$

where the functions ξ, η, ν also depend only on the three independent vectors of allocated directions and have the form

$$\begin{aligned}\xi&=\left[\left(x+y+z\right)\left(x-y-z\right)\left(-x-y+z\right)\right]^{1/3},\\ \eta&=\left[\left(x+y+z\right)\left(-x+y-z\right)\left(-x-y+z\right)\right]^{1/3},\\ \nu&=\left[\left(x+y+z\right)\left(-x+y-z\right)\left(x-y-z\right)\right]^{1/3}.\end{aligned} \tag{65}$$

Thus, in addition to (61) we have the following equations

$$\begin{aligned}&\left(-\frac{\partial}{\partial x}+\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)\left(\frac{\partial^3\varphi}{\partial\xi^3}+\frac{3}{\xi}\frac{\partial^2\varphi}{\partial\xi^2}+\frac{1}{\xi^2}\frac{\partial\varphi}{\partial\xi}\right)=0,\\ &\left(\frac{\partial}{\partial x}-\frac{\partial}{\partial y}-\frac{\partial}{\partial z}\right)\left(\frac{\partial^3\varphi}{\partial\eta^3}+\frac{3}{\eta}\frac{\partial^2\varphi}{\partial\eta^2}+\frac{1}{\eta^2}\frac{\partial\varphi}{\partial\eta}\right)=0,\\ &\left(-\frac{\partial}{\partial x}-\frac{\partial}{\partial y}+\frac{\partial}{\partial z}\right)\left(\frac{\partial^3\varphi}{\partial\nu^3}+\frac{3}{\nu}\frac{\partial^2\varphi}{\partial\nu^2}+\frac{1}{\nu^2}\frac{\partial\varphi}{\partial\nu}\right)=0\end{aligned} \tag{66}$$

and their solutions are functions

$$\begin{aligned}\varphi_1&=A_1\ln\frac{\xi}{B_1}+C_1x+C_2y+C_3z,\ \left(-C_1+C_2-C_3=0\right),\\ \varphi_2&=A_2\ln\frac{\eta}{B_2}+C_1x+C_2y+C_3z,\ \left(C_1-C_2-C_3=0\right),\\ \varphi_3&=A_3\ln\frac{\nu}{B_3}+C_1x+C_2y+C_3z,\ \left(-C_1-C_2+C_3=0\right).\end{aligned} \tag{67}$$

The gravitational redshift

Consider, according to (60) and the solution (63), the time coordinate interval between the sequence of impulses of some signal source at some fixed point $\mathbf{r}_1=\left(x_1, y_1, z_1\right)$ for R_1

$$\Delta t_1 = \frac{\Delta s}{\left[1 - \frac{4\varphi(R_1)}{c^2}\right]^{1/4}} \tag{68}$$

Similarly, at another point $\mathbf{r}_2$ for R_2, we have

$$\Delta t_2 = \frac{\Delta s}{\left[1 - \frac{4\varphi(R_2)}{c^2}\right]^{1/4}} \tag{69}$$

Coordinate frequency of signals $\nu_2 = (\Delta t_2)^{-1}$ in a point $\mathbf{r}_2$ connected with the frequency $\nu_1 = (\Delta t_1)^{-1}$ in the point $\mathbf{r}_1$ of approximate equality

$$\frac{\nu_2}{\nu_1} = \sqrt[4]{\frac{1 - \frac{4\varphi(R_1)}{c^2}}{1 - \frac{4\varphi(R_2)}{c^2}}} \approx 1 + \frac{1}{c^2}\left[\varphi(R_2) - \varphi(R_1)\right] \tag{70}$$

with an accuracy of $(1/c^2)$. From (70) we obtain the frequency shift of the signal corresponding to the values R_2 and R_1

$$\Delta\nu = \nu_2 - \nu_1 = \frac{\nu_1}{c^2}\left[\varphi(R_2) - \varphi(R_1)\right] = \frac{\nu_1}{c^2} \ln\frac{R_1}{R_2}. \tag{71}$$

If R_2 is a function of the coordinate of somebody outside the gravitating body, and R_1 – on the body, then the formula (71) shows the frequency shift to the red side at $R_2 > R_1$ since the $\varphi(R_2) < \varphi(R_1)$. According to (60), given the gravitational redshift of the spectral lines depends on three independent vectors of preferred directions in own three-dimensional space.

Similarly, taking into account (67) considered shift will also depend on other three preferred directions independent vectors. Here have selected only one solution, depending on the function R. The general solution of equation (58) is a linear combination of the four solutions (62) and (67).

Conclusion

The approximation of a weak gravitational field in Finsler space events Berwald-Moor possible to determine the gravitational redshift. The solution of equation (15) is a separate problem, as it does consideration of movements and trajectories of test particles in the approximation of a non-Newtonian classical physics. It seems it necessary to find a tensor, which in the linear approximation in the metric tensor, independent of time, lead to the equation (51) of the fourth order. The construction of such a theory requires special consideration. Therefore here are limited to only a question of geodesic lines and a weak gravitational field.

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Search Finsler metric effects: quantum interferometer to measure the local anisotropy

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The precise recording methods of metric anisotropy with co-operation for this purpose SQUID system. In quasi-static method of SQUID along with superconducting flux transformer, consisting of two electrically selfcontained at each other is perpendicular disposed superconducting turns, registers the effect redistribution between turns stored in them flux, arising from metrics anisotropy when slow rotation of transformer at 90º. In dynamic method owing to revolution of magnetostrictive cylinder (in the plane of perpendicular its axis) in space with anisotropic metrics of SQUID shall be filing the magnetic repercussion, responding the oscillations of longitudinal extension of cylinder.

Relativistic effects are closely connected to the natural anisotropy, possessed by the motion and gravity. In the special relativity theory this anisotropy manifests itself in the difference of Lorenz transformations along and across the motion direction, while in the general relativity theory it is present in the form of anisotropic influence of gravity on the appropriate components of the metric tensor, responsible for linear scale along and across the vector of gravity field strength. The natural anisotropy of relativistic effects is reflected in the geometry of the classical Michelson/Morley experiment, too. In this experiment the role of the velocity of the laboratory coordinate frame is played by the solar escape velocity, with which the Earth moves around the Sun (30 km/s). In this case, with respect to the Sun the interferometer was in the zero-gravity state. However, if an interferometer with mutually perpendicular arms, one of which is directed towards the centre of the gravitation field, will be actually at rest with respect to this centre, then the variation of the gravitational potential $\delta\varphi G$, corresponding to slow displacement of the device up or down by the distance δh, will cause the displacement of the resulting interference pattern. The effect will be, in principle, observable, but obviously hard to detect, since its order of magnitude will be $\frac{\delta\varphi_G}{c^2} = \frac{1}{c^2}\frac{\partial\varphi_G}{\partial z}\delta h$, where the strength of the gravitational field at the surface of the Earth is $\frac{\partial\varphi_G}{\partial z} \approx 9.8\, m/s^2$.

The required sensitivity may be provided by quantum interferometers [1], registering the phase change $\delta\Theta$, which is proportional to the effective elongation δl, expressed in the units of the operating wavelength, i.e., $\delta\Theta=2\pi\delta\ell/\lambda$, and the operating wavelength in the superconducting ring of a SQUID, depending on the accumulated magnetic flux Φ, can be essentially smaller than the typical one in common optical systems, $\lambda SQUID=2\pi R\Phi 0 /\Phi$ (R is the ring radius, $\Phi 0=\pi\hbar/e$). Consider a flux transformer (fig.1), incorporating two superconducting loops, closed on each other, with the inductances $L_{\perp}$ and $L_{\|}$ and the lengths $\ell_{\perp}$ and $\ell_{\|}$, the planes of which are oriented horizontally and vertically with respect to the direction of the gravitation field strength.

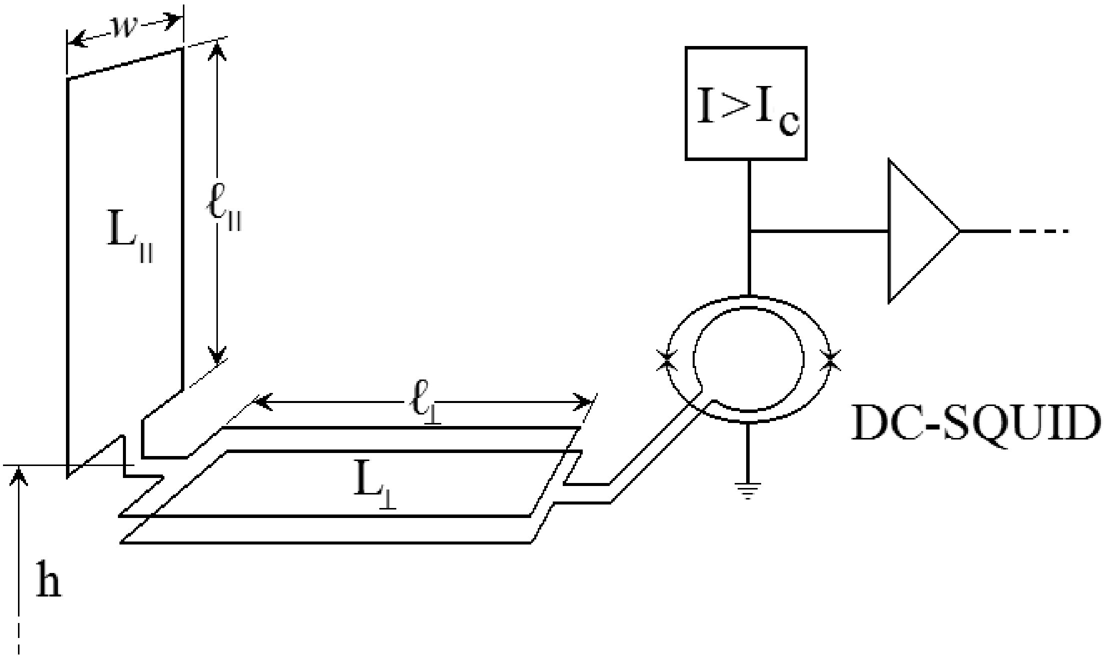

Figure 1. The scheme of quasistatic (the slow expansion above Earth surface) registration of changes of gravity potential by means of «superconducting Michelson interferometer», formed by closed-loop the superconducting circuit, including as of shoulders two turn $L_{\perp}$ and $L_{\|}$ which lay in horisontal and vertical planes. Redistribution of magnetic flux in turns, responding the anisotropic change of relative geometric dimensions shoulders $\ell_{\perp}$ and $\ell_{\|}$ caused by weak (~c-2) metrics variations in changing the gravity potential, is fixed by the operating coil of DC-SQUID. The crosses in the operating ring of the DC SQUID mark Josephson tunnel junctions – the basic elements of the superconducting quantum interferometer.

Assume that N flux quanta are stored in the transformer, then $N\Phi_0 = \Phi_{\|} + \Phi_{\perp} = (L_{\|} + L_{\perp})I$ (I is the current in the flux transformer) and, therefore, $\Phi_{\perp} = L_{\perp}I = L_{\perp}\dfrac{N\Phi_0}{L_{\|} + L_{\perp}}$. Expressing the inductance L of the long rectangular loop with the length, much greater than its width ℓ>>w, in terms of its specific value $L = \dfrac{\partial L}{\partial \ell}\ell$, assuming the specific inductances to be equal in both loops $\dfrac{\partial L_{\|}}{\partial \ell_{\|}} = \dfrac{\partial L_{\perp}}{\partial \ell_{\perp}}$, and neglecting the contribution of the ends, we get: $\Phi_{\perp} = N\Phi_0 \dfrac{\ell_{\perp}}{\ell_{\|} + \ell_{\perp}}$ and

$\delta\Phi_{\perp} = -\frac{N\Phi_0 \ell_{\perp}}{(\ell_{||} + \ell_{\perp})^2}\delta\ell_{||}$. Assuming that with no influence of gravitation taken into account the loops have equal lengths $\ell_{||} = \ell_{\perp} = \ell_0$ and using the law of linear scale transformation $\ell_{||} = \ell_0\sqrt{g_{zz}(z)} = \ell_0\sqrt{\left(1+\varphi_G / c^2\right)^{-1}}$ $\ell_{\perp} = \ell_0\sqrt{g_{\perp\perp}} = \ell_0$, describing the relativistic gravity effects in terms of the appropriate components of the metric tensor, we arrive at the expression for the flux variation in the horizontal loop $\delta\Phi_{\perp}$, arising when the superconducting interferometer is lifted by the height δh: $\delta\Phi_{\perp} = -\frac{N\Phi_0}{8c^2}\frac{\partial\varphi_G}{\partial z}\delta h$. For quantitative estimation let us substitute the Earth gravitation parameter $\frac{\partial\varphi_G}{\partial z} \approx 9{,}8\text{м}/s^2$ into the obtained formula and assume that the superconducting loops, in which 109 flux quanta are stored, are lifted up the height 800m. Then the flux will be redistributed between the loops so that $\delta\Phi_{\perp} \approx -10^{-6}\Phi_0 = -2{,}07\times10^{-21}Wb$. If the elevation was performed with the velocity 0.5 m/s, then this decrease of the flux will be registered by the SQUID in the frequency band δf=1/t=(0.5m/s)/(800m)=6.25×10–3Hz, so that $\sqrt{\Delta f} = 0{,}025Hz^{1/2}$. Thus, to make the registration of the effect possible, the flux fluctuations in the quantum interferometer should not exceed 10–6Φ0/0.025Hz1/2 = 4×10–5Φ0/Hz1/2, whereas the noise amplitude in modern SQUIDs is almost two orders of magnitude smaller.

Directing the turning axis of coils normally to Earth surface (fig.2), it is possible in this way to measure the metrics anisotropy also not engaged in gravitation.

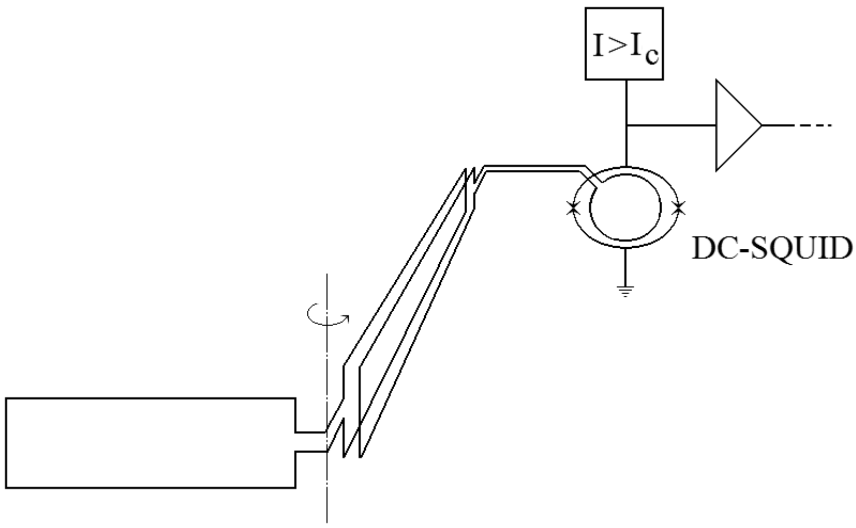

Figure 2. The quasistatic registration procedure of «transversal» anisotropy. It is intended that the turning of superconducting frames about the axis are produced rather slowly. Designations are alike as on fig.1.

The anisotropy is fixed by the SQUID because of redistribution of magnetic flux in perpendicular frame coils during their slow quasistatic turn with retention of total value $\Phi=\Phi1+\Phi2=const$. In principle notified the anisotropy can be caused by the effects of Finsler metrics [3-6] with which in modern cosmological theories, connects observed the matter distribution in the Universe.

Along with described above quasi-static registration procedure of anisotropy it is possible to view also dynamical the method (fig 3), in which mechanic the variation in dimension of rotating probe body, arising on twice the rotation frequency in lateral direction axis of such comparatively rapid rotation are fixed.

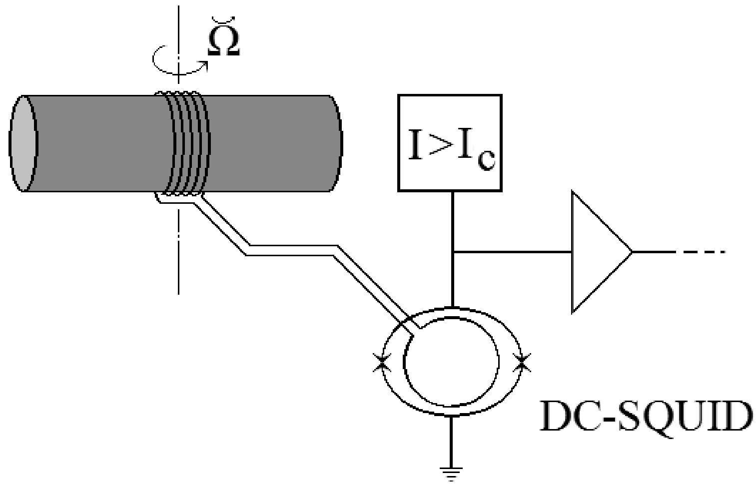

Figure 3. The dynamic registration procedure «transversal» anisotropy. The magnetostrictive cylinder grey coloration. It is intended that the rotation by about axis is done fast enough, so that in cylinder, both in elastic mechanical system at frequency $2\breve{\Omega}$ will come into being elongation oscillation.

Of course, that to mark the periodic alternations of cylinder elongation (fig 3) responding metrics anisotropies will be required the high sensitive system with regard to parameters comparable the gravitational wave detectors of Weber type. In works [2,7,8] usability of magnetostriction transducer «elongation→magnetization» connected with quantum interferometer as gravitational wave detector (fig 4). The operation of the SQUID/magnetostrictor system as pressure or elongation transducer, is based on the inverse magnetostriction effect (discovered by E.Villari in 1865), that is characterized by the quantity of magnetostrictive sensitivity $\Lambda^{(-1)} = \frac{\partial B}{\partial P}$, which demonstrate the quantitative connection of change of magnetic induction with elastic stress, causing this alteration in concrete material.

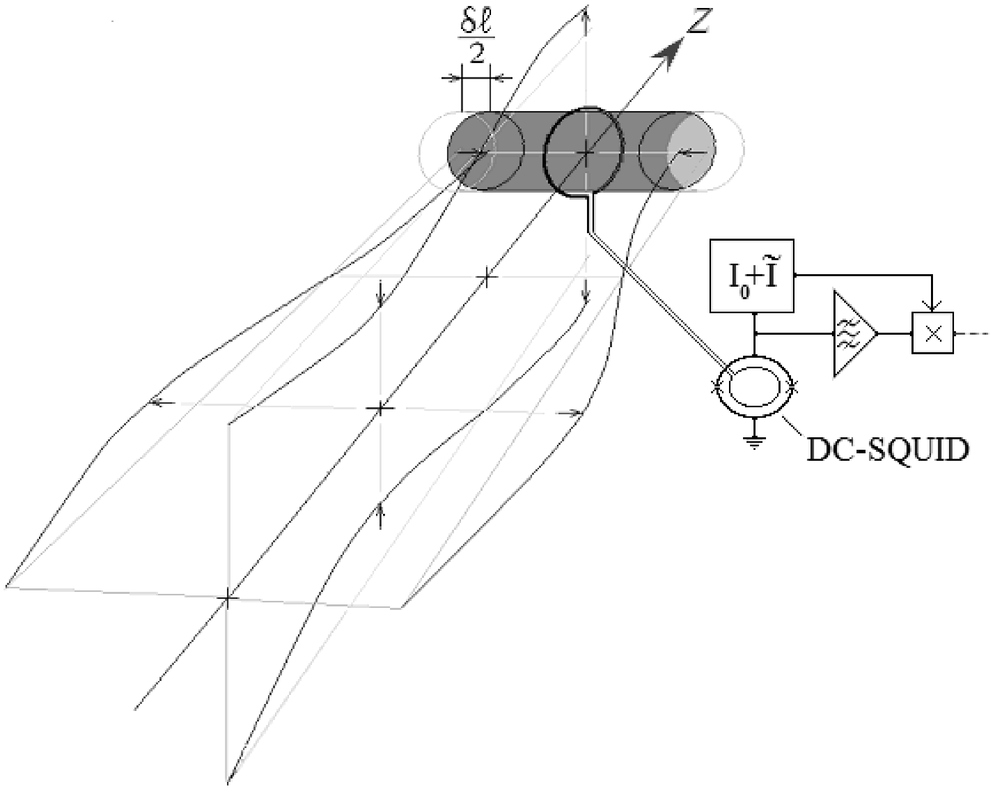

Figure 4. Qualitative relation between the geometry of gravitational wave propagation and the position of magnetostrictive antenna, the change of magnetic flux in which is detected by the SQUID. In the foreground the plane-polarised gravitational wave is schematically shown. Arrows point at the regions of decreasing and increasing dimensions of a virtual test body, caused by the influence of the gravitational wave. The proportion between the wavelength λ and the dimension ℓ of the cylindrical antenna is deliberately distorted (really λ is greater than ℓ by nearly 5 orders of magnitude); for clarity also exaggerated (by 20 orders of magnitude) is the variation of geometric dimensions of the test body in the field of the gravitational wave. The crosses in the operating ring of the DC SQUID mark Josephson tunnel junctions – the basic elements of the superconducting quantum interferometer.

Let us estimate the ultimate capabilities of this method in detecting the magnetic response by means of a superconducting quantum interferometer under the condition ΔΦ=δΦ, i.e., when the magnetic signal is equal to the resolution of the SQUID. The variation of the flux of the magnetic induction B through the section S of the magnetostriction cylinder is determined by the relation δΦ=ΔΦ=S ΔB=S Λ(-1)ΔP from where the ultimate sensitivity of the SQUID/magnetostrictor system used for pressure measurement is expressed in terms of the resolution of the used quantum interferometer in the form δP=δΦ/(SΛ(-1)). If $\langle\delta\Phi\rangle_{/\sqrt{Hz}} = 10^{-6}\Phi_0/\sqrt{Hz} = 2{,}07\times10^{-21}Wb/\sqrt{Hz}$, S=0.003m2 and Λ(−1)≈7×10–6T/Pa, then the ultimate sensitivity of the system is $\langle\delta P\rangle_{/\sqrt{Hz}} = 10^{-13}Pa/\sqrt{Hz}$. This pressure, which in principle can be still detected by the system in the unit bandwidth, corresponds to the ultimate detectable elongation of $\langle\delta\ell/\ell\rangle_{/\sqrt{Hz}} = 10^{-24}/\sqrt{Hz}$ in a magnetostrictor with the typical value of the Young modulus E=100GPa. Of course the dynamical registration procedure «transversal» anisotropic can be used and for solving much less extravagant missions, than search observed traces of Finsler metrics. Most important applied by the using direction of similar methodic is the gravi-exploration. In work [2] it is shown that with the same sensitivity of the SQUID/magnetostrictor system and the acquisition time 100s it is possible to detect the gravity field perturbation at the level of $\delta g \sim 10^{-13}(m/s^2)\big/\sqrt{Hz} = 10\,pGal\big/\sqrt{Hz}$, produced for example by a light tank (with the mass ~15t), at the distance of 10km. And finally, latest: the energy density ρE=1 keV/cm3 corresponds to the internal pressure Pint=ρE≈1.6×10-10Pa. In correspondence with the estimates, this is just the density ρE=1 keV/cm3 that should be possessed by the Dark Energy [9–13], the everywhere presence of which explains the additional acceleration in recession of galaxies, as compared to Hubble's law V=HR (the velocity of recession V is proportional to the distance R to the observed objects, H being Hubble's constant). One of the popular hypotheses about the nature of dark energy actually identifies it with the vacuum of quantised electromagnetic field. In principle, this model allows implicit registration of dark energy density variations by performing long-term observations of the Casimir effect, which consists in appearance of small difference of pressures, exerted by virtual photons from inside and outside the gap between two closely spaced parallel mirrors. The calculation, accounting for the resonance factors in the statistics of virtual photons that are created and annihilated in the quantum-field vacuum, shows that the pressure difference $\Delta P_V = \dfrac{\pi^2 \hbar c}{240 d^4} \approx \dfrac{1{,}2\times10^{-27}}{d^4{}_{[m]}}{}_{[Pa]}$ [14-17]. If d=50μm is the width of the gap that plays the role of a resonator for virtual photons, then this difference will, on average, amount to ~2×10-10~Pa. Thus, the SQUID/magnetostrictor system, used as a sensor for detecting the variations of the internal pressure Pint or the variations ΔPV, relevant to the Casimir effect, with the sensitivity $\sim 10^{-13}\,Pa\big/\sqrt{Hz}$, will allow laboratory studies of periodical variations of dark energy, corresponding to complex polycyclic motion of the Earth in the space.

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General principle of relativity in Finsler geometry and its generalizations

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Paradoxical situation has long been formed in the physical sciences. From the middle of the last century, in estimating the value of the general principle of relativity (GPR), the opinion of the physics community was divided into two oppositely point of view. We will not enumerate all the supporters and opponents of any of them, and for the sake of brevity, we combine them with the names of their representatives – the greatest physicists of the 20th century. Let's call the first one – the point of view of V. Gisnzburg (see, eg, (1)), and the second – the point of view of V. Fock (see, eg, (2; 3)). The first interprets the general theory of relativity (GTR) of A. Einstein as the most important achievement of physical thought of the 20th century. The second denies the role of GPR as a fundamental physical principle. Purpose of this work – give an idea of the current state of the question. The true meaning of the principle of relativity is revealed in the introduction of new geometries to the physical science, that are more general than the geometry of Riemann spaces, serving as the mathematical foundation of general relativity. These include the geometry of Finsler spaces and its generalizations — the geometry of spaces with an areal metric (see, eg, (10) – (13)).

Introduction

The general principle of relativity was formulated by A. Einstein in 1916 in the work (14): «The laws of physics have to be composed so they would be valid for arbitrarily moving coordinate systems». In the same work, and further, this provision has been used by him and other authors in this formulation: «The laws of nature has to be covariant with respect to arbitrary continuous transformations of coordinates». A. Einstein named this principle of general covariance as the general principle of relativity.

In the mid 50-ies of the last century in the physics community, there has been a serious split of opinions regarding the role and importance for the physical science of the general principle of relativity. Thus, V. Gizenburg in (1) called «... the general theory of relativity (GTR) is the greatest scientific achievement, created by the genius of Einstein». There he also formulated his views on the GTR, which he had repeatedly saying: «There is no doubt that the general theory of relativity will remain for centuries as one of the greatest achievements of human thought.»

In his turn, the V. Fock (2) in the same publication reported that «the theory of gravity misunderstood by its author.» Moreover, in his famous book (3), he makes even more radical conclusion: «The general principle of relativity is not possible as a physical principle which would have taken place in relation to the arbitrary reference systems.» Fock's statement caused, in it's time, downright painful response of some physicists.

In this context, we should mention that in the 70 – 80 years of the last century through the efforts of a number of russian and foreign experts some progress has been made in clarifying the role of the general principle of relativity in the physical sciences. We will not list the variants of approaches that being developed at that time and refer the reader to the review of N. Chernikov (4) and a very useful to a wide range of readers Y. Vladimirov's book (5).

Before going back to our days it would be appropriate to recall that 150 years ago R. Riemann, in his dissertation, presented to obtain the rank of assistant professor, gave an extremely profound idea (see (6)). Considering the definition of the metric as a measure of the distance between the «points» of an abstract geometric space, he noted that there may be two and only two possibilities. Either the space is discrete, then its metric lies within it and is given by counting of discrete elements. Or the space is a continuous space, then its metric can not be comprise directly in itself, but must be set from the outside.

According to A. Einstein, the metric of our world, ie the 4-dimensional space-time must be assigned to the second of these types of R. Riemann. However, the development of quantum physics in a new way has raised the question about the structure of space and time in the microworld. These days, it became apparent that in the scale of the microcosm space-time is discrete with the characteristic values determined by the Planck length l_{Pl} and time t_{Pl}. In other words in microcosm, apparently, the first of these features described by R. Riemann takes place. In the framework of the classical ideas discrete space can be associated with a set of crystal lattice sites. It is clear that such a structure cannot be anisotropic (7).

At present, we are witnessing that the in place of the Riemann geometry, which occupied a dominant position in physics (gravity, astrophysics, cosmology, etc.) for nearly a century, began to arrive new, more general geometric theories. We are talking about Finsler geometry (P. Finsler: 1918) (8), as it is currently presented and its further generalizations. We refer to the important generalizations of Finsler geometry, first of all, the geometry of spaces with areal metric (see, eg, (11) – (12) and references cited therein).

The interest in these geometries is caused primarily by the fact that the Riemann geometry is a special case of Finsler geometry. In turn, the interest in the geometry of spaces with areal metric arose in physics at most recent twenty years. This geometry includes the Finsler geometry as a special case and it is a typical example of non-commutative geometry. According to the classification of G. Riemann, it refers to the first type of geometries.

Beginnings of the geometry of spaces with areal metric described in the famous paper of an outstanding mathematician of the 20th century V. Wagner (15). Further development of the ideas of V. Wagner got in the works of G. Zhotikov (16) and N. Kabanov (17) and many of his disciples.

The purpose of this work is to look at the general principle of relativity from a height of geometries which include Riemann geometry as a special case. The result of such a view would be a modern point of view on the general principle of relativity and its role in modern physical theories.

The paper is organized as follows. Sections 2 and 3 provides the necessary information about the Finsler's geometry and the geometry of spaces with an areal metric. Section 4 discusses the geometric interpretation of the principle of gauge invariance. Section 5 introduces the concept of global space of observers. It's geometric model is a fiber bundle. The structure and properties of it are determined by the structure of the Lagrange function (Lagrangian) of physical problem. 4-dimensional space-time of both general and special theories of relativity is immersed in this space. Here also the role of the general principle of relativity discussed in terms of geometries which is more general than the Riemann geometry, and which is in its turn is a mathematical basis of the general theory of relativity (GTR). Finally, Section 5 we briefly formulate obtained results. Further throughout the paper we use the general system of units in which $h = c = 1$. Then an action is expressed by the dimensionless quantity.

We'll start with recalling some mathematical facts underlying the Finsler geometry and geometry of spaces with an areal metric.

Some facts of the Finsler's geometry

We come to Finsler's geometry by giving a geometric interpretation of the principle of least (extremal) action for an ordinary integral (see, e.g., (9), (10), (13) etc.).
This is done as follows. Suppose we are given the action functional of physical system

$$S[\gamma] = \int_{t_1}^{t_2} L(q, \dot{q})\, d\tau. \tag{1}$$

Here L —- Lagrange function (Lagrangian); $q = \{q^\alpha(\tau)\}$ — generalized coordinates, $\dot{q} = \{\dot{q^\alpha}(\tau)\}$, where $\dot{q^\alpha} = \dfrac{dq^\alpha}{d\tau}$ — generalized velocitis, $\alpha, \beta = 1, ..., n$. The integral is defined on the set of oriented curves γ,

$$q^\alpha = q^\alpha(\tau),\ \tau \in [t_1, t_2], \tag{2}$$

describing all the possible trajectories of the system.
The set of n variables $\dot{q}^\alpha$ is interpreted as a tangent vector to the curve γ and belongs to the tangent vector space $T^n(P)$ at the point $P(q^\alpha(\tau))$. The set of $2n$ numbers $(q^\alpha(\tau), \dot{q}^\alpha(\tau))$ defines a tangent line element at a certain point of the curve γ. Then the action S can be interpreted as the length of the arc of said oriented curve γ in the some abstract geometric space. An important example of the concept of geometric space is the space of configurations of the some dynamic system.

Let some dynamical system has n degrees of freedom. This means that each position can be determined by n independent parameters (generalized coordinates) $q^\alpha, \alpha = 1, ..., n$. Each position of the system (for fixed values of q^α is called its configuration. All the feasible set of states of the system is called the set of its configurations.

When we say that the system has n degrees of freedom, it is meant that each configuration of the system is defined by specifying an ordered system of n numbers. For example, a solid body moving in the ordinary Euclidean 3-space has six degrees of freedom: three coordinates of the center of gravity x, y, z and three Euler angles φ, ψ, θ for each configuration.

By introducing the concept of the configuration space the whole dynamics of the system under consideration is reduced to the study of motion of a point in a multidimensional space of configurations. If the classical dynamic system has n degrees of freedom, and t – invariant (proper) time, then the equations of motion have the form

$$q^\alpha = q^\alpha(t).$$

Geometrically, these equations describe a curve in the space of configurations, which we identify with the geometric space X^n. Let us now recall definition of a Finsler space.
Definition 1. *Geometric space X^n, in which the length of s curve arcs are defined by the expression (1) is called the Finsler space. The geometry of this space is called the Finsler geometry.*

The invariance of the action integral (1) with respect to transformation of general form

$$t \to t'(t);\ q^\alpha \to q^{\alpha\prime} = q^{\alpha\prime}(q^\alpha), (Det|\frac{\partial q^{\alpha\prime}}{\partial q^\alpha}| \neq 0) \tag{3}$$

impose restrictions on the structure of the Lagrangian $L(q, \dot{q})$. It should be:
1) a positive homogeneous one, of the first degree with respect to generalized speeds;
2) a non-negative ($L(q, \dot{q}) > 0$) for $\dot{q} \neq 0$) and 3) the quadratic form $g_{\alpha\beta}\xi^\alpha\xi^\beta$ from the variables $\xi^\alpha \neq 0$ где $g_{\alpha\beta} = \frac{1}{2}\frac{\partial^2 L}{\partial\dot{q}^\alpha\partial\dot{q}^\beta}$ is positive definite. Conditions 1) – 3) give the classical definition of Finsler metrics. In turn, condition of homogeneity 1) of the Lagrangian leads to the so-called homogeneous (relativistic) Lagrangian. We come to another definition of the Finsler space.
Definition 2. *Geometric space X^n with given Finsler metric in it is called a Finsler space.* Usually its denoted as F^n.

Equation

$$L(q^\alpha, x^\alpha) = 1,\ x^\alpha \in T^n(P),\ P \in X^n \tag{4}$$

determines in every local $T^n(P), P \in X^n$ the hypersurface, which is called the Lagrangian surface of a single action or a local indicator function of Finsler metrics.

The concept of *local indicatrix of metrics* is one of the key concepts in the geometric interpretation of the principle of least action. Very name «indicatrix» comes from the Latin words *indico*

— the pointing determining and signifies that the surface, the auxiliary surface, which characterizes the dependence of any properties of the environment from the direction. To construct the indicatrix from the single point (the center) radius vectors are drawn whose length is proportional to the value, which characterizing the property the environment in a given direction. For example, electrical conductivity, refractive index, modulus, etc.

Let us discuss the physical meaning of the concept of local indicatrix. Local indicatrix is a hypersurface[1] in each local tangent $T^n(P)$, $P \in X^n$. It characterizes the anisotropy of the rate of change of a single action and has a physical dimension[2] $[action : time = energy]$.

In Finsler's geometry Lagrange function acts as a metric function of the configuration space X^n. Each specific Lagrangian of the physical problem has its own local indicatrix, which determines the metric in the configuration space X^n. Thus, by specifying the local indicatrix, ie the surface of rate of change of a single action, is provided introduction of all the necessary procedures for the measurement of physical quantities at every point of space X^n. In other words, the equipping of the space X^n with the field of local indicatrixes means equipping each point in space X^n with ensemble of local observers equipped with a full set of necessary measurement tools and methods for the measurement of physical quantities.

Similarly, in the dual local tangent (more precisely, in the cotangent) space $T^{*n}(P)$, corresponding to the $T^n(P)\, P \in X^n$ with the family of hyperplanes, hypersurface is created

$$H(q^\alpha, y_\alpha) = 1, \;\; y_\alpha \in T^{*n}(P), \; P \in X^n, \tag{5}$$

where H is the Hamiltonian of the system, and y_α is generalized momentum in the $T^{*n}(P)$, wich is canonically conjugate to x^α в $T^n(P)$.

Hypersurface (5) is called the local surface of the Hamiltonian or local figuratrix of the Finsler metrics.

Equations

$$L(q^\alpha, x^\alpha) = 1, \; x^\alpha \in T^n(P), P \in X^n \tag{6}$$

$$H(q^\alpha, y_\alpha) = 1, \; y_\alpha \in T^{*n}(P), P \in X^n \tag{7}$$

define the contravariant and covariant vector metrics in F^n. In according to well-known *theorem of reciprocity*, the local surface of Hamiltonian Π_H (local figuratrix) is a one-polar surface to the surface of Lagrangian Π_L (local indicatrix) and vise versa.

Geometrization of action integral (1) takes the simple and obvious meaning, if from the expression for local indicatrix (6) and figuratrix (7) we go to their entris in the equivalent parametrical

[1]In geometry, a hypersurface is called surface of dimension $n-1$, immersed in n-dimensional space. The term hyperplane has a similar meaning.

[2]In the assumed system of units h = c = 1 its physical dimension is cm^{-1}.

forms:

$$x^\alpha = \ell^\alpha(q^\alpha, \Theta^i) = 1,\ i = 1, ..., n-1, \tag{8}$$

$$y_\alpha = \ell_\alpha(q^\alpha, \Theta^i) = 1,\ i = 1, ..., n-1. \tag{9}$$

Here Θ^i are there the parameters defining the position of a point or a tangent hyperplane on said surface $\Theta^i \in X^{n-1}(P), P \in X^n$. It follows that the state of the object (particle) is described by the coordinates of the position of point on the local indicatrix.

According to V. Wagner (9) the Finsler space F^n is convenient to consider as a fiber space. The basis of it is the space of configurations X^n and the layers are tangent spaces $T^n(P), P \in X^n$ with given them local indicatrisses $X^{n-1}, P \in X^n$. Thus we come to the fiber space $X^{n-1}(X^n)$.

This explains the physical meaning of space $X^{n-1}(X^n)$. This is a geometric model of the space of all arbitrary local ensembles of observers when describing the action functional with a normal integral (1) in the absence of connections imposed on the system. To the discussion of the concept of the space of all possible observers, we'll be back again in Section 5, taking into account the results obtained in sections 3 and 4.

Geometry of Riemann space, which we denote as V^n, is a special case of Finsler geometry of the space F^n. This leads to serious and far-reaching consequences. We recall in this connection that the mathematical foundation of the "classical" GTR is a 4-dimensional Riemannian space V^4.

Indeed, as stated above, Finsler geometry is reduced to field theory of the local hypersurface[3] (9). In general case, the local hypersurface (indicatrix) $X^{n-1}, P \in X^n$ may be, generally speaking, arbitrary. In turn, the Riemann geometry reduces to the field theory of local central hypersurfaces of the second order, ie, to the case when in each local tangent $E^n(P)$, $P \in X^n$ the indicatrix represents a hypersurface of 2nd order.

Let us now consider one of the generalizations of Finsler space. Let us return to the general Finsler space. Suppose now that on the system imposed p linkages where $p = r+1, ..., n$:

$$\Phi_p(q, \dot{q}) = 0. \tag{10}$$

Now we have to deal with the geometrization of the Lagrange problem known in variational calculus. The indicatrix of metric of Lagrange variational problem becomes r-dimensional local surface $X^r(P)$, where $r = 1, ..., n-1$.. Thus, we arrive at the concept of a differentiable manifold with a Lagrange metric. This problem is geometrical term reduced to a field theory of local surfaces

$$\ell^\alpha = \ell^\alpha(q^\beta, \Theta^i),\ (\alpha, \beta = 1, ..., n;\ i = 1, ..., r). \tag{11}$$

[3]In this context, the term field theory has a mathematical sense, for example: theory of the vector field. Not to be confused with the term field theory utilized in physics.

We come to the fiber space of type $X^r(X^n)$, ie, the space X^n with a specified therein metric Lagrange. Thus, we have one of the most important generalizations of the Finsler geometry — to the differentiable manifold X^n with Lagrange metric. In the particular case of the space X^n with Lagrange metric. If there is an $r = n - 1$, we return to the Finsler space F^n.

At the end of the section let's consider the simplest example of a metric function of a "Free" relativistic particle with no charge and spin. We now have to deal with the Minkowski space M^4. In other words, there is a special case: $n = 4; q^\alpha \equiv x^\alpha, \alpha = 0, 1, ..., 3; X^4 \equiv E^4$

$$L(x, \dot{x}) = m\sqrt{\dot{x}^2}. \tag{12}$$

Indicatrix of the metric for the Lagrangian (12) is a 3-dimensional hyperboloid, ie, 3-pseudosphere, centered on each point of the affine space E^4, which is a degeneration of space V^4

$$x^\alpha = \ell^\alpha(\Theta^i), i = 1, 2, 3. \tag{13}$$

This 3-hyperboloid is an example of surface of the 2nd order. He belongs to the local light cone with vertex at each point of $P \in E^4$, which is a degeneration of a Riemannian space V^4. Assignment 3-hyperboloid with vertex at each point in space turns the latter into a Minkowski space M^4.

In this example, Finsler space F^4 degenerates into a Riemannian space V^4, when the local hypersurface (indicatrix) of general form degenerate into the local hypersurface of 2nd order. It can be said that: a geometric interpretation of the action for the "free" relativistic particle (without charge and spin) results in Minkowski space M^4, ie, the space-time of STR. The space-time of GTR V^4 in the absence of curvature at each point, degenerates into an affine space E^4. Latter after setting at every point the indicatrix of the metric (13) is converted into a space-time M^4 STR.

Some facts from the geometry of spaces with areal metric

In the previous section, it was found that the Finsler geometry, which gives a geometric interpretation of ordinary variational problem, can be considered as the geometry of the space X^n in the tangent spaces $T^n(P)$ of which is given tangent $T^P, P \in X^n$ is defined hypersurface Π^{n-1}.

Geometric interpretation of variational problem, for multiple integrals or in other words, the geometric interpretation of the different theories of the field, requires formation of geometry of the space X^n, in tangent E^n of which is defined m-dimensional areal metric, i.e., defined dimension of m-dimensional volumes (areas) in m-dimensional oriented planes (see, for example (12), (15), (16), (16) and references therein).

In the Finsler geometry space X^n has the physical interpretation of extended configurational space where the q^α — generalized coordinates, $\alpha = 1, , ..., n.$. Now the space X^n carries a different physical meaning. It represents the common geometric space, which "points" are now becomes a set of n basic physical fields ψ^α, $\alpha = 1, ..., n$. Latter effect on m-dimensional differentiable manifold, X^m. Wherein satisfies $1 \leq m \leq n-1$.

The space-time of GTR is now considered as a 4-dimensional geometric space X^4 special type that is immersed into the space X^n. And, according to a theorem of Wagner-Kawaguchi ((16), (12)) if the numbers m and n – are relatively prime, then the m-dimensional areal metric X^n induces in the latter m-dimensional metric. The signature of the latter will be determined by the particular values of the numbers m and n.

Geometrization of the variational principle for the case of multiple integral leads to another, more general geometry. This geometry is called the geometry of areal spaces, or the geometry of spaces with an areal metric (15), (16), (17). It includes a Finsler geometry as a special case. A detailed outlining of essence can be found in the author's monograph (12). There you can find the necessary references to the work of other authors. According to the classification of G. Reimann, we are dealing with geometric spaces of the second type.

Geometrization of the principle of least action in the field theory can be mathematically correctly implemented without the involvement geometry of areal spaces. For this reason, our next task is to give a general summary of the geometry of spaces with a given metric. The concepts areal metric and space with an areal metric introduced as follows. We consider the problem of finding an extremum of m-fold integral

$$S = \int \cdots \int_m L(\psi^\alpha, \psi^\alpha_a) du^1 ... du^m \quad (a, b = 1, ..., m) \tag{14}$$

as a function of oriented m-dimensional surface

$$\psi^\alpha = \psi^\alpha(u^a) \tag{15}$$

with a fixed or mobile borderline. The value of the integral (14) with $\psi^\alpha_a = \dfrac{\partial \psi^\alpha}{\partial u^a}$ is called the area of the surface (15). Accordingly, the index m in (14) and in (15) satisfies to the condition $1 \leq m \leq n-1$.

<u>Definition 3.</u> *Space with m-dimensional areal metric called a space (differentiable manifold) X^n, in which the integral (14) is given and it is invariant under a valid parameterization of surfaces (15).*

For the case $m = 1$, we return to the geometrization of the ordinary action integral, or, in other words, the geometry of spaces with an areal metric for m=1 is reduced to the Finsler geometry. If $m = n-1$ (the case of hyperorder action integral) we come to Cartan geometry.

Studying the properties of m-dimensional areal metric in E^n is reduced to geometry of the $m(n-m)$-dimensional surface in the $\binom{n}{m}$-dimensional space M^n_m of all contravariant m-vectors of E^n. Here, the symbol $\binom{n}{m}$ indicates the number of combinations of n on m. Such surface is called a Grassmann indicatrix of m-dimensional areal metric in the local E^n.

The space M_m^n is the space of Klein, in which the transformation of admissible coordinate systems defined by the group of transformations which is a subgroup of the center-projective group in $\binom{n}{m}$ variables[4]. In the above-mentioned specific cases of $m = 1$ and $m = n - 1$, the space M_m^n degenerates into the E^n.

As in the case of Finsler geometry, the notion of indicatrix of m-dimensional areal metric appears very useful. Thus, a necessary condition of Weierstrass, sufficient condition of Weierstrass as well as conditions of the Legendre-Hadamard of extremum of the action functional (14) are interpreted geometrically with the indicatrix of areal metric.

The equations (8) and (9) of the previous section will now look like this:

$$L(\psi^\alpha, x^{\langle\alpha_1 \dots \alpha_m\rangle}) = 1,\ x^{\langle\alpha_1 \dots \alpha_m\rangle} \in M_m^n(P), P \in X^n, \tag{16}$$

$$H(\psi^\alpha, y_{\langle\alpha_1 \dots \alpha_m\rangle}) = 1,\ y_{\langle\alpha_1 \dots \alpha_m\rangle} \in M_m^{*n}(P), P \in X^n, \tag{17}$$

where the symbol $\langle\alpha_1 \dots \alpha_m\rangle$ indicates that indexes $\alpha_1 \dots \alpha_m$ in these expressions are lexicographically ordered. Accordingly, the expression for the local indicatrix (8) and figuratrix (9) will now be:

$$x^{\langle\alpha_1 \dots \alpha_m\rangle} = \ell^{\langle\alpha_1 \dots \alpha_m\rangle}(\psi^\alpha, \Theta^i) = 1,\ \ i = 1, ..., m(n-m), \tag{18}$$

$$y_{\langle\alpha_1 \dots \alpha_m\rangle} = \ell_{\langle\alpha_1 \dots \alpha_m\rangle}(\psi^\alpha, \Theta^i) = 1,\ \ i = 1, ..., m(n-m). \tag{19}$$

So, the geometrization of the variational problem on the unconditional extremum is reduced to theory of the field of local m(n-m)-surfaces, or, in other words, to the fiber space $X^{m(n-m)}(X^n)$. In turn, the Lagrange variational problem for multiple integrals, respectively, reduced to the field theory of local r-surfaces, where $1 \le r < m(n-m)$, or to the fiber space $X^r(X^n)$.

Both of these cases can be combined into one (see, e.g., (12), (16)) when the consideration is subject to a fiber space $X^r(X^n)$, and the dimension of the layer satisfies $1 \le r \le m(n-m)$.

On the geometric interpretation of the principle of the gauge invariance

Gauge principle, along with the variational principle is a cornerstone of modern physics. The terms "gauge symmetry" and "gauge transformations" were introduced by H. Weyl (H. Weyl) around 1920. At that time he was trying to formulate a theory that could unite electromagnetism with the general theory of relativity. He was the first to propose the theory that remains invariant

[4]about this the group see the next section.

to arbitrary expansion and contraction of space-time. In this theory, for each point in space-time was accepted different length and time scales. The work was written in German (18), and H. Weyl used the term «Eich Invarianz», which was originally translated as «scale invariance», but an alternate translation as «gauge invariance» is now generally accepted.

One of the simplest examples of gauge transformations are the gauge transformation in electrodynamics: the electromagnetic field tensor $F_{\mu\nu}(x)$ and Maxwell's equations do not change their form if the 4-vector potential of the electromagnetic field $A_\mu(x)$ transforms like

$$A_\mu(x) \to A_\mu(x) + \partial_\mu \lambda(x), \tag{20}$$

where λ – an arbitrary scalar function of the 4-coordinates x space-time, $\mu = 1, ..., 4$.

In fact, the original idea of H. Weyl was that the transformation (20) have to induce the transformation of the space-time degrees of freedom. However, after criticism of A. Einstain[5], H. Weyl was forced to abandon this attempt, while the formula (20) together with the formula

$$\psi(x) \to exp(\frac{ie}{h}\lambda(x))\psi(x),$$

where ψ – the wave function, and e – electron charge, included in many modern books on physics.

Let us now consider more general case of gauge transformations in the F^n. It is well known that the equations of motion obtained by variation of the action (1) does not alter their form (they are form-invariant) under the transformations of the Lagrange function $L \mapsto' L$:

$$'L = L - \frac{df(q)}{dt}, \tag{21}$$

where $f(q)$ – arbitrary function of the generalized coordinates.
Using the definition of the generalized momentum $p_\alpha = \dfrac{\partial L}{\partial \dot{q}^\alpha}$ and property of the function $L(q, \dot{q})$ (its positive homogeneity of the 1st degree with respect to $\dot{q}$), we have:

$$'p_\alpha = p_\alpha - \sigma_\alpha(q), \tag{22}$$

where σ_α is a covariant n-vector of pulse translation

$$\sigma_\alpha(q) = \partial_\alpha f(q). \tag{23}$$

Geometric interpretation of the gauge transformations (21) was given by V. Wagner (1945) in the (19). Detailed geometric theory of these transformations outlined by him in his paper (20).

[5]From a modern point of view of the criticism was obviously not fair.

Brief case is as follows. To ensure the invariance of the action integral (1), transformation of impulse translations (22) have to induce in the coordinate space conversion

$$'x^{\alpha} = \frac{x^{\alpha}}{1 - \sigma_{\beta}(q)x^{\beta}} \,. \tag{24}$$

In turn, the transformation (24) induce transformation of a pulse translations (24). Such interdependence occurs in projective geometry, thanks to it having the duality principle. The projective space can be viewed as a set of pairs of reciprocal (dual) counter- and covariant vectors. The set of transformations (22) and (24) V. Wagner (20) called the transformations of Caratheodory, referring to the fact that it is C. Caratheodory(21) was the first to apply the transformation (22) to conclude the sufficient conditions of extremum of the action functional (1). From this it is clear that the set of transformations (22) and (24)represent the geometrical interpretation of gauge transformations for the action (1). Sometimes, in order of generality, they include the discrete transformations of reflections from the local $E^n(P)$, $P \in X^n$. They are written in the form:

$$'p_{\alpha} = \varepsilon(p_{\alpha} - \sigma_{\alpha}(q)), \tag{25}$$

$$'x^{\alpha} = \frac{\varepsilon x^{\alpha}}{1 - \sigma_{\beta}(q)x^{\beta}}, \tag{26}$$

where the value $\varepsilon = -1$ corresponds to the operation of reflection from the center of the space. Conversion (25 – 26) are called proper if $\varepsilon = +1$ and improper, if $\varepsilon = -1$. The set of transformation (22) and (24) form a group, while the set of transformations (25) and (26) form a generalized group of Wagner. As parameters in both cases acts defining numbers (components) of a covariant vector of a pulse translations $\sigma_{\alpha}(q)$ in the local $E^n(P), P \in X^n$. This group is one of the subgroup of projective transformations in $E^n(P), P \in X^n$. In projective geometry, this group is a subgroup of the group of the central-projective transformations and called a group of homologous transformation, or briefly homology group (22).

Group of transformations (25), (26), as its subgroup defined by the transformations (22), (24) show the relationship between the changes occurring with the particle. Changes of the generalized momentum in the interaction with the environment (25) induces a change in its generalized coordinates according to (26). The converse statement is true: any change (movement) of the generalized coordinates (26) accompanied by the obtaining from the external environment or returning in it the appropriate portions of the action. So, we come to a conclusion that change in the state of motion of the body (particle) going by sharing portions (quanta) of the action with all its surroundings (environment).

The above tells us that giving a geometric interpretation of gauge transformations, we inevitably come to the introduction to the physics the ideas and methods of the projective geometry. If we consider the special case of Finsler space — Finsler space-time F^4, than the group of 4-pulse translations together with a group of rotation of the 4-dimensional space-time (the homogeneous Lorentz group) form a group of center-projective transformations in each local tangent $E^4(P)$, $P \in F^4$.

In spaces with the areal metric analogues of transformations (25)and (26)(25) become a convert

$$'p_{\langle\alpha_1\dots\alpha_m\rangle} = \varepsilon(p_{\langle\alpha_1\dots\alpha_m\rangle} - \pi_{\langle\alpha_1\dots\alpha_m\rangle}(\psi)), \tag{27}$$

$$'x^{\langle\alpha_1\dots\alpha_m\rangle} = \frac{\varepsilon x^{\langle\alpha_1\dots\alpha_m\rangle}}{1 - \pi_{\langle\beta_1\dots\beta_m\rangle}(\psi)x^{\langle\beta_1\dots\beta_m\rangle}}. \tag{28}$$

Now they are effect in each local area $M^n_m(P), P \in X^n$. As the transformation parameters serves covariant components of the m-vector $\pi_{\langle\alpha_1\dots\alpha_m\rangle}$ in each local M^n_m (16).

Group of transformations (25) (26)in the case of ordinary action integral, as well as a group of transformations (27) (28)in the case of a multiple action integral, give us a reason to speak about presence of more general types of symmetries in nature. Accordingly, we have more general conservation laws. These conservation laws will be valid for space-time that having a non-uniform and violated isotropic structure in any of its scale.

In conclusion, we note that the emergence of a new type of symmetry is very often means a radical change in the properties, as it is the result of the appearance of a new invariant.

The embedding GTR space-time in space of observers

In Section 2, giving a geometric interpretation of the principle of least action, we came to the Finsler geometry as the theory of the field of local surfaces in the space of of degrees of freedom X^n (the base of the fiber bundle) of the dynamical system, we came to the fiber space of type $X^{n-1}(X^n)$.

Geometrization of principle of least action in case of relations (Lagrange variational problem for the ordinary action integral) leads to the fiber space $X^r(X^n)$, where the dimension of the fiber r depends on the number of connections that are imposed on the system in the form of differential equations, and satisfies to the condition $1 \leq r < n-1$.

Both cases are reduced to a single geometric structure, in particular to the fiber bundle $X^r(X^n)$, where for the dimension of the fiber r following condition takes place $1 \leq r \leq n-1$.

Thus, the geometrization of all problems in the dynamics of many-particle systems, including systems with singular Lagrangians, reduces to the description of a particular case of a fiber space $X^r(X^n)$ under the above conditions.

In Section 3 we give a geometric interpretation of the variational principle for the case of m-fold integral that in physics takes place in the derivation of the equations of field theory. We have come to the geometry of spaces with areal metric and respectively, to the fiber space $X^{m(n-m)}(X^n)$.

Recall that the dimension of the base n is now the number of degrees of fields of theory, and m – the dimension of the integration surface X^m. They are related by $1 \leq m \leq n-1$. The same problem with the presence of constraints (Lagrange variational problem for multiple integrals) is reduced to the study of the fiber space $X^r(X^n)$.

Dimension of the fiber r depends on the number of bonds imposed in the form of differential equations, and can take any value in the range $1 \leq r < m(n-m)$. Both cases in this section are reduced to a single geometric structure, namely, the fiber space $X^r(X^n)$, where now the dimension of the fiber r satisfies $1 \leq r \leq m(n-m)$. We purposely not specify the numerical value of the dimension m of the integration surface $\psi^\alpha = \psi^\alpha(u^a)$ so far. As we proceed the discussion we will return to this issue.

Given the above, as well as for the generality of our next discussion, we will continue to deal with the general a fiber bundle $X^r(X^n)$. At the same time, depending on the specific problem being solved, we will remember the above considerations relating to the dimensions of the base X^n and laminary X^r spaces every time.

Consider the physical meaning of the fiber bundle $X^r(X^n)$. Partially this question has been raised in section 2. Now we will discuss this issue for the general case. Fixing the «point» of a base $P \in X^n$ gives us a local indicatrix (i.e., r-surface) with center $P \in X^n$. According to a theorem (16) (see also (12)), the objects of linear affine connections G^b_{ia}, G^q_{ip} and the binding tensor g^p_{ia}, g^a_{ip} determine a local r-surface with center $P \in X^n$ up to an arbitrary center-projective transformations. Let us recall that indices of these geometric objects run through values: $a, b = 1, ..., m$; $p, q, = 1, ..., n-m$; $i = 1, ..., r$. In other words, the setting up a specified set of basic geometric objects of the local indicatrix allows to set the ensemble of local observers at the point $P \in X^n$. The very same state of the investigated object (particle) is described by the state vector (the point) mmon the local indicatrix $X^r(P), P \in X^n$.

The set of all local indicatrixes $X^r(P), P \in X^n$ at all points of X^n forms a fiber bundle $X^r(X^n)$. We come to the important concept of *common geometric space of observers*.
The connection object in the fiber bundle $X^r(X^n)$

$$\Gamma^i = \Gamma^i(\xi^\alpha, \Theta^i). \tag{29}$$

defines the mapping of local r-surfaces $X^r(P), P \in X^n$ (local indicatrixes) along arbitrary lines (trajectories) of basis X^n. Depending on the task, the symbol of ξ^α for action (1) carries the meaning of generalized coordinates q^α, and for the action (14) — the component of the fundamental field ψ^α.

Connection (29) is defined in an invariant way through a fundamental system of geometric objects of the local indicatrix G^b_{ia}, G^q_{ip}, g^p_{ib}, g^a_{ip} ((12), (16)), and it is easily seen, have an important physical meaning.

In the traditional description of the dynamics and field theory all the local interaction of the system (particle) with its surroundings (the environment) come into consideration by including in the Lagrange function (Lagrangian of theory) or in the Hamiltonian (Hamiltonian of theory) the appropriate components responsible for these interactions. Now, with a geometric interpretation of the principle of least (extreme) action, all of these interactions "are included in the game" through a fundamental system of differential geometric objects of local indicatrix. We can say that a particular type of interaction define concrete form of the local indicatrix and themselves determined by it. Moreover, it appears that the composition of each of the ensembles of local observers determined by the fundamental system of differential geometric objects of local indicatrix.

The set of ensembles of local observers associated with each point P of the base space, forms the overall (global) space of observers along the entire X^n. In other words, the fiber bundle $X^r(X^n)$ is a geometric model of the global space of arbitrary observers (both inertial and non-inertial). Thus, the true arena of events in the physics must be the global space of observers, i. e., the space $X^r(X^n)$, and not the space-time of GTR which, under certain conditions, being an integral component of the latter.

As an example, let's consider what represents the general (global) space of observers of space-time of Einstein's GTR. The latter is a differentiable manifold X^4, endowed with a Riemannian metric with appropriate signature. In other words, we are dealing with a fiber space of type $X^3(X^4)$. Local indicatrix represented therein by 3-dimensional surface of the second order in each local tangent $T^4(P)$, $P \in X^4$.

On the other hand, going back to our arguments in end of Section 2, we arrive at the following conclusion. The Lagrangian (Lagrangians) of free particles and fields without sources correspond to local indicatrices of a partial form - local surfaces of 2nd order. The opposite assertion appears valid: assignment of an indicatrix of metrics in the form of a surface of the second order in each local tangent $T^4(P)$, $P \in X^4$ leads to the Lagrangians that describes free particle without charge and spin.

The latter circumstance gives us serious reasons to doubt the existence of gravitational waves in space-time of Einstein's GTR.

Thus, the general space of observers of the space-time of Einstein's GTR constitutes a complete set of locally inertial observers. Hence, we arrive at the conclusion, that the total space of observers of Einstein's GTR is a special case of Finsler space F^4. In this context, it is appropriate to recall, considerations of Y. Vladimirov, «The general theory of relativity (GTR) acquires meaning, inherent in its title, only when it is supplemented by method of specifying a reference systems» (5). This requirement was implemented in this paper.

Discussion of the results

Let us recall the main purpose of the work, which was formulated in the introduction. It consists of the following. Give a reasoned response to the question: how any two observers see the world and how the results of their individual measurements of the same phenomenon can be reconciled between them? The A Einstein's general principle of relativity? The results of the previous sections can provide answers to both questions.

Giving a geometric interpretation of the principle of least action (extremal) for the ordinary and multiple integrals, we found it necessary to introduce into the physics the ideas and methods of modern differential geometry, more general than Riemann geometry, which is the mathematical basis of Einstein's GTR. In turn, giving a geometric interpretation of the principle of gauge invariance, which along with the variational principle is a cornerstone of modern physics, we found it necessary to introduce into the physics the methods of modern projective differential geometry. Successful geometrization of these principles has led to the need for the introduction into physical science (which above all is an experimental science) of such an important concept as general space of observers. The concept of a common space of observers is more fundamental than the concept of space-time of GTR. The latter is a 4-surface immersed in a general space of observers, and its metric structure is determined by the structure of its accommodating space of observers.

Thus, the general principle of relativity, which in most textbooks and monographs on the GTR usually sounds like this: «All the laws of nature are independent of the choice (completely arbitrary) of frames of reference in which they are observed», what in Finsler geometry and in geometry of spaces with areal metric is realized automatically. As for the axioms of the special theory of relativity (STR) and its postulates, then the first of which is also performed automatically, while the 2nd of its postulate (constancy of the speed of light) turns out to be excessive.

This gives us the answer to the 2nd question: the general principle of relativity can not be considered as a fundamental principle of physical science, as it turns to be the result of two more fundamental physical principles: the variational principle and the principle of gauge invariance of physical laws.

It follows that the dispute between the two greatest physicists of XX-th century, ended today in favor of the point of view of V. Fock [3].

In summary, we again note the following. Geometry is not a passive arena in which the unfolding of physical effects and processes are taking place. It by itself defines them and is defined by them. As Kepler said, «ubi materia — ubi geometria» («where matter — there geometry»). In other words, the geometry and physics are integrated.

In conclusion, the author wants to thank his colleagues in the department of «Problems of Quantum Physics» Moscow Institute for Physics and Textology (MIPT)
I. E. Bulyzhenkov, A. A. Rukhadze, and L. I. Urutskoyev for their interest in this work and helpful discussions.

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Релятивистские модели в астрометрических наблюдениях периодического излучения пульсаров

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Предложен новый метод аппроксимации периодического излучения пульсаров на основе инвариантных в координатных системах уравнений интервалов наблюдаемых событий излучения, решениями которых являются наблюдаемые параметры вращения пульсаров. Численные значения периода вращения и производных, тождественные в координатных системах на совпадающие эпохи местного времени, обнаруживают свойство временной и пространственной когерентности излучения на вековом масштабе, согласующееся с постепенным замедлением вращения пульсаров вследствие потерь энергии на излучение. Интервалы, определяемые параметрами вращения с относительной погрешностью в пределах 10^{-18}-10^{-19} на 40-летней протяженности наблюдений, что на 4-5 порядков лучше традиционных статистических методов аппроксимации, представляют собой астрономические релятивистские шкалы координатного времени в пределах Солнечной системы, которые на 2-3 порядка превосходят стабильность квантовых эталонов времени.

Введение

Открытие пульсаров в 1967 году является одним из выдающихся достижений астрономии [1, с. 8]. К настоящему времени известно более 1800 пульсаров, представленных в наиболее полном, поддерживаемом в актуальном состоянии каталоге [2, интернет-ресурс]. Пульсары, отождествляемые с вращающимися нейтронными звездами (период 1,5 мс–8,5 с), состоят из вещества экстремально высокой плотности (10^{14}-10^{15} г/см^3) и генерируют при вращении импульсное излучение, преимущественно в радиодиапазоне. Высокостабильная повторяемость импульсов излучения свидетельствует, что мы имеем дело с жёсткой механической системой, а не, например, с газовым или плазменным конгломератом. При массе порядка массы Солнца радиус нейтронной звезды составляет всего около 10 км, что приближается к пределу, когда линейная скорость в экваториальной области $2\pi R/P$ сопоставима со скоростью света. Физическую модель излучения пульсара представляют обычно в виде магнитного диполя, вращающегося с замедлением за счет потерь энергии на излучение, мощность которых, в зависимости от момента инерции, периода вращения P и его производной, составляет приблизительно 10^{23}-10^{26} Вт, что в своем верхнем значении сопоставимо с мощностью, излучаемой Солнцем за счет ядерных реакций. Наблюдаемое замедление вращения выражается производной периода $\dot{P}$, с типичным значением порядка 10^{-19}-10^{-21} с·с$^{-1}$ для миллисекундных и 10^{-15}-10^{-16} с·с$^{-1}$ для секундных пульсаров.

Пульсары, благодаря своим уникальным физическим свойствам, привлекли внимание как потенциальные внеземные хранители времени, сопоставимые по точности и особенно долговременной стабильности с принятым в том же 1967 году на 13-й Генеральной конференции по мерам и весам (ГКМВ) атомным стандартом времени, в котором секунда СИ определена как фиксированное число периодов излучения при переходе между двумя сверхтонкими уровнями основного состояния атома цезия-133. Новое определение сменило действовавшую с 1956 г. эфемеридную секунду, основанную на астрономических наблюдениях орбитального движения Земли вокруг Солнца и выраженную в виде доли тропического года на фиксированную эпоху [3], а в качестве аргумента эфемерид было принято всемирное время UT [4, с. 13]. Однако, учитывая неравномерность вращения Земли и сравнительно низкую точность определения солнечной секунды, с 1960 г. UT было заменено на эфемеридное время ET, исключающее влияние неравномерности вращения Земли [4, с. 235]. На Генеральной Ассамблее Международного астрономического союза (МАС), тоже в 1967 году, было принято решение о прекращении использования эфемеридной шкалы по среднему солнечному времени и введении взамен нее шкал земного динамического времени (TDT, с 1991 г. используется аббревиатура *TT*, его реализацией является Международное атомное время TAI) и барицентрического динамического времени (*TDB*), в которых учитываются релятивистские эффекты.

Таким образом, вся история наблюдений пульсаров тесным образом связана с весьма динамичной эволюцией методов измерения времени и развитием эфемеридной астрономии, что, естественно, не могло не привести к появлению новых представлений и физических интерпретаций наблюдаемого периодического излучения пульсаров. В работе с помощью предложенной релятивистской аналитической модели интервалов исследованы устойчивые закономерности периодического излучения пульсаров, инвариантные относительно координатных преобразований. Получены зависимости наблюдаемых состояний процесса излучения от параметров вращения, сопоставимые в различных координатных системах. Показаны достижимые возможности астрономические мер времени-пространства, основанных на уникальных физических свойствах пульсаров.

Аналитическая модель интервалов периодического излучения пульсара

За основу модели было взято полиномиальное разложение моментов событий излучения пульсара, наблюдаемых в неподвижной точке Солнечной системы – ее барицентре [1, с. 75]:

$$t = t_0 + P_0 N + 0{,}5 P_0 \dot{P} N^2 + ... \qquad (1)$$

В соответствии с выражением (1) определяется барицентрический момент любого события *N=1,2…*, если известны значения периода *Po* на начальную эпоху *to* и его производная $\dot{P}$ (прямая задача). Для решения обратной задачи – найти значение наблюдаемого периода P^* и его производной – воспользуемся уравнением вида

$$PT_i = (1 + \alpha_i)(P^* N + 0{,}5 P^* \dot{P} N^2)_i , \qquad (2)$$

в котором PT_i есть численные величины интервалов выборочно наблюдаемых событий (*i=1,2,…* – номер наблюдения), отсчитываемые от выбранного начального события, *N* – полное число событий излучения в промежутке от начального до текущего события, $\alpha_i = \Delta P_i / P^*$ –

относительная величина вариаций наблюдаемого периода в этом промежутке. Интервалы PT_i в левой части уравнения (2) получены по планетным эфемеридам, определяемым движением небесных тел Солнечной системы, и их численные значения выражены в метрике общей теории относительности (ОТО). В правой части эти же интервалы определены аналитически через параметры вращения P^* и $\dot{P}$. Уравнение (2) решается методом линейного приближения параметров P^* и $\dot{P}$ к наблюдаемым интервалам PT_i по критерию наименьших квадратов, что соответствует минимальному значению коэффициента α_i, который отражает непараметризуемый остаток PT_i линейного приближения. Вычисленные по эфемеридам интервалы PT_i представлены либо в шкале земного динамического времени *TT* – топоцентрические интервалы, либо в шкале барицентрического динамического времени *TDB*, обозначим их *TB* – барицентрические интервалы. В этих системах наблюдаемые физические процессы имеют разные значения измеренных состояний – координат и времени: за счет перемещения земного наблюдателя относительно неподвижного барицентра, с учетом скорости, и из-за неодновременности наблюдаемых событий в разных точках пространства вследствие конечной скорости распространения светового (электромагнитного) сигнала от наблюдаемого объекта к наблюдателю. Эти расхождения приводят к нарушению синхронизации часов в разных координатных системах, и прямое сопоставление времени в них оказывается невозможным.

Г.Лоренц для преодоления этих ограничений ввел преобразования интервалов в 4-мерном координатном пространстве, такие чтобы для каждого наблюдателя, в какой бы системе он ни находился, отсчеты времени, выраженные в одних и тех же единицах, в одинаковых условиях совпадали [5, р. 81]. Преобразования Лоренца являются неотъемлемой частью специальной теории относительности (СТО). Согласно метрике СТО, пространство и время образуют единый четырехмерный континуум событий, уравнения физических процессов в координатных системах имеют одинаковый вид, а метрические свойства определяются интервалом

$$J = c^2T^2 - X^2 - Y^2 - Z^2, \qquad (3)$$

величина которого неизменна (инвариантна) в любой системе отсчета [6]. В дифференциальной форме инвариант (3) принимает вид:

$$(d\sigma)^2 = c^2(dT)^2 - (dX)^2 - (dY)^2 - (dZ)^2 \qquad (4)$$

Как следует из структуры инварианта *J*, записанного в ортогональных координатах, всегда можно ввести единое время *T* для всех точек трехмерного пространства, например, в топоцентрической системе. В барицентрической координатной системе отсчета единое время будет уже другим. Преобразования координат и времени Лоренц (1904 г.) определил следующим образом [6, с. 40]:

Прямые: $X' = \gamma(X - vT)$, $T' = \gamma(T - \frac{v}{c^2}X)$, $Y' = Y$, $Z' = Z$;

Обратные: $X = \gamma(X' + v\tau)$, $T = \gamma(\tau + \frac{v}{c^2}X')$, $Y = Y'$, $Z = Z'$; (5)

где $\gamma = \dfrac{1}{\sqrt{1 - v^2 / c^2}}$; $\dfrac{1}{\gamma} T' = \tau$; T' – измененное местное время.

Пуанкаре (1904 г.) показал, что преобразования (5) вместе со всеми пространственными вращениями образуют группу Лоренца и впервые ввел представление о четырехмерности ряда физических величин, преобразующихся как ct, x, y, z. Переход от системы координат x^{υ} к системе координат x'^{υ}, а затем к системе координат x''^{υ} эквивалентен прямому переходу от системы координат x^{υ} к системе x''^{υ}. Именно в этом смысле преобразования Лоренца образуют группу. В математическом выражении группа Лоренца – это совокупность элементов, в которой определена операция умножения. Элементы могут быть любой природы. Произведение двух любых элементов группы есть элемент той же группы [6, с. 84]. При этом физические явления можно описывать в системе x', y', z', t' точно таким же образом, как в системе x, y, z, t и что все это находится в точном соответствии с принципом относительности, сформулированном им для всех физических явлений в инерциальных системах отсчета. Свойство ортогональности пространства линиям времени позволяет рассматривать одномерную величину T и трёхмерную величину X как независимые времени-подобные и пространственно-подобные переменные соответственно, взаимосвязь которых полностью определяется четырехмерным инвариантом J.

Обобщая принцип относительности Пуанкаре, А.А.Логунов (1987 г.) распространил его также и на неинерциальные системы отсчета, показав, что выражение для интервала $(d\sigma)^2$ (4) инвариантно относительно произвольных преобразований координат. Метрический тензор пространства и при таких преобразованиях сохраняет общий вид, так что преобразования Лоренца (5), которые связывают одну инерциальную систему с другой, являются частным случаем таких преобразований [6, с. 62]. Преобразования координат, которые оставляют метрику форминвариантной, приводят к тому, что физические явления, происходящие в таких системах координат при одинаковых условиях, не позволяют отличить наблюдателю одну систему координат от другой. Как следствие, снимается ограничение в выборе только инерциальной, например, барицентрической, системы, топоцентрическая обладает теми же метрическими свойствами. Достигается прямое сопоставление отождествляемых с физическими свойствами наблюдаемых величин, которые при одинаковых условиях в любой координатной системе должны совпадать. В результате мы получаем возможность проверить метрическую эквивалентность предлагаемых математических моделей и тождественность наблюдаемых состояний физических явлений, опираясь на всю совокупность данных физического опыта.

Применительно к физическому процессу периодического излучения пульсаров, наблюдаемому в ускоренной топоцентрической и инерциальной барицентрической координатных системах, как показано в [7], следует:

- совпадение в координатных системах на одинаковые эпохи местного времени численных значений наблюдаемого периода вращения пульсара, отсчитываемого в единицах физической шкалы времени;

- инвариантность интервалов событий излучения пульсара, наблюдаемых в топоцентрической и барицентрической системах, что означает их совпадение на одинаковые эпохи уточненного по Лоренцу местного времени T';
- инвариантность единичного интервала шкал времени, по которым отсчитываются топоцентрические и барицентрические интервалы, т.е. естественные эталоны времени во всех системах отсчета одинаковы.

Эти признаки наблюдаемых величин определяют одинаковые условия, при которых физические процессы периодического излучения пульсара, протекающие в этих системах, тождественны для наблюдателя в любой из них. Перечисленные условия относятся в равной степени и к уравнению (2) интервалов наблюдаемых событий, если его записать с учетом признаков выбранной системы, соответственно топоцентрической и барицентрической. Уравнения интервалов TT_i и TB_i, отсчитываемых от общего начального до i-го наблюдаемого события в топоцентрической и барицентрической системах отсчета, соответственно, принимают вид:

$$TT_i = (1+\alpha_i)(P^* N_T + 0{,}5P^* \dot{P} N_T^2)_i \qquad (6)$$

$$TB_i = (1+\alpha_i)(P^* N_B + 0{,}5P^* \dot{P} N_B^2)_i \qquad (7)$$

Численные значения наблюдаемых параметров находятся в виде решения уравнений (6), (7), которое удовлетворяет условию наилучшего линейного приближения к интервалам наблюдаемых событий по критерию наименьших квадратов. Решением уравнений являются наблюдаемые параметры вращения $P^*, \dot{P}$ и вариации наблюдаемого периода α_i. Число событий, излученных в промежутке наблюдения, находится по среднему значению периода в этом промежутке, принимая во внимание равномерное увеличение периода с учетом производной:

$$N_{T_i} = TT_i / \overline{P}_{TT_i}\,; \quad N_{B_i} = TB_i / \overline{P}_{TB_i} \qquad (8)$$

Среднее значение периода в промежутке наблюдений:

$$\overline{P}_{TT_i} = P^* + 0{,}5\dot{P}\cdot TT_i\,; \quad \overline{P}_{TB_i} = P^* + 0{,}5\dot{P}\cdot TB_i \qquad (9)$$

Соотношения (8) определяют число событий в промежутке наблюдений, с учетом разницы координатного времени в топоцентрической и барицентрической системах.

Из соотношений (6) и (7) с учетом (9) находим отклонения интервалов в промежутке наблюдений в координатных системах из-за вариаций наблюдаемого периода:

$$\Delta TT_i = \alpha_i \overline{P}_{T_i} N_{T\,i}\,; \; \Delta TB_i = \alpha_i \overline{P}_{B_i} N_{B\,i} \qquad (10)$$

Отклонение интервалов между двумя произвольно выбранными наблюдаемыми событиями в любой координатной системе в промежутке (PTj, PTi, $i>j$) вычисляется интегрированием текущих отклонений наблюдаемого периода $\Delta P(t)$ в этом промежутке:

$$\Delta PT_{i-j} = \frac{1}{P^*}\int_{PT_j}^{PT_i} \Delta P(t)dt = \int_{PT_j}^{PT_i} \alpha(t)dt\,. \qquad (11)$$

При выборочных наблюдениях, заменяя непрерывную функцию $\Delta P(t)$ ее линейной аппроксимацией в промежутке между наблюдаемыми событиями, получаем отклонение интервалов в виде конечных разностей:

$$\Delta PT_{i-j} = \overline{P}_{i-j}(\alpha_i - \alpha_j)(N_i - N_j) \qquad (12)$$

где $\overline{P}_{i-j} = P^* + 0{,}5\dot{P}(PT_i - PT_j)$ – среднее значение наблюдаемого периода от $j\text{–}го$ до $i\text{–}го$ наблюдаемого события, N_i, N_j – общее число излученных импульсов, отсчитываемое от начального до $j\text{–}го$ и $i\text{–}го$ наблюдаемых событий соответственно.

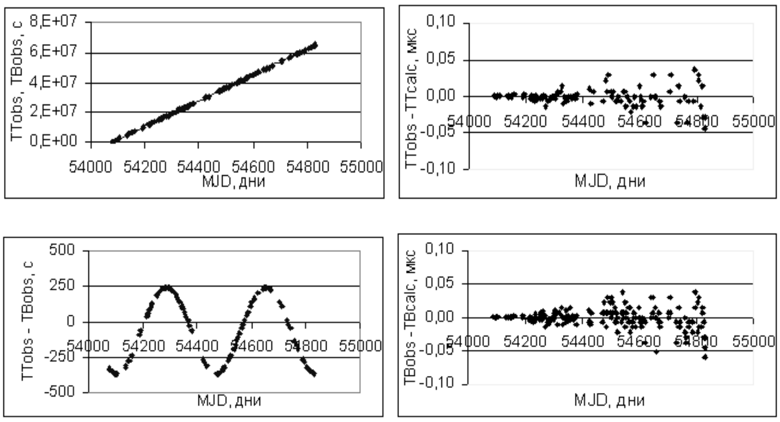
Как следует из выражения (12), отклонения интервалов в промежутке между двумя произвольно выбранными наблюдаемыми событиями определяется разностью коэффициентов линейного приближения на границах промежутка и числом событий излучения пульсара в пределах границ этого промежутка. При $j=0$ отклонения отсчитываются от начального наблюдаемого события: $\Delta PT_i = \alpha_i \overline{P}_i N_i$.

Метрическая эквивалентность эфемеридной и аналитической моделей

Интерпретируя левые части уравнений (6) и (7) как наблюдаемые топоцентрические и барицентрические интервалы *TTobs* и *TBobs* (интервалы *O*), соответственно, а правые – как те же интервалы, рассчитанные по наблюдаемым параметрам вращения *TTcalc* и *TBcalc* (интервалы *C*), сопоставим их разности *TTobs–TTcalc* и *TBobs–TBcalc*, с тем чтобы по величине расхождений (*O–C*) оценить степень приближения расчетных и наблюдаемых интервалов периодического излучения пульсара, выраженных независимо в метрике ОТО и СТО. В качестве иллюстрации на рис.3.1 на примере пульсара B0809+74 показаны полученные по эфемеридам интервалы *TTobs* и *TBobs* и их разности по двухлетним наблюдениям 2006-2008гг. на радиотелескопе БСА ФИАН (Пущино) [8]. Монотонно возрастающие топоцентрические и барицентрические интервалы на верхнем графике практически совпадают между собой, так как их циклические вариации из-за орбитального движения Земли вокруг Солнца, приведенные внизу, на несколько порядков меньше их абсолютных величин. По этим интервалам в соответствии с уравнениями (6) и (7) были определены значения периода вращения P^*_{TT}=1,29224151775083с на эпоху MJD$_{TT}$ 54080.0098 в топоцентрической системе, P^*_{TB}=1,29224151775088с на эпоху MJD$_{TB}$ 54080.0137 в барицентрической системе и производная периода $\dot{P}=1{,}676\cdot10^{-16}$, которая совпадает с указанным в каталоге [10] значением. По этим значениям параметров вращения были рассчитаны интервалы *TTcalc* и *TBcalc*. На рис.13,*б* приведены разности *TBobs–TBcalc*, которые показывают расхождения интервалов, выраженных в метрике ОТО и СТО, в обеих координатных системах.

Заметим, что разница указанных значений наблюдаемого периода в координатных системах обусловлена исключительно расхождением начальной эпохи наблюдаемого события излучения в координатных системах по шкалам местного времени. Для показанных на рис.1,*а* интервалов пульсара B0809+74 оно составляет –332,96872с. Такое соотношение эпох наблюдаемых событий и соответствующих им значений наблюдаемого периода подтверждает совпадение в координатных системах численной величины наблюдаемого периода вращения на совпадающие эпохи, вытекающее из соотношений (8) и (9). Действительно, расхождения наблюдаемого периода P^*_{TT} и P^*_{TB} в координатных системах определяются исключительно разницей эпох MJD_{TT} и MJD_{TB}, выраженных в шкалах местного координатного времени, с учетом производной периода $\dot{P}$:

$$P^*_{TB} = P^*_{TT} + \dot{P}\,(\mathrm{MJD}_{TB} - \mathrm{MJD}_{TT})\cdot 86400,\ \mathrm{c} \qquad (13)$$



а) наблюдаемые интервалы (*вверху*), их разность (*внизу*)

б) разность наблюдаемых и расчетных интервалов *TT* (*вверху*) и *TB* (*внизу*)

Рис.1. Наблюдаемые топоцентрические (*TT*) и барицентрические (*TB*) интервалы пульсара B0809+74 (*слева*) и их разность с расчетными значениями (*справа*)

В этом проявляется принцип относительности: физические процессы, наблюдаемые в топоцентрической и барицентрической координатных системах при одинаковых условиях, тождественны. Это необходимое условие выполнения фундаментального закона сохранения энергии вращения. В метрике СТО оно выполняется в результате преобразований Лоренца (5), которые определяют измененное местное время, для каждой координатной системы свое. Преобразования Лоренца учитывают перемещение и скорость наблюдателя в разных системах, неодновременность наблюдаемых событий в разных точках пространства из-за конечной скорости распространения радиосигнала, приводя тем самым физические процессы в одинаковые условия наблюдения. Это, в частности, означает, что интервалы наблюдаемых событий, определяемые одинаковым периодом вращения, также одинаковы в обеих системах на совпадающие эпохи местного времени, а поскольку местное время различно, то оно определяется в каждой системе разным числом наблюдаемых событий излучения N, отсчитываемых от общего начала.

Как следует из рис.1,*б*, расхождения интервалов, определяемых по эфемеридам и вычисленных по наблюдаемым параметрам вращения, носят случайный характер, имеют квазистационарный вид и находятся в одном диапазоне величин в обеих координатных системах. Стандартная статистическая оценка расхождения интервалов составляет 14,3 нс (*TT*) и 19,3 нс (*TB*) в двухлетнем промежутке наблюдений. Эти величины сопоставимы с суммарными погрешностями атомных эталонов времени и планетных эфемерид, не имеют отношения к параметрам вращения и могут быть учтены по аналитической модели. Интервалы, вычисленные по наблюдаемым параметрам вращения в соответствии с (6) и (7), определяются в шкалах координатного времени с наносекундной точностью и субнаносекундным разрешением. Таким образом, наблюдения пульсаров подтверждают тождественность релятивистских интервалов, определяемых численными планетными эфемеридами и наблюдаемыми параметрами вращения пульсара в любой выбранной системе отсчета и, следовательно, эквивалентность метрики ОТО и метрики СТО.

Параметры периодического излучения пульсаров на вековом масштабе

Итак, мы убедились в метрической эквивалентности эфемеридной и аналитической моделей интервалов периодического излучения пульсаров и тождественности релятивистских интервалов событий излучения, наблюдаемых в любой координатной системе. Теперь обратимся к аналитической модели интервалов для исследования собственных свойств пульсаров, в первую очередь наблюдаемых параметров вращения, которые определяют стабильность интервалов наблюдаемых событий периодического излучения. С этой целью наблюдаемые параметры вращения будем сопоставлять с ретроспективными данными для этих пульсаров, содержащимися в наиболее распространенных каталогах [2,10]. Численную величину периода вращения, которая указана в каталоге вместе с производной на некоторую фиксированную эпоху прошлого, по аналогии с (13) преобразуем с учетом производной к начальной эпохе наблюдений, по которым были определены параметры вращения. Приведенные к начальному наблюдаемому событию значения периода P будем определять из соотношения:

$$P = P_K + \dot{P}(MJD_0 - MJD_K)\cdot 86400,\ \text{с} \qquad (14)$$

где MJD_0 – эпоха начального наблюдаемого события, выраженная в долях текущих суток (дробная часть) на дату наблюдения (целая часть),

MJD_K – эпоха, на которую определен период P_K по каталогу.

Так, например, пересчитанные в соответствии с выражением (14) значения периода пульсара B0809+74 с учетом указанного в каталоге [10] значения периода P_K=1,29224132384*с* на эпоху MJD_0 40688.97 и производной $\dot{P}=1{,}676\cdot 10^{-16}$ с·с$^{-1}$ составляют P^*_{TT}=1,29224151775083*с* на эпоху MJD_{TT} 54080.0098 в топоцентрической системе и P^*_{TB}=1,29224151775088*с* на эпоху MJD_{TB} 54080.0137 в барицентрической системе. Сопоставляя полученные результаты, видим, что пересчитанные значения периода PSR B0809+74 из эпохи почти 40-летней давности (12.04.1970 г.) совпадают с периодом вращения на эпоху начального наблюдаемого события на БСА в 2007-2009гг. в топоцентрической и барицентрической системе. Эти результаты свидетельствуют о согласованности периода вращения на вековом масштабе, сопоставимом с исторической продолжительностью наблюдений, и, учитывая тождественность наблюдаемого периода в координатных системах, являются прямым подтверждением когерентности периодического излучения пульсара. Свойство когерентности означает, что события, наблюдаемые в любой координатной системе, синфазны с периодическим (во времени) и волновым (в пространстве) процессом излучения. Отсюда следует, что независимо от эпохи, выбранной для продолжения наблюдений после паузы продолжительностью даже в несколько десятков лет, мы будем получать в любой координатной системе последовательность интервалов с точно предсказанной фазой наблюдаемых событий, которая определена по установленным в прошлом параметрам вращения, численные значения которых согласованы также и на эпоху текущих наблюдений. Таким образом, события, наблюдаемые в любой координатной системе, привязаны к фазе периодического (во времени) и волнового (в пространстве) процесса излучения пульсара, которая определена согласованными параметрами вращения на эпоху прошлого, настоящего и будущего.

Интервалы событий когерентного излучения в любой выбранной координатной системе определяются в соответствии с аналитической моделью (2) наблюдаемыми параметрами вращения пульсара:

$$PT_{ког\,i} = (P^* N + 0{,}5 P^* \dot{P} N^2)_i \,. \qquad (15)$$

Соответственно находятся немоделируемые отклонения интервалов, обусловленные внешними факторами, не связанными с вращением пульсара:

$$\Delta PT_i = PT_i - PT_{ког\,i} = \alpha_i (P^* N + 0{,}5 P^* \dot{P} N^2)_i \,. \qquad (16)$$

На рис.2 показаны относительные вариации наблюдаемого периода (слева), численно равные коэффициенту линейного приближения $\alpha_i = \Delta P / P$ в уравнении (2), и вычисленные по ним в соответствии с выражениями (10) и (16) отклонения топоцентрических интервалов ΔTT (*справа*). Вариации наблюдаемого периода находятся в диапазоне порядка $\pm 1 \cdot 10^{-15}$, что сопоставимо с погрешностями шкалы атомного эталона времени. Среднеквадратическая величина отклонений интервалов составляет около 16 *нс* в пределах двухлетнего промежутка наблюдений. Сравнивая графики отклонения топоцентрических интервалов на рис.2 и

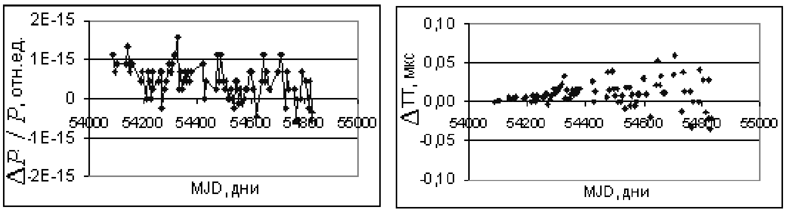
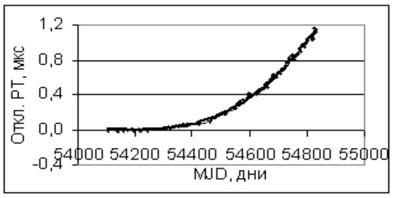


Рис.2. Вариации наблюдаемого периода (*слева*) и интервалов (*справа*) в топоцентрической системе

разности наблюдаемых и расчетных интервалов ТТ на рис.1,*б*, отметим их выраженное сходство, как по распределению во времени, так и по диапазону значений. Отсюда можно сделать вывод, что, во-первых, случайные вариации на этих графиках определяются внешними факторами, не связанными с вращением и когерентным излучением пульсаров. Во-вторых, можно ожидать, что в этих вариациях доминируют общие причины, которые проявляются как в наблюдаемых по эфемеридам, так и в рассчитанных по наблюдаемым параметрам вращения интервалах. К ним, очевидно, относятся текущие отклонения атомной шкалы времени, по которой отсчитываются интервалы наблюдаемых событий в выбранных системах, и погрешности планетных эфемерид. Эти погрешности, выявляемые аналитической моделью интервалов, могут быть учтены в качестве поправок при определении интервалов по наблюдаемым параметрам вращения.

Благодаря свойству когерентности излучения пульсаров точность определения периода вращения по уравнениям (6), (2.) на 2-3 порядка выше по сравнению с данными в каталогах, поэтому интервалы периодического излучения пульсара, вычисляемые по наблюдаемым параметрам вращения, определяются с субнаносекундным ($<10^{-9}$ с) разрешением. Относительная погрешность интервалов после исключения случайных погрешностей, обусловленных внешними факторами, не связанными с вращением пульсаров, находится в пределах 10^{-18}-10^{-19} на 40-летнем промежутке наблюдений. Для сравнения, это на 2-3 порядка превосходит достижимую стабильность современных квантовых эталонов времени.

Субнаносекундное разрешение аналитической модели позволяет в отклонениях интервалов уверенно обнаружить и определить по ним численное значение второй производной периода, несмотря на то, что ее вклад в эти отклонения всего лишь около 1 мкс на двухлетнем промежутке наблюдений. Так, например, по наблюдениям пульсара B0834+06 в 2007–2008гг. [8] были определены период вращения $P^*=1{,}27377145381349c$ и производная $\dot{P}=6{,}79918\cdot10^{-15}$ с·с$^{-1}$ на эпоху MJDo 54103.96609. Наблюдаемый период отвечает условию согласованности по критерию когерентности с приведенной в каталоге [10] величиной $P\kappa=1{,}27376417152\ c$ на эпоху MJD 41707.5, и совпадает с указанной в этом каталоге производной. На рис.3 (*слева*) показаны отклонения интервалов, вычисленные по соотношениям (10). Отклонения, в отличие от PSR B0809+74 на рис.2 (*справа*), имеют выраженный тренд, величина которого составляет около 1,2 мкс на двухлетнем промежутке.



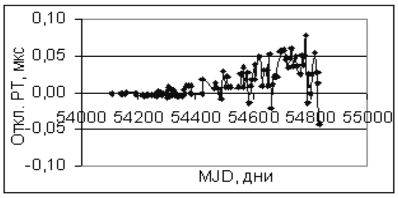


Рис.3. Отклонения интервалов PSR B0834+06, обнаруживающие вторую производную периода (*слева*), и они же с учетом второй производной (*справа*)

Характер тренда, который аппроксимируется полиномом 3-го порядка, свидетельствует об обнаружении второй производной периода вращения $\ddot{P}$, которая не учтена в модели интервалов (6), (7). Она имеет постоянную величину и связана с $\dot{P}$ соотношением:

$$\dot{P}=\dot{P}_0+\ddot{P}\cdot PT\ , \qquad (17)$$

где $\dot{P}_0$ – значение первой производной на эпоху начального наблюдаемого события,

PT – промежуток между начальным и текущим наблюдаемыми событиями.

После подстановки (17) в модель (2) при известных величинах $P^*, \dot{P}$ линейным приближением интервалов находим вторую производной периода $\ddot{P}$=1,1615·10^{-29} с$^{-1}$, которая соответствует наблюдаемому тренду в отклонениях интервалов. На графике рис.3 (*справа*) показаны отклонения интервалов с учетом второй производной. Среднеквадратическое отклонение составляет 21,7 *нс* – оно того же порядка, что и у PSR B0809+74 на рис.2 (*справа*). Вторая производная, постоянная на всей протяженности наблюдений, не нарушает когерентности периодического излучения пульсара.

Согласованность периода вращения пульсара и производных

Постепенное замедление вращения пульсаров, обусловленное потерями энергии на излучение, оценивается величиной показателя торможения *n*, который, согласно модели пульсара как излучающего магнитного диполя, определяется:

$$n = 2 - P\ddot{P} / \dot{P}^2 , \qquad (18)$$

и его величина, как показано в [11, с. 61], находится в пределах 1< *n* <3. Поскольку соотношение (18) включает все три наблюдаемых параметра вращения, которые согласованы по критерию когерентности, то можно сопоставить, насколько эти параметры соответствуют магнитодипольной модели, учитывающей торможение пульсара, которое определяется производными периода. Это особенно важно в отношении второй производной, которая определяется аналитической моделью по очень небольшому, всего около 1 мкс на двухлетнем промежутке, систематическому тренду интервалов, обусловленному этой производной, и вероятность ошибок здесь намного выше, чем, например, при определении первой производной. У наблюдаемых пульсаров B0809+74, J1509+5531, B1919+21, B0329+54, B2217+47, B0834+06 показатель торможения оказался больше 1, но не более 2. Верхнее значение относится к пульсарам B1919+21 и B0809+74, у которых на двухлетнем промежутке наблюдений не обнаруживается значимой второй производной, и она принята нулевой. У остальных пульсаров вторая производная положительна, а величина *n* находится в диапазоне 1,65-1,72 с погрешностью порядка единицы младшего знака, что не влияет на оценку целочисленных пределов *n*. Результаты наблюдений подтверждают, что параметры вращения, согласованные по критерию когерентности, согласованы также и по критерию торможения у всех перечисленных здесь секундных пульсаров.

Для сравнения были определены показатели торможения по параметрам вращения, взятым для этих же пульсаров из каталога [2] (в более раннем каталоге [10] сведений о вторых производных нет). Результаты сравнения *n* приведены на рис.4. Сопоставление параметров вращения показывает, что производные наблюдаемого периода когерентного излучения пульсаров, полученные по аналитической модели интервалов и определяющие замедление вращения за счет потерь энергии на излучение, соответствуют магнитодипольной модели у всех рассмотренных здесь секундных пульсаров, при неотрицательных значениях второй производной и показателе торможения в диапазоне 1<*n*<2. Если же для расчета показателя торможения взять параметры $\ddot{P}$, полученные преобразованием вторых производных частоты $\ddot{\upsilon}$, взятых из каталога [2], то значения n оказываются за пределами указанного диапазона для всех наблюдаемых пульсаров, причем различия могут достигать 4-5 порядков. К тому же у PSR

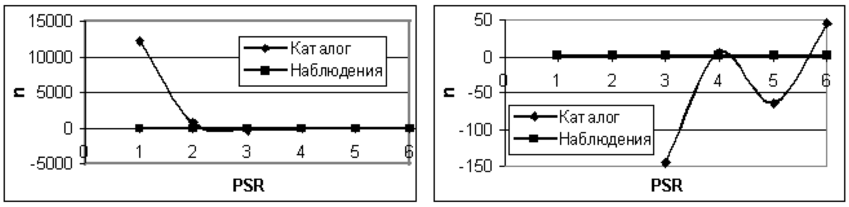


Рис.4. Показатели торможения по наблюдениям и каталогу. Пульсары пронумерованы от 1 до 6 в порядке перечисления в тексте, по убыванию n. *Слева* включены все 6 пульсаров, *справа* – за исключением первых двух .

B1919+21, B2217+47 показатель торможения в этом каталоге отрицательный. Аномальные отклонения второй производной в каталоге [2], не укладывающиеся в систематику потерь вращательной энергии, в работе [12, p. 1042] объясняются немоделируемыми шумами в барицентрических остаточных уклонениях наблюдаемых событий, по которым определялись вторые производные. Таким образом, аналитическая модель релятивистских интервалов не только обнаруживает согласованность наблюдаемых параметров вращения пульсаров по критерию когерентности, но и приводит наблюдаемый период, первую и вторую производную в соответствие с магнитодипольной моделью излучения для всех наблюдаемых пульсаров.

Заключение

Релятивистские модели интервалов периодического излучения пульсаров, инвариантные относительно координатных преобразований Лоренца, подтверждают эквивалентность пространственно-временных соотношений, выраженных как в метрике ОТО по численным планетным эфемеридам Солнечной системы, так и в метрике СТО через параметры вращения пульсара. Параметры вращения, которые являются решениями уравнений, тождественны в координатных системах. По релятивистским моделям обнаруживается свойство когерентности периодического (во времени) и волнового (в пространстве) излучения пульсаров на вековом масштабе, согласующееся с постепенным замедлением вращения пульсаров вследствие потерь энергии на излучение. Наблюдаемые параметры вращения определяют интервалы когерентного излучения с субнаносекундным разрешением и относительной погрешностью в пределах 10^{-18}-10^{-19} на 40-летней протяженности наблюдений. Таким образом, релятивистские интервалы, полученные по наблюдаемым параметрам вращения, представляют собой астрономическую эталонную меру координатного времени в пределах Солнечной системы, которая на 2-3 порядка превосходит стабильность современных квантовых эталонов времени.

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The relativistic models in the astrometric observations of periodic pulsar radiation

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A new method of the approximation of periodic pulsar radiation based on the invariant in the coordinate systems equations for intervals of radiation events, is presented, together with its solutions, which are the observed parameters of the rotation of pulsars. The numerical values of the rotation period and the derivatives, which are identical within any coordinate systems on coincide epoch of the local time, show the properties of time and spatial coherence of radiation at the decade scale, which are consistent with a gradual slowing of the rotation of pulsars due to energy losses by radiation. Intervals to be determined by the parameters of rotation with the relative inaccuracy within range 10^{-18}-10^{-19} for 40-year length of the observation that the 4-5 orders of magnitude superior to the traditional statistical methods of approximation, are the precise astronomical reference measure of coordinate time within the solar system that are 2-3 orders of magnitude exceeds stability of quantum standards of the time.

ТЕОРЕТИКО-ГРУППОВЫЕ ОСНОВЫ ИНЕРЦИАЛЬНОЙ НАВИГАЦИИ

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Применяющийся в теории инерциальной навигации математический аппарат развит в XIX веке, сама инерциальная навигация как раздел механики сформировалась в XX веке и есть все предпосылки для включения в XXI веке основ инерциальной навигации в курс физики для средней школы. В тексте доклада основное внимание уделено двум тесно связанным друг с другом темам: группы пространственно-временной симметрии (группы Галилея и Пуанкаре, их подгруппы и расширения) — естественный инструмент классификации и метод построения физических теорий; постановка задачи инерциальной навигации на основе теоретико-группового подхода.

Ключевые слова: инерциальная навигация, методика преподавания, кватернионы, бикватернионы, группа Галилея, группа Пуанкаре, конформная группа.

1. ВВЕДЕНИЕ

Работа носит обзорно-методический характер и рассчитана на широкий круг читателей, в том числе на тех, кто, имея базовые знания по физике и математике, ничего не слышал об инерциальной навигации. Поэтому начнём введение с краткого ответа на типовые вопросы [1].

«Что такое инерциальная навигация?» — задача инерциальных навигационных систем состоит в определении положения объекта без использования внешней информации («в слепом полёте»).

«Приведите пример инерциальной навигационной системы» — инерциальная навигационная система человека. Дело в том, что любой высокоразвитый активно двигающийся живой организм обладает инерциальными датчиками[1)], позволяющими некоторое время сохранять пространственную ориентировку при отсутствии внешней информации (получаемой, например, с помощью зрения).

«Как соотносятся инерциальная навигация и физика?» — кратко ситуацию можно охарактеризовать следующими тремя утверждениями: инерциальная навигация раздел механики;

[1)] «У человека, например, в вестибулярных аппаратах имеются три биодатчика угловых ускорений; один двухосный биоакселерометр и один многофункциональный биодатчик, выполняющий функции одноосного биоакселерометра и вибрационного скоростного гироскопа» В.П. Селезнёв, Н.В. Селезнёва [2, с. 77].

механика раздел физики; но, на сегодняшний день, инерциальная навигация не раздел физики.

«Сколько времени потребуется, чтобы инерциальная навигация стала разделом физики?» — неизвестно.

Естественный для системы физического образования XX века исходный пункт при изложении основ инерциальной навигации — второй закон Ньютона [3, с. 53]:

$$m\vec{w} = \vec{F}.$$

Здесь $\vec{w}$ — вектор ускорения (который часто обозначают буквой $\vec{a}$); остальные обозначения в комментариях не нуждаются. Переход к уравнениям инерциальной навигации осуществим в несколько шагов. Прежде всего напомним, что в механике Галилея–Ньютона ускорение (полное) равно второй производной по времени от радиус-вектора:

$$\ddot{\vec{r}} = \vec{w} = \frac{\vec{F}}{m}.$$

Следующий принципиальный шаг состоит в разделении полного ускорения на две величины разной физической природы [4, с. 12]:

$$\ddot{\vec{r}} = \vec{g} + \vec{p}.$$

Здесь $\vec{g}$ — вектор гравитационного ускорения, а $\vec{p}$ — вектор негравитационного (кажущегося) ускорения, измеряемый инерциальными датчиками — акселерометрами [5, с. 76] (его тоже часто обозначают буквой $\vec{a}$). Дальнейшие усложнения выписанного уравнения связаны в основном с необходимостью учёта вращения объекта, положение которого определяет установленная в нём инерциальная навигационная система. Идея определения положения тела путём двойного интегрирования показаний акселерометров при известном распределении (карте) гравитационных ускорений, родилась и доведена до технической реализации в XX веке. «Инерциальная навигация в наши дни — один из ярких примеров того, как идея, первоначально казавшаяся совершенно фантастической, находит свое реальное воплощение и прокладывает дорогу к широкому практическому применению» А.Ю. Ишлинский [6, с. 369].

Попытка включить инерциальную навигацию в физику была предпринята полвека назад Р. Фейнманом: «После лекции о вращающихся системах была прочитана лекция об инерциальной навигации, но, к сожалению, при издании ее опустили» [7, с. 14]. До вузовских учебников по общей физике инерциальная навигация пока так и не дошла. Автором разработана методика преподавания вводного курса по физико-математическим основам теории инерциальной навигации [8]. В её основе лежит использование широко применяющихся в физике групп пространственно-временной симметрии. Центральное понятие — преобразование, связывающее инерциальные системы отсчёта. Элементы спецкурса, в том числе представленные ниже, после соответствующей адаптации могут быть использованы при изложении раздела «Механика» авторами учебников по курсу общей физики.

2. ГРУППЫ

Преобразования, с которыми имеют дело в геометрии и физике, образуют группы. Группы преобразований (в физике их часто называют группами симметрии) содержат в качестве элементов-преобразований: тождественное преобразование; обратное преобразование для каждого элемента и композицию каждой пары элементов. В механике обычно имеют дело с непрерывными группами преобразований. Количество независимых вещественных чисел (параметров), характеризующих элемент непрерывной группы, называется её размерностью.

О теоретико-групповом подходе к геометрии сегодня можно прочитать даже в школьном учебнике, правда, не в любом [9]. Наиболее последовательно теоретико-групповой подход к геометрии выражается следующими словами А. Пуанкаре: «То, что мы называем геометрией, есть не что иное, как изучение формальных свойств некоторой группы, так что мы можем сказать: пространство есть группа» [10]. Например, в основу построения евклидовой геометрии может быть положена шестипараметрическая группа Евклида, которую в механике называют группой перемещений твёрдого тела.

В физике теоретико-групповой подход играет важнейшую роль: «В основе специальной теории относительности лежит математическое понятие группы» В. Паули [11, с. 51]; «Развитие физики в последние годы обратило, в известном смысле, соотношение между уравнениями движения и группами симметрии. Теперь группа симметрии физической системы выступает на первый план, представления этой группы и ее подгрупп несут самую фундаментальную информацию о ней. Таким образом, группы оказываются первичным, наиболее глубоким элементом физического описания природы. Самые понятия пространства и времени играют при этом роль "материала" для построения представлений групп, обычное же место, отводимое им в физике, объясняется лишь историческими причинами» Ю.Б. Румер, А.И. Фет [12, с. 8]; «На ландшафт современной физики надо смотреть с высшей точки зрения: не из оврага истории, а с вершины принципов симметрии» Л.Б. Окунь [13].

«Может возникнуть вопрос: почему все-таки именно теоретико-групповую структуру предлагается взять за основу? Ведь современная физика использует и другие весьма глубокие математические концепции, например функционально-аналитические, топологические, а также алгебраические, выходящие за рамки теории групп. Выбор именно теоретико-групповой концепции определяется не только тем, что она в XX в., как мы пытались показать, выдвинулась в лидеры среди других математических теорий, находящих применение в физике. Дело в том, что она является адекватным математическим выражением фундаментальных физических концепций, лежащих в основе структуры физических теорий, как они понимаются в настоящее время. Речь идет о понятиях и принципах *симметрии*, *инвариантности*, *относительности* (кстати говоря, здесь мы не будем различать эти понятия, считая их в основном совпадающими). Именно эти принципы, с одной стороны, связаны с самыми глубокими основами *научного* познания, экспериментального и математического по своему существу, а с другой — могут быть сформулированы в достаточно строгой форме, именно посредством теоретико-групповой структуры.

В самом деле, вспомним аргументацию "эрлангенского" подхода к геометрии, принадлежащую самому Клейну и особенно четко выраженную Картаном: только задание некоторой фундаментальной группы позволяет развить геометрию как науку (в том смысле, что ее утверждения обладают определенной степенью общности и могут быть возведены в ранг научного закона). Еще более четко такого рода аргументация может быть проведена в отношении физики: результаты измерений предполагают наличие определенной системы отсчета, а требование некоторого уровня общности соответствующих утверждений приводит к необходимости введения целого класса эквивалентных систем отсчета; принцип отождествления этих систем и есть то, что мы понимаем под принципом относительности (симметрии, инвариантности), и он обладает структурой группы, поскольку отношение равенства, лежащее в основе этого принципа, имеет теоретико-групповую структуру» Вл.П. Визгин [14, с. 95].

В школьном учебнике о теоретико-групповом подходе к физике сегодня прочитать можно[2)], но тоже далеко не в каждом.

Для описания преобразований удобно использовать гиперкомплексные числа. Начнём с трёхпараметрической группы вращений (твёрдого тела). Элемент этой группы (поворот) обозначим $\Theta_{\vec{\vartheta}}$, где векторный параметр $\vec{\vartheta}$ определяет величину и направление поворота (трём параметрам группы соответствуют три компоненты этого вектора). Повороту ставится в соответствие число — нормированный кватернион:

[2)] «Итак, если классическая механика представляет собой теорию движений тел, основанную на группе Галилея, то *специальная теория относительности — это такая физическая теория, группой симметрии которой является группа Пуанкаре*» С.В. Громов [15, с. 142].

$$\Theta_{\vec{\vartheta}} \quad \longleftrightarrow \quad e^{i\vec{\vartheta}/2} = \exp\left(i\frac{\vec{\vartheta}}{2}\right) = \cos\frac{\vartheta}{2} + i\sin\frac{\vec{\vartheta}}{2} \qquad (\text{здесь } i^2 = -1).$$

Обратному повороту соответствует число $\exp(-i\vec{\vartheta}/2)$, тождественному преобразованию — число $\exp(0) = 1$. Композиции поворотов ставится в соответствие произведение кватернионов:

$$\Theta_{\vec{\vartheta}_1} \circ \Theta_{\vec{\vartheta}_2} = \Theta_{\vec{\vartheta}_{12}} \quad \longleftrightarrow \quad e^{i\vec{\vartheta}_1/2} \circ e^{i\vec{\vartheta}_2/2} = e^{i\vec{\vartheta}_{12}/2};$$

$$\Theta_{\vec{\vartheta}_2} \circ \Theta_{\vec{\vartheta}_1} = \Theta_{\vec{\vartheta}_{21}} \quad \longleftrightarrow \quad e^{i\vec{\vartheta}_2/2} \circ e^{i\vec{\vartheta}_1/2} = e^{i\vec{\vartheta}_{21}/2}.$$

Здесь важно отметить, что результирующий поворот зависит от последовательности составляющих поворотов, то есть в общем случае $\vec{\vartheta}_{21} \neq \vec{\vartheta}_{12}$. Умножение кватернионов в точности соответствует сложному закону композиции пространственных поворотов, чем и объясняется эффективность применения кватернионов для их описания.

Перейдём к рассмотрению шестипараметрической группы Евклида (перемещений твёрдого тела). Кроме поворота появляется параллельный перенос $R_{\vec{r}}$. Переносу тоже ставится в соответствие число — бикватернион Клиффорда специального вида:

$$R_{\vec{r}} \quad \longleftrightarrow \quad e^{\varepsilon i\vec{r}/2} = \exp\left(\varepsilon i\frac{\vec{r}}{2}\right) = 1 + \varepsilon i\frac{\vec{r}}{2} \qquad (\text{здесь } \varepsilon^2 = 0).$$

Композиции переносов соответствует перенос, не зависящий от порядка выполнения составляющих переносов (закон композиции переносов прост: их векторные параметры складываются), поэтому кружочек не используется:

$$R_{\vec{r}_1} R_{\vec{r}_2} = R_{\vec{r}} \quad \longleftrightarrow \quad e^{\varepsilon i\vec{r}_1/2} e^{\varepsilon i\vec{r}_2/2} = e^{\varepsilon i\vec{r}/2} = e^{\varepsilon i(\vec{r}_1+\vec{r}_2)/2}.$$

В общем случае перемещение представляет собой композицию переноса и поворота, которой ставится в соответствие умножение связанных с ними чисел. Здесь порядок, в котором производятся преобразования (соответственно, умножаются числа), важен:

$$R_{\vec{r}} \circ \Theta_{\vec{\vartheta}} = \Theta_{\vec{\vartheta}} \circ R_{\vec{r}\,'} \quad \longleftrightarrow \quad e^{\varepsilon i\vec{r}/2} \circ e^{i\vec{\vartheta}/2} = e^{i\vec{\vartheta}/2} \circ e^{\varepsilon i\vec{r}\,'/2}.$$

Параметр поворота инвариантен, а параметр переноса в общем случае изменяется, $\vec{r}\,' \neq \vec{r}$. Бикватернионы Клиффорда адекватно описывают все особенности композиций евклидовых движений, чем и определяется основная область их применения в классической механике.

Рассмотрим шестипараметрическую группу Лоренца. В ней кроме поворота появляется буст $V_{\vec{\psi}}$ с параметром скорости $\vec{\psi}$. Бусту ставится в соответствие число — бикватернион Гамильтона специального вида:

$$V_{\vec{\psi}} \quad \longleftrightarrow \quad e^{\vec{\psi}/2} = \exp\frac{\vec{\psi}}{2} = \operatorname{ch}\frac{\psi}{2} + \operatorname{sh}\frac{\vec{\psi}}{2}.$$

Композицию бустов с непараллельными параметрами нельзя заменить одним бустом, приходится вводить дополнительный поворот [11, с. 52]:

$$V_{\vec{\psi}_1} \circ V_{\vec{\psi}_2} = V_{\vec{\psi}} \circ \Theta_{\vec{\vartheta}} \quad \longleftrightarrow \quad e^{\vec{\psi}_1/2} \circ e^{\vec{\psi}_2/2} = e^{\vec{\psi}/2} \circ e^{i\vec{\vartheta}/2}.$$

В общем случае преобразование Лоренца представляет собой композицию буста и поворота, которой ставится в соответствие умножение связанных с ними чисел. И здесь важен порядок, в котором производятся преобразования (умножаются числа):

$$V_{\vec{\psi}} \circ \Theta_{\vec{\vartheta}} = \Theta_{\vec{\vartheta}} \circ V_{\vec{\psi}'} \quad \longleftrightarrow \quad e^{\vec{\psi}/2} \circ e^{i\vec{\vartheta}/2} = e^{i\vec{\vartheta}/2} \circ e^{\vec{\psi}'/2}.$$

Параметр поворота инвариантен, а параметр буста в общем случае изменяется, $\vec{\psi}' \neq \vec{\psi}$. Бикватернионы Гамильтона адекватно описывают все особенности композиций лоренцевых преобразований, чем и определяется основная область их применения в релятивистской механике.

На первый взгляд осталось только сопоставить сдвигу во времени T_t со скалярным параметром t скалярное число, чтобы получить кватернионное представление 10-параметрической группы Пуанкаре, преобразование общего вида из которой можно записать в виде композиции четырёх элементарных преобразований $T_t R_{\vec{r}} \circ V_{\vec{\psi}} \circ \Theta_{\vec{\vartheta}}$. Но всё не так просто. При попытке переставить местами числа, сопоставленные переносу и бусту, приходится вводить [16] новое преобразование $\Phi_{\vec{\varphi}}$:

$$e^{\varepsilon i\vec{r}/2} \circ e^{\vec{\psi}/2} = e^{\vec{\psi}/2} \circ e^{\varepsilon i\vec{r}'/2} e^{\varepsilon\vec{\varphi}/2} \quad \longleftrightarrow \quad R_{\vec{r}} \circ V_{\vec{\psi}} = V_{\vec{\psi}} \circ R_{\vec{r}'} \Phi_{\vec{\varphi}}.$$

Собрав воедино формулы для умножения и перестановки чисел, сопоставленных элементарным преобразованиям, и выделив главное, получим выписанную ниже таблицу незамкнутости для 13-параметрической кватернионной группы [17]:

			T	R	Θ	V	Φ
$\exp(\varepsilon i t/2)$	$\longleftrightarrow$	T					
$\exp(\varepsilon i\vec{r}/2)$	$\longleftrightarrow$	R				Φ	
$\exp(i\vec{\vartheta}/2)$	$\longleftrightarrow$	Θ					
$\exp(\vec{\psi}/2)$	$\longleftrightarrow$	V		Φ		Θ	R
$\exp(\varepsilon\vec{\varphi}/2)$	$\longleftrightarrow$	Φ				R	

(в её клетки ставятся только символы элементарных преобразований, новых по сравнению с теми, которые складываются или переставляются). Отметим, что любые два из следующих трёх требований совместимы со специальной теорией относительности: 1) переносам соответствуют числа вида $\exp(\varepsilon i\vec{r}/2)$; 2) бустам соответствуют числа вида $\exp(\vec{\psi}/2)$; 3) композиции преобразований соответствует произведение сопоставленных им чисел. Но в кватернионной группе нет подгруппы, изоморфной группе Пуанкаре.

3. ЗАДАЧИ

Постановку всех рассматриваемых далее задач можно свести к следующему символическому уравнению инерциальной навигации [18]:

$$\Lambda_{IE(\tau+d\tau)} = \Lambda_{IE(\tau)} \circ \Lambda_{E(\tau)E(\tau+d\tau)}.$$

Здесь I — базовая инерциальная система отсчёта, относительно которой рассматривается движение объекта; $E(\tau)$ и $E(\tau + d\tau)$ — две бесконечно близкие инерциальные системы отсчёта, проходимые объектом; соответственно, $\Lambda_{E(\tau)E(\tau+d\tau)}$ — бесконечно малое преобразование, их связывающее. Выписанная формула — частный случай общей формулы композиции преобразований группы, отличающийся тем, что одно из преобразований бесконечно малое. В соответствии с постановкой задачи инерциальной навигации бесконечно малое преобразование $\Lambda_{E(\tau)E(\tau+d\tau)}$ должно включать перенос во времени, но не должно содержать пространственного переноса (поскольку объект неподвижен в связанной с ним системе отсчёта). Задачу инерциальной навигации в общем виде можно сформулировать следующим образом: по известному положению объекта $\Lambda_{IE(\tau)}$ в момент τ и измеряемому (с помощью жёстко связанных с объектом инерциальных датчиков) приращению положения $\Lambda_{E(\tau)E(\tau+d\tau)}$ требуется найти новое положение объекта $\Lambda_{IE(\tau+d\tau)}$ в момент $\tau + d\tau$. Повторяя этот элементарный процесс, можно, зная начальное положение и получая информацию от инерциальных приборов, рассчитывать текущее положение объекта.

Одна из важнейших навигационных задач — определение времени на борту движущегося объекта. Для этой задачи:

$$\Lambda_{IE(\tau+d\tau)} = T_{t+dt}, \quad \Lambda_{IE(\tau)} = T_t, \quad \Lambda_{E(\tau)E(\tau+d\tau)} = T_{d\tau}.$$

Соответственно, символическое уравнение инерциальной навигации принимает следующий вид:

$$T_{t+dt} = T_t T_{d\tau}.$$

Из этого теоретико-группового уравнения следует (числовое) уравнение $dt = d\tau$ для параметров.

Следующая практически важная задача — определение ориентации объекта (рассматриваемого как твёрдое тело) с использованием датчиков угловой скорости $\vec{\omega}$. Для этой задачи кроме сдвигов во времени используются повороты:

$$\Lambda_{IE(\tau+d\tau)} = T_{t+dt}\Theta_{\vec{\vartheta}+d\vec{\vartheta}}, \quad \Lambda_{IE(\tau)} = T_t\Theta_{\vec{\vartheta}}, \quad \Lambda_{E(\tau)E(\tau+d\tau)} = T_{d\tau}\Theta_{\vec{\omega}d\tau}.$$

В результате получается теоретико-групповое уравнение

$$T_{t+dt}\Theta_{\vec{\vartheta}+d\vec{\vartheta}} = (T_t\Theta_{\vec{\vartheta}}) \circ (T_{d\tau}\Theta_{\vec{\omega}d\tau}),$$

из которого кроме тривиального $dt = d\tau$ получается ещё и существенно более сложное (поскольку $d\vec{\vartheta} \neq \vec{\omega}d\tau$) уравнение инерциальной ориентации (его проще записывать с использованием кватернионов).

Следующий естественный шаг состоит в расширении фундаментальной группы до 10-параметрической группы Галилея. Теоретико-групповое уравнение получается таким:

$$T_{t+dt}R_{\vec{r}+d\vec{r}}V_{\vec{v}+d\vec{v}} \circ \Theta_{\vec{\vartheta}+d\vec{\vartheta}} = (T_tR_{\vec{r}}V_{\vec{v}} \circ \Theta_{\vec{\vartheta}}) \circ (T_{d\tau}V_{\vec{a}d\tau}\Theta_{\vec{\omega}d\tau}).$$

В нём проявляются уже все особенности общей постановки задачи инерциальной навигации. Для получения информации об изменении связанной с объектом системы отсчёта используется традиционный набор приборов: часы, акселерометры и датчики угловой скорости. Некоммутативность композиции преобразований приходится учитывать уже при разложении преобразования, связывающего две системы отсчёта, на элементарные (в данном случае перенос во времени T, перенос в пространстве R, нерелятивистский буст V и пространственный поворот Θ). Из этого теоретико-группового уравнения получается следующая система уравнений инерциальной навигации (нерелятивистских и без учёта гравитации):

$$\frac{dt}{d\tau} = 1, \qquad \frac{d\vec{r}}{d\tau} = \vec{v}, \qquad \frac{d\vec{v}}{d\tau} = \mathrm{Q} \circ \vec{a} \circ \mathrm{Q}^{-1}, \qquad \frac{d\mathrm{Q}}{d\tau} = \frac{1}{2}\mathrm{Q} \circ (i\vec{\omega}),$$

где $\mathrm{Q} = \exp(i\vec{\vartheta}/2)$ — кватернион поворота.

Последний шаг, который остаётся сделать для полноценной формулировки задачи инерциальной навигации в рамках теоретико-группового подхода — включение в фундаментальную группу элементарного преобразования, соответствующего гравитационному ускорению (ускорению тяготения):

$$T_{t+dt}R_{\vec{r}+d\vec{r}}V_{\vec{v}+d\vec{v}} \circ \Theta_{\vec{\vartheta}+d\vec{\vartheta}} \circ G_{\vec{g}+d\vec{g}} = (T_tR_{\vec{r}}V_{\vec{v}} \circ \Theta_{\vec{\vartheta}} \circ G_{\vec{g}}) \circ (T_{d\tau}V_{\vec{a}d\tau}\Theta_{\vec{\omega}d\tau}G_{\vec{n}d\tau}).$$

В состав инерциальной навигационной системы приходится включать ещё прибор, измеряющий скорость приращения гравитационного ускорения [19, с. 27]. «Выше описана принципиальная возможность создания инерциальных систем без заранее заданной карты для пространственного поля ускорений тяготения. Еще недавно были исследователи, которые сомневались в принципиальной возможности построения подобной инерциальной системы» Л.И. Седов [19, с. 28]. Приведём соотношения, которые характеризуют гравитационное преобразование, входящее в расширенную группу Галилея:

$$G_{\vec{g}} \circ T_t = T_t R_{\vec{r}} V_{\vec{v}} G_{\vec{g}}, \qquad \vec{r} = \vec{g}t^2/2, \quad \vec{v} = \vec{g}t;$$

$$G_{\vec{g}} R_{\vec{r}} = R_{\vec{r}} G_{\vec{g}};$$

$$G_{\vec{g}} V_{\vec{v}} = V_{\vec{v}} G_{\vec{g}};$$

$$G_{\vec{g}} \circ \Theta_{\vec{\vartheta}} = \Theta_{\vec{\vartheta}} \circ G_{\vec{g}\,'}, \qquad \vec{g}\,' = e^{-i\vec{\vartheta}/2} \circ \vec{g} \circ e^{i\vec{\vartheta}/2};$$

$$G_{\vec{g}_1} G_{\vec{g}_2} = G_{\vec{g}}, \qquad \vec{g} = \vec{g}_1 + \vec{g}_2.$$

При учёте релятивистских эффектов расширять приходится не группу Галилея, а группу Пуанкаре, причём сразу до 15-параметрической конформной группы. Теоретико-групповое уравнение инерциальной навигации при этом выглядит так:

$$T_{t+dt} R_{\vec{r}+d\vec{r}} \circ V_{\vec{\psi}+d\vec{\psi}} \Gamma_{\alpha+d\alpha} \Theta_{\vec{\vartheta}+d\vec{\vartheta}} \circ G_{\vec{g}+d\vec{g}} W_{w+dw} =$$

$$= (T_t R_{\vec{r}} \circ V_{\vec{\psi}} \Gamma_\alpha \Theta_{\vec{\vartheta}} \circ G_{\vec{g}} W_w) \circ (T_{d\tau} V_{\vec{a}d\tau} \Gamma_{\mu d\tau} \Theta_{\vec{\omega}d\tau} G_{\vec{n}d\tau} W_{\nu d\tau}).$$

Здесь появились ещё два символа элементарных преобразований: Γ — масштабное преобразование и W — преобразование, связанное с гравитационным изменением масштабов. При работе с конформной группой наиболее эффективным оказывается использование бикватернионов Гамильтона [20].

«Итак, математика двумя различными путями вела физику к принятию конформной группы ("эрлангенская" схема и анализ свойств инвариантности уравнений Максвелла), но отсутствие достаточно глубоких оснований физического характера и успех альтернативного способа расширения Р-группы (ОТО) препятствовали этому принятию» Вл.П. Визгин [21, с. 205]. Потребовавшаяся для инерциальной навигации расширенная группа Галилея (группа Галилея–Ньютона) и есть то основание, которое позволяет согласовать конформную группу с классической механикой. Выпишем для сравнения таблицы незамкнутости элементарных преобразований для расширенных групп Галилея и Пуанкаре:

	T	R	Θ	V	G
T				R	R, V
R					
Θ					
V	R				
G	R, V				

,

	T	R	Θ	V	G	W	Γ
T				R	R, V, W, Γ	Γ	
R				T	Θ, Γ	T, V, G, Γ	
Θ							
V	R	T		Θ	W	G	
G	R, W, V, Γ	Θ, Γ		W			
W	Γ	T, G, V, Γ		G			
Γ							

.

4. ЗАКЛЮЧЕНИЕ

Приведём напутствие редактора сборника статей Г. Минковского, вышедшего в 1910 году: «И пусть каждый по мере своих сил способствует осуществлению смелой мечты Минковского о том, чтобы в сознании человечества для будущих поколений пространство и время низвелись до роли теней, и живым осталось бы только *пространственно-временное преобразование*» О. Блюменталь [22, с. 15–16].

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Метод неассоциативной алгебры при построении теории гравитации

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Пространство-время рассматривается как дуальное пространство пространству полей. Тогда пространство функций, на котором определяются физические поля являются амплитудами состояния f(x) частицы. Так как вероятностная интерпретация квадрата модуля f(x) следует из дуальности f(x) и f*(x) (поле f*(x) – пространство функций в дуальном пространстве), то задача нахождения представления волновой функции частицы сводится к теореме Фробениуса об алгебрах с делением. Как логическое следствие на основе расширения алгебры полевых взаимодействий до неассоциативной алгебры октонионов предлагается модель ОТО и обобщение ОТО до модели с кручением при учёте киральности поля.

Введение

Единственным обобщением алгебры кватернионов, имеющим операцию деления, является алгебра октонионов, по терминологии Артура Кэли, или алгебра октав, по терминологии Джона Грейвса. Известно [1], что первая публикация, описывающая эту алгебру, принадлежит А. Кэли и датирована 1845 годом – через два года после открытия кватернионов Гамильтоном.

Видимо первая попытка применения алгебры октонионов к физической теории была сделана в работе [2], где вводилась октонионная квантовая механика. Однако серьёзного развития идеи этой работы не имели, возможно из-за недостаточного развития математических и физический концепций тех лет, а возможно в математической абстрактности используемого в статье алгебраического аппарата октонионов. Немного ранее, в работах Цорна [3], были предложены матричные представления алгебры октонионов, но элементами матриц Цорна были специальные объекты: числа и векторы. Поэтому эта работа указывала на путь представления неассоциативной алгебры матрицами, но предлагаемый вид матриц был очень абстрактным. Не замеченной физиками и недостаточно оценённой, по мнению автора, явилась замечательная статья J. Debout и R. Delbago [4], которая буквально «вписывала» октонионы в физическую теорию: было найдено представление октав матрицами Дирака с особым правилом умножения.

Актуальность проведённых автором исследований расширения групповой теории Вайнберга-Салама слабых взаимодействий на неассоциативную алгебру обусловлена необходимостью поиска новых подходов для построения теории, совмещающих в себе как общекоординатные, так и калибровочные преобразования. При этом имеется надежда включить группу общекоординатных преобразований как проявление неассоциативности взаимодействия.

Основные успехи в теории элементарных частиц связаны с использованием теории представлений групп. В данной работе модель взаимодействий строится не на базе заложенных симметрий, а посредством расширения алгебраической структуры поля. С этой целью в Стандартной Теории Вайнберга-Салама (СТ) предлагается заменить киральную симметрию $U(1) \times SU(2)$ на неассоциативную алгебру [5]. В силу неассоциативности алгебры происходит потеря свойств калибровочной симметрии, которая, будем считать, обусловлена структурой вакуума на планковском уровне, вне которых они восстанавливаются. Дополнительные эффекты, которые возникают как следствие неассоциативности, предлагается интерпретировать, в частности, проявлением гравитации.

Волновая функция частицы как функция на многообразии

Определим существование некоторого векторного поля $A^d = A^d_\mu e^\mu, \mu = 0, 1, 2, 3$ аксиоматически. (В СТ вводится поле Хиггса $\varphi(x)$. Тогда вектор $\partial\varphi = \partial_\mu \varphi e^\mu$ может играть роль этого векторного поля. Однако в приводимых ниже рассуждениях конкретный вид векторного поля не предполагается.)

Возможность описывать движение в пространственно-временном многообразии Ω рассматриваем как метки поля A^d. Могут существовать и другие поля A, которые в пространстве-времени проявляются как векторные. Пусть физическое поле A^d воздействует по крайней мере само на себя, индуцируя «точечные сгущения» x на $\Omega \ni x$, которые позволяют различать соседние точки многообразия Ω («точечными сгущениями» называем результат самодействия поля A^d и его взаимодействие с полями A). Так как «точечные сгущения» индуцируют метки, то относительно них с помощью линейной формы будем строить дуальное векторное пространство $T^*(x)$. Векторное поле A^d на многообразии Ω понимается обычным образом: существует однопараметрическая функция $\tilde{\varphi}(x(\tau))$ на физическом многообразии Ω, относительно которой задаётся векторное поле как $d/d\tau$. В дуальном векторном пространстве определяем ортогональный базис линейных форм $dx = *A^d$ из правила $< dx, A^d >= I$, где $< dx, A^d >$ – линейная форма, I – единичная матрица. Здесь форма $dx = (dx^0, \ldots, dx^3)$ индуцирует координаты $(\xi^0, \ldots, \xi^3)$. (Физически речь идёт о координатах $(ct, x, y, z) = (\xi^0, \ldots, \xi^3)$.) Формально строим кинематические соотношения между координатами вида

$$d\xi^i/d\xi^0 = v^i, (i = 1, 2, 3) \tag{1}$$

и экспериментально устанавливаем динамические уравнения в случае консервативных систем вида

$$mdv^i/d\xi^0 = \delta^{ij}\partial_j V^{ext}(x), (i = 1, 2, 3, I = (\delta^{ij})) \tag{2}$$

(здесь $V^{ext}(x)$ – внешнее потенциальное поле, например, ньютоновское гравитационное или кулоновское электрическое) соответствующие ньютоновской механике и обобщаем всё на специальную теорию относительности, вводя лоренцевую инвариантность интервала

$$ds^2 = c^2dt^2 - dx^2 - dy^2 - dz^2 \tag{3}$$

С точки зрения предлагаемого подхода в (2) речь идёт о некотором токе J как о векторе, который определяется идентификацией «точек сгущения». В базисе $e_\mu, \mu = 0, 1, 2, 3$ он имеет вид:

$$J = J^\mu e_\mu = m\frac{d\xi^\mu}{ds}e_\mu \tag{4}$$

«Замечая», что «точечные сгущения» обладают зарядом, строим напряжённость электромагнитного поля F, как антисимметричный тензор второй степени. Из опыта убеждаемся в точности формы F, то есть $dF = 0$. Равенство $dF = 0$ понимаем как отсутствие источников поля, с одной стороны, с другой стороны, это означает существование 1-формы ω, для которой $d\omega = F$. В частном случае электромагнитного поля тензор первой степени ω – это электромагнитный вектор-потенциал $A^{el-m} = \omega$ и $dA^{el-m} = F^{el-m}$. Будем рассматривать электромагнитный вектор-потенциал A^{el-m} и векторное поле A^d как два линейно-независимых векторных поля линейного пространства T. Так как вектор, дуальный dx – это вектор $\partial/\partial x = \partial_x$, то векторами пространства T в некотором представлении являются векторы ∂_x и A, которые взаимодействуют с «точками сгущения» – состояниями, обозначаемыми в дальнейшем как Φ (или Ψ). Тогда можно говорить о взаимодействии полей A^d и A с «точечными сгущениями» Φ в виде $A \cdot \Phi$ и $A^d \cdot \Phi$. Вектор, дуальный вектору A^d – это вектор dx. Вектор, дуальный вектору dx это вектор ∂_x. Таким образом можно установить изоморфизм между ∂_x и A^d. Вектор ∂_x задаётся на пространстве функций, поэтому будем считать, что в выбранном нами представлении $A^d \cdot \Phi = \partial_x\Phi$.

Состояние Φ является формальным представлением «точечных сгущений» – состояния частицы. Следовательно определено дуальное пространство состояний $*\Phi$, для которого $<*\Phi, \Phi >= C$. (Можно договориться, что $C = 1$.) Например, в пространстве функций равенство $< *\Phi, \Phi >= 1$ могло бы выглядеть как $\int(*\Phi(x))\Phi(x)dx = 1$.

Пусть речь идёт об одной частице в объёме V. Тогда состояние Φ не должно зависеть от координаты x, если по-прежнему понимать наличие аргумента x в выражении $\Phi(x)$ как указатель на локализацию частицы в точке с координатой x. Состояния Φ определяются как точечные сгущения или как частицы и не могут быть связаны с внутренней структурой частиц, поэтому не имеет смысл их различать. В этом случае получаем «хороший» (с точки зрения предъявляемых на данный момент требований к функциям $\Phi(x)$ и $*\Phi(x)$) вид этих функций: $\Phi(x) = *\Phi(x) = 1/\sqrt{V}$. Фактически имеем равенство:

$$< *\Phi, \Phi >= \int (*\Phi(x))\Phi(x)dx = \int \Phi(x)\Phi(x)dx = \int (*\Phi(x)) * \Phi(x)dx = 1 \qquad (5)$$

Данное равенство замечательно тем, что предлагает способ симметризации состояний частиц и состояний античастиц.

$$|\Phi \cdot *\Phi| = |\Phi| \cdot |*\Phi| = 1 \qquad (6)$$

(6) и (5) позволяют интерпретировать величину $|\Phi(x)|^2$ как вероятность нахождения частицы в состоянии Φ в точке x, а само значение (6) как полную вероятность. В теории вероятностей математическое ожидание непрерывной случайной величины $\hat{\xi}$, имеющий закон распределения $\tilde{F}_{\tilde{\xi}}$, когда $\tilde{F}'_{\tilde{\xi}}(t) = \rho_{\tilde{\xi}}(t)$, находится по формуле

$$M(\hat{\xi}) = \int \hat{\xi}(x)\rho_{\xi}(x)dx, \qquad (7)$$

поэтому все характеристики частиц находятся как среднии по ансамблю. При этом частицы рассматриваются как безструктурные. В статистической физике такой подход реализуется в операторном формализме физических величин над вероятностным распределением частиц, например, в [6].

Однако можно наделить структурой частицы дуального пространства. Если под состоянием Φ понимать пространство состояний, соответствующее частицам, то с современных позиций логично назвать пространство состояний $*\Phi$ состоянием античастиц. Действительно, если говорить, что «точечные сгущения» порождают частицы, обладающие некоторыми свойствами, то сохранение этих свойств требует состояния античатиц, которые описываются «одновременно» с состоянием Φ.

Состояние Φ инвариантно относительно ряда преобразований (по крайней мере относительно лоренцевых преобразований) так как под ним понимается описание, например, частицы в обычном пространстве-времени. Введём физическое поле $*F$ в дуальном пространстве. Для него $d{*}F = 4\pi J$, где J – ток, индуцированный «точечными сгущениями». Появление тока J обусловлено появлением «точек сгущения», то есть состояний Φ, поэтому теперь $d * F \neq 0$. Вектор тока J определим как $J = (d * \Phi)\Phi - *\Phi(d\Phi)$. Такое определение согласовано с тем, что $dd * F = d^2 * F = 0$. Таким образом имеется следующая совокупность утверждений:

$$dF = 0 \qquad (8)$$

$$d * F = 4\pi J \qquad (9)$$

$$J = (d * \Phi)\Phi - *\Phi(d\Phi) \qquad (10)$$

$$*\Phi \cdot \Phi = 1 \tag{11}$$

Рассмотрим локализованное состояние частицы в объёме V. Например, $\Phi(x) = C_0 e^{-kx}$. Логично понимать свободное движение частицы и античастицы как состояния, которые описываются одинаковым образом. Следовательно состояния частиц и античастиц описываются одной и той же функцией. Тогда $*\Phi(x) = \Phi(x)$. Такой подход не является конструктивным так как возвращает к уже изученному движению частиц в статистической физике, когда они считаются бесструктурными. Следовательно необходимо изменить структуру функции состояния $\Phi(x)$. Но имеется и конструктивный подход, сохраняющий симметрию состояний частиц и античастиц, но расширяющий и углубляющий эту симметрию: рассмотреть различные алгебры, для которых выполняется равенство (6), то есть сформулировать **гипотезу состояния частицы:** *алгебра представления состояния частиц a и b, с нормами $|a|$ и $|b|$ соответственно, удовлетворяет равенству*

$$|a \cdot b| = |a| \cdot |b| \tag{12}$$

По Теореме Гурвица: *Любая нормированная алгебра с единицей (то есть выполняется условие (12)) изоморфна одной из четырёх алгебр: действительных чисел, комплексных чисел, кватернионов или октав.*

Физическая теория как соответствие выбранной алгебре

В предлагаемом подходе естественно понимать статистическую механику как вещественное представление пространства состояний, так как в основном речь едёт о механических движениях, не учитывающих существование различных свойств у частиц и античастиц.

Из (11) следует, что состояние вида $\Phi(x) = Ce^{ikx}$, удовлетворяет «гипотезе представления состояния частицы». В этом случае величиной, дуальной времени dt, является $\frac{\hbar}{i}\partial_t$. Заметим естественное введение мнимой постоянной i. Понятно, что постоянная Планка вводится из более глубоких соображений – здесь достаточно понимание того, что дуальный элемент ∂_t определяется с точностью до постоянного множителя.

Предложенное обобщение на комплексные числа является единственным минимальным обобщением по теореме Гурвица.

Будем считать определённой линейную комбинацию векторных полей A^d и A вида $A^d \cdot e_1 + A \cdot e_2$ в некотором изоморфном представлении (q – заряд частицы)

$$\frac{\hbar}{i}\partial_x + qA(x) \tag{13}$$

Симметрия $U(1)$ для нормы $\Psi^*(x)\Psi(x)$ индуцирует симметрию $U(1)$ для выражения

$$\Psi^*(x)(\frac{\hbar}{i}\partial_x + qA(x))\Psi(x) = \Psi^*\nabla\Psi \tag{14}$$

при этом выражение $\frac{\hbar}{i}\partial_x + qA(x)$ в смысле геометродинамики эквивалентно ковариантной производной в главном расслоении.

Первоначально электромагнитный вектор-потенциал A^{el-m} был определён из точности тензора напряжённости поля F^{el-m}. Теперь, однако, понятно, что первичной характеристикой является всё-таки вектор-потенциал A^{el-m}. Тем самым тензор F^{el-m} необходимо определить как производную от A. С этой целью заметим, что в пространстве Минковского имеется билинейная квадратичная формы с метрикой Минковского, поэтому может быть определена площадка $\Sigma = dx \wedge dx$, при обходе которой ковариантным образом ($\frac{\hbar}{i}\nabla\Psi = A\Psi$) индуцируется величина [7]

$$\Delta\Psi(x) = F\Sigma\Psi \tag{15}$$

Задача совместного состояния тензорного поля F и ему сопряжённого $*F$ по определению является согласованной, поэтому может быть определён скаляр $S = *F \wedge F$. Таким образом получаем ещё несколько скалярных инвариантов

$$S_{el.-m.} = *F \wedge F \tag{16}$$

$$S_{sp} = (\nabla * \Psi)\Psi - *\Psi(\nabla\Psi) \tag{17}$$

$$S_{sc} = *(\nabla\Phi)\nabla\Phi \tag{18}$$

Тут же возникает замечание: «точки сгущения», с которыми ассоциируются состояния Ψ, оказываются вторичным элементом по отношению к векторному полю. Векторное поле A^d оказывает воздействие на состояния Ψ. Первоначальные рассуждения были нужны, чтобы понять требования на состояния Ψ. Естественным является такое же требование на векторные поля A как элементов той же алгебры. Поэтому «алгебра векторных полей A должна принадлежать одной из алгебр: вещественной, комплексной, кватернионной или алгебре октав». В нашем случае какому-либо из представлений этих алгебр. Это утверждение назовём «Гипотезой об алгебре поля». Но векторное поле A является «первичным» объектом. Следовательно «Гипотеза о состоянии частицы» возможно неверна на некоторой алгебре, но нарушать «Гипотезу об алгебре поля» оснований нет.

Вектор поля A^d не имеет заранее определённой структуры. Его проявление определяется самодействием и взаимодействием с полями A. Результат взаимодействия с математической точки зрения является элемент алгебры. Далее нашей задачей было построение непротиворечивой схемы над этой алгеброй и изучение её свойств. По теореме Гурвица можно сделать следующий шаг, не противоречащий общему подходу, излагаемому в этой работе. Считать, что алгебра – это алгебра кватернионов. Тогда элементом пространства состояний является алгебраический элемент, изоморфный алгебре кватернионов. Формально в качестве такого элемента можно было бы взять любое представление, но ввиду того, что уже «придумано» хорошо согласующееся с опытом описание модели над комплексным полем (в соответствии с предлагаемыми рассуждениями комплексные величины описываю внутреннюю степень свободы, идентифицирующуюся как заряд для представления частиц и античастиц) будем требовать, чтобы в качестве коммутативного поля над функциями состояния выступало комплексное поле C. По этой причине выберем в качестве такого представление представление матрицами $SU(2)$. Тогда вектор поля A должен быть элементом этой алгебры. Следовательно, будем считать, что

$$A = A^a_\mu(x) \cdot \sigma^a e^\mu \tag{19}$$

σ^a – это единичная матрица и три матрицы Паули. Коэффициенты $A^a_\mu(x)$ выбираем вещественными, так как с ними ассоциируем физические величины.

Следовательно пространство состояний является элементом пространства, на котором действует группа $SU(2)$. Но тогда форма dx имеет вид:

$$dx = dx^{\mu(a)} \cdot \sigma^a e_\mu \tag{20}$$

Фактически мы имеем спинорное представление пространства-времени вида [8]

$$\begin{pmatrix} T+Z & X+iY \\ X-iY & T-Z \end{pmatrix} = T\sigma^0 + X\sigma^1 + Y\sigma^2 + Z\sigma^3 \tag{21}$$

В [8] показано, что изоморфное отображение спиноров в группу вращений $SO(3)$ можно установить, вводя изотропный флаг $\mathbf{K}$ и пространственно-подобный вектор $\mathbf{L}$ (в обозначениях [8]). Тогда плоскость $\mathbf{\Pi}$, натянутая на векторы $\mathbf{K}$ и $\mathbf{L}$:

$$\mathbf{\Pi} = a\mathbf{K} + b\mathbf{L}, \quad b > 0 \tag{22}$$

отвечает лоренц-преобразованиям и бустам пространства-времени

Действия (16-18), обобщённые на кватернионную структуру поля и состояния, когда поля (13) берутся в представлении трёх матриц Паули и единичной матрицы 2×2 в киральном случае позволяет построить модель электрослабых взаимодействий. (Киральность лагранжиана СТ по мнению Пенроуза (там же) обусловлена именно ограничением $b > 0$, в случае же предлагаемого подхода эта догадка получает дополнительные основания в силу появления дополнительного картановского взаимодействия спина и гравитации [9]). Правда для перенормируемости теории в полное действие необходимо включить самодействие состояний Φ в виде потенциала Хиггса

$$V_{Higgs} = m^2|\Phi|^2 - \frac{f}{4}|\Phi|^4 \tag{23}$$

что в общем-то не нарушает общий взгляд на построение теории, излагаемый здесь. В результате мы приходит к киральной модели слабых взаимодействий $SU(1) \times SU(2)$ теории Вайнберга-Салама.

Следующий шаг – обобщение теории на алгебру октонионов. В матричном представлении форма на простанстве-времени имеет вид (по аналогии с (20)):

$$dx = dx^{(a)\mu} \cdot \Sigma^a e_\mu, \quad a = 0, 1, \ldots, 7, \mu = 0, 1, 2, 3. \tag{24}$$

Таким образом на ядерных расстояниях в таком подходе [10] возникают основания говорить о постранстве-времени с большим, чем четыре количеством измерений, но этот вопрос в данной работе не исследуется. Обсудим другую серьёзную проблему – неассоциативность алгебры. Оказывается невозможным извлечение симметрии из каждого из выражений действий (16,18). Только действие (17) может быть представлено в ассоциативной форме.

На заре развития теории электромагнетизма основными понятиями электромагнитной теории были измеримые поля $\vec{E}$ и $\vec{B}$. Если пойти тем же путём и считать первичным именно эти поля, то действие (16) оказывается квадратичным по физическим полям $\mathbf{E}$ и $\mathbf{B}$:

$$F = dA + [A, A] = F_{ik} dx_i \wedge dx_k =$$

$$= ((A^a_{k,i} - A^a_{i,k})\Sigma^a + A^a_{k,i} A^b_{i,k} \varepsilon^{abc} \Sigma^c) dx_i \wedge dx_k = F^a_{ik} \Sigma^a dx_i \wedge dx_k \tag{25}$$

– использовано обозначение образующих алгебры Σ^a со структурными постоянными ε^{abc}. Неассоциативность определяется на трёх элементах алгебры, для которых в общем случае:

$$a \cdot (b \cdot c) \neq (a \cdot b) \cdot c \tag{26}$$

но (25) линейно по физическому полю F^a, поэтому квадратичен лагранжиан (16). С другой стороны, алгебра октонионов со своими структурными постоянными образует групповую симметрию G_2 и содержит симметрию $SU(3)$. В этом смысле октонионная структура поля в (16) должна проявляться в том числе и как структура, содержащая физическую симметрию $SU(3)$ [11]. Но симметрия $SU(3)$ наблюдается как цветная симметрия в теории сильных взаимодействий. По-видимому исследования, аналогичные сделанным в [8] позволили бы пролить свет на ряд удивительных фактов в теории сильных взаимодействий, но пока такая работа не проводилась и здесь этот аспект дальше не исследуется.

С точки зрения предлагаемого здесь подхода, в соответствии с теоремой Гурвица, рассмотрим обобщение теории на неассоциативную алгебру октонионов, явно содержащую неассоциативность поля.

Опыт обобщения теории на алгебру кватернионов показывает, что конструктивным является представление именно матрицами (2×2). Октонионы тоже могут быть представлены матрицами, но с особым правилом умножения. В [5] рассмотрено обобщение лагранжиана слабых взаимодействий Стандартной Теории на алгебру октонионов, если рассматривать матричное представление октонионов над комплексным полем. Там же был построен соответствующий лагранжиан. Лагранжиан строился таким образом, чтобы он переходил в лагранжиан Стандартной Теории в случае равенства нулю констант взаимодействия со старшими полями (полями $A^a, a = 4, 5, 6, 7$, соответствующими расширению на алгебру октав). Там же показано, что в результате обобщения на алгебру октонионов возникают новые массивные частицы, которые взаимодействуют с материальными полями (под материальными полями понимаются спинорные поля). Оказывается [12], что в силу неассоциативности полей, вместо одного массивного поля появляются два поля – безмассовое и массивное. При этом безмассовое поле можно интерпретировать как поле, индуцирующее гравитацию и кручение [9]. Тогда оказывается, что масса новых частиц планковская, а гравитацинная постоянная равна обратному квадрату массы этой частицы (с точностью до константы примерно равной 0.01). Важным обстоятельством является то, что мы получаем уравнения Эйнштейна именно в том виде, в каком на основе догадки написал Эйнштейн $G_{ik} = \kappa T_{ik}$, где роль тензора Эйнштейна G_{ik} играет неассоциативная часть лагранжиана. В [9] показано, что кручение, которое появляется в теории мало и пока не наблюдаемо. Отметим [9], что предлагаемый здесь подход называется О-теорией.

Заключение

В данной работе предлагается самосогласованный подход для построения теории как соответствие некоторой алгебре. Имеется хорошее согласие с идей, что алгебра кватернионов в определённом представлении соответствует СТ. Поэтому имеет смысл рассмотреть ближайшее обобщение этой идеи на алгебру октав Кэлли, что и предлагается в данной работе. Интересным обстоятельством является отмеченная возможность включения в такую модель сильные взаимодействия.

Благодарности

Автор выражает благодарность участникам семинара лаборатории теоретической физики им. А. А. Фридмана за полезные обсуждения. Работа выполненена при частичной финансовой поддержки министерства образования и науки Российской Федерации.

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Sensitivity of the GW bar detector's optical-electronic readout

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The small vibration transducer on Pound–Drever–Hall technique is considered in radio-physical and mechanical aspects. Reference to laser optics had allowed obtaining expression for the signal conversion gain. Photodetector, photo current and preamplifier noises are discounted in the analysis of device sensitivity. The spectral densities of laser intrinsic frequency and power fluctuations are entered, measured and discounted. The back action stochastic force of the sensor is compared with Nyquist force of the bar.

Lambda-instability of Keplerian orbits and its observable manifestations

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This work is devoted to the investigation of the effects caused by the Lambda-term in Einstein equations (commonly called the "dark energy") on the long-term evolution of Keplerian orbits. The appropriate astrophysical examples can be the planetary systems, binary stars, *etc*.

A mathematical framework for our consideration is the so-called Kottler metric of General Relativity for a point-like mass embedded in the Lambda-background, which was transformed to the Robertson–Walker coordinates to provide the adequate cosmological asymptotics at infinity [1]. As was already shown in our earlier work [2], the action by Lambda-term results in the secular increase of the mean orbital radius, which might be observed, for example, in the parameters of the Earth–Moon system [3, 4].

A further investigation of the same equations of motion have shown that, along with the above-mentioned gradual Lambda-perturbations (which should be interesting, first of all, for the planetary dynamics), there is also a specific Lambda-instability of the orbits, resulting in a disruption of the orbit and an escape of the low-mass body with a considerable velocity. This effect was revealed quite unexpectedly, when we studied the corresponding set of equations by the numerical methods for the case of sufficiently large perturbations.

The above-mentioned phenomenon can have important astrophysical applications, *e.g*., to explain the anomalous peculiar velocities of pulsars and some other types of stars with respect to the background stellar population. In fact, such velocities were observed for a long time, but they have been commonly attributed either to the "sling effect" after explosion of one star in the binary system or to the "reactive force" in the case of asymmetric explosion of a supernova. As follows from our consideration, onset of the Lambda-instability may be yet another mechanism for explanation of the same phenomenon. Its main advantage is that it does not require any stellar explosions at all and, therefore, can operate for the stars of any kind.

Because of the mathematical complexity, the calculations performed by now were done only for the case of a body with infinitely small mass (*i.e*., a test particle) moving in the gravitational field of the central massive component, embedded in the Lambda-background. However, it should be expected that the same instability will exist also in the case of two components with comparable masses.

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Вращение галактического диска без темной материи в семимерном пространстве-времени

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В статье ставится задача объяснить аномальное увеличение скорости движение звезд в галактическом диске без использования темной материи. Для этого рассматривается семимерное пространство-время, в котором в качестве дополнительных координат выступают углы Эйлера. Подобный подход, позволяет не только объяснить кривую скорости галактик, используя лишь светящуюся материю, но и объясняет эмпирический закон торможения Скуманича.

Введение

В 1937 году Фриц Цвикки опубликовал работу "On the Masses of Nebulae and of Clusters of Nebulae в которой на основе наблюдений относительных скоростей галактик в скоплении Волос Вероники получил результат, согласно которому видимая, светящееся материя скопления оказалась значительно ниже массы скопления, рассчитанной исходя из линейных скоростей членов скопления. Это означает, что сумма всей наблюдаемой массы скопления оказывается недостаточной, чтобы удерживать составляющие части галактики от разлета в разные стороны (1).

В 1959 году Луис Волдерс обнаружил, что Галактика Треугольника (спиральная галактика M33) не вращается в соответствии с динамикой Кеплера. Позднее в 70-е годы (2) подобное явление было обнаружено во многих других спиральных галактиках. Модель Кеплера подразумевает, что вещество (такое как звезды или газ) в дисковой части спирали должно вращаться вокруг центра галактики в соответствии с механикой Ньютона. То есть средняя орбитальная скорость звезд после определенного расстояния будет уменьшаться обратно пропорционально квадратному корню от радиуса орбиты. Однако наблюдения кривой скорости движения звезд не подтверждают динамику, построенную на модели Кеплера. Кривая скорости не уменьшается обратно пропорционально квадратному корню от расстояния, а является практически неизменной или даже возрастает. Объяснение этого явления, с минимальным изменением физических законов вселенной состоит в том, что существует материя, которая находится на значительном расстоянии от центра галактики и не излучает свет. Эта дополнительная масса называется - темной материей.

Однако введение темной материи выглядит, как искусственная попытка подогнать наблюдение под существующую теорию. В данной работе строится модель гравитационного взаимодействия внутри галактики, которая позволяет получить необходимую кривую вращения звезд, внутри галактического диска, используя только светящуюся материю.

В геометризированных теориях гравитации описывается движение точек и векторов, которые обладают тремя степенями свободы. Но протяженные твердые тела, обладают шестью степенями свободы, каждая из которых способна приводить к изменению положения тела (3). Поэтому для выполнения условия паритетности между уравнениями поступательного и вращательного движения перейдем от четырехмерного пространства-времени, с которым сейчас оперирует общая теория относительности, к семимерному пространству-времени, где три дополнительных измерения ориентируют твердое тело в пространстве. Подобный подход позволит рассматривать вращение тел как обычное движение в многомерном пространстве описываемое уравнениями движения.

В работе (4) показано, что для объяснения динамики не только поступательного, но и вращательного движения тел в гравитационных полях можно пользоваться семимерным пространством-временем, в котором помимо времени и трех пространственных координат присутствуют три координаты ориентирующие тело в пространстве $x^4 = \varphi$, $x^5 = \psi$, $x^6 = \theta$, являющимися углами Эйлера. Используя уравнения геодезических такого семимерного пространства-времени можно получить не только уравнения поступательного движения, но и уравнения гироскопов описывающих вращательное движение (5).

Метрика шарообразного тела, с моментом инерции J относительно главных осей, и массой m, в семимерном пространстве-времени имеет вид:

$$g_{00} = 1,$$

$$\begin{aligned} g_{\alpha\alpha} &= -1, \\ g_{44} &= -\frac{J}{m}, \\ g_{45} = g_{54} &= -\frac{J\cos\theta}{m}, \\ g_{55} &= -\frac{J}{m}, \\ g_{66} &= -\frac{J}{m}, \end{aligned} \tag{1}$$

где $\alpha = 1, 2, 3$ пробегает по пространственным координатам.

Гравитационные уравнения в семимерном пространстве-времени, как показано в работе (6), могут быть записаны в виде:

$$R_{mn} = k\left(T_{mn} - \frac{1}{2}g_{mn}T\right) + \Lambda_{mn}, \tag{2}$$

где Λ_{mn} - дополнительный тензор нулевой энергии, введение которого необходимо по той причине, что не все компоненты тензора кривизны R_{mn} для метрики (1) в отсутствии материи $T_{mn} = 0$ обращаются в ноль.

Получение уравнения движения звезд внутри галактического диска

В качестве модели спиральной галактики будем использовать закон распределения плотности материи ε от расстояния до центра галактики x^1 который соответствует наблюдательным данным о светимости скоплений:

$$\varepsilon(r) = \varepsilon_0 e^{-r/r0}, \tag{3}$$

где r_0 - радиальный параметр галактики, ε_0 - нормировочный множитель, который отражает наблюдения светимости участков галактик.

Тензор энергии-импульса пылевого облака, которое представляет собой галактика, возьмем в виде:

$$T^m_n = \varepsilon \cdot \tilde{u}^m \tilde{u}_n. \tag{4}$$

В связи с тем, что каждая галактика является замкнутой системой, то в семимерном пространстве-времени для тензора энергии-импульса должно выполняться условие закона сохранения (6):

$$\left(T^{mn} + \frac{3}{4} g^{mn} T\right)_{;n} = 0. \tag{5}$$

Так же необходимо отметить, что все последующие рассуждения будут касаться только периферийных областей галактик, на которых наблюдается нарушение законов Кеплера.

Уравнения гравитационного поля, будем записывать в виде (2). При этом необходимо обратить внимание на то, что каждая частица (звезда) составляющая гравитирующее пылевое облако (галактику), не только обладает массой и движется вокруг ядра галактики, но и способна вращаться относительно своей собственной оси. Пренебрегая возможной прецессией и нутацией, будем считать, что каждая звезда обладает собственной угловой скоростью вращения $\tilde{u}^4 = \Omega$ отличной от нуля.

Подставим тензор энергии-импульса пылевого облака (4) в уравнение (5) и рассмотрим радиальную составляющую:

$$0 = -\frac{3}{4} \frac{\partial}{\partial x^1} (\varepsilon c^2 - \varepsilon R^2_{in} \Omega^2),$$

где R_{in} - радиус инерции звезды. Подставляем распределение плотности (3) находим, что

$$R^2_{in} \Omega^2 = c^2 + C \cdot e^{x^1/r0}, \tag{6}$$

где C - константа. Получившееся равенство (6) отражает изменение с расстоянием нормированной на массу кинетической энергии вращения тел вокруг собственной оси. При этом на участке $x^1 << r_0$, кинетическая энергия остается практически не изменной.

Для нахождения поправок к гравитационным уравнениям рассмотрим разложение тензора кривизны на величины четвертого порядка малости получаем, что нулевая компонента тензора кривизны принимает вид

$$R_{00}^{IV} = -\frac{1}{2}\frac{\partial g_{00}^{IV}}{\partial x^\mu}\frac{\partial g^{0\kappa\mu}}{\partial x^\kappa} - \frac{1}{2}g^{0\kappa\mu}\frac{\partial^2 g_{00}^{IV}}{\partial x^\kappa \partial x^\mu} - \frac{1}{2}\frac{\partial g^{II\kappa\mu}}{\partial x^\kappa}\frac{\partial g_{00}^{II}}{\partial x^\mu} - \frac{1}{2}g^{II\kappa\mu}\frac{\partial^2 g_{00}^{II}}{\partial x^\kappa \partial x^\mu} -$$

$$-\frac{1}{2}\frac{\partial g^{IV\kappa\mu}}{\partial x^\kappa}\frac{\partial g_{00}^{0}}{\partial x^\mu} - \frac{1}{2}g^{IV\kappa\mu}\frac{\partial^2 g_{00}^{0}}{\partial x^\kappa \partial x^\mu} -$$

$$-\frac{1}{4}\left(g^{0\kappa\lambda}g^{0\mu\beta}\frac{\partial g_{00}^{II}}{\partial x^\beta}\left(\frac{\partial g_{\lambda\mu}^{II}}{\partial x^\kappa} + \frac{\partial g_{\lambda\kappa}^{II}}{\partial x^\mu} - \frac{\partial g_{\mu\kappa}^{II}}{\partial x^\lambda}\right) + g^{II\kappa\lambda}g^{0\mu\beta}\frac{\partial g_{00}^{II}}{\partial x^\beta}\left(\frac{\partial g_{\lambda\mu}^{0}}{\partial x^\kappa} + \frac{\partial g_{\lambda\kappa}^{0}}{\partial x^\mu} - \frac{\partial g_{\mu\kappa}^{0}}{\partial x^\lambda}\right)\right) -$$

$$-\frac{1}{4}g^{0\kappa\nu}\left(\frac{\partial g_{\nu\lambda}^{0}}{\partial x^\kappa} + \frac{\partial g_{\nu\kappa}^{0}}{\partial x^\lambda} - \frac{\partial g_{\lambda\kappa}^{0}}{\partial x^\nu}\right)\left(g^{0\lambda\mu}\frac{\partial g_{00}^{IV}}{\partial x^\mu} + g^{II\lambda\mu}\frac{\partial g_{00}^{II}}{\partial x^\mu} + g^{IV\lambda\mu}\frac{\partial g_{00}^{0}}{\partial x^\mu}\right) +$$

$$+\frac{1}{4}g^{0\kappa\beta}\frac{\partial g_{00}^{II}}{\partial x^\beta}\frac{\partial g_{00}^{II}}{\partial x^\kappa},$$

где A^{IV00} или A_{00}^{IV} - IV порядок разложения нулевой компонеты тензора. Разложение тензора свертки энергии-импульса по второму порядку малости позволяет получить

$$S_{00}^{II} = \frac{1}{2}T^{II00} + g_{00}^{II}T^{000} - \frac{1}{2}g_{\alpha\beta}^{0}T^{II\alpha\beta}.$$

Таким образом, уравнения поля $R_{00}^{IV} = æS_{00}^{II}$, с учетом записанных выше значений и возможных упрощений по порядку малости величин получаем

$$g^{0\kappa\mu}\frac{\partial^2 g_{00}^{IV}}{\partial x^\kappa \partial x^\mu} = æg_{\alpha\beta}^{0}T^{II\alpha\beta}.$$

Пусть все тела, составляющие пылевое облако, способны вращаться относительно своих осей с некоторой угловой скоростью Ω, тогда правую часть полученного уравнения можно записать в следующем виде

$$g_{\alpha\beta}^{0}T^{II\alpha\beta} = -\varepsilon R_{in}^2\Omega^2,$$

где R_{in} - радиус инерции тел составляющих пылевое облако. Объединяя эти два уравнения, получаем

$$\nabla^2 g_{00}^{IV} = -æ\varepsilon R_{in}^2\Omega^2.$$

Решением данного дифференциального уравнения является компонента тензора метрики

$$g_{00}^{IV} = \frac{æ}{4\pi x^1}\int R_{in}^2\Omega^2\varepsilon(x')d^3x'.$$

Считая, что R_{in} и Ω не зависят от x', получим полную компоненту метрики пылевого облака

$$g_{00} = 1 - \frac{æc^2}{4\pi x^1}\int \varepsilon(x')d^3x' + \frac{æc^2}{4\pi x^1}\int R_{in}^2\Omega^2\varepsilon(x')d^3x'. \tag{7}$$

Интеграл второго слагаемого (7) при подстановке (3) дает значение массы галактики в сфере ограниченной радиусом r, которая убывает с расстоянием. Интеграл в третьем слагаемом представляет собой плотность кинетической энергии вращения.

Подставляем (3) и (6) в (7) получаем потенциал гравитационного поля галактики

$$g_{00} = 1 + \frac{æC\varepsilon_0(x^1)^2}{3}. \tag{8}$$

Найдем скорость орбитального движения частицы вокруг пылевого облака. Согласно уравнениям движения, формула скорости орбитального движения имеет вид

$$V = \sqrt{\frac{c^2x^1}{2}\frac{\partial g_{00}}{\partial x^1}}. \tag{9}$$

Подставляя потенциал (8) в формулу скорости орбитального движения (9), обнаруживаем, что

$$V = x^1\sqrt{\frac{æC\varepsilon_0c^2}{3}}, \tag{10}$$

то есть на больших масштабах, скорость вращения будет пропорциональна расстоянию до центра галактики, что и наблюдается в кривых вращения галактик.

Также следует отметить тот факт, что у разных галактик функция скорости вращения может быть различной Рис. 1. Подобные вариации возникают только за счет различного распределения плотности материи в галактике $\varepsilon(x^1)$. Так в шаровых скоплениях, у которых распределение плотности идет по закону Гаусса, подобного (3), скорость вращения увеличивается, как предсказывает (10). У спиральных галактик закон распределения плотности более сложный, более сложную кривую представляет собой и график скорости вращения.

Интересно отметить также тот факт, что из полученного уравнения (6) следует, что с уменьшением расстояния от центра галактики угловая скорость вращения звезды должна уменьшаться. Известно, что для звезд главной последовательности, снижение скорости вращения может быть записано математическим соотношением, называемым законом торможе-

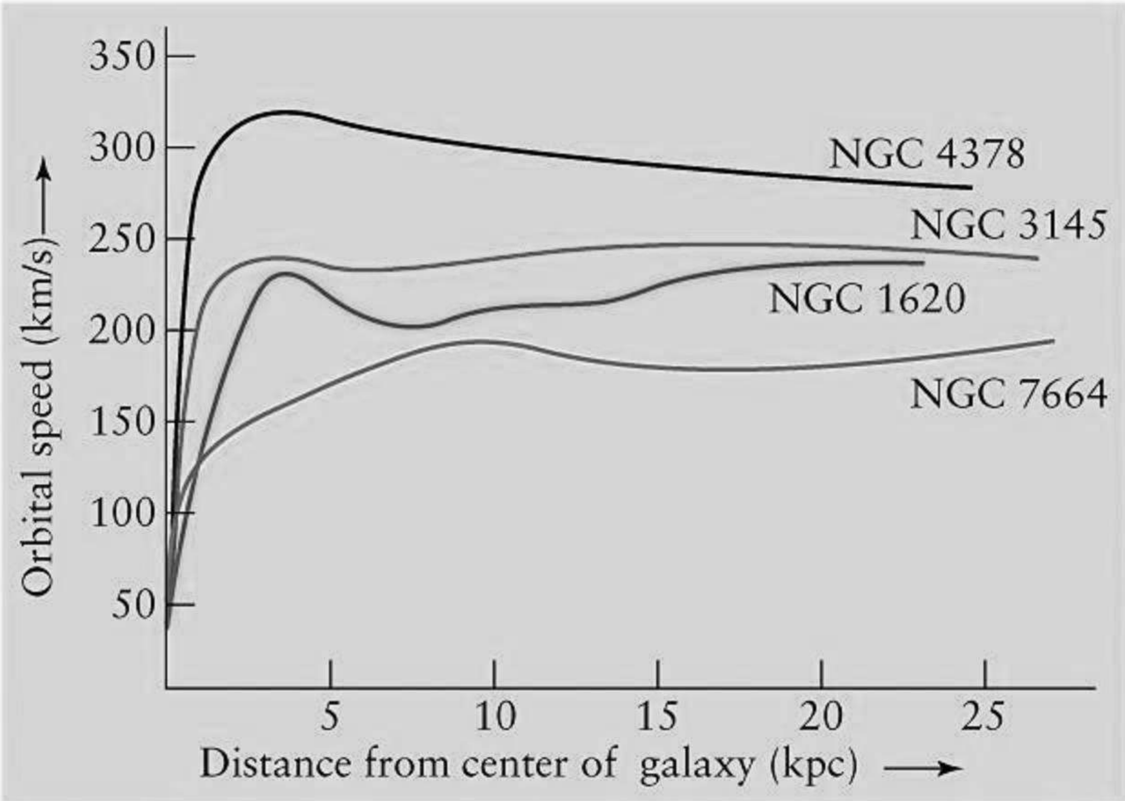

Рис. 1. Графики скоростей вращения галактик

ния Скуманича, открытым в 1972 году (7):

$$\Omega_e \sim t^{-1/2},$$

где Ω_e - угловая скорость вращения звезды на экваторе и t - возраст звезды (8). Звезды формируются на окраине галактик и по мере старения приближаются к ее центру $x^1 \sim 1/t$, с учетом закона Скуманича это означает, что чем дальше звезда находится от центра галактики, тем выше ее угловая скорость вращения вокруг собственной оси. Что полностью соответствует уравнению (6).

Выводы

Подводя итоги работы необходимо отметить две основных вывода: первое, была получена модель, позволяющая описывать кривую скорости вращения галактик (10) без привлечения темной материи; второе, для корректной работы модели требуется, чтобы кинетическая энергия вращения звезд вокруг своих осей подчинялась бы уравнению (6), в котором угловая скорость вращения звезд вокруг своих осей уменьшается по мере приближения к центру галактики, что соответствует эмпирическому закону Скуманича. Таким образом, можно сделать вывод, что в работе была построена не противоречивая модель вращения галактик, которая использует только светящуюся материю.

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Macroscopic theory of the dark sector

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Self-consistent account of the most simple non-gauge vector fields with as simple a Lagrangian as possible (1), provide an adequate tool for macroscopic description of the main observed consequences of the existanse of dark energy and dark matter. In the scale of the whole Universe it gives a broad spectrum of regular scenarios of its temporal evolution completely within the frames of the Einstein's General relativity. In the scale of a galaxy it allows to derive the galaxy rotation curves analytically from the first principles completely within the Einstein's general relativity. Now, when we have the general expression (3) for the energy-momentum tensor describing the dark sector, it is possible to analyze its role in various astrophysical phenomena.

In my report on the conference PIRT-2011(1) I have shown that a vector field ϕ_I with as simple a Lagrangian as possible,

$$L = a(\phi^M_{;M})^2 - V(\phi_M \phi^M), \tag{1}$$

(a is a constant), provides an effective tool for macroscopic description of the main observed features of the dark sector. Now I am going to aply it to two major problem: – galaxy rotation curves and evolution of the Universe.

Vector fields in general relativity

The covariant field equations

$$a\phi^K_{;K;I} \left(= a\frac{\partial \phi^K_{;K}}{\partial x^I} \right) = -V'\phi_I \tag{2}$$

and the energy-momentum tensor

$$T_{\mathrm{dm}\ IK} = g_{IK}\left[(\phi^M_{;M})^2/a + V\right] + 2V'\left(\phi_I\phi_K - g_{IK}\phi^M\phi_M\right), \tag{3}$$

following from the Lagrangian (1), describe the behavior of the vector field ϕ_I in the background of any arbitrary given metric g_{IK} (2). Here $V' \equiv \frac{dV(\phi_M\phi^M)}{d(\phi_M\phi^M)}$.

In applications of the vector fields to elementary particles in flat space-time the divergence $\frac{\partial\phi^K}{\partial x^K}$ is artificially set to zero (4):

$$\frac{\partial \phi^K}{\partial x^K} = 0. \tag{4}$$

This restriction allowed to avoid the difficulty of negative contribution to the energy. In the electromagnetic theory it is referred to as Lorentz gauge. However, in general relativity (in curved space-time) the energy is not a scalar, and its sign is not invariant against the arbitrary coordinate transformations. From my point of view, (4) looks like "throwing out the baby along with the diapers". Considering vector fields in general relativity, it is worth getting rid of the restriction (4), using instead a more weak condition of regularity.

Properties of the potntial $V\left(\phi_K\phi^K\right)$ following from the requirement of regularity.

The sign of the scalar $\phi_K\phi^K$ is invariant against the arbitrary transformations of coordinates. Therefore, there are three different independent vector fields: $\phi_K\phi^K < 0$, $\phi_K\phi^K = 0$, $\phi_K\phi^K > 0$. If $\phi_K\phi^K \neq 0$, then in general relativity one can choose a frame where either $\phi_0 = 0$ when $\phi_K\phi^K < 0$ (space-like vector), or $\phi_{I>0} = 0$ if $\phi_K\phi^K > 0$ (time-like vector). When the field is small, and the second and higher derivatives of the potential can be omitted, the field equations (2) are

$$a\frac{\partial\phi^K_{;K}}{\partial x^I} = -V'(0)\,\phi_I.$$

If there is no direct interaction between space-like and time-like fields, then the fields remain independent, and the requirement of regularity leads to different conclusions. In particular, the sign of $\frac{V'(0)}{a}$, chosen in accordance with the requirement of regularity, is different for space-like and time-like fields.

In a flat space-time with no other physical objects a regular solution should be finite everywhere: $|\phi_I| < \infty,\ -\infty \le x^K \le +\infty$. A time-like vector and a space-like vector obey different equations:

$$\frac{d^2\phi^0}{\left(dx^0\right)^2} = -\frac{V'(0)}{a}\phi_0 = -\frac{V'(0)}{a}\phi^0, \qquad (g_{00} = 1)$$
$$\frac{\partial^2\phi^1}{\partial\left(x^1\right)^2} = -\frac{V'(0)}{a}\phi_1 = \frac{V'(0)}{a}\phi^1, \qquad (g_{11} = -1)\,.$$

The solutions are finite if $\frac{V'(0)}{a} > 0$ for a time-like vector, and if $\frac{V'(0)}{a} < 0$ – for a space-like one.

The parameter

$$m^2 = \begin{cases} +\frac{V'(0)}{a}, & \phi_K\phi^K > 0 \\ -\frac{V'(0)}{a}, & \phi_K\phi^K < 0 \end{cases} \tag{5}$$

is usually considered as the squared mass of a vector field. In accordance with the requirement of regularity, the sign plus corresponds to a time-like vector $\left(\phi_K\phi^K > 0\right)$, and sign minus – to a space-like one $\left(\phi_K\phi^K < 0\right)$.

The third case $\phi_K\phi^K = 0$, $V'(0) = 0$,– a massless vector field, – corresponds to dark energy.

The field equation (2) reduces to $\frac{\partial \phi^K_{;K}}{\partial x^I} = 0$, and therefore $\phi^K_{;K} = const.$

Galaxy rotation curves

Applying general relativity to the galaxy rotation problem it is reasonable to consider a static centrally symmetric metric

$$ds^2 = g_{IK} dx^I dx^K = e^{\nu(r)} \left(dx^0\right)^2 - e^{\lambda(r)} dr^2 - r^2 d\Omega^2 \tag{6}$$

with two functions $\nu(r)$ and $\lambda(r)$ depending on only one coordinate - circular radius r. Real distribution of the stars and planets in a galaxy is neither static, nor centrally symmetric. However this simplification facilitates analyzing the problem and allows to display the main results analytically. If a galaxy is concentrated around a supermassive black hole, the deviation from the central symmetry caused by the peripheral stars is small.

So far there is no evidence of any direct interaction between dark and ordinary matter other than via gravitation. The gravitational interaction is described by Einstein equations

$$R_{IK} - \frac{1}{2} g_{IK} R + \Lambda g_{IK} = \varkappa \left(T_{\mathrm{dm}\ IK} + T_{\mathrm{om}\ IK}\right) \tag{7}$$

where

$$T_{\mathrm{om}\ IK} = (\varepsilon + p)\, u_I u_K - p g_{IK} \tag{8}$$

is the ordinary energy-momentum tensor of macroscopic objects.

In the background of the centrally symmetric metric (6) the vector ϕ^I is longitudinal. In accordance with the field equation (2) its only non-zero component ϕ^r depends on r.

In the "dust matter" approximation $p = 0$, and the only nonzero component of the energy-momentum tensor (8) is $T_{\mathrm{om}\ 00} = \varepsilon g_{00}$. Whatever the distribution of the ordinary matter $\varepsilon(r)$ is, the covariant divergence $T^K_{\mathrm{om}\ I;K}$ is automatically zero. If $p = 0$ the energy $\varepsilon(r)$ can be an arbitrary given function.

$V_0 = V(0)$ together with the cosmological constant Λ affects the rate of expansion of the Universe. In the scale of galaxies the role of expansion of the Universe as a whole is negligible, and one can set $\widetilde{\Lambda} = \Lambda - \varkappa V_0 = 0$ in the Einstein equations. Omitting the second and higher derivatives of the potential $V(\phi_M \phi^M)$, we have the Einstein equations as follows (see (3), page 382 for the derivation of the left-hand sides):

$$-e^{-\lambda}\left(\frac{1}{r^2} - \frac{\lambda'}{r}\right) + \frac{1}{r^2} = \varkappa T^0_0 = \varkappa \left[(\phi^M_{;M})^2/a + V' e^{\lambda} \left(\phi^r\right)^2 + \varepsilon\right] \tag{9}$$

$$-e^{-\lambda}\left(\frac{\nu'}{r} + \frac{1}{r^2}\right) + \frac{1}{r^2} = \varkappa T^r_r = \varkappa \left[(\phi^M_{;M})^2/a - V' e^{\lambda} \left(\phi^r\right)^2 - p\right] \tag{10}$$

Here prime $'$ stands for $\frac{d}{dr}$, except $V' = \frac{\partial V\left(\phi_M \phi^M\right)}{\partial(\phi_M \phi^M)}$. Together with the field equation

$$\left[(\phi^r)' + \left(\frac{2}{r} + \varkappa r e^{2\lambda} (\phi^r)^2 V' + \frac{1}{2} \varkappa r e^{\lambda} (\varepsilon + p) \right) \phi^r \right]' = -m^2 e^{\lambda} \phi^r \tag{11}$$

we have the set of three equations (9−11) for three unknowns λ, ν, and ϕ^r. ϕ^r is the space-like vector, and in accordance with the requirement of singularity, (5), $m^2 = -\frac{V'(0)}{a}$. The boundary conditions, $\phi^r = \frac{1}{3}\phi_0' r$, $\lambda = \frac{1}{3}\varkappa\left(\varepsilon_0 - \phi_0'^2\right) r^2$, $r \to 0$, are determined by the requirement of regularity in the center. Here $\varepsilon_0 = \varepsilon(0)$, and $\phi_0' \equiv \phi^K_{;K}(0)$ is a parameter characterizing the field ϕ^r. The equations (9−11) are derived with no assumptions concerning the strength of the gravitational field.

In a static centrally symmetric gravitational field ν' determines the centripetal acceleration of a particle ((3), page 323). In case of the dust matter approximation ($p = 0$) the equation (10) determines ν'.

The curvature of space-time caused by a galaxy is small, $\lambda \ll 1$, the field ϕ^r is weak, and

$$\nu' = \varkappa r \left[m^2 (\phi^r)^2 + (\phi^M_{;M})^2 \right] + \frac{\lambda}{r}, \tag{12}$$

In the linear approximation the solutions of the vector field equation (11) and of the Einstein equations (9), (10) at $\lambda \ll 1$ are:

$$\phi^r = \frac{\phi_0'}{m^3 r^2} \left(\sin mr - mr \cos mr \right), \qquad \phi^M_{;M} = \phi_0' \frac{\sin mr}{mr}, \tag{13}$$

$$\lambda(r) = 2\left(\frac{V_{\rm pl}}{c}\right)^2 \Psi(mr) + \frac{\varkappa}{r}\int_0^r \varepsilon(r) r^2 dr, \qquad V_{\rm pl} = \sqrt{\frac{\varkappa}{2}}\frac{c\phi_0'}{m}, \qquad \lambda \ll 1. \tag{14}$$

$$\nu'(r) = \frac{2}{r}\left(\frac{V_{\rm pl}}{c}\right)^2 \left[f(mr) + \Psi(mr) \right] + \frac{\varkappa}{r^2}\int_0^r \varepsilon(r) r^2 dr, \qquad \lambda \ll 1.$$

where $f(x)$ and $\Psi(x)$ are universal functions:

$$f(x) = \left(1 - \frac{\sin 2x}{x} + \frac{\sin^2 x}{x^2} \right), \qquad \Psi(x) = \frac{1}{x}\int_0^x \left(\frac{\sin^2 y}{y^2} - \frac{\sin 2y}{y} + \cos 2y \right) dy. \tag{15}$$

The last term in (14) gives the Newton's potential.

The balance of the centripetal $\frac{c^2\nu'}{2}$ and centrifugal $\frac{V^2}{r}$ accelerations determines the velocity V of a rotating object as a function of the radius r of its orbit:

$$V(r) = V_{\rm pl}\sqrt{f(mr) + \Psi(mr) + \left(\frac{c}{V_{\rm pl}}\right)^2 \frac{\varkappa}{2r}\int_0^r \varepsilon(r) r^2 dr}, \qquad V_{\rm pl} = \sqrt{\frac{\varkappa}{2}}\frac{c\phi_0'}{m} \tag{16}$$

Function $\sqrt{f(x)+\Psi(x)}$ is presented in Figure 1.

Far from the center $f(mr) \to 1$. The dependence $\sqrt{f(mr)+\Psi(mr)}$ turns at $r \gtrsim m^{-1}$ from a linear to a plateau with damping oscillations. The plateau appears entirely due to the vector field. In the limit $\varepsilon \to 0$ the transition to a plateau due to the dark matter is a universal function.

The plateau value V_{pl} (16) is connected with the parameter ϕ_0'/m, and the period of oscillations is $\frac{2\pi}{m}$. The form of a plateau allows to restore the value of the parameter $\phi_0' = \phi^M_{;M}(0)$ at $r \to 0$ in the boundary conditions. The origin of specific values ϕ_0' and m of a particular galaxy depends on what happens in the center. As long as the gravitation is weak, in the linear approximation ϕ_0' and m are free parameters. It looks like for each galaxy the values of V_{pl} and m are driven by some heavy object (may be a black hole, may be a neutron star) located in the center (by the way, supporting the central symmetry of the gravitational field).

The existance of dark matter, described by a vector field with the Lagrangian (1), actually justifies the empirical Milgrom's hypothesis of MOND (5) – the Newton's dynamics really gets modified. Naturally, basing only on the intuitive arguments, it was scarcely possible to guess that the transition to a plateau is accompanied by damping oscillations.

While the role of dark matter is characterized unambiguously by the two parameters ϕ_0' and m, the situation with the density of the ordinary matter $\varepsilon(r)$ in galaxies is not that clear. Radiation coming from the galaxies does not carry information about cooled non-emitting stars and planets. Just the opposite: the deviation from the universal curve could provide us with the average distribution of the ordinary matter in a galaxy.

Figures 2 show the difference between previous MOND (2*a*) and current (2*b*) fitting.

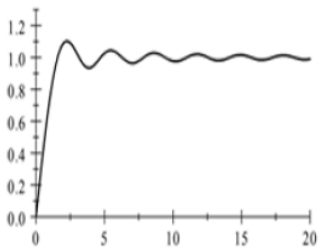

Figure 1. $\sqrt{f(x)+\Psi(x)}$ in (16) – contribution of dark matter.

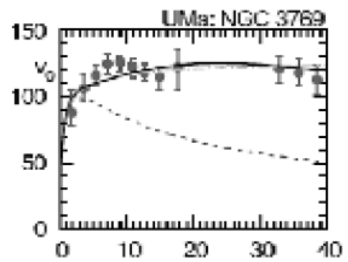

Figure 2*a*. Fitting via MOND. Dashed curve is the Newtons's dynamics.

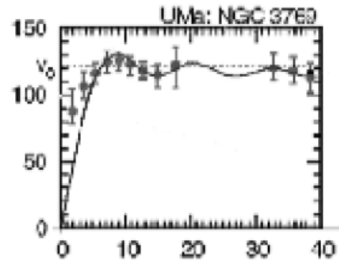

Figure 2*b*. Points with error bars are observations, Solid curve is fitting by (16).

Frankly speaking, I did not expect such a coincidence. Deviation at small radii is a hint of the presence of an additional strongly gravitating compact object located at the center. If a heavy object in the center really exists, it supports the central symmetry, and the gravitational field becomes only slightly distorted by other stars and planets of the galaxy. See more about fitting in (6).

Evolution of the Universe

According to observations the Universe expands, and its large scale structure remains homogeneous and isotropic. Consider the space-time with the structure $T^1 E^{d_0}$ and the metric

$$ds^2 = g_{IK} dx^I dx^K = (dx^0)^2 - e^{2F(x^0)} \sum_{I=1}^{d_0} (dx^I)^2 \tag{17}$$

depending on only one time-like coordinate $x^0 = ct$. Here $d_0 = 3$ is the dimension of space. However, the derivations are applicable for arbitrary $d_0 > 1$. The uniform and isotropic expansion is characterized by the single metric function $F(x^0)$, and the rate of expansion $\frac{dF}{dx^0} \equiv F'(x^0)$ is the Hubble parameter changing in time. Its present value is called the Hubble constant H. The Ricci tensor is diagonal:

$$R_{00} = -d_0(F'^2 + F''), \tag{18}$$

$$R_{II} = e^{2F}(F'' + d_0 F'^2), \qquad I > 0. \tag{19}$$

Massless field

The solution of the field equation (2) for the massless field is $\phi^K_{;K} = \Phi_0/a = const$. The energy-momentum tensor (3),

$$T_{IK} = g_{IK}\left(\Phi_0^2/a + V_0\right), \quad V' = 0, \tag{20}$$

acts in the Einstein equations (7) as a simple addition to the cosmological constant:

$$R_{IK} - \frac{1}{2} g_{IK} R + \widetilde{\Lambda} g_{IK} = 0, \quad \widetilde{\Lambda} = \Lambda - \varkappa\left(\Phi_0^2/a + V_0\right). \tag{21}$$

Here V_0 is the value of the constant potential $V\left(\phi_L \phi^L\right)$ in the case of massless field ($V' = 0$). The contribution of the zero-mass field to the curvature of space-time remains constant in the process of the Universe evolution.

The metric

$$ds^2 = (dx^0)^2 - e^{\pm\sqrt{-\frac{8\widetilde{\Lambda}}{d_0(d_0-1)}}(x^0 - x^0_0)} \sum_{I=1}^{d_0} (dx^I)^2, \quad d_0 > 1 \tag{22}$$

is the self-consistent regular solution of the Einstein equations (21), provided that

$$\widetilde{\Lambda} < 0. \tag{23}$$

$F(x^0)$ is a linear function; x_0^0 is a constant of integration. The metric (22) is called de Sitter (or anti de Sitter, depending on the sign definition of the Ricci tensor). It describes either expansion (sign $+$), or contraction (sign $-$) of the Universe at a constant rate. In the case of sign $+$ the rate of expansion

$$H = \sqrt{-\frac{2\widetilde{\Lambda}}{d_0(d_0-1)}} \tag{24}$$

is the Hubble constant. In our 3-dimensional space $H = \sqrt{-\frac{1}{3}\widetilde{\Lambda}}, \qquad d_0 = 3.$

Massless and massive fields

There is a principle difference between the massless and the massive vector fields. The massive field vanishes with time in the process of expansion, while the massless one does not (7). In the scale of the Universe the massive vector field is longitudinal: the only nonzero component ϕ_0 is directed along and depends upon the same time-like coordinate x^0. In accordance with the requirements of regularity (5) $\frac{V'(0)}{a} = m^2$. The energy-momentum tensor (3) for the massive field reduces to (7)

$$T_{IK} = a\left[g_{IK}\left(\phi_0' + d_0 F'\phi_0\right)^2 + \left(2\delta_{I0}\delta_{K0} - g_{IK}\right)m^2\phi_0^2\right]. \tag{25}$$

Here the prime denotes the derivative d/dx^0 (except that $V' = \frac{dV}{d(\phi_L\phi^L)}$).

Massive and massless vector fields are independent of one another. Therefore there are two different vectors, ϕ_E and ϕ_M, for the massless and massive fields, respectively. The total energy-momentum tensor is the sum of (20) for $\phi = \phi_E$ and (25) for $\phi = \phi_M$. The massless field enters the Einstein equations

$$\frac{1}{2}d_0\left(d_0-1\right)F'^2 + \widetilde{\Lambda} = \varkappa a\left[\left(\phi_0' + d_0F'\phi_0\right)^2 + m^2\phi_0^2\right], \tag{26}$$

$$\left(d_0-1\right)F'' + \frac{1}{2}d_0\left(d_0-1\right)F'^2 + \widetilde{\Lambda} = \varkappa a\left[\left(\phi_0' + d_0F'\phi_0\right)^2 - m^2\phi_0^2\right] \tag{27}$$

only via the cosmological constant $\widetilde{\Lambda}$ (21), corresponding to $\phi = \phi_E$. $\widetilde{\Lambda}$ remains constant throughout the whole history of the Universe. The massive field is described by the function $\phi_0\left(x^0\right)$, corresponding to $\phi = \phi_M$. $\phi_0\left(x^0\right)$ enters the Einstein equations (26-27) directly (7). Regular solutions of these equations exist if $\widetilde{\Lambda}$ and a are of the same sign. It follows from (30) below.

The vector field equations (2) reduce to the only one equation

$$\left(\phi_0' + d_0F'\phi_0\right)' + m^2\phi_0 = 0. \tag{28}$$

Extracting (26) from (27), we have

$$F'' = -\frac{2a}{d_0 - 1} \varkappa m^2 \phi_0^2. \tag{29}$$

The fourth order set of equations (28-29) with the boundary condition

$$\phi_0'^2 + m^2 \phi_0^2 = \frac{\widetilde{\Lambda}}{\varkappa a} \quad \text{at} \quad F' = 0, \tag{30}$$

following from (26), has the same solutions as the third order set of the Einstein equations (26-27). In the regular expanding solutions at $x^0 \to \infty$, in accordance with (23), $\widetilde{\Lambda} < 0$. Hence, a is also negative, and F'' (29) is positive: the massive longitudinal vector field makes the rate of expansion $F'(x^0)$ a monotonically growing function. Without ordinary matter the second derivative (29) does not change sign. x^0 is a cyclic coordinate, and it is convenient to set the origin $x^0 = 0$ at a point where $F' = 0$. In view of the $x^0 \to -x^0$ invariance of the equations, it is clear, that the rate of expansion $F'(x^0)$ is a monotonically growing function between its limiting values $F'(-\infty) = -H$ in the past and $F'(+\infty) = H$ in future. The scale factor $R = e^F$ decreases with time while $F' < 0$, reaches its minimum, and grows when F' becomes positive.

One of the two constants ϕ_0 and ϕ_0' at $x^0 = 0$ remains arbitrary within the boundary condition (30). In the case $\phi_0'(0) = 0$ the field $\phi_0(x^0)$ is a symmetric function, and in the case $\phi_0(0) = 0$ it is an antisymmetric one. In these both cases $F'(x^0)$ is antisymmetric. If both constants ϕ_0 and ϕ_0' at $x^0 = 0$ are not zeroes, a regular solutions still exist, but there is no symmetry with respect to $x^0 \to -x^0$.

The derivations are performed in (8) in dimensionless variables $\phi = \sqrt{-\frac{2a\varkappa}{d_0(d_0-1)}}\phi_0, \quad z = Hx^0$. The equations (28,29)

$$\left(\phi_{,z} + d_0 F_{,z}\phi\right)_{,z} + \mu^2\phi = 0, \tag{31}$$

$$F_{,z,z} = d_0 \mu^2 \phi^2 \tag{32}$$

contain only one dimensionless parameter

$$\mu = \frac{m}{H}. \tag{33}$$

(In dimensional units $\mu = \frac{mc^2}{\hbar H}$). Subscript $_{,z}$ denotes d/dz. The boundary relation (30) in terms of $\phi(0)$, $\phi_{,z}(0)$, and $F_{,z}(0)$ is

$$\phi_{,z}^2(0) + \mu^2\phi^2(0) = 1, \quad F_{,z}(0) = 0. \tag{34}$$

$F(z)$ enters the equations (31,32) only via the derivatives, but not directly. For this reason

$F_0 = F(0)$ remains arbitrary. With no ordinary matter taken into account, the constant F_0 only determines the scale of the space coordinates, and it does not change the structure of the metric and vector field.

Massless field, massive field, and ordinary matter: Regular scenarios of the Universe evolution

Regular cosmology

With the ordinary dust matter taken into account, we now have

$$F_{,z,z} = d_0\mu^2\phi^2 - \frac{d_0\Omega}{2}e^{-d_0F}. \tag{35}$$

The parameter Ω,

$$\Omega = -\frac{\varkappa\rho_0}{\widetilde{\Lambda}} = \frac{2\varkappa\rho_0}{d_0(d_0-1)H^2}, \tag{36}$$

denotes the ratio of the present energy density of the ordinary matter to the density of the kinetic energy of expansion. Now the metric function $F(x^0)$ enters the equation (35) directly. The vector field equation (31) remains the same. The equation (35) resembles the Newton's law: "acceleration" $F_{,z,z}$ is proportional to the "repulsing force" $d_0\mu^2\phi^2$ minus the "attracting force" $\frac{d_0\Omega}{2}e^{-d_0F}$. Altogether the regular solutions of the set (31,35) with the boundary conditions

$$\phi_{,z}^2(0) + \mu^2\phi^2(0) = 1 + \Omega e^{-d_0F_0}, \quad F_{,z}(0) = 0, \quad F(0) = F_0, \quad \widetilde{\Lambda} < 0, \tag{37}$$

contain five dimensionless parameters: $\mu, \Omega, F_0, \phi(0)$, and $\phi_{,z}(0)$. In view of the bounding relation (37), four of them remain independent. These equations are easily integrated numerically. In the most interesting cases the analytical solutions are found (8).

In the case of small $\mu \ll 1$ we exclude ϕ and come to the single equation for F :

$$F_{,z,z} = d_0e^{-2d_0(F-F_0)} - \frac{d_0\Omega}{2}e^{-d_0F}. \tag{38}$$

Its solution with the boundary conditions (37) is

$$F(z) = F_0 + \frac{1}{d_0}\ln\left[\left(1 + \frac{1}{2}\Omega e^{-d_0F_0}\right)\cosh(d_0z) - \frac{1}{2}\Omega e^{-d_0F_0}\right], \quad \mu \ll 1. \tag{39}$$

For the rate of evolution $F_{,z}(z)$ and for the scale factor $R(z) = e^{F(z)}$ we get

$$F_{,z}(z) = \frac{\sinh(d_0 z)}{\cosh(d_0 z) - \left(1 + \frac{2}{\Omega} e^{d_0 F_0}\right)^{-1}}, \tag{40}$$

$$R(z) = \left[\left(e^{d_0 F_0} + \frac{1}{2}\Omega\right)\cosh(d_0 z) - \frac{1}{2}\Omega\right]^{\frac{1}{d_0}}. \tag{41}$$

Analytical solutions (39-41), derived for $\mu \ll 1$, are as well applicable in the vicinity of the transition for $\mu \sim 1$ if $|F_0| \gg 1$. It is because for very large negative F_0 the width of the contraction-to-expansion transition Δz is very narrow:

$$\Delta z \sim \frac{2}{d_0\sqrt{\Omega}} e^{d_0 F_0/2}, \quad F_0 < 0, \quad |F_0| \gg 1.$$

The repulsing term $\sim e^{-2d_0 F}$ in (35) increases faster as $F \to -\infty$ than the compressing term $\sim e^{-d_0 F}$. It is the reason why a regular bounce replaces the singular Big Bang independently of how big the negative F_0 is. After the bounce the repulsing term decreases faster, than the compressing one, leading to matter domination over the field.

Oscillating solutions

Without the ordinary matter the relation (34) can be satisfied only if $\widetilde{\Lambda} < 0$, provided that $a < 0$. Appearance of the term $\Omega e^{-d_0 F_0}$ in (37) admits the solutions with positive $\widetilde{\Lambda}$. If $\widetilde{\Lambda}$ changes sign, then H (24) becomes imaginary. The equations (31,35) are invariant against $H \to iH$, but the boundary conditions (37) are not:

$$\phi_{,z}^2(0) + \mu^2\phi^2(0) = -1 + \Omega e^{-d_0 F_0}, \quad F_{,z}(0) = 0, \quad F(0) = F_0, \quad \widetilde{\Lambda} > 0. \tag{42}$$

The necessary condition for regular solutions with $\widetilde{\Lambda} > 0$ is the existence of an extremum moment ($F_{,z}(0) = 0$) with the energy density of the ordinary matter exceeding the kinetic energy of expansion:

$$\Omega e^{-d_0 F_0} = \frac{\varkappa\rho(0)}{\widetilde{\Lambda}} > 1, \quad F_{,z}(0) = 0, \quad \widetilde{\Lambda} > 0.$$

In the case $\widetilde{\Lambda} > 0$, $\mu \ll 1$ (as well as for $F_0 < 0$, $|F_0| \gg 1$, $\mu \sim 1$) the scale factor $R(z)$ and the rate of evolution $F_{,z}(z)$,

$$R(z) = e^{F_0}\left[\frac{1}{2}\Omega e^{-d_0 F_0} - \left(\frac{1}{2}\Omega e^{-d_0 F_0} - 1\right)\cos(d_0 z)\right]^{\frac{1}{d_0}}, \qquad F_{,z}(z) = \frac{\sin(d_0 z)}{\left(1 - \frac{2}{\Omega} e^{d_0 F_0}\right)^{-1} - \cos(d_0 z)},$$

are periodical functions with no singularities. Without the massive field ($\phi = 0$) the solutions with positive $\widetilde{\Lambda}$ are possible only if the parameters are fine tuned $\left(\varkappa\rho(0) = \widetilde{\Lambda} \right)$:

$$F(z) = F_0 + \frac{1}{d_0} \ln \cos^2 \frac{d_0 z}{2}, \qquad F_{,z}(z) = -\tan \frac{d_0 z}{2}, \qquad R(z) = e^{F_0} \cos^{\frac{2}{d_0}} \frac{d_0 z}{2}.$$

These "fine tuned" $\left(e^{F_0} = \left(\frac{2\varkappa\rho_0}{d_0(d_0-1)H^2} \right)^{1/d_0} \right)$ singular solutions have periodical singularities at $z = z_n = \frac{\pi}{d_0}(1+n)$, $n = \pm 1, \pm 2, \ldots$. In the vicinity of each singular point $z_n = H x_n^0$, as well as at $H \to 0$, the Hubble constant H drops out, and the scale factor (in the ordinary units $x^0 = z/H$) reduces to

$$R\left(x^0\right) = \left(\frac{d_0 \varkappa \rho_0}{2(d_0 - 1)} \right)^{1/d_0} \left| x^0 - x_n^0 \right|^{2/d_0}, \quad \left| \frac{x^0}{x_n^0} - 1 \right| \ll 1. \tag{43}$$

At $d_0 = 3$ (43) reproduces the scale factor of the Friedman-Robertson-Walker (9) singular cosmology. The longitudinal vector field $\phi \neq 0$ removes the singularities.

Summary

The non-gauge vector field with as simple a Lagrangian (1) as possible provides the macroscopic description of the major observed properties of the dark sector within the Einstein's theory of general relativity.

In the galaxy scale the space-like vector field (as a dark matter) with the energy-momentum tensor (3) allows to describe analytically the galaxy rotation curves in detail. In the scale of the whole universe the vector fields with the same Lagrangian (1) provide various possible regular scenarios of its evolution. The singular Big Bang turns into a regular inflation-like state of maximum compression with the further accelerated expansion at late times. The zero-mass vector field acts as the dark energy, and the massive time-like field – as a dark matter. Thus, there is no need in any modifications of the general relativity, either to explain the observable plateau in rotation curves, or the features of the evolution of the universe.

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On Schwarzschild superspace

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Interests of modern theoretical physics require the use of geometric spaces with exotic features as an arena of action. Most of all, this is due to the success of particle physics, where the methods of Clifford algebras are applied. In this case it is possible to bring to the study of physical phenomena powerful mathematical tools. The theory of supermanifolds is related with the concept of supersymmetry developed in fundamental works of Golfand and Lichtman, Volkov and Akulov, Wess and Zumino. The supersymmetry relates space-time symmetries and internal symmetries. It is the basis of new theory of gravity – supergravity as well as superstring theory, which is a candidate for a unified theory of all the fundamental interactions. This work is devoted to the construction of supersymmetric cosmological models in the framework of a consistent supersymmetric approach developed in the works of A.V. Aminova and S.V. Mochalov.

Consistent approach to the supersymmetric theory of gravity means that the supergeometry is defined by supersymmetry properties. This approach requires the development of group-invariant methods of supergravity. In this direction we not only have almost no concrete results, but the very principles that should guide the relationship between supersymmetry and supergeometry were not clearly formulated. The first steps in this direction were made in (4). The given work continues that line. We regard supersymmetry as an automorphism of a supergeometric structure, in particular, as infinitesimal supertransformation preserving a metric of a superspace. The metric is defined as an invariant of a Lie supergroup of supertransformations in the spirit of Kleyn‘s program, the idea of which is to consider the symmetry, or a group of transformations as a basis for determining the geometry of space. In this paper supergroup-invariant methods are applied to the physically significant case of a spherically symmetric world. The supersymmetric generalization of spherically symmetric world was obtained. We consider the important case of supersymmetric Schwarzschild space defined by the equation $Ric = 0$. We found curvature tensor of spherically symmetric superspace and investigated the (super)field equations. It was shown that they reduce to the system of ordinary differential equations for the four functions $\mu(r), \lambda(r), \tau(r)$ and $\kappa(r)$ on radial variable r that determine the fermion part of the supermetric. We considered the case $\lambda(r) = \tau(r) = 0$ and obtained expressions for $\mu(r)$ and $\kappa(r)$. Asymptotical properties were also discussed.

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Termination of the physics relativization and logical reloading in the space-time geometry

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In the beginning of the twentieth century the relativity theory had not been completed in the sense that dynamic equations were relativistic, but the particle state remained to be nonrelativistic. Consecutive relativistic approach admits one to construct unified formalism of the particle dynamics which can be applied for deterministic and stochastic motion of particles. This formalism admits one to found the quantum mechanics and to explain quantum phenomena without a use of quantum principles. Refusing from the constraint on continuity of the space-time geometry and using the metric approach to geometry, one explains stochastic motion of elementary particles and constructs the skeleton conception of particle dynamics. The skeleton conception admits one to investigate the elementary particle structure (but not only to systematize the elementary particles, ascribing quantum numbers to them)

Introduction

In the beginning of the twentieth century the relativity theory had not been completed in the sense that dynamic equations were relativistic, but the description of the particle state remained to be nonrelativistic. Nonrelativistic concept of the particle state is a point in the 3-dimensional space or in the phase space. The real relativistic definition of the particle state looks otherwise.

Conventionally the special relativity principle is formulated as the Lorentz-invariance of dynamical equations. On the other hand, a general physical principles can be hardly formulated as a statement connected with such details of description as a coordinate transformation. We formulated the relativity principle as follows. *The space-time is described by one space-time structure* ST. It means that the space-time geometry is described by the only quantity: space-time distance ρ, or only by the world function $\sigma = \frac{1}{2}\rho^2$. In the non-relativistic physics the space-time is described by means of two independent quantities (structures): spatial distance S and and temporal interval T. Among three structures: T, S, and ST only two of them are independent. Such a formulation of the relativity principle is more general, because it is valid not only for the space-time geometry of Minkowski. It is valid for any space-time geometry, including a discrete space-time geometry. Besides, this formulation is coordinateless.

One cannot be sure that the space-time geometry is continuous in microcosm. Restricting our consideration by the continuous space-time geometries, we are mistaken. This mistake is justified by the fact that the formalism of a discrete geometry has not been developed, and one believes that the space-time geometry cannot be discrete. In reality, a discrete geometry, as any geometry, is a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. But the Euclidean geometry is to be described in terms of distance ρ and only in terms of distance, because other concepts of $\mathcal{G}_{\mathrm{E}}$ contain a reference to continuity of $\mathcal{G}_{\mathrm{E}}$, and they cannot be used for a construction of a discrete geometry.

The simplest discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ is described by the world function

$$\sigma_{\mathrm{d}}(P,Q)=\sigma_{\mathrm{M}}(P,Q)+\frac{\lambda_0^2}{2}\mathrm{sgn}\left(\sigma_{\mathrm{M}}(P,Q)\right),\quad \forall P,Q\in\Omega \tag{1}$$

where Ω is a set of all points of the space-time, σ_{M} is the world function of the Minkowski space-time geometry $\mathcal{G}_{\mathrm{M}}$, and λ_0 is the elementary length. In the inertial coordinate system the world function σ_{M} has the form

$$\sigma_{\mathrm{M}}(x,x')=\frac{1}{2}g_{ik}\left(x^i-x'^i\right)\left(x^k-x'^k\right),\quad g_{ik}=\mathrm{diag}\left(c^2,-1,-1,-1\right) \tag{2}$$

In the discrete space-time geometry a pointlike particle cannot be described by a world line, because any world line is a limit of the broken line, when lengths of its links tend to zero. But in the discrete geometry $\mathcal{G}_{\mathrm{d}}$ there are no infinitesimal lengths, because all lengths are longer, than λ_0. In $\mathcal{G}_{\mathrm{d}}$ a pointlike particle is described by a world chain (broken line) instead of a smooth world line. Description of a pointlike particle state by means of the particle position and its momentum becomes inadequate. The reason lies in the fact that in the continuous (differential) space-time geometry the particle 4-momentum p_k is described by the relation

$$p_k=g_{kl}\frac{dx^l}{d\tau}=g_{kl}\lim_{d\tau\rightarrow 0}\frac{x^l(\tau+d\tau)-x^l(\tau)}{d\tau} \tag{3}$$

where $x^l=x^l(\tau)$, $l=0,1,2,3$ is an equation of the world line. The limit in the formula (3) does not exist in $\mathcal{G}_{\mathrm{d}}$, and the 4-momentum p_k is not defined (at any rate in such a form). In general, the mathematical formalism of a differential geometry, based on the infinitesimal calculus (differential dynamic equations), is inadequate in the discrete space-time geometry, where infinitesimal distances are absent.

In the case of arbitrary space-time geometry the particle state is described by two space-time points. The two points P,Q determine the vector $\mathbf{PQ}=\{P,Q\}$, which can be interpreted as the particle momentum. In the case of a discrete space-time geometry $\mathcal{G}_{\mathrm{d}}$ the vector $\mathbf{PQ}$ can be also interpreted as a momentum, but its presentation in the form (3) is impossible.

In the arbitrary space-time geometry the pointlike particle is described by a world chain $\mathcal{C}$

$$\mathcal{C}=\bigcup_{s}P_{s},\quad \left|\mathbf{P}_{s}\mathbf{P}_{s+1}\right|=\mu=\text{const},\quad s=...0,1,... \tag{4}$$

Here μ is a geometric mass of the particle (length of the world chain link), which is connected with the particle mass m by the relation

$$m=b\mu \tag{5}$$

where b is an universal constant.

In $\mathcal{G}_{\mathrm{d}}$ only coordinateless description is possible [1], which is produced in terms and only in terms of the world function σ_{d}, or in terms of the space-time distance ρ_{d}, because a use of all geometric concepts of the Riemannian geometry (except for distance) contains a reference to continuity of the geometry. In the coordinateless description the scalar product $(\mathbf{PQ}.\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ has the form

$$(\mathbf{PQ}.\mathbf{RS})=\sigma\left(P,S\right)+\sigma\left(Q,R\right)-\sigma\left(P,R\right)-\sigma\left(Q,S\right) \tag{6}$$

and

$$\left|\mathbf{PQ}\right|^{2}=(\mathbf{PQ}.\mathbf{PQ})=2\sigma\left(P,Q\right) \tag{7}$$

The coordinateless definitions of the scalar product $(\mathbf{PQ}.\mathbf{RS})$ and of the vector length $|\mathbf{PQ}|$ coincide with their conventional definitions in the proper Euclidean geometry. They can be used in any space-time geometry $\mathcal{G}$, which is completely described by its world function σ. Such a space-time geometry will be referred to as a physical geometry.

Equivalency $(\mathbf{PQ}\text{eqv}\mathbf{RS})$ of two vectors $\mathbf{PQ}$ and $\mathbf{RS}$ is defined by two coordinateless relations

$$(\mathbf{PQ}\text{eqv}\mathbf{RS}):\quad (\mathbf{PQ}.\mathbf{RS})=|\mathbf{PQ}|\cdot|\mathbf{RS}|\wedge|\mathbf{PQ}|=|\mathbf{RS}| \tag{8}$$

The discrete space-time geometry is multivariant in the sense, that there are many vectors $\mathbf{PQ},\mathbf{PQ}',\mathbf{PQ}'',...$ at the point P which are equivalent to the vector $\mathbf{RS}$ at the point R, but vectors $\mathbf{PQ},\mathbf{PQ}',\mathbf{PQ}'',...$ are not equivalent between themselves.

In the proper Euclidean geometry $\mathcal{G}_{\mathrm{d}}$ the equivalence relation (8) is single-variant, and there is only one vector $\mathbf{PQ}$ at the point P which is equivalent to the vector $\mathbf{RS}$ at the point R.

If the world chain (4) describes a free particle, its links satisfy the relations

$$\left(\mathbf{P}_{s}\mathbf{P}_{s+1}\text{eqv}\mathbf{P}_{s+1}\mathbf{P}_{s+2}\right),\quad s=...0,1,... \tag{9}$$

These relations are multivariant in $\mathcal{G}_{\mathrm{d}}$. It leads to a wobbling of the world chain. This wobbling means that the particle motion is stochastic (random). Amplitude of wobbling is restricted by the elementary length λ_0 in $\mathcal{G}_\mathrm{d}$ for timelike vectors, But this amplitude is infinite for spacelike vectors. In the geometry of Minkowski $\mathcal{G}_\mathrm{M}$ the wobbling is absent for timelike vectors ($\lambda_0 = 0$), and amplitude of this wobbling is infinite for spacelike vectors.

In the nonrelativistic approximation a statistical description of timelike world lines in $\mathcal{G}_\mathrm{d}$ leads to the Schrödinger equation [2], if the elementary length

$$\lambda_0^2 = \frac{\hbar}{bc} \tag{10}$$

where b is the universal constant defined by (5). A single particle with the spacelike world chain (tachyon) cannot be detected, because of the infinite amplitude of its wobbling. However, gravitational field of the tachyon gas can be detected (dark matter) [3, 4].

It is very important that the statistical description of wobbling world chains is produced relativistically, when the pointlike particle state is described by two points (but not by a point in the phase space). In this case the statistical ensemble is a dynamic system of the type of a continuous medium, and one may introduce the wave function as a method of the continuous medium description [5]. In the nonrelativistic description the particle state is a point in the phase space. In this case the statistical description is a probabilistic construction describing evolution of the particle state probability [6, 7, 8].

Let us stress that the statistical ensemble as a dynamic system (but not as a probabilistic construction) is a result of a correct (relativistic) definition of the particle state (but not a result of some new hypothesis). Description of the particle motion by means of the world chain (4) is a corollary of the consecutive application of the relativity principle.

Unified formalism of particle dynamics

The main optical element of the experimental setup is a rotating optical disc. Since the optical disc is a wedge-shaped, then when disc rotates the fringes move in the plane of the photodetector by a harmonic law. Specifications and a more detailed description of the SADE experimental setup can be found in [**?**]. When using spatial interference pattern shift measurement methods there is a loss of interference pattern contrast due to rotation of the disc caused by disc wedge shape. Wedge shape is hard to remove production defect. Instead of trying to create the ideal discs without the wedge shape, which is an expensive process, the discs are made slightly wedge-shaped. Owing to the high resolution of temporary measurements with respect to spatial, this approach allows us to get the best value for S/N ratio.

Foundation of the quantum mechanics on the basis of the stochastic particle dynamics is obtained as corollary of unified formalism of the particle dynamics [9]. Stochastic particle $\mathcal{S}_\mathrm{st}$ is not a dynamic system, and there are no dynamic equations for $\mathcal{S}_{\mathrm{st}}$. However, statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$, i.e. a set of many independent stochastic particles $\mathcal{S}_{\mathrm{st}}$, is a dynamic system of the type of the continuous medium. Deterministic particle $\mathcal{S}_{\mathrm{det}}$ as well as statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{det}}]$ are dynamic systems, and there are dynamic equations for them. At the conventional approach to the particle dynamics, when the basic element of dynamics is a single particle, one cannot construct a unified dynamic conception for stochastic and deterministic particles, because there are no dynamic equations for a single stochastic particle. However, after the logical reloading, when the statistical ensemble becomes a basic object of the particle dynamics, one obtains dynamic equations for statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$ of stochastic particles $\mathcal{S}_{\mathrm{st}}$ and for statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{det}}]$ of deterministic particles $\mathcal{S}_{\mathrm{det}}$ [9].

For instance, the action for the statistical ensemble of stochastic particles $\mathcal{S}_{\mathrm{st}}$ has the form

$$\mathcal{A}_{\mathcal{E}[\mathcal{S}_{\mathrm{st}}]}[\mathbf{x},\mathbf{u}] = \int\int_{V_\xi} \left\{\frac{m}{2}\dot{\mathbf{x}}^2 + \frac{m}{2}\mathbf{u}^2 - \frac{\hbar}{2}\boldsymbol{\nabla}\mathbf{u}\right\}\rho_1(\boldsymbol{\xi})\, dt d\boldsymbol{\xi}, \qquad \dot{\mathbf{x}} \equiv \frac{d\mathbf{x}}{dt} \tag{11}$$

The variable $\mathbf{x} = \mathbf{x}(t,\boldsymbol{\xi})$ describes the regular component of the particle motion. The independent variables $\boldsymbol{\xi} = \{\xi_1, \xi_2, \xi_3\}$ label elements (particles) of the statistical ensemble $\mathcal{E}[\mathcal{S}_{\mathrm{st}}]$. The variable $\mathbf{u} = \mathbf{u}(t,\mathbf{x})$ describes the mean value of the stochastic velocity component, $\hbar$ is the quantum constant, $\rho_1(\boldsymbol{\xi})$ is a weight function. One may set $\rho_1 = 1$. The second term in (11) describes the kinetic energy of the stochastic velocity component. The third term describes interaction between the stochastic component $\mathbf{u}(t,\mathbf{x})$ and the regular component $\dot{\mathbf{x}}(t,\boldsymbol{\xi})$ of the particle velocity. The operator

$$\boldsymbol{\nabla} = \left\{\frac{\partial}{\partial x^1}, \frac{\partial}{\partial x^2}, \frac{\partial}{\partial x^3}\right\} \tag{12}$$

is defined in the space of coordinates $\mathbf{x}$.

Formally the action (11) describes a set of deterministic particles $\mathcal{S}_{\mathrm{d}}$, interacting via the force field $\mathbf{u}$. The particles $\mathcal{S}_{\mathrm{d}}$ form a gas (or a fluid), described by the variables $\dot{\mathbf{x}}(t,\boldsymbol{\xi}) = \mathbf{v}(t,\boldsymbol{\xi})$. Here this description is produced in the Lagrange representation. Hydrodynamic description is produced in terms of density ρ and velocity $\mathbf{v}$, where

$$\rho = \rho_1 J, \quad J \equiv \frac{\partial(\xi_1,\xi_2,\xi_3)}{\partial(x^1,x^2,x^3)}, \quad v^\alpha = \frac{\partial(x^\alpha,\xi_1,\xi_2,\xi_3)}{\partial(t,\xi_1,\xi_2,\xi_3)}, \quad \alpha = 1,2,3 \tag{13}$$

Nonrotational flow of this gas is described by the Schrödinger equation [9].

The dynamic equation for the force field $\mathbf{u}$ is obtained as a result of variation of (11) with respect to $\mathbf{u}$. It has the form

$$\mathbf{u} = \mathbf{u}(t,\mathbf{x}) = -\frac{\hbar}{2m}\boldsymbol{\nabla}\ln\rho \tag{14}$$

The vector $\mathbf{u}$ describes the mean value of the stochastic velocity component of the stochastic particle $\mathcal{S}_{\mathrm{st}}$. In the nonrelativistic case the force field $\mathbf{u}$ is determined by its source: the fluid density ρ.

In terms of the wave function the action (11) takes the form [9]

$$\mathcal{A}[\psi,\psi^*]=\int\left\{\frac{i\hbar}{2}(\psi^*\partial_0\psi-\partial_0\psi^*\cdot\psi)-\frac{\hbar^2}{2m}\nabla\psi^*\cdot\nabla\psi+\frac{\hbar^2}{8m}\rho\nabla s_\alpha\nabla s_\alpha\right\}\mathrm{d}^4x \tag{15}$$

where the wave function $\psi=\left\{\begin{smallmatrix}\psi_1\\ \psi_2\end{smallmatrix}\right\}$ has two complex components.

$$\rho=\psi^*\psi,\qquad s_\alpha=\frac{\psi^*\sigma_\alpha\psi}{\rho},\qquad \alpha=1,2,3 \tag{16}$$

σ_α are 2×2 Pauli matrices

$$\sigma_1=\left(\begin{array}{cc}0&1\\1&0\end{array}\right),\qquad \sigma_2=\left(\begin{array}{cc}0&-i\\i&0\end{array}\right),\qquad \sigma_3=\left(\begin{array}{cc}1&0\\0&-1\end{array}\right), \tag{17}$$

Dynamic equation, generated by the action (15), has the form

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\nabla^2\psi+\frac{\hbar^2}{8m}\nabla^2 s_\alpha\cdot(s_\alpha-2\sigma_\alpha)\,\psi-\frac{\hbar^2}{4m}\frac{\nabla\rho}{\rho}\nabla s_\alpha\sigma_\alpha\psi=0 \tag{18}$$

In the case of one-component wave function ψ, when the flow is nonrotational and $\nabla s_\alpha=0$, the dynamic equation has the form of the Schrödinger equation

$$i\hbar\partial_0\psi+\frac{\hbar^2}{2m}\nabla^2\psi=0 \tag{19}$$

Thus, the Schrödinger equation is a special case of the dynamic equation, generated by the action (11) or (15). Thus, linearity of dynamic equation in terms of the wave function is a special case of dynamic equation for the statistical ensemble of stochastic particles, although it is considered usually as a principle of quantum mechanics.

Reason of the elementary particles stochasticity

Stochasticity of elementary particles and wobbling of their world chains are conditioned by discreteness of the space-time geometry, more exactly by its multivariance [1]. A discrete geometry is constructed as a generalization of the proper Euclidean geometry $\mathcal{G}_{\mathrm{E}}$. But for such a generalization one needs to produce a logical reloading and to present $\mathcal{G}_{\mathrm{E}}$ in terms of the world function [10, 11]. A use of the discrete space-time geometry admits one to formulate the skeleton conception of elementary particles, where the particle state and all parameters of an elementary particle are described by the particle skeleton [12]. The skeleton is several space-time points, connected rigidly. Distances between the skeleton points determine parameters of the particle. World chain (4) with the two-point skeleton describes the simplest case of elementary particle. In this case there is only one parameter of the skeleton. It is the length $\mu = |\mathbf{P}_s\mathbf{P}_{s+1}|$ of the world chain link. According to (5) the particle mass is a geometrical quantity. In other words, description of a particle motion is geometrized completely.

A generalization of two-point skeleton of a pointlike particle arises at consideration of the Dirac equation [13, 14, 15]. Analyzing the Dirac equation from the viewpoint of quantum mechanics, one meets abstract dynamic variables (γ-matrices), whose meaning is unclear. Analyzing the Dirac equation and using the united formalism of particle dynamics (without a use of quantum principles), one concludes that world line of the Dirac particle is a helix with timelike axis. Helical motion of a free particle is possible, if its skeleton contains three (or more) points [16]. Helical motion of a particle explains the particle spin and magnetic moment, whereas at the quantum approach spin and magnetic moment are simply quantum numbers, whose nature is unknown. Thus, the skeleton conception of elementary particle dynamics admits one to investigate structure and arrangement of elementary particles.

The skeleton conception is obtained as a direct corollary of physical principles without a use of artificial principles and hypotheses alike the quantum mechanics principles. It is the main worth of the skeleton conception. The skeleton conception is obtained as a result of correction mistakes in the conventional theory: (1) nonrelativistic concept of the particle state and (2) unfounded restriction by the continuous space-time geometry. Correction of these mistakes leads to the skeleton conception without any additional suppositions.

A use of the skeleton conception admits one to explain the dark matter as a tachyon gas and to explain impossibility of a single tachyon detection [3, 4]. These phenomena cannot be explained from the point of view of quantum approach.

A use of the logical reloading is followed by essential change of a mathematical formalism. This change of formalism is perceived hardly by people, using conventional formalism. For instance, the discrete geometry $\mathcal{G}_\mathrm{d}$ described by the world function (1) is uniform and isotropic. Indeed, the world function σ_M (2) of the geometry of Minkowski is invariant with respect to Poincare group of transformations. The world function σ_d (1) is a function of σ_M. It is also invariant with respect to Poincare group of transformations. It means that the discrete geometry (1) is uniform and isotropic. This fact contradicts to conventional approach to the discrete geometry which is considered as a geometry on a lattice. Geometry on a lattice cannot be uniform and isotropic. Besides, in the discrete geometry (1) there is no definite dimension (maximal number of linear independent vectors). At the conventional approach to geometry such a situation is impossible, because any construction of a geometry begins from a fixation of the geometry dimension in the form of a natural number.

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On the Problems of Experimental Verification of General Relativity

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General issues pertaining to experimental verification of the General Relativity Theory are discussed, with a special emphasis placed on the problems of gravitational waves radiation and detection. A number of new effects connected with the exposure of a free mirror of the Fabry-Perot cavity to laser radiation which can affect the sensitivity of laser gravitational antennas are pointed out.

Experimental verification of General Relativity (GR) is of paramount importance among the most important problems of contemporary physics, which is due to the fact that GR gives an insight into the structure of the universe and the way it has been developing in time. In spite of the fact that substantial observational data related to the universe at tremendous scales of space and time has been collected, some issues are far from being ultimately solved. However, for the present the approach based on the ideas and equations of GR turns out to be the only serious approach to the description of the universe's behavior. This explains why the experimental verification of GR remains a crucial task of present-day physics.

It is important to note the following well-known conclusions from GR that have been experimentally substantiated:

1. Understanding of cosmological red shift which follows from the isotropic model of the universe derived from the solutions of GR equations (A.A. Friedmann, 1922), according to which the relative magnitude of atoms' oscillation frequency turns out to be proportional to the distance to an observed light source (Hubble's red shift).
2. Explanation of the bending of a light ray in a static gravitational field, or gravitational lensing, an effect occurring when the gravitational field of a massive star or galaxy acts as a huge lens focusing the light radiation energy from a distant star or galaxy making it easier to observe distant objects.
3. Explanation of the perihelion shift of Mercury, which proves that Newtonian mechanics does not give a complete and ultimately accurate description of planets' movement in a centrally symmetric field of gravity.
4. Experimental validation of the Lense –Thirring effect obtained relatively recently using Earth applications satellites ([1], see also [2-4]). This effect is significant because in contrast to Newtonian mechanics GR predicts the changes in the space-time metric due to the rotation of a central mass and, as a consequence, occurrence of interactions between a spinning central mass and a test particle orbiting it. 'Spin-orbital' and 'spin-spin' coupling result in additional precession of the spin axis (of the gyroscope). It is this effect that was found experimentally in [2]. It is worth noting that the Lense–Thirring effect makes it possible to distinguish between GR and linear gravitational theories [4].
5. Indirect proof of binary star's gravitational waves radiation (see survey [5]).

From among the above mentioned observational and experimental justifications of GR the indirect proof of gravitational waves radiation is of special significance, great importance having been attached to their detection in the course of the last quarter-century, with many research teams working on the problem. Unparalleled installations for the direct detection of gravitational radiation have been created, two ideas being fundamental for them, i.e. that of a mechanical resonator, in which acoustic oscillations are generated induced by gravitational waves (first put forward and implemented by J. Weber [6]) and that of an optical interferometer, in which deflection of interference fringes occurs under the influence of gravitational waves (suggested by M.E. Gertsenshtein and the author of this paper in 1962 [7]). Unique installations based on interferometers have been created: two LIGO installations in USA, VIRGO in Italy, GEO600 in the Federal Republic of Germany and TAMA in Japan. Apart from that, individual elements for the LISA geosynchronous space based antenna with the interferometer arm of 5 million kilometers are currently under development. The European Space Agency and NASA are working on the project of underground interferometer (known as *Einstein Telescope*) with a 3 kilometer arm; the possibility of the creation of similar optical gravitational antennas in Australia is also under discussion. Experimental research showed that the sensitivity of existing gravitational antennas is not high enough yet for the detection of gravitational waves from cosmic sources, however, there is no doubt that in the not so distant future, once these unique installations' sensitivity is increased, gravitational waves will be detected [8,9].

In this paper, the author would like to focus on some aspects connected with gravitational radiation and its detection which should be taken into consideration when creating new detection techniques and updating the existing unique installations.

Let us first look at the high-frequency gravitational wave radiation by ultrarelativistic particles. This problem was examined quite a while ago (e.g. see our paper [10]), in contrast to the well-known papers on this subject, let us point out a certain peculiarity of gravitational wave radiation which was found out in the process of working on paper [10] but has not been reflected in literature.

The problem of gravitational wave radiation by an ultrarelativistic particle was formulated by V.L. Ginzburg as far back as 1961 with respect to particle motion along a circular orbit in a strong magnetic field similar to synchrotron radiation in a cyclotron. The interest in this problem was due to the following considerations. Electromagnetic wave emission when an electron (or a proton in a collider) is in circular motion in a strong magnetic field resulting in the loss of energy E by the particle is known to be defined by the following relation [3]:

$$\frac{dE}{dt} = -\frac{2}{3}\frac{e^2 c}{a^2}\left(\frac{E}{mc^2}\right)^4 , \tag{1}$$

where e is the electron charge, c is the velocity of light, m is the electron mass, a is the orbital radius, which depends on the particle energy and the magnetic field strength

$$a = \frac{mcv}{eH\sqrt{1 - v^2/c^2}} = \frac{m^2 c^4}{eHE} . \tag{2}$$

Here H is the magnetic field strength and v is the particle velocity. It follows from formulae (1) and (2) that energy losses of the particle due to electromagnetic waves emission are in proportion to the square of the particle energy. On the other hand, the electromagnetic fields equations (i.e. Maxwell's equations) for determining the electromagnetic field of particle radiation written in four-dimensional generally covariant way will be as follows [3]:

$$\frac{\partial F^{ik}}{\partial x^k} = \frac{4\pi}{c} j^i , \tag{3}$$

where F^{ik} is the electromagnetic field tensor, $F^{ik} = \frac{\partial A^k}{\partial x^i} - \frac{\partial A^i}{\partial x^k}$ (A^k is the electromagnetic field vector power), j^i is current. Equation (3) in a plane metric can be rewritten directly for the vector potential as follows:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) A^k = \frac{4\pi}{c} j^k \tag{4}$$

Let us now write the GR equation for gravitational radiation. It is commonly known that this equation is given by [3]:

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{c^2}\frac{\partial^2}{\partial t^2} \right) h_i^k = -\frac{16\pi\kappa}{c^4} T_i^k , \tag{5}$$

where h_i^k is the gravitational field amplitude tensor, T_i^k is the tensor of matter and κ is the gravitational constant. As we can see equations (4) and (5) are very similar to each other, the difference being that for an electromagnetic field the source in the right-hand member of the equation is in proportion to the velocity since current is proportionate to the particle velocity, while for a gravitational field it is proportional to the square of the velocity or energy as the energy-momentum tensor includes quantities proportional to the energy itself. Therefore it would seem that since a gravitational wave source includes higher velocity (or energy) values, energy losses by gravitational wave radiation according to formula (1) should contain the energy which is higher than the losses by electromagnetic wave radiation according to formula (1). If this was the case, it would mean that there exists a certain, rather high, value of particle energy, which when exceeded would result in the gravitational waves intensity exceeding the intensity of electromagnetic waves emission. However, that is not the case. Analysis given in [4] shows that the ratio of gravitational waves intensity to the intensity of electromagnetic waves of an ultrarelativistic particle at any values of particle energy remains constant, does not depend on energy and is determined by the ratio of the particle's gravitational radius to the electromagnetic one. The reason behind it turned out to be the 'double' transversality of a gravitational wave's amplitude (only spatial components of tensor h_i^k are responsible for gravitational radiation; indices i and k assume values 1, 2, 3, which corresponds to the fact that gravitational waves radiation is governed only by spatial components of the metric tensor). It is worth mentioning paper [5], in which it was shown that there is also a substantial contribution of magnetic field energy fluctuations into relativistic particle gravitational radiation (in point of fact, in the ultrarelativistic case, when the velocity of an electron approximates the velocity of light, the mass of the electron becomes arbitrarily large, in which case radiation is formed by the quadruple of a 'heavy' electron and the magnetic field along with the accelerator mass).

Thus the generation of high-frequency gravitational waves by relativistic particles turns out to be quite problematic. That is why it is interesting to consider other possible mechanisms, and it appears that parametric mechanisms based on the assumption of the equality of propagation velocity of gravitational and electromagnetic waves might appear to be the most promising ones. Although there is no direct evidence of propagation velocities equality, there are [1]observations of changes in the revolution period of a binary star (both seem to be neutron stars forming PSR B 1913+16 pulsar) and good agreement with the well-known formula describing changes in revolution period [5] has been obtained:

$$\frac{dT}{dt}=-\frac{192(2\pi)^{8/3}}{10\,c^5}\frac{k^{5/3}}{T^{5/3}}\frac{m_1 m_2}{(m_1+m_2)^{1/3}}, \tag{6}$$

where m_1 and m_2 are stellar masses. There is good agreement between the observations of period temporal variations with formula (6), with a margin of error not exceeding 0.2%, conventional value of the velocity of light being used for gravitational wave propagation velocity. (A similar result has been obtained recently for another pulsar [12]). It is significant that formula (6) contains the gravitation wave propagation velocity raised to the fifth power, if it were different from the velocity of light, the agreement would not be so good. Therefore, light wave and gravitational wave velocities can be considered coincident. This is an important fact as it makes it possible to suggest new parametric mechanisms of high-frequency gravitational waves radiation and detection [13]. The underlying concept behind a parametric mechanism of gravitational waves detection is as follows. Let there be a strong electromagnetic wave characterized by a high degree of coherency, i.e. laser radiation. (It is known that there exist extended sources of laser radiation in space [14]). Then it can be shown that as follows from equation (5) there appears a weak gravitational wave the frequency of which will be equal to the doubled frequency of the original electromagnetic wave . A gravitational wave is known to be equivalent to the environment with a certain refraction index which is in proportion to the amplitude of the gravitational wave; consequently the initial electromagnetic wave will undergo diffraction on the spatial periodic structure of the refraction index. This would result in the emergence of an electromagnetic wave at the tripled frequency, which can be recorded. Therefore surveillance over distant sources' spectra and detection of triple-frequency harmonic radiation will give evidence of the aforementioned mechanism of energy conversion of an electromagnetic wave field into a gravitational wave and the subsequent process of electromagnetic wave diffraction.

As there is work underway to upgrade gravitational antennas based on interferometers for the purpose of their sensitivity enhancement, it is necessary to point out the following.

It is a known fact that to enhance these antennas' sensitivity the mirrors forming a Fabry-Perot (Michelson) cavity should be free, that is suspended on fine threads these mirrors treated as pendulums apart from everything else should possess very high mechanical performance. That is why there are a lot of papers devoted to different mechanisms having an effect on the quality factor of such a pendulum. This paper looks at another mechanism which can have a significant effect on the quality factor of such a pendulum.

To obtain a high mirrors' reflection index their surfaces are covered with multilayer dielectric coatings (e.g. in VIRGO 44 half-wave layers of SiO2 and TaO5 are used), which ensure a high reflection index, and as a consequence, high density of light energy within the Fabry-Perot cavity. Light wave energy distribution in the vicinity of the mirror's reflecting surface is shown in Fig. 1. It is obvious that at the distance in the order of $N\lambda$, where N is the number of layers and λ is the laser emission wavelength, the wave field drastically decays. Such inhomogeneous field distribution results in the emergence of the force acting upon the mirror (see section 16 of book [3]). This force results in surface charges redistribution, the effect known as photo-charge effect (see [15] and [16]). The charge distribution is equivalent to the mirror's polarization, i.e. the mirror's acquiring electric dipole moment. The action of the external radiation falling upon the mirror's edge is similar to that of the external electrostatic field which causes polarization. In its turn, the mirror's acquiring electric dipole moment results in secondary interaction forces between the mirror and the neighboring setup components, which leads to reduced quality factor of the pendulum setup (the mirror's suspension mount).

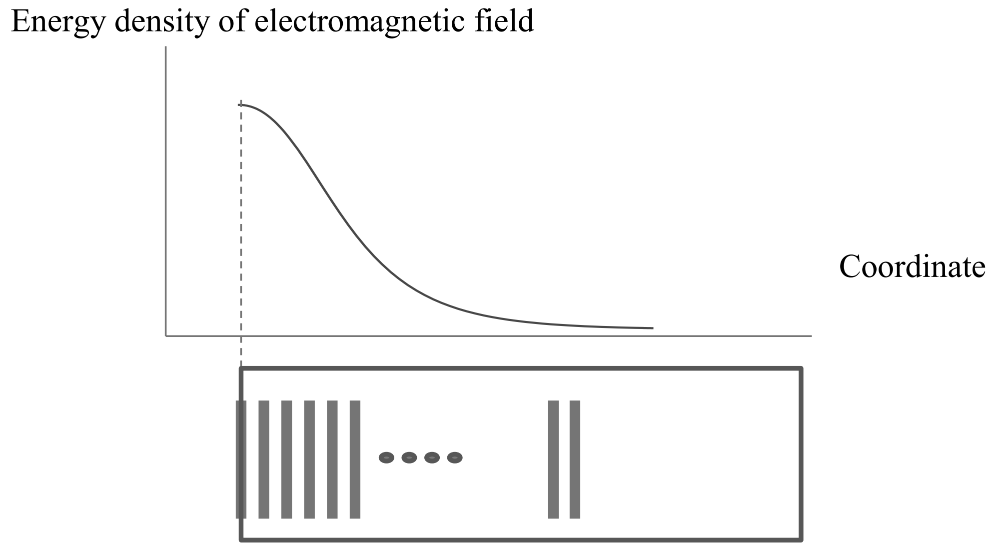

Fig. 1: Laser radiation energy density distribution near reflecting surface

Apart from the aforementioned effect of powerful laser radiation on the mirror surface leading to polarization given free charges available for distribution, there exists another effect, which does not seem to have been mentioned previously, that is the effect of elastic strain in the mirror induced by laser radiation. Indeed, the electric field of a laser wave causes Maxwell elastic strain tensor σ_{ik} to be different from zero (ε is the dielectric permeability of the medium, i.e. the mirror; E_i is the wave field):

$$\sigma_{ik} = \varepsilon \frac{E_i E_k}{8\pi} - \varepsilon \frac{E^2}{8\pi} \delta_{ik} \tag{7}$$

Averaging formula (7) over the wave period and considering that the field depends on the mirror – depthward coordinate, let's say if the direction is *x*, for the wave incident normal to the mirror surface the averaged elastic strain tensor will be equal to:

$$\langle \sigma_{ik} \rangle = \begin{vmatrix} -\frac{\varepsilon}{8\pi}(\langle E_y{}^2 \rangle + \langle E_z{}^2 \rangle) & 0 & 0 \\ 0 & \frac{\varepsilon}{4\pi}(\langle E_y{}^2 \rangle - \langle E_z{}^2 \rangle) & \frac{\varepsilon}{4\pi}\langle E_y E_z \rangle \\ 0 & \frac{\varepsilon}{4\pi}\langle E_y E_z \rangle & \frac{\varepsilon}{4\pi}(\langle E_z{}^2 \rangle - \langle E_y{}^2 \rangle) \end{vmatrix} \tag{8}$$

It follows thence that elastic strains in the mirror depend on radiation polarization, energy density distribution at the beam cross section and of course on coordinate *x*. The mirror deformation can now be found from the solution of the relevant elastic problem:

$$\lambda_{iklm} \frac{\partial u_{lm}}{\partial x_k} = \frac{\partial \langle \sigma_{ik} \rangle}{\partial x_k}, \tag{9}$$

where λ_{iklm} is the elastic modulus tensor, u_{lm} is the strain tensor. Equation (9) makes it possible to determine the mirror deformation if we know laser radiation energy density spatial distribution. Apart from the obvious effect of the mirror surface distortion subject to forces $\frac{\partial \langle \sigma_{ik} \rangle}{\partial x_k}$ it is necessary to point out the following. Real-world experiments usually use laser radiation time duration modulation (to improve the conditions of mirror position recording). It means that force $\frac{\partial \langle \sigma_{ik} \rangle}{\partial x_k}$ is already time dependent, so we can no longer confine ourselves to static consideration of the problem and it is necessary to take account the term pertaining to time in equation (9). That is to say the time duration modulation of laser radiation will cause elastic mode excitation (acoustic waves) with all the consequences that come with it. It must be borne in mind in the design of experiments.

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Proceedings of International Scientific Meeting
PIRT-2013, 1 – 4 July 2013

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